Ordinary Differential Equations

Prof. A K Nandakumaran
Department of Mathematics
Indian Institute of Science, Bangalore

Module - 5
Lecture - 24
General Systems and Diagonalizability

Welcome to the new module on linear systems and of first order differential equations and particularly, it is qualitative analysis. So far, in our all earlier modules, the topics we have completed the topics, which are basically, covered in a university syllabus. Now onwards, two of the main modules; one of the stability analysis basically, what we call it qualitative analysis, and qualitative analysis of linear systems; first we will do. Then in another module, we will learn the qualitative analysis of the non-linear system. So, you can view this module, is a precursor to the module on non-linear systems, and stability analysis, that trajectory behaviour around an equilibrium point, etcetera.

So, that is the main aim of this module. It is not just the representation of the solutions, which we can still obtain, using the classical theory, which we have already completed. We can use the existence uniqueness theory of the system of the things, which we have studied in the previous module. We can apply in a similar fashion here, but the more important aspects about the study of the behaviour nature of the equilibrium points, behaviour of the trajectories, because the solutions of the trajectories of ODE systems are solutions, you can think it as a motion of certain particles.

So, we want to understand that. First, we will do in this module about the linear systems and later, in another module about the non-linear systems. You will also see the power of linear algebra and its diagonalization, the jordan decomposition, its usefulness in analysing these equations. So, what we have done in one of our earlier modules; we have studied the first and second order linear systems. We can also consider the n th order linear systems, and what we will see quickly, now every n th order linear system is a special case on general n th order system.

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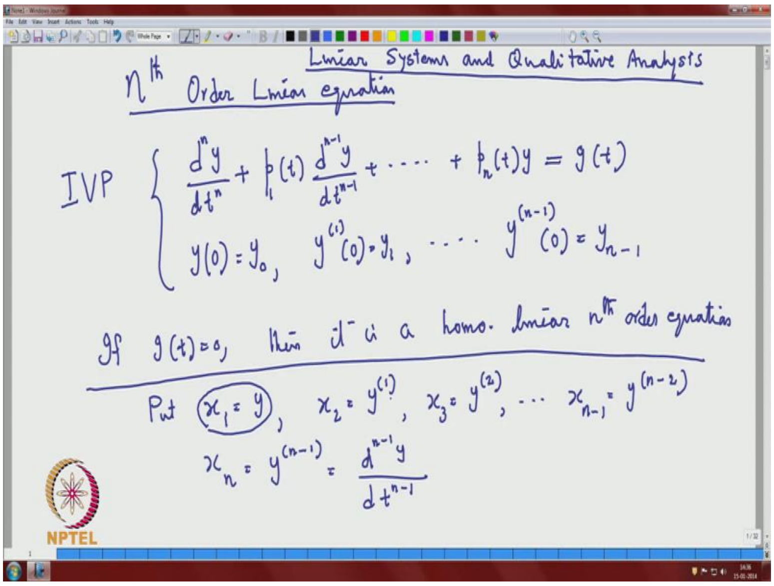

How does the general n th order linear systems; let us look at the n th order linear system, linear equations. So, let me give before that; it is a linear system; a title to this module; linear systems and qualitative analysis. So, that is the title of this particular topic. How does a general n th order linear equation will look like? A regular linear equation will have the following form; you will have the n th equation, which are coefficient one; d th power y by d t power n plus, let me use a correct one; p 1 t into d power by n minus 1, y by d t power n minus 1. As if you go here, you will get final term p n of t of y is equal to some g t.

If you are looking for an initial value problem, you can define as you have seen, will be seen rather, for n equal to 2, for this second order equation; not only the initial value problems; you can also define the boundary value problem. You will see few lectures on boundary value problems of the second order equations in a different module. So, for an initial value problem, you need n conditions; for example, y at 0 is equal to y naught y 1; y 1 at 0 means y prime at 0 is equal to y 1, etcetera. up to y n minus 1 of 0 is equal to say, n condition you need it.

So, that is an initial value problem, If g t equal to 0, this is called a homogeneous; g t equal to 0, then it is a homogeneous linear n th order equation. What I am saying is that such a linear n th order equation can be converted into a first order system; how do you do that one? You put x 1 is equal to y 1; x 2 is equal to y 1 x, not y 1; x 1 is equal to y; x

2 is equal to y prime. That is nothing but yes, x 3 is equal to y prime; x 3 is equal to y second derivative, etcetera. up to x n minus 1. You will define to be x n minus 1 to be y n minus 2, and your last term x n is equal to the n minus one th derivative; that is equal to d power n minus 1 y by d t power n minus 1. So, if we use this one, and we can use in that equation here, if you look at that x 1 dot is equal to y dot. So, the derivative of x 1 here is nothing but y 1. So, you can write down that way.

