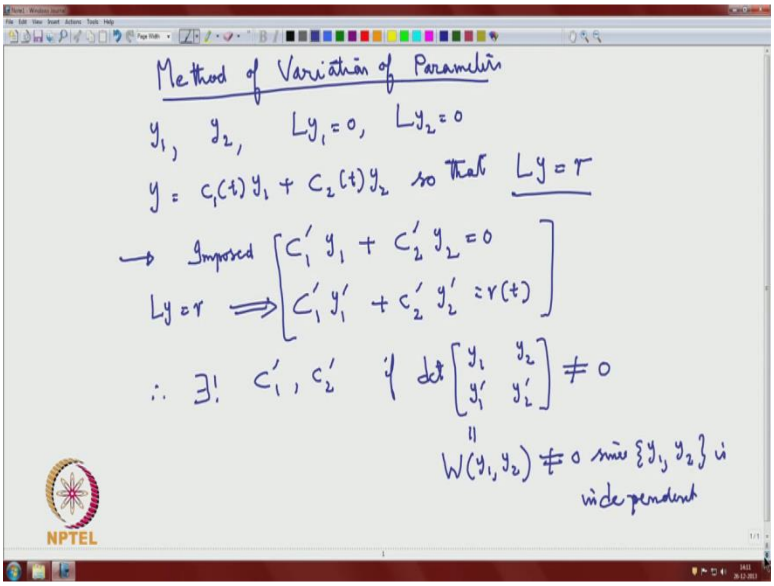
Ordinary Differential Equations
Prof. A K Nandakumaran
Department of Mathematics
Indian Institute of Science, Bangalore

Module - 3
Lecture - 14
Second Order Linear Equations Continued

So, in the last lecture, I could not complete the method of variation of parameters. So, let me quickly recall what I have done in the last few minutes of the lecture; previous lecture and then let me complete it.

(Refer Slide Time: 00:39)



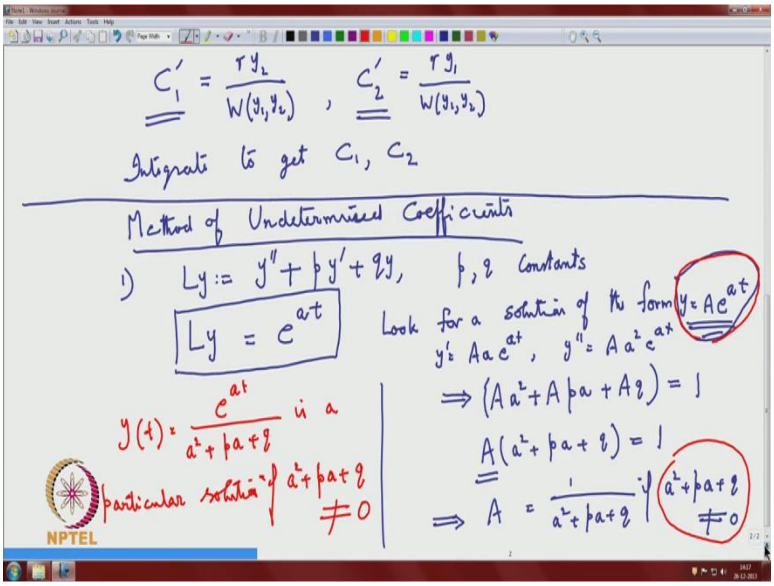
So, what we are doing is that, method of variation of parameters. So, what are the things we have given? We have given 2 solutions: y1 and y2, which are solutions to homogeneous equations. Therefore, Ly1 is equal to 0 Ly2 equal to 0. And then, our idea is to look for a solution y of the form, c 1 t y1 plus c 2 t y 2, so that y is a solution to the nonhomogeneous equation. So, y satisfy L y equal to r. What we have requested you to do? A computation. And after doing the computation, we have imposed a condition because, we have to determine c 1 and c 2 as functions of t. So, we require 2 conditions.

Looking at the differential computation of L y, first do the computation of L y by computing y prime, y double prime and substituting in l y and which should be equal to l of y equal to r. We have seen a term, so we imposed a condition; c 1 prime y1 plus c 2 prime y2 is equal to 0. Once you impose this condition, then L y equal to r will reduced to the condition c 1 prime y1 prime plus c 2 prime y2 prime equal to r t.

So, you see. So, we have 2 equations in 2 unknowns. Here c 1 prime and c 2 prime are the unknowns, y1 y2 y1 prime and y2 prime are given to you to determine. So, that therefore, there exists unique c 1 prime c 2 prime if, the corresponding determinant of this matrix: y1 y2 y1 prime y2 prime is not equal to 0. But what is this? This determinant is nothing but the Wronskian of y1 y2 and this is non-zero since, y1 y2 independent solutions.

So, we are through, in the sense that this is non-zero, so there exists unique y1 and y2. In fact, you can write down the solutions quickly. In fact, you can solve it. You can find the set to a linear system with 2 unknowns.

(Refer Slide Time: 03:42)



So, let me write down your c 1 prime. So, you will get your c 1 prime is equal to just do it, but do the computation you will get r y2 by W y1 y2 and c 2 prime. If there are any mistakes do the computation and we will see there are any mistake r y1 by I think the computation is correct, but you can compute it and check for any errors. So, you get c 1 prime, you get c 2 prime; integrate to get c 1 and c 2.

So, this is a reasonable a very general method. And what it shows that, to determine a solution to your homogeneous system, again we need to know to independent solutions.

But when the equation with constant coefficients, with certain right hand side we can write down the equation in a neat way. This is another method. The method of undetermined coefficients, we introduce that also; method of undetermined coefficients. This is for some special class, because we want to complete the nonhomogeneous term with constant coefficient. When it is a homogeneous equation with constant coefficient, we already know how to write down the equations completely, depending on the roots of the characteristic equation.

