Ordinary Differential Equations
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Lecture - 2
Examples

Welcome to this second lecture. In the first lecture, we have explained various aspect of differential equations including the importance of studying ODE as a topic of analysis that is what we concentrated in the last lecture, and that is what we are going to do throughout the lecture. What we have especially emphasized is that the methods available to study ordinary differential equations is very limited, and there are very handful of examples where you can actually develop methods.

In fact, most of the physical problems, and the corresponding differential equations cannot be solved in the sense that of implicit explicit that means the, your solution y as function of x and implicit relation between y and t, which is in general not possible in most of the interesting differential equations. It is in this respect only, we explain the concept the enlarge the concept of solutions to differential equations. We the solution does not you know it is no longer means that when there is a solution, the solution can be explained or given in an implicit or explicit form.

There are other also other concept of differ x solutions, which we may not come into that, but it is all are important especially when you study control theory and other physical problems. We will not come to that one, but for us when we there is a solution excess. And that is in this is the scenario, where you need to understand study the solutions and the qualitative study begins there, especially the qualitative say existence uniqueness, and then of course more qualitative analysis about differential equations.

In the next 2 to 3 lectures including this lecture, we are going to present few real life examples, these examples are not new, but it is a important examples it is available in the literature. And we will give some details about some of the examples in this first to 3 lectures, and other details of some other examples you will see, when we proceed the course in our lectures will explain to it. So, you will see also in the through this examples, just producing even solutions explicitly or implicitly is not enough, because the solutions may not reveal it is a features.

So, the idea if you want to understand you are to even after getting the solution, we will see that you are to study the solutions in a sense of analysis. But before coming to the example, I would like to explain 1 or 2 issues which I learned from experience by going and teaching students of BSC students especially. There are certain issues which are not clear, not only to the students and this also not explain to the students by the teachers properly. And we want to spend some time may be 10 minutes on that, and if those who are already think in that, that is trivial they can just get out, just by pass this part of the thing and they can get do that examples directly.

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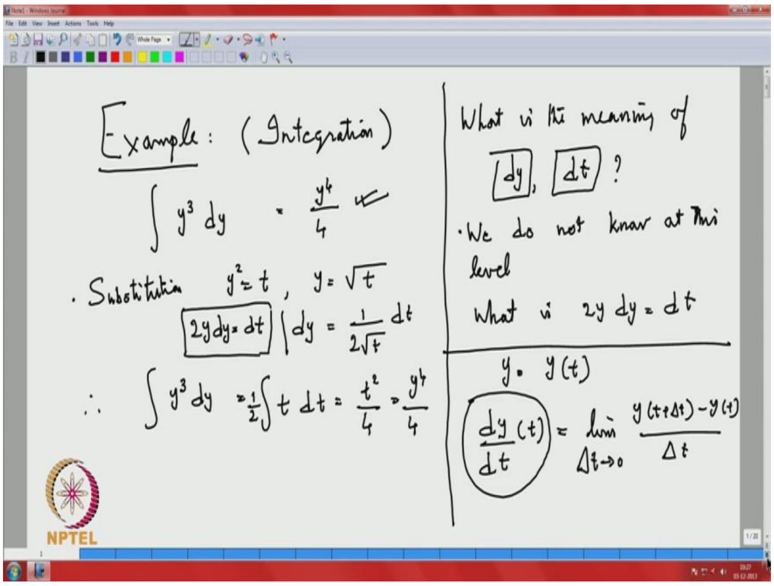

So, let me start with an example, what you want to do it? This is an example from integration, so I am going to from the integration theory as we know that. Today's starter from the integration theory, so I want to start with a very simple example, which you do with in your plus 2 type, and that is where or all trouble quite often starts certain steps are not clear to it. Look at the integral you want do it y cube d y, you want to integrate the, there are 2 ways of doing it.

We know the what this integral is, so you immediately write y power 4 by 4 that is fine, that is no problem that is good, but you do not know this thing. The another way is which is important, here you are able to do it, because you know the anti derivative you can do that one, but when you do not know anti derivative the integration is done via substitution. So, what you do it, you will substitution you will do substitution, what do you do is that? Suppose you I put y square equal to t one substitution, we want to unique or equivalently y equal to root t.

The immediate step you write is, you immediately say an next step you never think of that, you will immediately write 1 by 2 root t d t. Then the next step in the solution you write is, therefore this is what the steps you are write in your 12th standard exactly I am going to write. You know want to know that, y cube d y is equal to y square is your t and your y d y y square is t, so d y is equal to otherwise we will write here 2 y d y this equal to d t, so the ((Refer Time: 06:10)) that one, so one of this both ways we can write it. So, 2 y d y is equal to d t and immediately said that, so y d y y square is substitute as t, y d y is half d t, you write half d t and you write t square by 4 this is the procedure you follow which is nothing but y 4 by 4 hence. So, what is not clear which I never got an answer from the BSC or other students, when you ask what do you mean by this? This is where the first difficulty in understanding coming to you, what do you mean by what is the meaning of d y, d t.

If you do not know the meaning of that is a first step in we never most of the time we never get a satisfactory answer to that. It effect we do not understand this, we do not know it we do not know at this level, that is important why I say that at this level when we study advanced differential advanced topics in mathematics like differential geometry etcetera. Then you do interpret this in a different way, but that is not the think we have to understand at this stage. So, when you do not know d y and d t, how do you how so the question is that what is this? What is 2 y d y is equal to d t. So, I want to spend bit of time here 2 minutes here.