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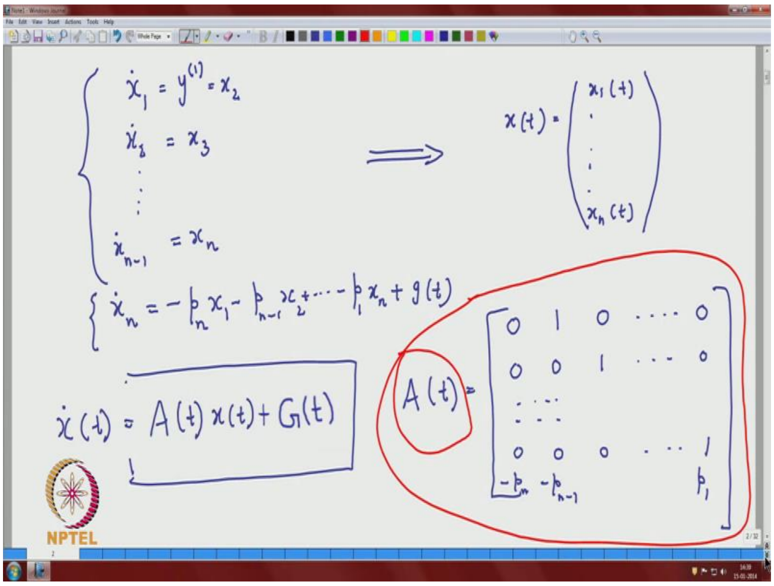

So, if you look at that one, x 1 dot is equal to d x 1 by d t, exactly, will be y 1. I am using two notations; dot and 1, both are for the derivatives. So, we can write down this is y 1; that is equal to x 2. So, if you write x 2 dot, will be x 3. So, if you go like that, you will get x n minus 1 is equal to dot x n minus 1 dot is equal to x n and x n dot has to be recovered from your equation; x 1, at that if you look at here, this is what your thing. That will be, if you take this to the right hand side. You can at write everything as, write down that you will get minus p n into x 1 minus p n minus 1 into x 2.

If you go like that, the last part will be p 1 into x n and then, you will also have your g t here. So, this is from the equation. This is first n minus 1, from the definition; the last 1 will be from the equation. So, if you put this together here, and if you define your x t now, is equal to x 1, etcetera. x n, all evaluated at t and you can write down your equation. If you combine all these, you will write your equation as x dot of t is equal to, you can write your matrix a t into your x t, you see. What is a t? That a t is the matrix, is a special form of the matrix; you have a special 0, 1, 0, 0, then 0, 0, 1, like that; 0, 0, the last 1 here.

Here you will have minus p n, minus p n minus 1, etcetera. up to p 1, you will have; this is correct. So, if you multiply this 1, a t into x t, you have this kind of system. So, that gives you the linear system of equation with a t in this special form; see a t is of this 1. This is your a t; you have your a t like that, but this allows us to consider any general a t. You do not have to consider that way; so, general linear system.

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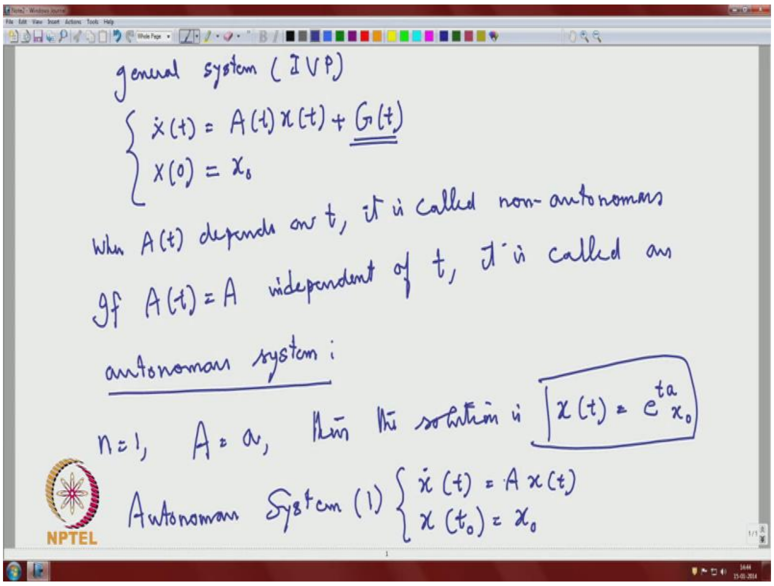

Thus, a general system can be written as initial value problem in this form; x dot of t is equal to a t x t; a t need not be in the above form; it can be, a t will consist, the entries of a t will consists of functions, need not be the form given by the earlier thing; x at 0 or you can put it any point. x at 0 is equal to x naught. So, this is a general linear system. Of course, in general, they are not going to concentrate on system, such as if this system, when a depends on t, yes, plus some g t will also come; a non-linear term where, you can have some, this gives you a non homogeneous. When a t depends on t, it is called non autonomous. So, if a t is equal to a, independent of t, it is called an autonomous system. As you see even, in our first order and second order equation, for example; even, when we are studying second order equation, when the second order coefficients are independent of the functions; you have seen the advantages and able to solve the equations, completely. So, there is a larger advantage and many other even, in the qualitative analysis, you see the difference, when you study non-linear analysis; part of this problem. So, for autonomous system, it is much more easier to study than non autonomous system. So, most of these lectures though, in this module, we will concentrate only, on the autonomous systems, and we try to represent the solutions and the stability analysis of that system.

So, our major aim, if you look at n equal to 1; that means, a is equal to a. Then, the solution is; you know already; then, the solution is x t is equal to e power t a into x naught, you see. So, our aim immediately, we will see that we want to have, we want to represent the solution for the system, if possible of the form; when it is for an autonomous system; what is our autonomous system? Autonomous system is x dot of t is equal to a of x t where, a is independent of t; that is more important thing, and you will study the initial problem at any initial value is x naught, which is also a vector. So, this system, we are always going to refer to as 1. The important point here is that system 1 is, a is independent of t.