So, consider 1 special cases; 2 special cases. So, we will take L y is equal to these are with the constant coefficient, but certain calculations is also true without the constant coefficient. But for the simplicity, we will assume p q are constants. Some of the parts will work even with that, but you write with a b c, but a is non-zero which can be divided, you can write it in this form. So, we are interested in… this is the L y definition. So, a special right hand side; L y equal to e power a t form.

So, we are interested in solving want to solve. This is why we want to solve this. This is the equation we want to solve; l of y is equal to e power. Again this suggest by the same arguments which I have given in the class. Here there is a y here which is missing. See y y prime are all proportional and y prime is proportional to y, then y double prime is also proportional to y and that is why he will proposed a solution of the form e power r t. But here on the right hand side e power a t is available. So, we want to get canceled; the best way 1 of the best motivation to look for a solution of that form.

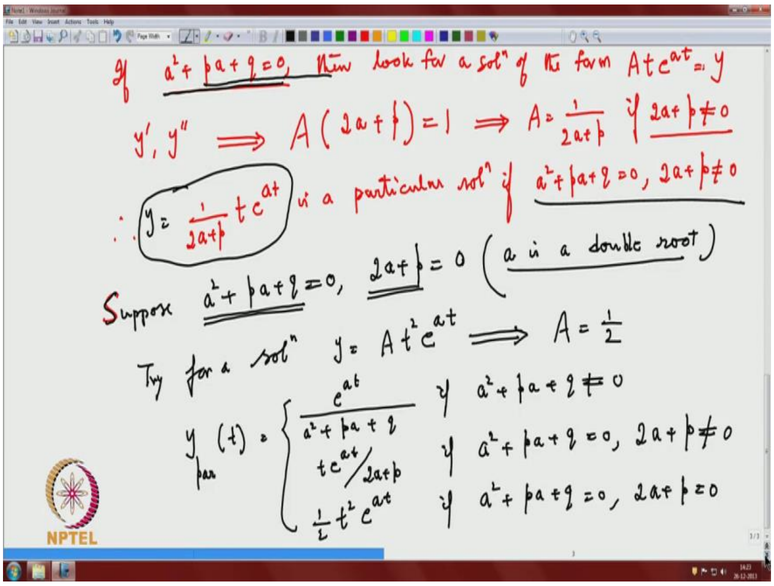
So, look for a solution of the form y is equal to, some constant we do not know, a e power a t. So, if you compute y prime. So, you will have y prime is equal to A a e power a t, y double prime is equal to A a square e power a t. So, if you substitute there e power a t will cancel. This will give you immediately A a square plus p here, p is constant A p a plus q a q into equal to e power a t, that right hands e power a t, that will get once. So, that is A into a square plus p a plus q is equal to 1.

So, our idea is to find an A, so that, y equal to a e power a t equal to 0. So, we need A to be divided. So, this implies A you can take it to be 1 over a square plus p a plus q. And this is possible if that is non-zero. So, that puts a condition; if you want to find a solution that the immediately not equal to 0. So, this immediately suggests that, we have a solution of this form if A is not a root of this equation. This is the thing which we have actually done, which we will see again it is a kind of resonance type same. So, if you have a right hand side e power a t then, such an e power a t should not be…

So, we are proposing the solution to this equation of the form e power a t to a nonhomogeneous equation. Only that e power a t should not be a solution to the homogeneous equation. That is the minimum thing it is telling. So, exactly telling that, e power a t should not be a solution to the homogeneous equation, is the condition that a square plus p a plus q is not equal to 0.

So, that gives a solution. So, y t equal to e power a t by a square plus p a plus q is a particular solution. We are looking for particular solutions, if a square plus p a plus q not equal to 0.

(Refer Slide Time: 10:14)



So, again... So, what happens if a square plus p a plus q equal to 0, then look for a solution, now you have an idea. Then look for a solution of the form; some constant A into t e power a t. So, you are looking for it, we have known that why it is a solution. Some c t into e power a t form and that is t form. So, now proceed it. So, you are looking for the solution of the form; this y our y looking for it.

So, compute again, do a simple computation. You compute y prime in this case y double prime. Then that will imply, do the computation. I will skip the computation. What you will get is that A into 2 a plus p equal to 1. You will arrive at this formula. So, that will give you A is equal to 1 over 2 A plus p, of course, then in that case 2 a plus p should be non 0. If 2 a plus p non-zero, so you see first you got an solution.

Therefore, 1 by 2 a plus p into t e power a t is a particular solution in this particular case, if this has to be 0. It is in that with that condition you are getting it, p a equal to q equal to 0. And you also need this condition, but you need a non 0 condition in that case. So, now, what happens if both are 0? So, 1 more case left out. Suppose. So, let us go this. Suppose, these 2 are happens, that is, a square plus p a plus q is equal to 0, 2 a plus p not equal to 0, that is also equal to 0.

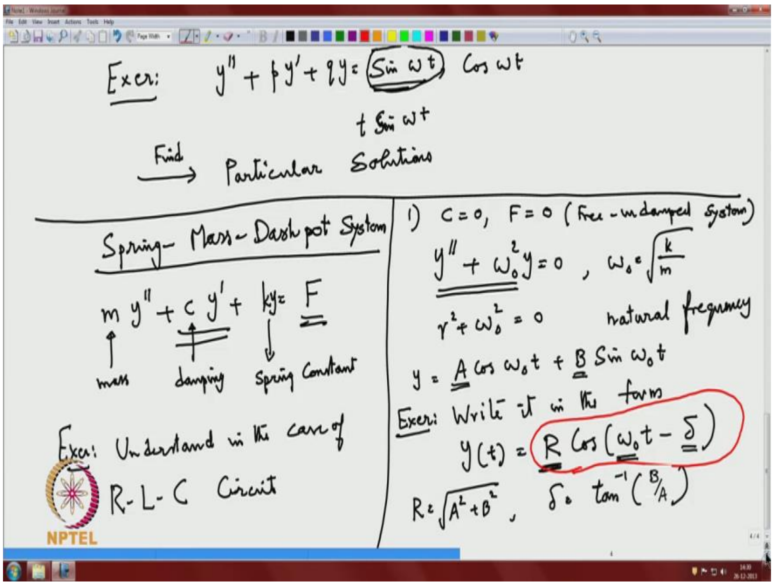
So, what will happen? What is this situation first of all? This is the more interesting you see; 2 a plus p is nothing but the derivative of these with respect to a. That means, it is nothing but a is a double root. That is the situation. So, here when a is a root; that means, the input function, in the engineering terminology, such kind of rights hands sides are always called input function or a forcing function.