So, here is the what is your trouble you are doing, what you know really is. So, you have to understand that when you treat y as a function of y (t), that is what we are doing it. We know what is the y by d t together, this you as a function of t you know what is this? You do not know what is d y probably, you do not know what is d t? This is say symbolic representation; this is a notation to understand the meaning of this limit delta t tends to 0 y of t plus delta t minus y of t by delta t. So, you have the meaning for as a whole of these things, but you do not have the individual meaning to it. So, that is a what you have to, so that is what were.

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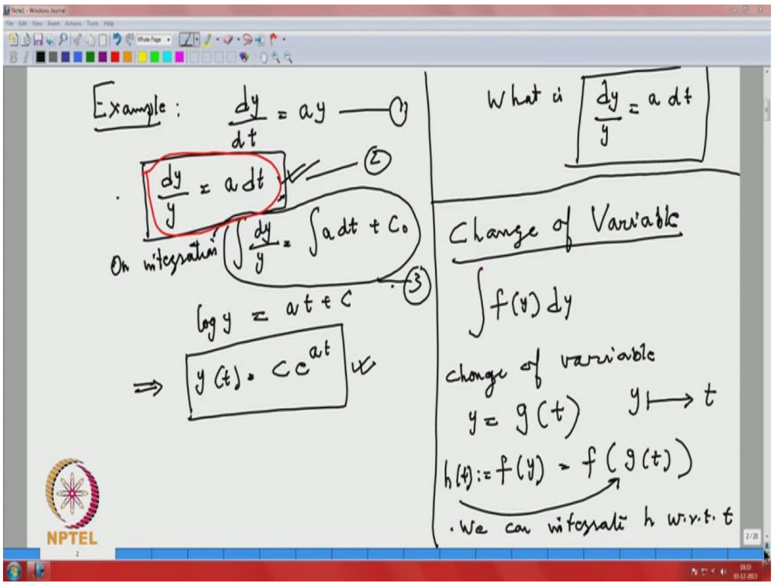

I will come to that example together. So, similarly let me start with one more example where a similar lack of understanding this I want to do it, because differential equation. Similar thing how you proceed here, you have d y by d t equal to a of y in the similar way, what you write is that the next step you write is that. All of you will write d y by d t, so take d y by y is equal to a d t this will be it immediately.

And then say that on integration you immediately write integral of d y by d y is equal to integral of a d t plus some constant integrate you get your log y, these are the steps written quite often a t plus C it want. And finally with one more step you write y (t) is equal to C e power a t good the answer is fine, see the whole thing is that in the previous example than it is right, even the answer is correct here. So, again the issue is that, what do you mean by d y by y is equal to y, and this is whatever sees an arbitrary constants here.

So, what is the meaning again, what is d y is equal to a d t divide by y is equal a d t to do that, because it explain this one if you want to understand this one. If you want to understanding you have to understand d y, and here is a in total result be quite often by pass, but that is one after important aspect when you want to study the not only the integration theory quite a many other thing. Whenever you develop an integration theory, you have to understand one of the important result what is called the change of variable formula.

So, we will introduce in our preliminary, change of variable is what is the underline principle behind all these things to explain to use that. So, what I am trying to say is that all these are all, so when you recall the first example, when you have so when you call when you writing this one, this is only a symbolic representation of this fact that is what I am saying that. So, I said this has no meaning, so you have an expression here say 1, you have an expression here 2, you have an expression here 3, what I am saying that 1 in 3 have meanings, 2 is only a representation of the fact 3, what you really need to prove?

Derive 3 from 1 and this is the part here the part d y by d I equal to y, you have to view it as a step 2 represent 3. Well this is where is given from the what is called from the change of variable formula, how does I change a variable formula looks like, which is what we are going to explained, but you will see it in the preliminary set. Suppose you want to integrate integral of suppose you want to integrate f of y d y.

And you are making change of variable that means when you are change of variable, here y you view with as the variable, here the view y is the independent variable as far as this is const. And then you are making y going to equal to as a making a change of variable as a function g (t), you have t you see for making. So, you are having the with independent variable y, you are making to a new variable t via this relation. So, in that case f of y will be f of g of t you see, hence this can be viewed as a function of t, you see you have a function of t. And hence, we can integrate h with respect to t.

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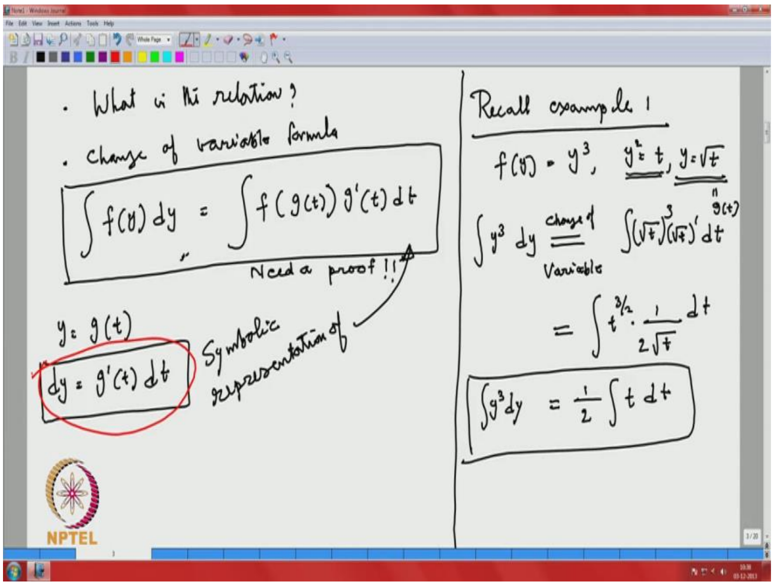

So, the question is that, what is the relation? This is given by the change of variable formula, this is what is given by change of variable formula, which it think that mean you if you integrate this one f of y d y is equal to nothing but integral f of g of t g prime of t d t, this is nontrivial so it is not immediate need a proof of course, need a proof. And this is what you will be seeing in the did a proof, it is not clear this is what I want to say that. So, whenever you making a an integration is introduced, you have to always prove certain types of change of variable formula.