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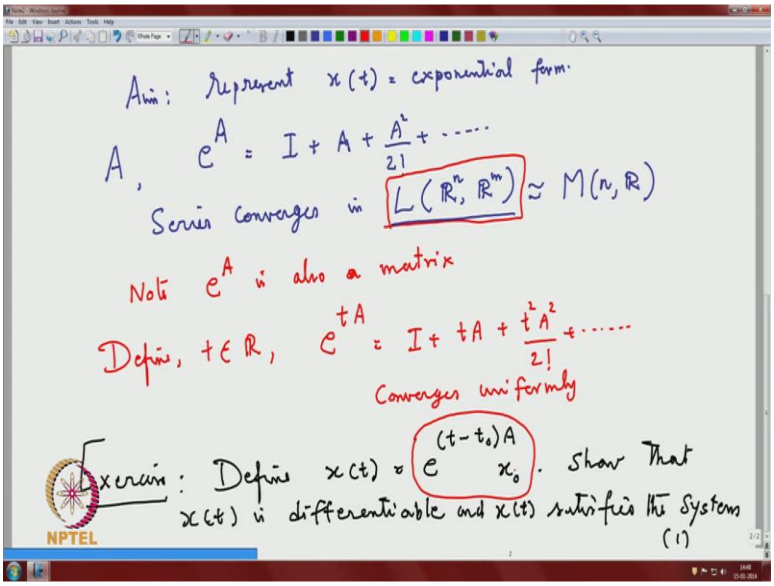

So, the whole aim is to represent this solution, x t is of the form, some exponential form. Here is, what exponential form here, we what we want to recall that definition of an exponential of a matrix, which we have studied already in the basic part of the module. So, if you have a matrix a, if a is a matrix, then e power a can be defined to be I plus a plus, a square by 2 factorial plus, etcetera. Where, the series converges; that is what, series converges in the set of all linear operators r n to r n. So, in this case, given a matrix, one can view it as a linear operator from the space r n to the space r n, and this is also, identified with a set of all n by n matrices. That is what we are trying to set in this case. So, on this set of all linear operators from r n to r n, we have the corresponding itself vector space, which has a topology. Under that, you will talk about the convergence of this infinite series, and this e power a; note, e power a is also a matrix; that is a important part of it.

If you can define e power a in particular, we can also define for any t, belongs to r, e power t a; that is nothing but identity. Again, it is symmetric plus, t a plus, t square a square by 2 factorial and so on. It is again, an infinite series and this series converges uniformly, in this case, because there is a t parameter. Again, e power t is a matrix here, which you want to do that. So, here is an exercise; I want to give you, start with an exercise now, which you can do it easily. Define x t is equal to e power t minus t naught of a x naught; not that e power t minus t naught into a, is a matrix. So, that matrix will act on the vector x naught and the x t is defined there. So, the exercise is that show that x t is differentiable in t and x t satisfies 1; the system 1. What is the system 1? This is the system 1. So, you just compute x dot of t. You can see that x dot of t is nothing but a of x t and definitely, x at t naught t is easy to see that, x at t naught is, when t substituted for t naught, the first term here, will be identity. So, x at t naught will be x naught.

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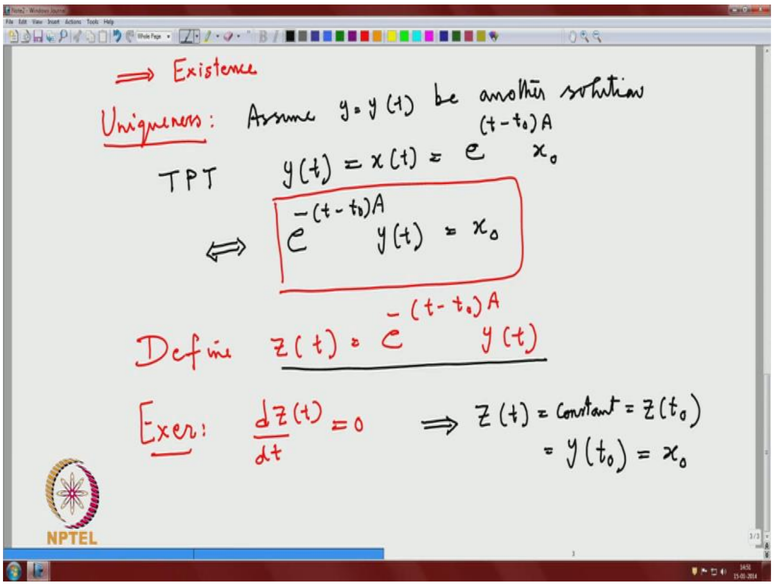

So, that way, we have the existence. So, this proves, this gives existence; that is what I am saying. So, you have proved the existence directly, by construction. Of course, as I said, the existence and uniqueness can also, be proved using the general existence uniqueness theorem. So, the existence, which we already have it and we want to give you uniqueness; a quick uniqueness proof. You can get the uniqueness using the general theory, but I want to give you a general easy thing. So, assume y be another solution; y equal to y t be another solution. Then, what you have there? To prove that, you have to prove that y t is equal to x t; that is what you have; x t is defined already. What is x t; x t is nothing but e power t minus t naught of a, into x naught; that is what you have to prove it.

Equivalently, proving this is equivalent to that; I can take this out; e power minus of t minus t naught, into a at y t; I will take that to the left side; is equal to x naught. So, what you want to prove is that, this is what you have to prove it. This, you have to prove it; that means, left side is a function of t; right is just x naught, which is a constant. So, you want to show that the left side is a constant, and that constant is nothing but x naught. So, define in that case, just the left hand side as z t. You define z t is equal to e power minus t, minus t naught of a y t, which I can define. What I have to prove is that z is a constant in t, and that constant is nothing but x naught. So, to prove that constant, here is an exercise again, for you. These are all simple exercises; show that d z by d t is equal to 0; that is an easy exercise. Once you show that d z by d t is equal to 0, then this exercise will give you z is equal to t, is a constant, but what is that constant? But that constant has the d z at t naught; z t is a constant, already proved; say, it has to have the same value at same naught.