So, if e power a t, if the forcing function, has this e of the form e power a t, then a is not a root of the auxiliary equation. Then, you have an immediate solution of that form. On the other hand, if a is a root of this auxiliary equation or a characteristic equation, but if a is not a double root, then you have again solution of this form is a y equal to e power a t a particular solution of this equation. If, is a double root, that means these 2 conditions are trying.

So, try for a solution of the form next level. Try for a solution of the form y t is equal to A t square e power a t. All this will be independent and then you can see, do a computation again. A simple computation you can do and you can see that A is nothing but a half. So, a complete independent solution can be obtained. Therefore, you can get your particular solution of this case; y particular of t is equal to e power a t by a square plus p a plus q, if a square plus p a plus q not equal to 0. You will have this solution e power a t t e power a t by 2 a plus p if a square plus p a plus q is equal to 0, but 2 a plus p not equal to 0. And you have this situation half of t square e power a t if both are 0. So, it completes, the complete analysis you have.

So, you have your particular resolution. You also know how to compute your complementary functions, what are called the solutions to your homogeneous system.

(Refer Slide Time: 15:48)



The same procedure can be adopted. So, I will leave it for you as an exercise, to leave it if you want to compute of the form equation of the form other types of signals. Input functions or forcing functions are available say for example, suppose you have sine omega t or cos omega t, one of them... You can do that and you can actually determine the solution of the form.

So, what you would look for is that, a solution first you determine a solution of this form. If omega is not a root, again that kind of same homogeneous system, if it is not a solution you can proceed with a same form and you can… If that is not a solution, next time you try omega sine omega t. So, the exercise is to find the particular solution. You may find a particular solution. So, this completely analysis the situation, when your solution, when you have a equations second order equation with constant coefficients, even with particular signals on the right hand side; like e power omega t sine omega t and other kinds of things, which you can do it completely.

So, what I will do is that, I will in this particular lecture, I would like to recall once again, which we have done it in the first module namely; the spring mass system. Because now, you know we have already study the spring mass system, but now you know how to write down the solution. I will not give the details here, but to complete the thing and you want you to understand this equations is in the better way, you recall your spring mass system. So, this is a quick recovery spring mass dashpot system. Because, now you we will understand exactly what we want to do that 1.

So, for more details you go back to that problem. So, what is the spring mass system? m y double prime plus c y prime plus k is equal to some external force F. And we have analyzed this in various situations. What is m? m is the mass, this is c is the damping and k is the spring constant. What I would like the students to do, when they read these things, try to understand this equation even in the case of… Exercise is; understand in the case of R L C circuit. Because it is the same equation which we have studied completely, but now since we know the solutions, we can write it here.

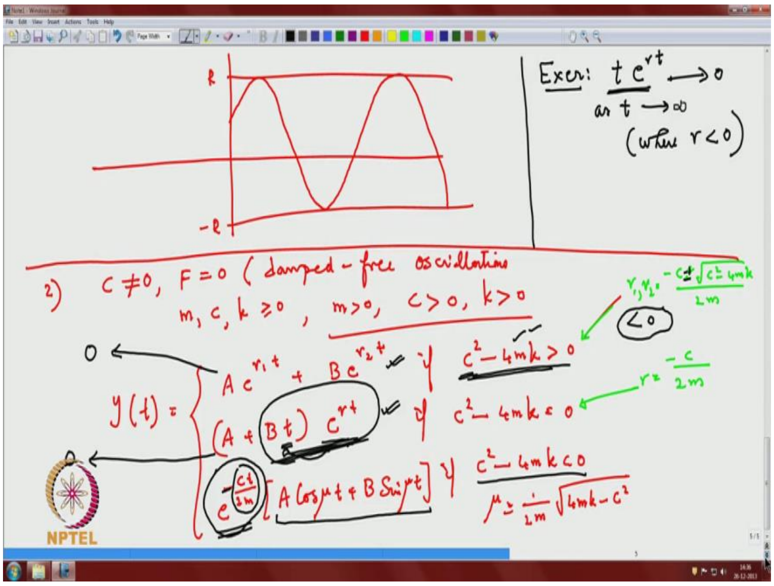
So, we have analyzed various cases. So, let me quickly recall all the cases. What is the first case we have studied? When c equal to 0 F equal to 0, this is called no damping and no free oscillations, is the case of a free undamped system. There is no damping system. So, that system moves; either it is at rest or if you have the system starts moving, it keeps on moving. The phenomena of Newton's law.

So, in that case what we have done is that c is equal to 0, c is equal to 0, F is equal to 0 you can divide by m. So, you will have y is prime square is equal to omega naught square into… there is a y here omega naught square y equal to 0. What this omega naught? omega naught is called the square root of k over m. This is called the natural frequency of the system. This is what we have done it. And this immediately if you write down, the auxiliary equation now you see, the auxiliary equation will be m square. I will use the r r square plus omega naught square which has complex roots, you see omega naught. So, you get complex roots and your solution will be a cos omega naught t plus b sine omega naught t.