Unless we do that one, we will not be able to proceed further, we not be able to integrate quite often many interesting functions. So, the change of variable formula is very, very important, when you study any integrate whenever you that is a you might have study on just not just one integration. Many other types of view integration will come across with that, each time you have to do a change of variable formula, this also important when you have a multi valued function. Suppose, if you say function of different variable y 1 etcetera y n, how do you change it.

There will be change of variable formula is much more complicated, you will have in terms of here certain Jacobin, etcetera then. And then so when you whenever you make a substitution y equal to g of t, the normal procedure you immediately write d y is equal to g prime of t d t, what I am trying to convey to you is that, this is only a symbolic representation of this fact. So, whenever I am telling that you can integrate in to whenever you have function f n, whatever if f equal to 1 will have integral of d of y is equal to integral of g of t, this is the fact used here.

So, as said this has no meaning, if at all you are giving to going to meaning is that you have to give this meaning to this fact, this what. If this change of variable you will then a in the a preliminaries we may say if you more words about it there will be of course, we have some function why you get a function of t, but you have to re substitute to t in terms of y, all that facts are that essentially that one. So, when they are you will have the limits for this integration then there is no problem, if not limits you have to re substitute back that what it.

Now, let us recall the facts, now recall our example recall example 1. And I do not want to use it, because I do not have anything meaning to that. I want to see prove everything just you seen my integral we cannot example 1, 2 examples we are and integration theory example, what is the first example? You will have f of y is equal to y cube, and then you have to substitute y square is equal to t or y is equal to root t. And then I want to understand the integration now with this relation to that in. So, I am not going to write d y equal to something, because ((Refer Time: 18:08)) I am not going to use d y equal to d whatever 1 by root t d t that only because that has no meaning.

So, I want to understand the integral of that one, so what is your y cube d y here is equal to change of variable, here is a change of variable is the change of variable, what do you want to string? Instead of the change of variable I want to substitute y square equal to t f of that, when you are put y is equal to root t, I will get root t power cube. And then what is my g of t? Here this is my g of t you see, so I want g prime of t, so I want root t prime d t. So, when I write this one, so I have not used anything I have use the change of variable formula, I if I substitute that one I will get it t power 3 by 2 1 by 2 root t d t is nothing but, so I have arrived that half of t half t d t.

So you see, so arrived that formula integral of y cube, I proved this third equation without by passing that thing you apply just that, and then you proceed it. So, this is your first example which you there. So, wherever you do it we keep we will also write this think now onwards. So, whenever the write something like this d y equal to something like g prime of t, keep it in your mind all the time that there is something relation to this one, some sort of that is the meaning of it. So, you need to prove it, you need to require a change of variable formula at of point of time to conclude that one you see.

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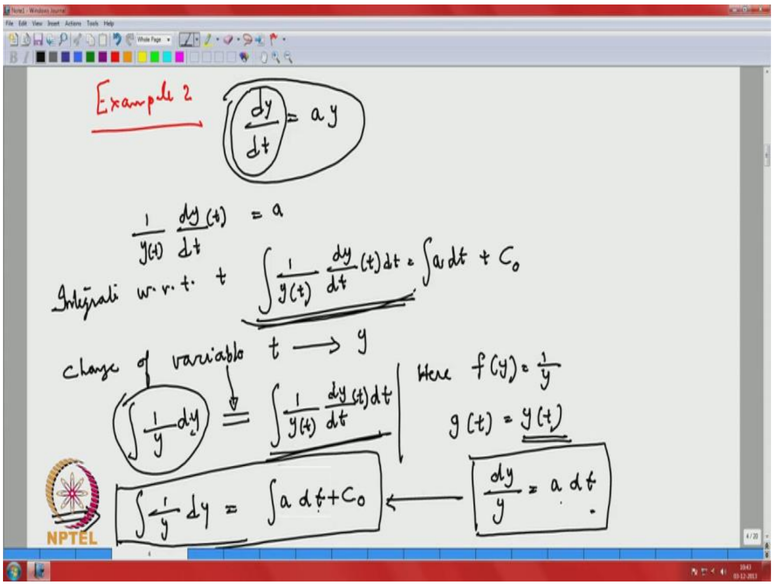

Now, we will come to the other example as saw also differential the example 2 which we have done example 2, what is the example 2? You have your d y by d t is equal to a of y you see, I do not want to write d y equal to d y by y equal to a d t. Eventually we will do it as a, but we have it in mind, so I want to write this is d y by d t is meaningful I told you, because it is a function of t, so there is no problem. So, I can write y is meaningful, so I will write 1 over y d y by d t is equal to this all function of t, so y is the function of t d y by d t is a function of t, you will have a is a constant which is a function of t.