But z at t naught, by the definition here, put t equal to t naught here. So, you will get nothing but y at t naught, but what is given to you; y is a solution to your initial value problem, and y at t naught is nothing but your x naught. So, you have your uniqueness in a quick way, without appealing to the general theory. So, we have proved the existence and the uniqueness; existence by construction and uniqueness by direct application; that linear system, the autonomous linear system has a unique solution.

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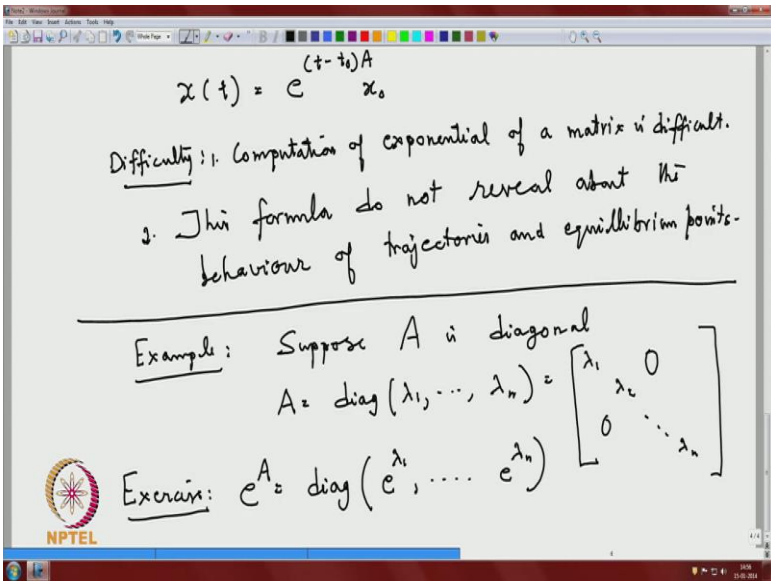

We may wonder what more to be done here; the issues that you already proved the existence and uniqueness. Again, if you go back to our second order linear equations, just obtaining solutions are not enough. Just like here, we have the representation of the solution, x t in the form. We have the representation of this solution x t in the form, e power t minus t naught a x naught; you have that one. So, the two of the difficult issues to remark; two difficulties here; one difficulty; the computation of exponential is very difficult. Though, you have a representation, but the computation is of exponential of a matrix is difficult in general. That is one point. The second point is that as I remarked again, x t, you have to think it in dynamical system, is a solution to a dynamical system. In other words, x t, you can think it as a motion of a particle. So, the important issue is that how these particles behave, especially, near an equilibrium point, which we are going to explain to you, and you will also learn about the equilibrium points, later in the non-linear theory. So, what we are more interested is that the behaviour of x t, especially, near an equilibrium point, especially in particular, the stability analysis of the trajectory.

So, this formula does not reveal anything about x t. So, the second difficulty is that this formula does not denote, reveal about the behaviour of the trajectories and equilibrium points, which you are going to study; trajectories and equilibrium points. You see that is a whole thing. So, one is especially, the computation of the whole thing, and second one is the trajectory behaviour, and the whole module on this one and the non-linear theory is to understand this trajectory; how does it behave; what are the equilibrium points;

whether, you have the stability near any equilibrium; these are the questions from physics and engineering point of view, but then there are some matrices where, you can have a computation easy. Example, if you want to see, example; suppose, a is diagonal; that means, I write a diagonal matrix in this form. Suppose, it is a diagonal matrix; I will read only the diagonal entries lambda 1, etcetera. lambda n; this is nothing but , what I mean by that is nothing but lambda 1, lambda 2, only along this diagonal, lambda n. All the other elements are 0.

So, I want you give, it is an exercise from again, linear algebra, which you would have done already. I will also give what happens to the trajectories of the solutions with this a as the matrix. The thing is that you can immediately, compute e power a; e power a will be again, diagonal, but then, entries with e power lambda 1. So, this is a small exercise, which you can immediately, do it. What you have to do is that you have to compute a square, a cube, etcetera. If you compute a square, you will get the diagonal lambda 1 square, lambda 2 square, etcetera. lambda n square. For any a power n, it will be diagonal of lambda 1 power n, etcetera. lambda n power n. Then you add it and you get the matrix e power a immediately, but what does this mean, when a is diagonal, as far as our linear system is concerned?

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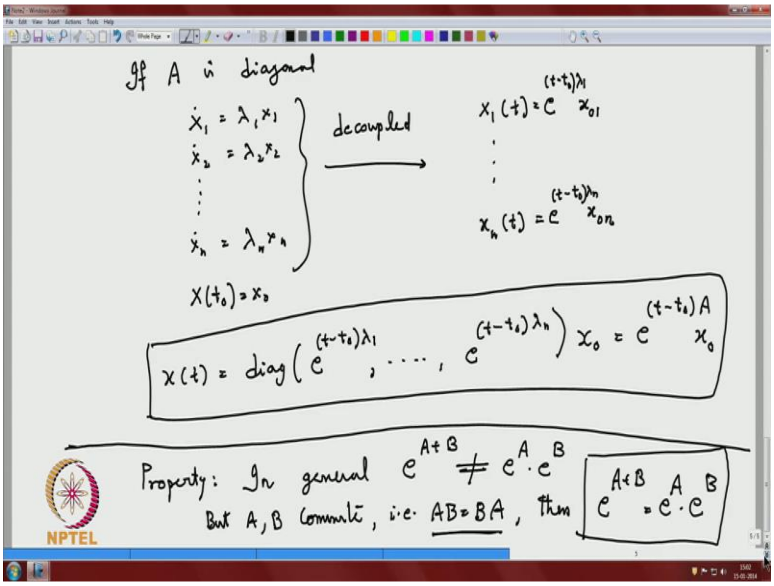