So, an exercise which probably at that time I have not given which I do not remember probably it is given. Write it in the form; this is better way to understand the system, write it in the form. Try to do this form y t is equal to R cos omega naught t minus delta. So, you have these equations, where R is equal to square root of a square plus b square and delta is equal to tan inverse of minus b by a. Note that omega naught is given by k by m. So, it is not a constant. The constants are…

So, for mathematician, this is a general solution because, it consists of 2 arbitrary constants; R and delta. Here the arbitrary constants are given by A and delta. So, this if you look at it, R gives you the amplitude, cosine varies from between minus 1 and 1. So, this will vary between, it will be oscillates fully between R and R with a phase difference delta.

(Refer Slide Time: 22:24)



So, if you plot this curve, you will get the solution it will be plotting, there will be a phase difference. So, this will be your minus R, this will be your R with a phase difference. So, it will oscillate and the oscillation never stops as expected. The case 2 which we have considered in this is that is a first case. So, there is no damping. So, there is no external force.

So, now we are taking the case c naught equal to 0. So, you are provided some damping to the system, like your spring and putting it oil or something like that. So, you are providing a damping. And damping is understanding damping, in the case of mechanical system as well as in the electrical system which is nothing but the resistance there, is important. So, you have F equal to 0. This is damped free oscillations damped, but free oscillations.

So, you see, in this case it is a homogeneous equation. And homogeneous equation you know how to write down here, because M C K are nonnegative. In fact, M and C in this case we are assuming, M is always positive we are assuming, C positive in this case. So, everything is… of course, K is also positive. So, your assuming everything is positive in this case. So, in this case you can write down your solution, which we have already obtained, depending on the root of that.

So, you will have a general solution of the form e power r 1 t plus b e power r 2 t. When is that? The roots are distinctive. If I write down that, 1 it will be c square minus 4 m k positive and it will be A plus B t into e power r t if c square minus 4 m k equal to 0. And it will be e power minus c t by m 2 m will be, A cos sum mu t, mu t can be exactly read on plus B sine mu t if c square minus 4 m k negative. And where what is mu, mu is nothing but 1 by 2 m square root of 4 m k minus…This is what in the last class we have seen it already. You see you can write down. And where r 1 is in this case, what is in this case?

So, in this case r 1 r 2 given by minus c plus or minus square root of c square minus 4 m k by 2 m. Here your r the root should be minus c by 2 m you see. And look at it, it is an interesting observation. c square minus 4 m k is the positive case, in that case it is the positive quantity ,whether you put minus sign or plus sign, it will in absolutes value it will be.

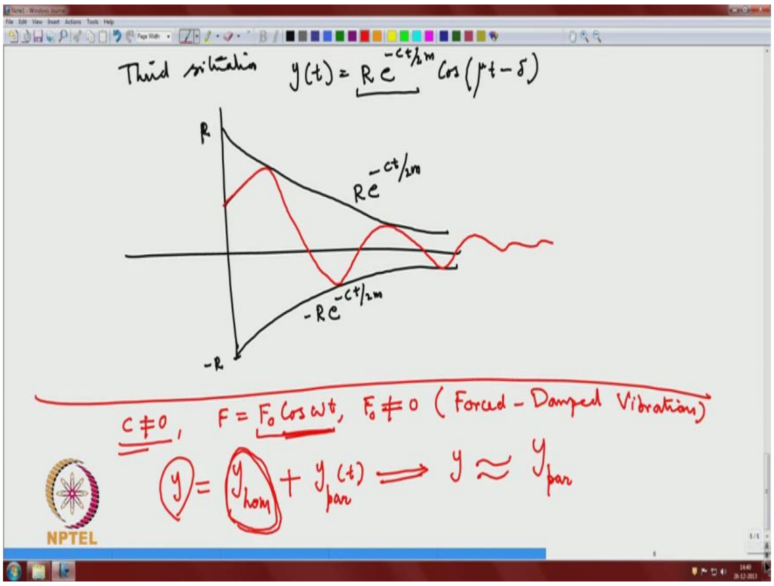
So, in this case not both r 1 and r 2 are negative. This we explain or let me repeat it because, now it will be clear. Its negative, since r 1 and r 2 are negative, this will go to 0 this will go to 0 exponentially. The second case here; this term will go to 0, but then there is a t term, but then t into e power r t, where does it will go? The exercise is that this will also go to 0.

So, here is a simple exercise. Most of you will have a tendency to say that, exponential goes faster than polynomial which is a fact. But what is the meaning of exponential goes thing. So, you have to prove this, just mere words of telling that, exponential goes faster than polynomial is not enough. You have to show that; t e power r t goes to 0 as t tends to infinity, where r is a negative quantity. So, you prove this, otherwise that is not true. So, proving this fact is the meaning of that, exponential goes faster than the polynomial, the third cases.

So, there are; in the first 2 cases this case and this case, there are no oscillations and this term goes faster; that means, this has to be m and k are fixed quantities. All this sign changes because of c and c is the 1 which we are controlling in the damping. So, when you want to make c square minus 4 m k positive, it means that you have to add more damping to the system. And if you reduce that damping, you will reach a situation here. But even if you reduce the damping, it still goes to 0, this term is still goes to 0, but slightly slower way, but still exponentially.

But if you reduce the damping still further, you will reach a situation here, but some damping is required. But because, it is c is positive still there is a negative quantity sitting here, you see c is positive, so this is minus of that. So, still negative quantity sitting here. So, this exponential term still there, but the difference when your damping is reduced further and further, you will have a term of this form, which will have an oscillation. This will behaves like an oscillation, it will have an oscillations behaving. But then this will kill. So, the oscillations will kill thing and we can write that third case.