So, I can integrate, when I integrate with respective to t, what do I get it? I get a 1 over y of t d y by d t, which is a function of t d t equal to integral of a of t a of d t, some constant of integration. Now, change of variable, what is the change of variable here, again change of variable here t 2 y you can make change of variable any where you like it, I want here a change not earlier it was y of t. So, I want a t going to y, I want to view instead of t as my independent variable, I want to see y as my independent variable. And I want to see as you know very well 1 over y is the function coming into picture, so I want to integrate 1 over y.

Now, I am viewing by as independent variable and I am differentiate it. So, my f of y is 1 over y here f of y is 1 over y, and my g of t is equal to the. So, here f of y is equal to 1 over y and my g (t) is nothing but y of t, this is what you are doing it. So, if I do that one by change of variable again, change of variable formula again here I have my thing I need view this as a function of t. And I want to differentiate this differentiating is nothing but my d y d t that is it, y prime of t I want it. And I can integrate that d t I have t into d t, you see and this is nothing but, this left hand side of this one.

So, hence that implies if I substitute this for this term, I will write the integral over 1 over y d y is equal to integral of a d t plus C naught you see. So, you arrived up this formula with a change of variable. And this again symbolically represented by d y by y is equal to a d t you see, I want to say that this is it this cannot be derived from here, this formula cannot be derived from here this not directly derived is the fact of integration parts. So, this is the one important thing I would like to explain.

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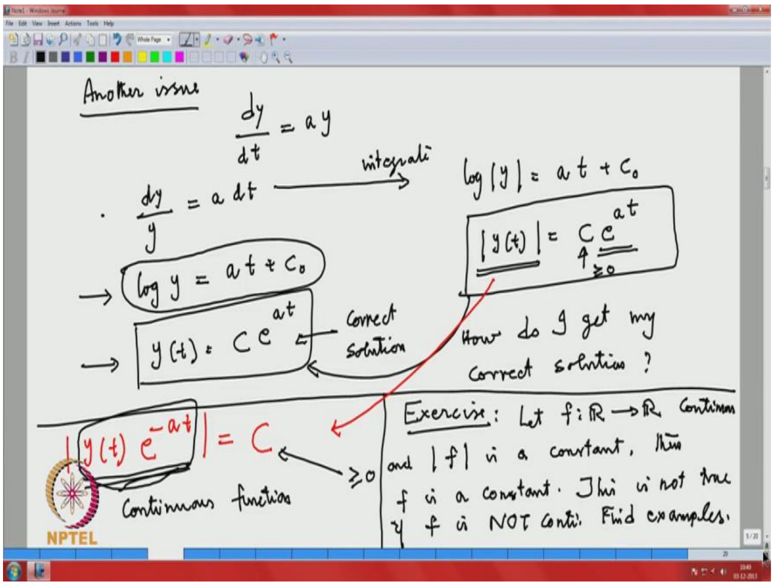

And one more fact which may be looks simple, but I have seen lot of student making mistakes, another issue probably trivial to many of you, but I feel that I want to clarify this one because we are going to do this again and again. The other issue which you are again with three these examples let me explain to you this is equal to a y. So, now symbolically let me write it, because the meaning of this is understood. Now, I can write this with when I write this one I have something else in my mind, the thing is that there is in integration for the change of variable formula.

So, symbolically we are going to do this again and again, but and then you the next step most of the time we write log y or log natural you can use it, log y is equal to a t plus C naught. And then that you will write eventually y of t is equal to some arbitrary constant into e power a t, fortunately you get the correct solution no problem correct solution. But, this is not entirely correct integrating though you have a solution here, when you integrate here, when you integrate this one, what you get is really log mode y is equal to a t plus C naught this is what you will get it.

And then from there you will get modulus of y (t) is equal to C e power a t that is what you will get it. So, how do you arrive mode y is immovable able to differentiate. So, how do I get my correct solution from here, so that is a mistake how do I get my correct solution, so here is something, which I want to tell you may be able to follow. Write this may be use a different color, write this modulus of y (t) into e power minus a t this is are simple things, but the moment you understand the simple things. The difficulty the difficult things are the combination of simple and easy things is nothing like difficulty pars, all your simple lines easy things.

If you put it together licks things looks very difficult you want to change it, since your job, any students job or any teachers job is to understand things the easiest way possible. So here you have a constant, here is again one need to use one of the important thing, what a called the continuity of the function. So, this function being differentiable is a continues function, and another thing since this is a modulus, this formula is value C is greater than C equal to C naught. So, here it is self, this is a positive quantity this is a positive quantity, hence this is valid of course this is coming something like e power C naught, so it is greater than equal to 0.

You will be immediately see that this is not for any constant for negative thing, but this is a continuous function, here is a first small exercise for you, exercise which you should do it. So, whenever we are giving this course we will be leaving small lot of gaps in the proofs, we will be leaving small, small exercises for you, and that you how to work out before going for to next 2 lectures. So, study the entire course like lecture by lecture, solve the problems solve more problem get into the books try to understand more example we may present only 1 or 2 example, but you try to get more and more example and then you present it.

The example let f from R to R continuous and modulus of f is a constant is a constant this is very simple example the moment you learn continuity you will be learning, but if you right you follow that f is a constant. Then f itself is a constant that is a thing, if the module if you have a continuous function this modulus is constant then the function, this is not true if f is not continuous this is not true. If f is not continuous find example not continuous, find examples trivial example are available, but you can find that one with this observation, this is a continuous function you see. And whose modulus is constant, hence this continuous function y (t) into e power minus a t itself it is a constant.