If a is diagonal, what does that diagonal system means to us; diagonal, then it just says that x 1 dot is equal to lambda 1 x 1, x 2 dot is equal to lambda 2 x 2. If you rewrite the system, the x n dot is equal to lambda n x n, you see. So, it is a decoupled system, you see, there is no connection between, there is no interaction, coupling between two variables. Any of the variables do not interact with other variable, and each one is a simple equation; x 1 dot is equal to lambda 1 x 1, and since, the x 2 etc, x n are not coming into picture. This immediately, this is a decoupled system; this is what it says decoupled. So, there is no problem. Immediately, this is decoupling system, will give you the solution x 1 t is equal to e power lambda 1 t, into the initial value of the first component; that is all. If you go this way, you get your x n t is equal to e power lambda n of t; the n th component of your initial value.

If you write this in the form, this is equivalent to saying that if you write this e power x t is, yes, of course, you have to change here. If I look my initial conditions at t naught, is equal to x naught. So, I have to change accordingly. I will change here, accordingly. This will be e power t minus t naught, into lambda 1. So, I will change here. This is, what I wrote was for the initial value at the origin, into lambda n x naught; this is just a multiplication. So, if I write these solutions, you have your x t is nothing but your diagonal of e power t minus t naught lambda 1 with entry is this one. That is equal to e power 2 minus t naught of lambda n; this is a matrix. This is your matrix operator on x naught. So, that is nothing but your e t minus t naught a, into x naught, you see. So, you have your solution and you have your complete solution, if a is a diagonal matrix. Another interesting property with this, which will be useful for our analysis; we are eventually, want to do more analysis. If the given matrix is diagonal, the corresponding ODE is a decoupled system. Since, it is a decoupled system, it is solving each equation each n of the equations independently, because it is a first order equation in one variable. Each one can be solved separately, and then you can write down that solution.

So, as long as the matrix is diagonal, you have no problem of solving it. Another property, which I would like to recall here; the property is that in general, for exponential map e power a plus b, not equal to e power a into a power b. So, this property of real numbers is not true for general matrices, but if a and b commute; that is a b equal to b a, then e power a plus b; this will be useful in our analysis later; e power a into e power b. This you can verify. If a and b commute, you can prove that result. So, this exponential, one of the fundamental properties of an exponential function in one variable is not true for exponential functions of the matrix, exponentials of the matrices, but if there is a commutation; that is not contradicting; for real numbers, this is always true; a b is equal to b f of real numbers and that is the thing. The another important thing, I will recall today, some more interesting things, which is necessary here, for another property which, we want to call a property.

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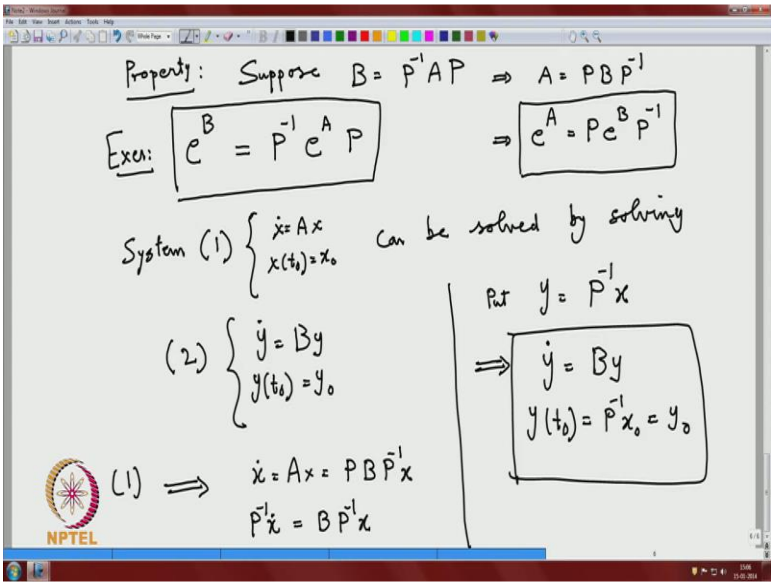

Suppose, b is of the form p inverse of a p; such matrices are called similar matrices. As know from linear algebra, linear similar matrices represent the same linear transformation. It is only on according to different basis, you will have different matrices, but it will represent essentially, the same linear transformation. Suppose, b is equal to p inverse a p, then what is e power b? I want to know. This is a simple result again. I will leave it as a small exercise for you, if you have not done these things in the linear algebra. This may be the correct time to do that one. This will be p inverse of e power a p, you see. So, you have a nice representation. We can do this other way also. So, if this is equal to, this symbolize, of course, a is equal to p b, p inverse and that also implies e power a.