(Refer Slide Time: 28:38)



Third situation you can write it a third situation. You can still write the third situation as; y t is equal to r e power minus c t by 2 m same way, do it as an exercised cos of mu t minus delta you see. So, this is your now amplitude. The amplitude decreases oscillates if I, when there is c equal to 0, this term is not there. So, its oscillations are not reducing. But since c is positive, the oscillations are still there, but its amplitudes reduces.

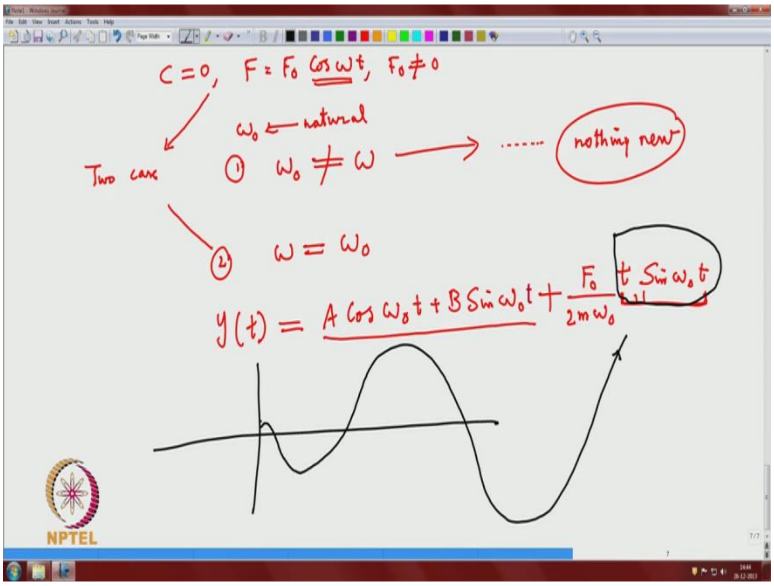
So, if you plot this graph here; you can plot 2 curves here, what the curve s also you can plot it. This curve is nothing but minus R e power c t by 2 m. This curve is nothing but R e power minus c t by 2 m. This is at the point minus R. This is at R and this starts oscillating it touches, see oscillates like that, it decays, like that it goes to 0. So, that is a situation when there is a damping.

Now, the final situations, let me not do it completely, but let me do the next case. Suppose c naught equal to 0 and F naught equal to 0, but I do not want to take arbitrary a, F it is an input function. So, let me try with F naught is equal to cos omega naught t with f naught not equal to 0. So, you an external frequency coming here now, but then there is a damping is provided. So, this is called forced damped vibrations, you see forced damped.

So, what you can do is that. So, it is a case of nonhomogeneous situation and nonhomogeneous situation with c naught equal to 0. You know for the homogeneous part, you know how to write your homogeneous solution, which I already described in the just previous part. And plus you can write down a solution particular solution, we can write down, because it is of the form cos omega t you can do that. And what we have observed is that as t tends to infinity this goes to 0. So, as t tends to infinity will we will have the; y will behave like y particular solution you see.

So, because y homogeneous part, this part goes to 0 as t tends to infinity. So, the behavior of your solution will behave like y. And this is not thing it is a according to that, you can exactly write down your equation, write down your solution. So, what I suggest you to now solve this equation, because it is a cos omega t. Using method of undetermined coefficient, you can find your y particular. So, this we y (( )) we already determine. So, you determine y particular to find the solutions. And see that, there will be oscillations and decay everything were according to that situation. The one interesting case is that, we have already seen that when there is no damping.

(Refer Slide Time: 32:38)



When there is no damping and no external force acting on it, there itself you have vibrations, which is not decayed. Now we will have the most probably; either unwanted or desirable, according to the situation. In mechanical system it is undesirable, because if there is no damping, at the same time an external force coming and acting on the system. So, you will have a situation where c is equal to 0 and F is equal to F naught cos omega t cos sine omega t with F naught not equal to 0. When c equal to 0, you have your solutions and this gives this again reduces to 2 cases, because when you have a omega t here, when c equal to 0 there is a natural frequency omega naught.

So, if you looking for; this is a natural frequency, if you just the previous introduction, when you have a solution here cos omega t and if this omega happen to be the solution of this here in the homogeneous equation, then the solution will be differ. So, if omega case 1; in 2 cases the natural frequency omega naught is different from omega. Then there is no problem, you can write down the solution and nothing interesting than what we have seen. Nothing interesting, nothing new. Let me write it; you will write down the solution you will have some oscillation.

So, if you even if there is no damping, it will be you have a homogeneous system which is oscillating, which we have already seen that a homogeneous solution, then you will have a component of a particular solution and there absolutely no problem. The second case, when you have an equation; omega is equal to omega naught. But then cos omega t cannot be a multiple cos omega t sine omega t cannot be the solution on to your homogeneous system.

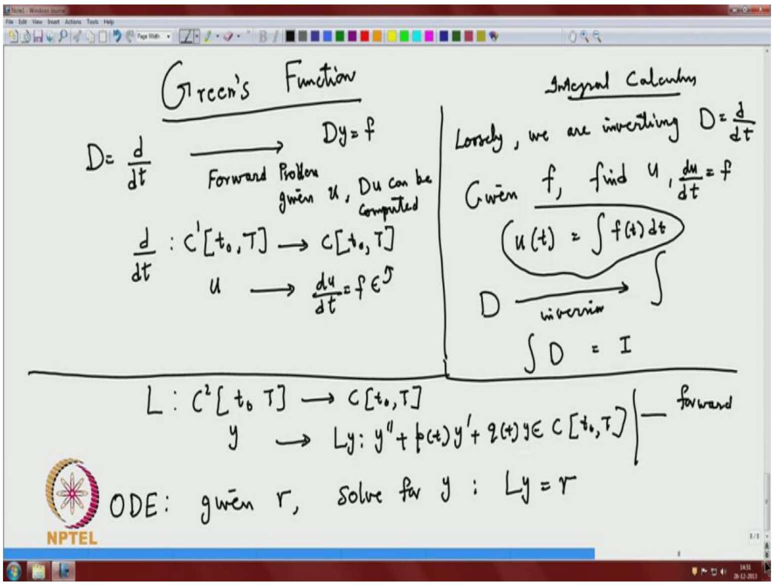
So, the solution in this case will look like little more interesting. You will have a solution due to the homogeneous part cos omega naught t plus c sine omega naught t and then the particular solution will have a…Another fact the particular solution will be of the form, let me write down F naught by 2 m this is these are small constant. What the fact is that, you have to try a solution. If omega is a solution to a that cos omega t is a solution to your homogeneous system, you have to look for a particular solution of the form t cos omega t or t sine omega t and that is what it gives you. You will have a sine omega naught t.