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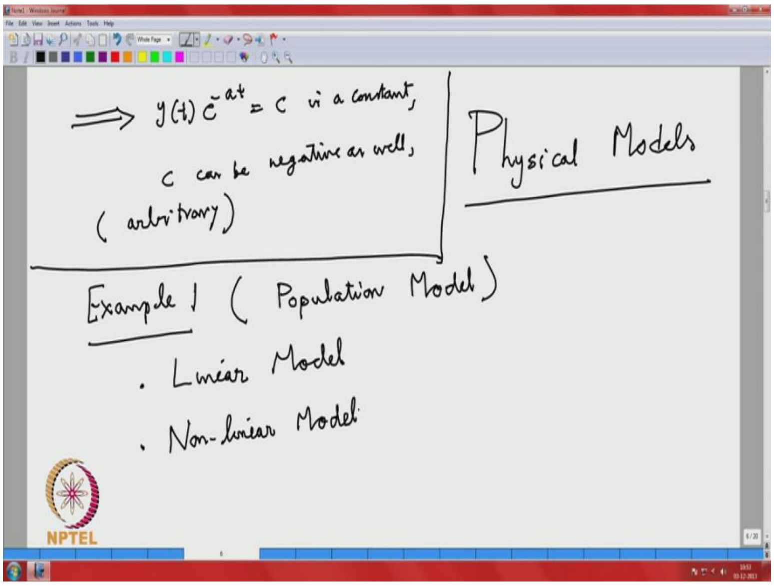

So, you will have, so that implies the example the exercise implies y (t) e power minus a t equal to c is a constant, what is a difference from earlier? This constant can be negative, where C can be negative as well can be negative as well, so that is why call it very arbitrary. So, this to importance node observations, which you have to one, has to be very clear and you have the correct solution you see. We see these kind of small facts in your study, not only in ordinary differential equation with other equations as well. And for a new way they are going to give you examples in the as I said in the next few lectures 2 to 3 lectures. And will start with some physical models starting with physical models.

So, today now to complete this an in just to one example, example 1 you have the population model this is what we are going to do it. And our it is easy for you to solve those who are studied in a differential equation they will able to solve it immediately, but the purpose is not that we want to say, how we use the analysis to derive more about the knowledge of the trajectory population model. So, there are two things one is the linear model. So, will start with the linear model, and then we will see that linear model is not always correct not always be the good model. Then we will go to the one of the famous non-linear model given one by the biologist will come to that.

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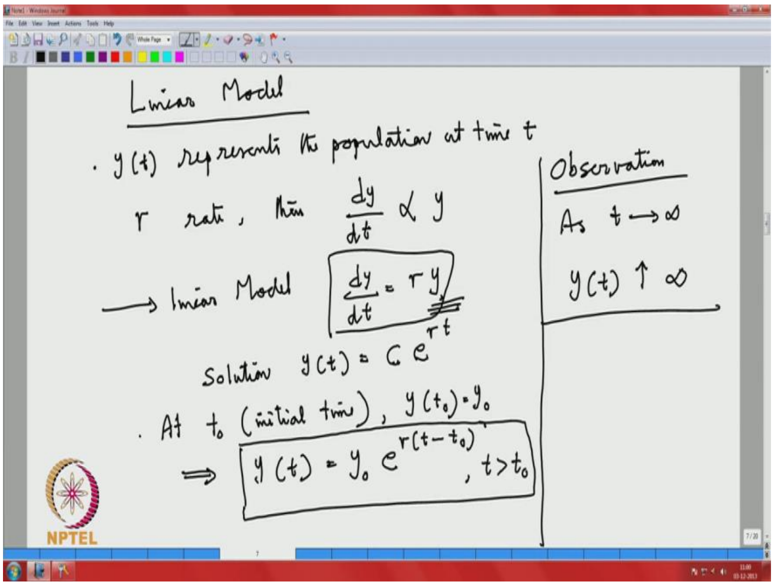

So, will go to that one, so will have the linear model. So, suppose my y (t) represents the population at time t, so let me population at time t. So, how do you suppose is a population is growing just based on the population no influence from anywhere else, you have enough place, not computation, no natural disaster nothing it depends only on the population of the number of people available. So, it is a natural way to thing that, suppose you have 2 cities or 2 systems, in which one system you have n number of population n is the population; other one is the 2n is the populations. Then after an year if there is a growth, if it is a normal procedures.

The extra population in the first systems and if you compare the extra population in the second system, the second system will have the double the extra population then the first population you see. So, the simplest and easiest model is that your rate of change will depending on the population, how much it will grow is what is called the population rate. So, there will be a not be the populated rate, then a simpler model will be d y by d t the rate of change of population, with this phenomenon with there is no other things will be proportional to y it is a that is easiest model.

And that gives you your linear model d y by d t is equal to some r into y. And you have seen you already solve instead of r with a, you already solve this equations. So suppose, so you will get the solution you see, solution y (t) is equal to some constant into e power of r t you see, you have e power r t. So, if at t naught your initial time, wherever you can put your initial time, you know the population say suppose the population y t naught is equal to y naught, then that will imply if you substitute at t naught, you will get y naught. So, your population will be y at t will be y naught e power r into t minus t naught.

You can evaluate and do that, this are the small steps, but I always suggest to the students that you better work out, you may looks trivial just substitute in then do that, that is way you how to learn the mathematics. So, you have your solution is easy way, you have this solution completely. Now, can immediately see observation that is what I said, you got the solution down stop there, but always analyze the solution what happens?