So, you can write down, because p is an invertible matrix; p e power b p inverse. This is one important property, which now we are going to use it. Under this linear transformation, this helps. So, you have a linear system x dot equal to a x, but then this allows you to solving for x dot equal to a x that namely, the system x dot equal to a x, can be obtained by solving another system, corresponding to b. So, the system 1 can be solved. Let me recall system 1. System 1 means a x equal to x dot equal to a x with x at t naught is equal to x naught. This is my system 1, can be solved by solving system 2 namely, corresponding to b. So, I introduce a system, b equal to y, y at t naught equal to something, whatever it is, y at t naught is equal to some y naught, which we will show. How do you achieve this one? So, recall. What is 1? 1 gives x dot is equal to a x, but what is a x? It is nothing but p b, p inverse of x. Now, put, yes; this, if you call it here, you will get p inverse of x dot is equal to b p inverse of x; not that p is a invertible matrix with constant coefficient.

So, p is independent of t. So, you can take inside x. You put y is equal to p inverse of x; that implies, y dot is equal to b y, you see. What is y at t naught; y at t naught is nothing but p inverse at x at t naught; that is x naught, and this is your y; this, you call it to be y naught. So, you can solve your system. If you have two matrices of similar, the solution of the system, corresponding to one matrix, can be obtained by solving the system, corresponding to another equivalent. So, such two matrices are called the linearly equivalent.

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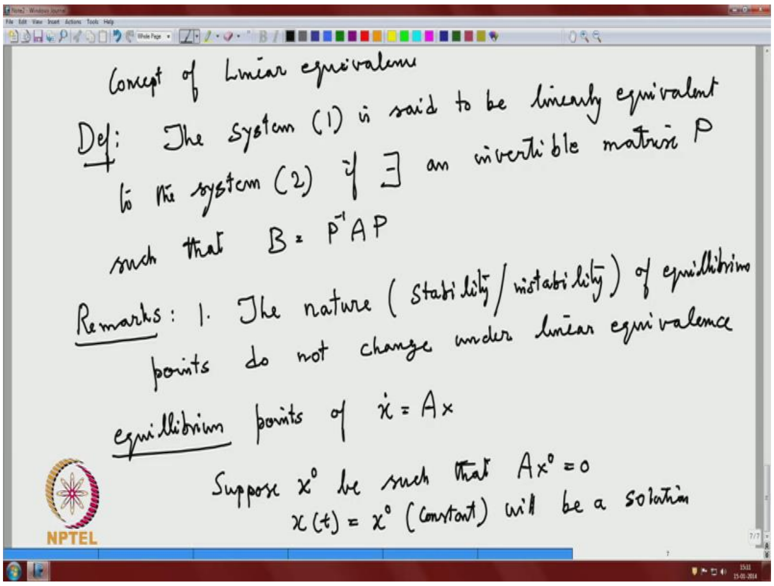

This leads to the concept of linear equivalence, So, you have a definition for this one. Two systems, the system 1; that is x dot equal to a x is said to be linearly equivalent to a system 2 that if there exists an invertible matrix p, such that b is equal to p inverse of a p. So, that is the point. So, whenever, two systems are linearly equivalent, you can go from one system to another system. What is so important about it? The important thing, which you are going to see soon, in next lecture or coming lectures; one property, the one important thing is that under linear equivalence, the nature of the equilibrium points or the stability of the trajectories, do not change. For example, if a particular point is equilibrium point, there will be a corresponding equilibrium point to the other system. This equilibrium point is stable. You will see what are the kinds of stability and unstability available soon, and then the other point will also be thing, and the behaviour of how the trajectories will even, nature of the behaviour will also, remain the same.

So, the nature; the first important property. Right now, you may not know it, as we have not defined any sort of, any concept of stability, but the important point, which we are going to see the nature. For example, stability or instability of equilibrium points; we will see what is equilibrium point; equilibrium points, do not change under linear equivalence. What are these equilibrium points you have done? So, recall, what is equilibrium points? We will see again, as I said, more on non-linear systems about it; correct definitions again, equilibrium points. Equilibrium points are actually, solutions of the linear system; equilibrium points of x dot equal to a x. So, equilibrium point, though we call it point, it is our solutions, steady state solutions. Suppose, a x equal to 0. Suppose, x b such that; let me calculate up; not to confuse with initial values; x naught b, such that a x naught is equal to 0; that means, it satisfy the right hand side, vanishes at that point. This symbolize, if I define x t is equal to the constant x naught, constant will be a solution. That means, if you have any equilibrium point, and if at t naught, if the solution is at that point, then the solution will remain at that point all the time; it will not move from there. So, that is why, it is a steady state solution. Whenever, you have an equilibrium point in the solution, starting from there, it will not change there.

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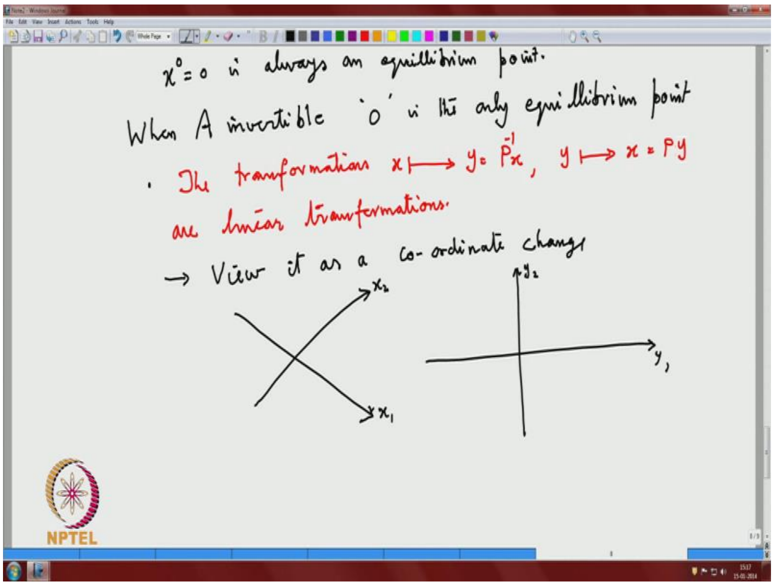