Now, look at this fact; this will oscillate without reducing your vibration amplitude. But here you will see a factor; this is not only oscillating as t increases this amplitude of this factor, the amplitude of this portion increases. So, if you have, plot your graph this 1 you will have something, initially there may be some vibration as t the vibration is. This is what is called the undesirable oscillation and I explained in my first module. Unwanted vibrations in the case of mechanical systems and the Tacoma bridge collapse etcetera, which we have discussed.

On the other hand, the same system when you are trying or an RLC circuit, even if a small current comes and these equation represent the second order linear equation with constant coefficient, represent the equation for I. And then what is if you have an external frequency coming which coincides with your natural frequency, that gets its amplitude gets becomes larger and larger and you can get the signal and that is exactly what you by in the tuning.

So, many examples in this situation can be done worked out. So, just solving equations using the methods or theory is not enough, 1 need to interpret the solutions according to the problem type of problem 1 is studying. Here this more or less completes, which I want to tell in this module. But before completing this particular module on second order linear equations, I want to introduce one of the important concept, not only for this initial value problem, it will also come in boundary value problem and this is also appears extensively in partial differential equations, which this is called the a concept of Green's function.

(Refer Slide Time: 37:56)



So, I am not going to do it too much of Green's function here. My idea is just to motivate you why Green's function the Green's function comes naturally in this set up. So, Greens' function, basically used to represent solutions e to get representation formula,. But why we do expect such a green's function and what is a green's function in the setup of the second order equations. So, let us, let me recall your operator once more, a little later I will call it.

So, let us start with a differential operator d by dx. So, this is a differential operator which we are studying, now you are comfortable with that 1. And you have the finding derivative, it is a forward problem. So, you have already seen that forward problem. That means, you have your d u D y equal to f given f, given f given u find D u can be computed. That is a forward problem, no doubt in that all. So, you have a... So, you can think that d by d x is a mapping probably. As I said this more I motivation, 1 can make it precisely defined.

So, you can think it as a for example, you can say that this is a function from c 1 of some interval t naught to T to go into c of t naught to t you see. So, you take a differentiable continuously differentiable function, differentiate. So, if you take u here, your d u by d x equal to f belongs to here you see. On the other hand, if you do this is the differential calculus. On the other hand, when you go to an integral calculus, you are trying to invert it. We are trying to loosely speaking loosely, we are trying to, we are inverting, and we are inverting d by d x the operator d by d x. That is what you are doing it.

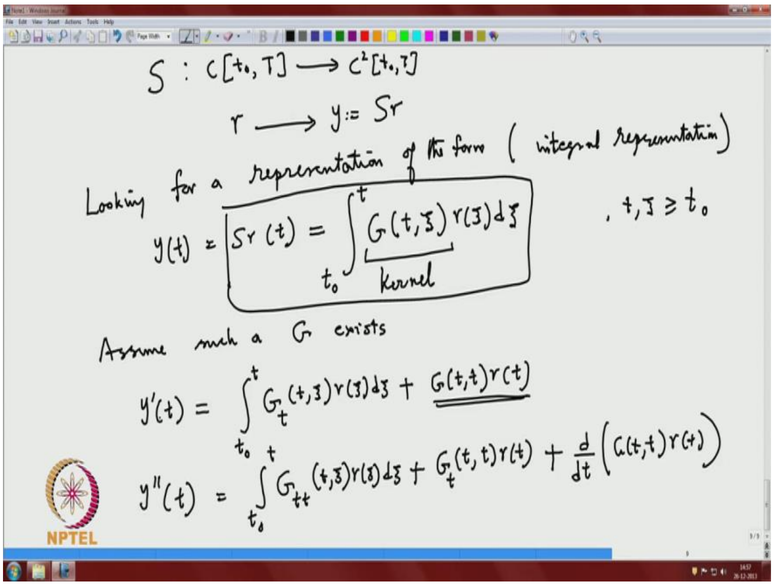
So, these are different interpretations. We are trying to invert means; given f find u, that is what you are trying to do it. Find u so that d u by d x is equal to f you see. And that is what we eventually calculated your calculated u t in terms of… I have used t, since I am throughout using t, let me use t only. So, let me use d by d t d by d t d by d t. So, you have d by d t, you have d by d t. And this is formally represented by f of t d t. This is a formal representation.

So, you have in some sense d going to an inversion of integral. So, in other sense, if you have, your fundamental theorem essentially tells you that integral of D you are trying to get some identity. That is what you are basically trying to do that 1; an integral calculus problem. So, thing is that, whether we can do some representations. So, you are trying to representing your solutions u is of this form.

So, now you have a general operator L. I have a general operator L. I can think that L is an operator; say from c 2 to t naught to T going to say c of t naught to T, where L of u you are y here taking, going to L of y. By definition what is l of y? Is equal to y double prime, p q are given to you p m q are given to you y prime plus q t y. This belongs to c naught t naught to t. So, you have an operator L mapping from the thing here.