And especially when you create a model of a physical phenomenon dais down observations delta and all kinds of thing, what you say that your model is actually true, whether you are an actually your able to predict your thing. Whether you are a model go strong at some point of time and it does not really give you the does not really represent your physical phenomenon. You have to that is what is after the analysis which I represent that the schematic diagram last week. You have to after doing the analysis, you have to go back to the modeling once more, and you may have been you may need more data, it is what I explain thing of it.

So, you see that an immediate observation here, this you can prove mathematically of course in c c as t tends to infinity your population y (t) increases to infinity, but this is not a decide state of a year, nowhere in the universe you can have a population as time goes the population will goes to infinity, what will happen is that? When the population increases to infinity, when the population becomes bigger and bigger there will be much more competition between the species.

We are analyzing here only about one species there will be sees there are systems which we may explain next one example later more than one species, where even with just one species we are analyzing this problem. So, within the system that 2 species 2 persons or to back it can be a bacteria whatever it will, there will be competition between for food, space everything all kinds of competition are. So, we are looking into a model, where there is only a competition between, still we are not taking into account other aspect like national natural disasters and things like that.

And that can also and also the population can come from other places to this one, all kind of things can happen we do not take. So, we are going to give you a non-linear model now. As this model is not suitable when this model is not suitable, when the population is not the population this model is reasonably, when you are population is not too big. And it is a non factor, but so we are going to give a non-linear model.

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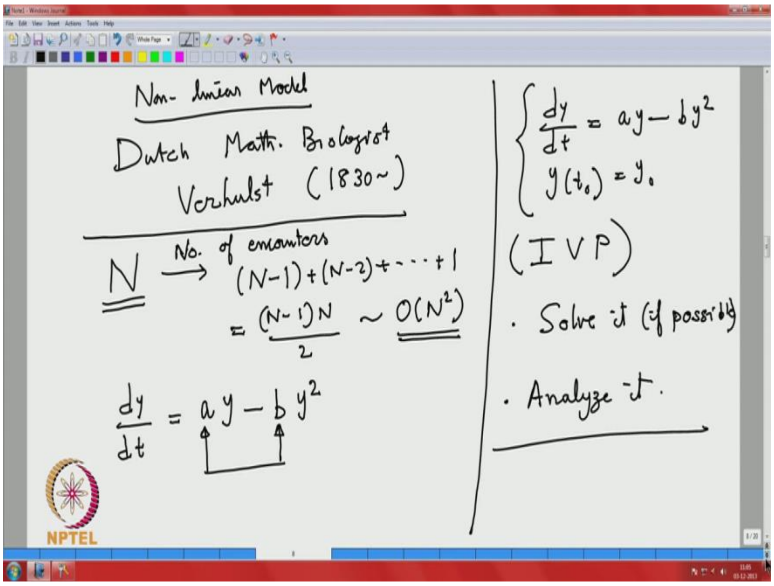

So, going to introduce a non-linear model with just one fact, that you are taking into account the competition between the species within them, not the competition with the species with another species, so the competition. And this is given by one of the famous by Dutch mathematician mathematical biologist Dutch mathematical biologist is a verlulst is called what is the spelling I think verlulst is given probably in 18 something like 30s or something approximately.

So, how do you take into account, so let us say anything to understand anything to model, you have to having intuitive feeling about. Suppose you have a population with N people, what are the possible numbers of competitions a person can meets all other people. So, a person can meet that number of encounters, this called basically I am trying to give a statistical average, what are the number of encounters? A simple thing will get it, can the number of encounter is that the first person can meet a minus 1, person may be the next person can meet N minus 2 like that.

So, the number of encounters will be thing, if everyone meets every body, that want when that is where you have to understand the population. And then you have to put a factor, this is something like up equal to N minus 1 into N by 2 that is in the order of N square, so the this is. So, N is the number of people in a population, the number of all encounters between each person meets one is order of N square. So, it is a the message is that it is of order square, but not that everybody will meet, so you have to understand when you are modeling it, they have to understand on what fraction, and this have to understand from a particular thing.

And what verlulst given is a model then d y by d t, you have already you have the factor r, let me use it a now that r is same as a here a y. But then due to this encounter, you are saying that 2 persons meets encounter they get killed, may be 2 bacteria’s meet they attack because both of them will be trying for the same space or same food. So, they one of them they do the fighting, and one of them will survive. So, it will review the population it will be in the order of y square, but not that everyone will meet, so there will be a factor.

So, as I said for a particular problem, whether it is a bacteria growth, human population growth, study how to chose a and b depends on the particular population which we are not going to give it here. It has to do a statistical analysis you have to collect the data how over a long period of time, and you how to evaluate n p for a particular population. Even for a particular population it may work for a certain period of time, so you have to do an update may be you make a remark again later at that one.

So, you have a non-linear model, you see this is a non-linear model given by d y by d t is equal to a of y minus b of y square. And suppose your relation at initial time y naught is equal to y naught, so you have a very nice initial value problem you see. So, two steps we want to do it here, so as I said we are not going to give you all the details about the problem, I would request the student to go through it and fill up the gaps, whichever is missing there.

So, first you want to solve it if possible, always that is the thing solve it if possible that is the first thing, then to the interpretation just solving is not enough. After that I want to understand just like in the previous example, which you have seen, which you like previous example your population is growing, I want to understand a mathematical study of that equation after getting the solution. The solution may not reveal the, this one and analyze it. And this is what we are quickly going to do that one.