For example, for linear system x naught equal to 0, is always an equilibrium point. When a is invertible and this is the case, when we are going to consider; when a is invertible, 0 is the only equilibrium point, because a x not equal to 0, if a is invertible; x naught will be 0. If a is invertible, 0 is the only equilibrium point. You will see more examples in the next class. I want to set the kind of thing. Therefore, if you start a solution at the origin at time t naught, the solution will remain there. Here is where, your concept of stability are based. In applications, you can never start from a particular point; you will always going to make error. So, the question is that if you start a solution at the origin, at an equilibrium point, then the solution will remain there. Suppose, you make an error and the solution is not starting at the origin, but then a solution is starting say, at a point, initial point x naught, which is close to the origin.

So, the question is that whether, trajectory will deviate from there, very faraway, causing instability, because this is important in the point of view of applications. You will never be able to start at a particular point; you are always going make error. These kinds of concepts also; you have to know that when a solution, starting near an equilibrium point, will remain there, will go to the equilibrium point where, it will move away from that and leading to various definitions, various types of stability, especially, when x t is a vector. It will have different components. Certain components will go to the origin. Certain components will have the stability. Certain components will not have the stability and all that can happen. That is what basically, in the qualitative analysis, we are going to see that. The second point; so, this is about the equilibrium point. We are going to, initially, we will give few examples even, when a is not invertible and then, you can see the various equilibrium point and how the solution behave. Our more interest is that when a is invertible; in that case, we have only one equilibrium point near 0, and we are going to study the stability analysis near that point, near the equilibrium point. This is the first point. Second point of interest; the second point is about the change of variable.

Basically, we are making a change. If you go back to our earlier analysis, the previous analysis, if you go back to this kind of previous transformation, you see, this is the transformation we made to converge the system, x dot equal to a x to a system is equal to y. So, there are two linear transformations, basically. So, you have that transformations, x going to y, is equal to p x; that is one transformation, and the other one; y going to x, is equal to p, which one is the correct one. Both are fine; y is equal to p inverse of x; does not matter; you can replace that x equal to p y, are linear transformations; this is important. These are linear transformations; not some arbitrary transformations.

Why it is linear, because a t is a matrix, because it is linear transformation; you can view this as a; that is an important one; can view it as a coordinate change. So, basically, when you are considering coordinate change, when you have some coordinates, you will be basically, looking at two coordinates change, either, x going to y or y going to x. So, you can see that if this is y 1 y 2, you will have a coordinate change, x 1 x 2. So, effectively, when we are considering under linear equivalence, x dot equal to a x and y dot equal to p y; we are making a coordinate change. Under this coordinate change, when you have; for example, if a is invertible and x naught is an equilibrium point, and you have y naught is equal to p inverse of x naught, will be an equilibrium point. That is our only equilibrium point. We can get that equilibrium points and the stability for the system, x naught is stable, which we will see; y naught will also be stable. That is what I say; it does not change. There may be a difference in movement of that trajectory, but the stability analysis; you cannot convert a stable equilibrium point to an unstable equilibrium point under linear equivalence.

Fundamentally, it is a linear, is a coordinate change; that is what under this linear transformation, one has to understand. That is not surprising, because this linear equivalence is between two similar matrices. As you see that in the similar matrices, represent the same linear transformation and hence, you should be able to do the same thing. So, what is our aim now?

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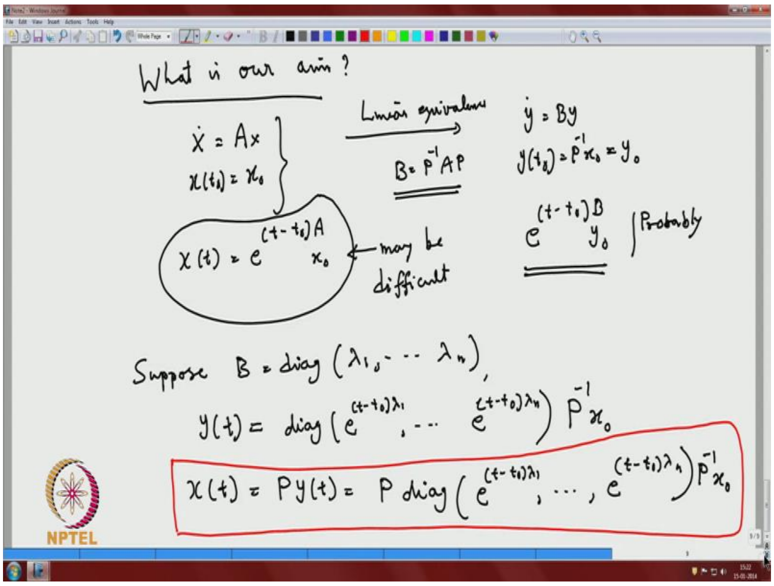

With this as general remarks about, is basically, what is our aim? We should not forget about our aim. What are we trying to do this one? See, what we have seen is that you are interested in solving a linear system, x dot equal to a x, and that you have a representation. You have a solution, x at t naught is equal to, is x naught; you have a linear system. What we have so far discussed is that if there is a linear equivalence; that means, you have b is equal to p inverse of a p, and you have the corresponding system, y dot equal to b y, and y at t naught is equal to some p inverse of x naught; that is your y. Here, the solution, as I said that one of the major difficulties is the computation of the solution.