So, what is the meaning of solving it? So, what is the ODE we are trying to do so far? ODE: given r continuous given r, solve for y right solve for y satisfying L y. So, this is the forward problem basically. Given thing is forward, that is not a problem. So, when do we solving is that, your trying to in the integral calculus setup in the more general ODE. Your trying to invert your ODE, your trying to solve for L of y is equal to r. So, I can introduce a mapping.

(Refer Slide Time: 43:35)



So, I can introduce a mapping S from c of t naught to t. You do not have to worry about the spaces right now, but that is not the issue here. t naught to t, for given r your finding the solution y which is nothing but S r I call this is equal to S r. So, if you put initial condition, you know that there is a solution given here. We put the, see in particular if you put conditions y 0 is equal to 0, y prime of 0 equal to 0, this S will be linear. If you put other conditions, you would not get linearity, but that is ok.

So, if you put that conditions, you have a basically linearity. You are trying to invert this kind of equations. So, our question is that, analogous to this 1 given u you are getting a representation. So, we are looking for a representation. So, looking for a representation of the form you want to represent y, y as a function of t that is nothing but S r as acting as a function of y. So, this is an operator, you want to get a representation of this operator S in an integral form, can you have an integral form G of t of psi d psi r d psi. So, I get a representation in the integral form.

So, it is an a slightly more general, you cannot have an easy way integral form, integral representation. In turn, in particular not in particular, if you realize that, 1 of classical theory is finding solutions to integral operator, but there is a certain Kernel is given. So, given a Kernel this we can call it a Kernel. In general, we call it here a Green's function. So, when you given a 2 variable function and you can introduce the integral operators which are linear integral operators. In fact, the development of a linear theory was actually begun from the integral operations test theory.

So, integral operator looking for such a function is possible. So, you have a is it possible to find a 2 variable function G, which is a function of t and psi defined for our t psi greater than or equal to say t naught. And can we find such a G, so that you have a solution representation for G. And that G should be independent of course, the forcing term.

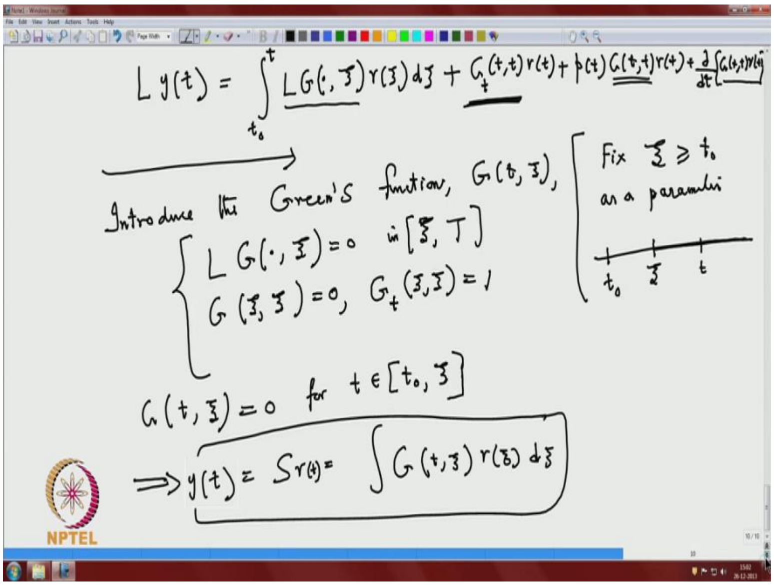
So, let us see, let us try to compute formally which also can be justified. Suppose assume such a G exists, how the assume such a G exists. Don't the worry about the computational aspects of with, which 1 can justify and prove the precise result. As I said the motivation of Green's function is more important, we are not going to spend time. So, you compute your y prime t, when you compute your y prime of t. So, this, we will define say probably from t naught to make a precise t. And you should know how to take the derivative. This is the simple formula, which probably we may do it in our preliminaries. When you do the computation, you can take your t naught to t.

Now, let me write this is with respect to t, you are differentiating with respect to t, t of psi r psi d psi and then there will be a boundary term. How do you find the boundary term? We just put psi equal to t and differentiate. So, that is nothing but G t t r t, that is the boundary term it is easy to calculate. You put limits and differentiate the limits. That is the general formula, when such a you can integrate from alpha t to beta integral alpha t to beta t and 2 variable function how to take derivative.

Now, compute your y double prime of t. When you compute your y double prime of t, you have t naught to t, I will g t t of t psi r psi d psi plus you have already computed g t and put evaluation at t r t that is this is fixed. So, that now here there is a difference, you should not be differentiating with respect to a first variable. You have to differentiate this as a now, when you are differentiating, you do treat this as a single variable function and you have to differentiate.

So, there is a difference between when I write here d t of G t t r t. So, do not write G t at t t; that means, when you write G t at t t means that you are differentiating only with respect to the first variable and substituting for psi equal to r t. Here it is not like that you are substituting and then differentiating.

(Refer Slide Time: 49:22)



So, you do some a bit of calculations. So, you have calculated y prime of t, y double prime of t. Substitute in the equation. So, you will have L of y, where l of y of t L of y of t. You can actually do this justifications t naught, because here assuming G, we are not introduced g yet. So, you will have L of g, the corresponding L of g. Of course, with respect to the first variable, the second variable psi is nothing but a integration parameter. But the differentiation and the differential operator is with respect to the first variable not the other 1 r psi d psi plus you will have few terms here. For example, you will have G t t r t plus you will have a term p t into G t t r t plus, you will have a d d by d t of G t t r t. So, you have that kind of term, you can do that 1.