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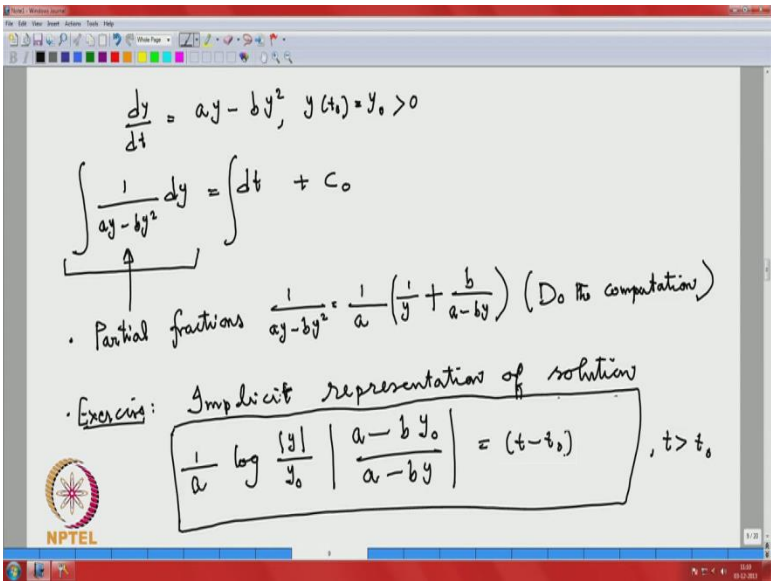

So, you have the problem d y by d t, again let me write initially we spend go little more slowly y square y at t naught is equal to y naught this all of you know it, you can use my change of variable concept or now you are know that, what does that mean? You can do the same thing, you can write down your 1 of a of y minus b of y square b y this is formal, but I can still write formally, if you do not like now. Now, after seeing that writing this is you have to be you have to see this with pinch of salt, when you are say that there is an integration formula. You can divide with the d y by d t equal to 1, and do the same procedure to do that one.

So, you after that you integrate and put a constant no problem, this is fine how to integrate is All right. Now, how do you integrate this one, in the of course you would have a study how to integrate this one, this integration is done via partial fractions you can write down. So, you would go into this you have to it, b y square you can write it as probably, you can write it as I do not do the computation, but I you should do the computation and you study this course 1 over y b a minus b of y do the computation, do the this is an excise whatever I am leaving a part of the exercise which you should do it, instead of writing the exercise that at the end of it will write the exercises here.

So, do the computations do that work, once you do this one, so these are in the 1 by y 1 by a minus b y form. Now you write it everything 1 minute will continue, do the computation so this not minus and this is plus do this one. So, this is in the log form, this is in the log form make sure that everything is writing in the modulus form do not make the mistakes like writing 1 by log y 1 by log of a minus b y, what you have to do is that.

So that is a next exercise for you, exercise for you to get it you produce a solution and implicit form implicit representation. You prove that representation of solution, how do you get in implicit representation of this solution 1 over a log mode y by mode y naught, y naught is positive because that is given to you, y naught is a positive quantity which is given to you one can be 0 also no problem, but here is greater than 0 of into modulus of a minus b y naught by a minus b y to get that one is equal to t minus t naught.

So, you have a solution representation using quit often you stop at this stage, I want do not want you to do that, this is for t greater then t naught. This does not say anything about you do not understand, how you are y behaves though you may complete that I have a solution to your problem, I have an implicit relation connecting between y and t, but you have to do much more in this analysis and that is what we are going to do a bit more thing.

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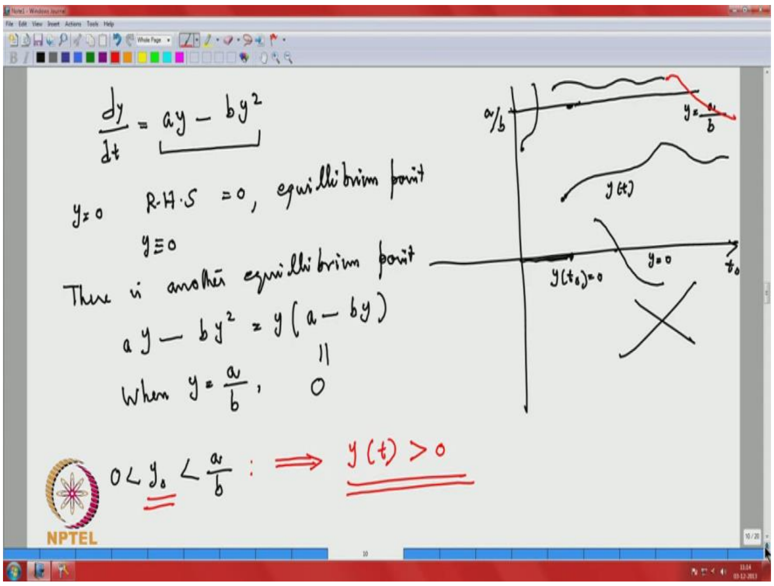

We have to see few things about it, quickly you will learn this facts later d y by I am telling you if you facts, but you write it properly to complete it, d y by d t is equal to a y minus b y square. Look at this expression, when y equal to 0 R H S is equal to 0, that means this such points are call equilibrium points. And whenever you have y equal to 0 in the way if you. So, if you have solution y at t naught starts at 0, then d y by d t will be 0 and it remains to be 0. So, you get y equal to 0, I said identical solution it does not move.