Even though, you have a solution x t is equal to, let me recall once again, because this is an important point; x naught is equal to a into x naught; the computation of this is difficult. Suppose, you are able to transform this under linear equivalence, you look for a linear transformation, so that, the computation of e power t minus t naught, into b into y naught, is called y naught. So, this may be easy; this may be difficult, probably easy, you see. This is the situation, we are looking at it. So, the question is that is it possible to find that linear equivalence, so that, the solution to y power dot is equal to b y, is easy to compute. When I say that solution to y dot equal to e power b y, is easy to compute; this is easily, computable. We have seen one situation where this can be computed easily, when b is diagonalizable. So, this leads the question of diagonalizability of a matrix. So, that is where, we are looking at it. So, that is where, we we are looking, so far, we had discussed this linear equivalence, so that, we are looking for a matrix b, corresponding to a, using the linear equivalent, so that, b is diagonal, you see.

Suppose, d is diagonal; that is a question. Suppose, b is diagonal; lambda 1, etcetera. lambda n. Then, what is solution of y t? The solution y t is, immediately, you can write it; y t is equal to the diagonal of that one. So, diagonal e power t minus t naught, lambda 1, etcetera. e power t minus t naught. lambda n of; this is a matrix, acting at y naught; what is y naught; p inverse of x naught. So, this is what is your y t and what is your x t; x t is nothing but p of y t; that is what about our transformation. So, you have your solution immediately, p diagonal e power t minus t naught, lambda 1, etcetera. e power t minus t naught, lambda n of p inverse of x naught, you see. You have your complete solution here. You can write down, if you can question. So, that is why, this is the question of diagonalizability, immediately.

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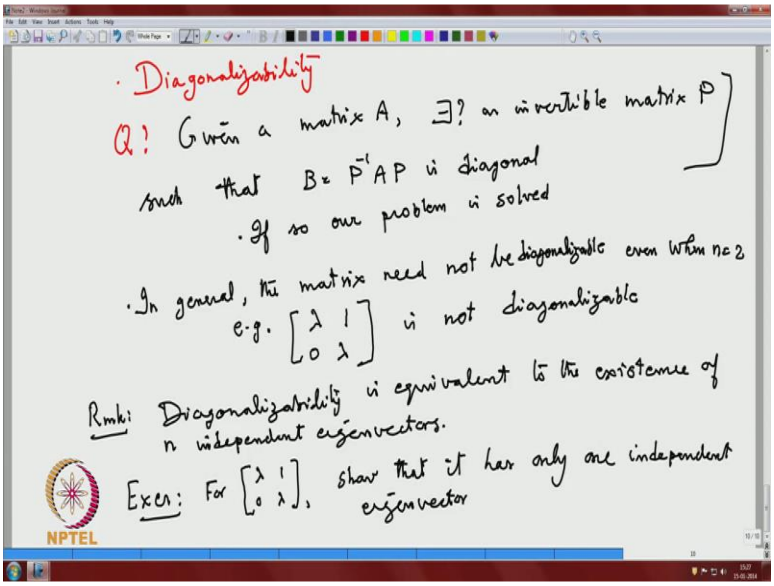

The importance of, you would have seen the diagonalizability importance, not only here; when you are studying the linear algebra whether if you can solve a linear system if the matrix is diagonal, because there again, is a decoupling system, exactly, what you are showing that. So, the question is that given a matrix A, does there exists an invertible matrix p, such that b is equal to p inverse of a p is diagonal. If so our problem is solved Our computational problem is solved, because now, the computation of e power is easy, because b is diagonal, but unfortunately, in general, the matrix need not be diagonal. Example; even when n e equal to 2, that is a simple diagonal, need not be diagonalizable; that is what I said; not diagonal, when n e equal to 2. If such a matrix exists, it is called the diagonalizable. Example, you take lambda 1, 0; lambda is not diagonalizable. How do we prove that? What is the kind of diagonalizable? We see even, such a matrix, simple matrix, simple looking matrix is not diagonalizable.

Again, let me recall from linear algebra; diagonalizability is equivalent to the existence of n independent eigenvectors. So, remark, recall; diagonalizability is equivalent to the existence of n independent eigenvectors. This, we have already seen in linear algebra. We also recalled some of these things in our basics; independent eigenvectors. So, here is an exercise for you for the problem, for the above matrix; for this matrix, show that it has only one eigenvector, one independent eigenvector. You see that. So, when you are studying the linear systems, our main aim is that given a matrix; can we have a diagonalizability property? The moment your given matrix is diagonalizable; that means, you can find a matrix b, so that it is a corresponding b; completely, the system is halt. If your system is not diagonalizable, what can we do about it? This is where the linear algebra and the Jordan decomposition comes into help, and we will use that to understand that how much we can do it, if the matrix is not diagonalizable.

What we will do in the next lecture, we will do a complete analysis in a 2 by 2 system, because 2 by 2 system, the possibility, what are all the possibilities, is easy to classify. Even, in higher dimensions, a Jordan decomposition tells you the best possible way you can do it, but we will spend more detailed study in 2 by 2 systems, and in the process, we introduce the concept of phase plane and phase portrait, and then the various stability. So, we will have a complete analysis in the next lecture.

Thank you.