So, when you do this 1. So, this motivates us to define, we wants this to be r. The best way to define r is that, for each parameter you have to introduce the parameter properly. You define that to be 0, for each of that parameter appropriately. So, you have to introduce that parameter that gives you. You want to be r t. So, you want this as a to be 0 for every t, you want this G t t to be 1, you want this G t of this is equal to 1, so that you get your r t. And this you remains 0 once that remains 0 d by d t of that also will be 0.

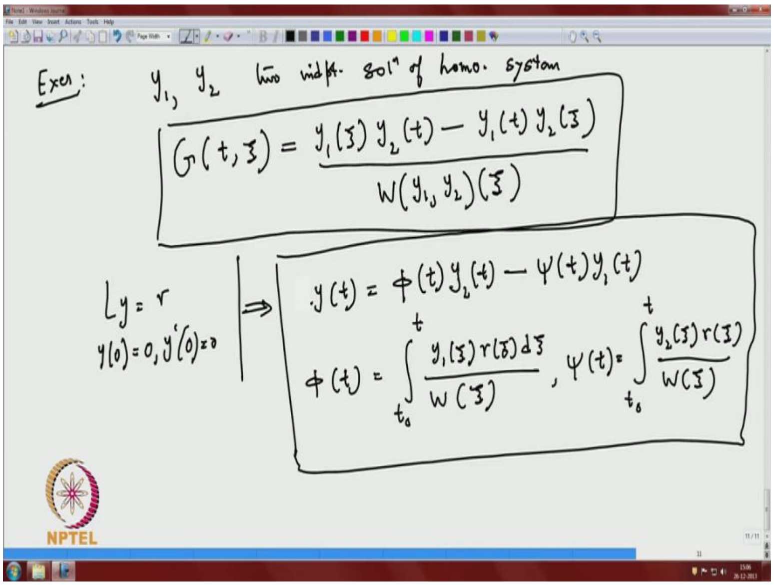
So, this forced to introduce the Green's function. Introduce the green's function G t psi. I introduce this in a very nice clever way, 1 has to introduce. I introduced g t psi for psi greater than or equal to t. So, I will… So, I will introduce G t psi for fix psi. So, we have to introduce you fix psi fix psi as a parameter. So, I am introducing fix psi greater than or equal to t naught as a parameter and want to treat it as a parameter. So, what I will do is that, so you have t naught here. I have some psi here. On this portion I will introduce the problem.

So, I will introduce the problem L g. This is differential operator is with respect to t parameter and psi coming because; I am introducing that problem in 4 psi to t or whatever it is. So, you see. So, I need conditions such psi. So, what are the conditions I will put as psi? I will put G t at psi psi equal to 0, I want G psi equal to 0 G t at psi psi is equal to 1. So, you see r is not coming into picture here. So, you introduce this problem.

Now, this problem by uniqueness, you have that solution, unique solution which is smooth enough. So, that everything can be justified whatever I have done. So, you have introducing this problem. On the left side, you have to introduce for a entire interval, you define G t psi equal to 0 because, this is it is defined only for psi; greater than t greater than or equal to psi it is. So, the problem is t's here. So, you want this 1. So, G t psi is equal to 0 for t in t naught to psi. So, you can complete it.

So, you have a Green's function. So, you are defining on that interval defining that 1. Then you actually prove now everything. So, you have your solution operator y. So, that will give you your solution operator y is equal to S r that is equal to integral of. So, it is how this quite often this is introduced the as an arbitrary way. But this is naturally, we are demanding a representation G t psi S r acting at t while acting at t r psi d psi. So, you have a nice representation of this thing.

(Refer Slide Time: 54:50)



And you can actually do little more using the variation of parameters etcetera. In fact, 1 can do a computation little more. Suppose y1 y2 be 2 independent solutions of homogeneous system. Then you know see look at here; even though it is a homogeneous even though it is a homogeneous system, you have an homogeneous parameter. So, even the solutions of this can be computed using the variation of parameters. Exactly what we have done compute the nonhomogeneous system, even for this equation you can use the variation of parameters using the 2 independent solutions using the…

So, these are all I will leave it as a simple exercise. You can do that 1; y1 and y2 it is a homogeneous solution, then you can actually compute G of t psi to be nothing but y1 of psi y2 of t careful with psi and t minus y1 of t and y2 of psi you see. So, you have a nice representation of that divided by so you have to divide W of you calculate the Wronskian of 2 and evaluated at psi. That is the thing you have to do that.

So, you have a representation of the solution. So, if you write down now your solution. So, if you can write down for your solution with L y equal to 0 y at 0 is L y is equal to r with y at 0 is equal to… Please note that the solution satisfies the homogeneous conditions. So, you will get y 0 is equal to 0, y prime of 0 is equal to 0. So, we can write down the solution here; y evaluated at t in a nice form. So, you are integrating only with respect to psi.

So, you can write down and you have an… you are integrating with respect to psi. So, you can this will come out; y2 will come out. So, we will have phi t into y2 of t minus y1 will come out. The remaining integral will be psi t into y1 of t and what is your phi t? phi t is nothing but L y t Gt psi r psi. So, this factor you have to take it. So, is equal to integral t naught to t, you get y1 of psi r psi d psi by W evaluated at psi y1 y2 and your psi t will be integral of t naught to t evaluated at y2 of psi r psi you have w of psi.

So, you have a good representation of your t, so the Green's function. The main idea of this talk part is to introduce the Green's function. So, you can introduce and but what I say that this looks simple, but understanding the Green's function, fundamental solutions are fundamental in the other situations. So, you can see this probably when we deal with boundary value problems, we will come to the Green's function little more. But so there is a beautiful representation using Green's function.

With this we complete the; this particular module on linear second order and linear first order equations. In another module, we study the n'th order linear equations and n'th order linear system. There our main focus will be and to understand the stability of the equilibrium points, hence we will be using the linear algebra extensively.

Thank you. Bye.