So, it is an equilibrium points of the solution starts at the origin at 0 at t naught, if a solution starts from 0 the solution will remain to be 0 all the time. The interesting thing is that there is another equilibrium points, what is that equilibrium point? This can be a y minus b y square can be written as y into a minus b y. So, when b equal to y equal to a by b, one more point here normally b is very small compare to a that is again a thing a fact which you have to tell. Then again this is 0, so you have a second equilibrium point when y equal to a by b.

So if you have if you try to draw these thing here, so if you are population. So, the a b be will play going to play in important role here. So, if you have your population, if you have population starts at t naught this is the t naught, in if this is your y equal to 0 graph that is nothing but y equal to 0 graph. So, if you are y t naught is equal to 0, you are y (t) will remain to be 0 all the time. The same thing when you have a curve, this is the curve y (t) equal to a by b y equal to a by b curve, so this is also in equilibrium point.

So, at t naught if your solution is there, the solution were remain to be there. And then it is interesting fact that two trajectories all that you will be learning here in a two trajectories, the two trajectories will not meet. So, whenever your initial population is somewhere between 0 and a by b the entire trajectory will be somewhere here, we do not know. We are going to understand more of this graph, similarly if your trajectory somewhere here that trajectory will be here.

So what is shows that, what I am trying to say that if your trajectory y naught starting between the population 0 in the a by b 0 y naught less than a by b. I the exercise is that you have to explain this thing whatever I have explain do it and try it properly. And this trajectory, so if you have a proper thing that trajectory will not come here, this will be learning. You cannot cross that trajectory, because the moment you come here it will mean the you cannot come there, you cannot cross this trajectory.

Similarly, cannot even go here such trajectories will not exist. If you take here the trajectories will not cross here, if you start at initial point here. So, that immediately shows even though immediately in it is correct. So, the eventually what I am trying to conveys to you is that, your trajectory the solution y (t) will always remain to be positive between, so you see. Even though intuitively clear, yes the population goes, but if you have a nontrivial population to start with as it y naught, you start your population will always be positive, and that is the fact you seen it.

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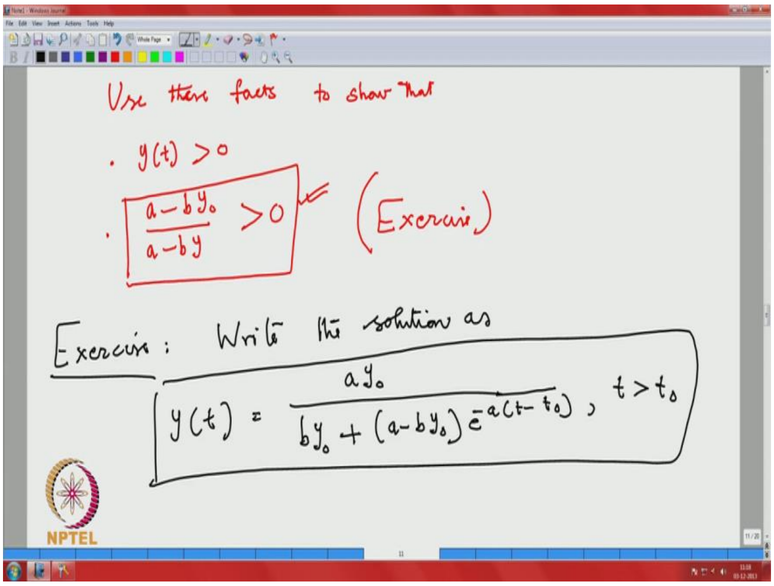

So, from your solution, so what I want you to prove eventually is that use this facts, whatever I have explain use that fact explained. Use these facts to show that y (t) you already seen y (t) is always positive one thing. The second part is more important a minus b y naught by a minus b y is also positive; this is a important thing you want you to prove it. This is what I explain from the earlier graph, this graph what I say that if this is one fact there is more information, because this is y (t) equal to a by b is also a equilibrium point. If y naught is between a by b y (t) will not only positive, y (t) will be remaining in a by b here.

Similarly, if y (t) is greater than a by b, y naught is greater than a by b, y (t) will also remain greater than a by b. Hence the solution which you see it, the solution you see here the solution which we get it, this will and this will retain the sign, and hence the whole thing will have the same positive sign it will retain the sign, because it way. And this is what you have to prove it, it prove this one. So, prove this is an exercise for you, once this is positive the entire modulus can be removed from the solution. If you look at here the entire this is positive, this is positive, the entire modulus can be removed and you can write your solution.

So, that is where next exercised for thing for you the exercise. So, is a 1 by 1 exercise write the solution now in explicit form, earlier you got it an implicit form write the solution. Write the solution as y of t equal to a y naught by b y naught plus a minus b y naught into e power minus a into t minus t naught prove this, write down this thing for t greater than t naught you see. So, you have an explicit representation. And still this not enough for us, we want to understand it little more about it. And probably we will do that in the next lecture, we will a little more time another 5 minutes I will spend on this equation in the next lecture.

And one thing you can immediately sees that y (t) as t tends to infinity this fact goes to 0, once this goes to 0 a y naught and y naught will get cancel, y naught and y this will go to 0, and y naught and y naught will cancel and you see the limiting behavior. So, the limit goes to y (t) goes to a by b, you see that is what is wherever you even if it is a very small population, the population will go to infinity. We will see these facts in the next lecture, and will do further examples.

Thank you.