Ordinary Differential Equations

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Module - 3
Lecture - 11
Exact Equations

Welcome back again to the second lecture in this module of linear first and second order equations. The first lecture, we have completely studied the first order linear equations and we have essentially, seen that the first order linear equations, both homogeneous and non-homogeneous can be essentially, converted into the integral calculus problem. This is actually, a general feature of that another class of problem called exact differential equations. So, basically we have seen that first order linear equations can be put it in the exact differential form, if necessary, by multiplying by a suitable function called integrating factor.

There is a complete study of the first order linear equations, but such an easy way of determining the solutions to second order linear equations, is not available, but before going to the second order linear equations, we will introduce in this lecture, the concept of exact differential equations. There is another interesting class which probably, most of you are familiar called a separable equation. The separable equations also can be integrated out straight away and it falls in the category of exact differential equation. So, I will give you one or two quick examples of separable equations and then, we will move on to the exact differential equations in this lecture.

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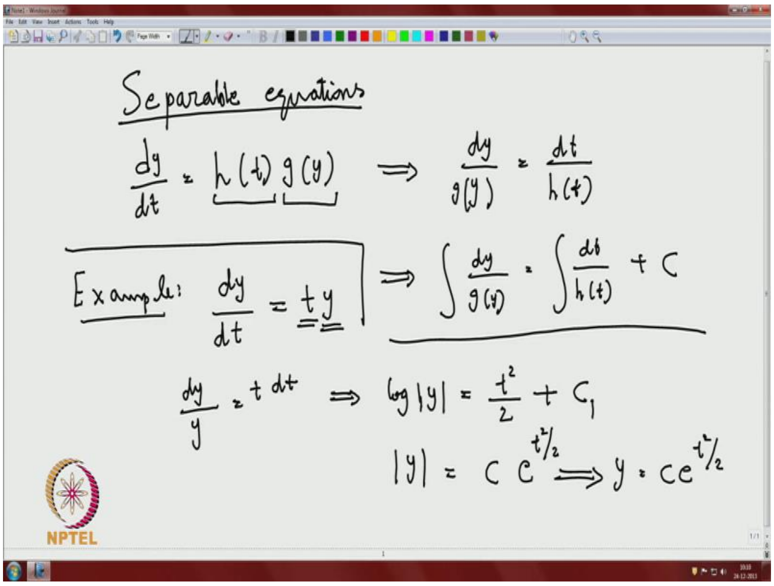

Separable equations: A differential equation of the form, if you have, if you can write a equation of the form dy by dt, is equal to h of t into g of y. So, this equation need not have to be linear at all. This equation need not have to be linear, but it is in the separable form ht; the dependent variable and independent variables are separated out. So, you can write formally this one in the form; dy by dy, dy by g of y is equal to dt of h of t. Then, one can integrate, because this is an integral. One can actually, integrate and then, find the solutions you can. So, this implies, you can have integral of dy by dy, g of y, is equal to integral of dt by ht, plus a constant, and which can be solved. So, there is also an easy class of equations from the non-linear equations in general, which can be solved easily.

So, we will start with familiar examples, which all of you are already familiar. For example, you want to have an example; simple example, other complicated examples, we can do it, but the idea is not all the time to make the ideas clear, rather than the getting into the complications of the integrals here. So, look at the easier equation t of y, which you have already seen it. This can be solved immediately, by dy by dy, is in the separable form. Here, ht is equal to t, g of y is equal to y. So, dy by y is equal to is equal to tdt, actually, now tdt; that implies, you have to always do this procedure of log mod y in general. Log mod y is equal to t square by 2, plus a constant and then, you do the normal procedure mod y is equal to; it is a constant, say c 1, you can put it. You can write this mod y is equal to some constant c into e power t square by 2, but then, the same procedure, which I have told; this can be brought it here, and it is a continuous function, whose modulus is constant, implies finally, y is of the form c e power t by square by 2; these, all of you are familiar and it is much more easy.

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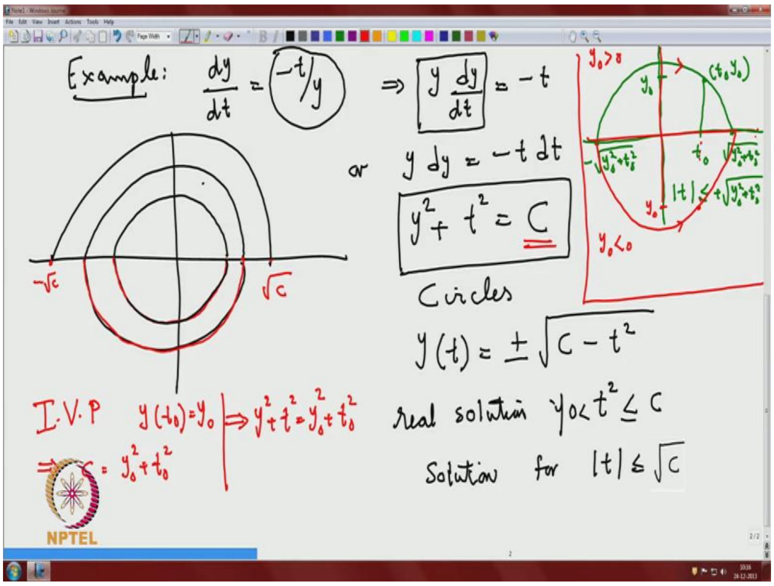

We will go the next example, another example, interesting. We will have one or two examples in this situation. Example: When you go to a slightly different equation of the form dy by dt, there is a change; dy by dt equal to minus t by y. Of course, this is not in the category of the normal equations, because this function as a function of t and y, is not continuous, there. So, this is a kind of, this is in the category of singular differential equation. So, if you write this equation, this is of the form y dy by dt is equal to minus t or in the formal notation of a symbolic integral, this is nothing, but minus t dt. Naturally, you can expect trouble at this, at the origin, because as I told you, the moment there are coefficients, which vanishes, it comes what are called the singular equation. In general, we do not treat it as singular equations, but in other modules, you will see some special class of singular equations, if our time permits. So, if you go here, naturally, you can expect trouble near the origin, which is exhibited. That is why we want to show this example.

If you integrate that one, you will get y square plus some constant, plus t square equal to constant, you see here. That is the general solution for this first order equation, but we want to understand this graph. These are nothing, but circles, circles you see. This equation of the circle is in the implicit form. So, I want to understand the solution curves here. So, if you plot, if you are trying to solve this equation, it is not possible to solve uniquely. You will have different solutions and you have to understand.

You have yt, if you write it, this will be square root of plus or minus c minus t square, you see. This, if you want to have a real solution, then if t has to be; that means, t square has to be less than c or less than equal to c, you see; 0 less than t square less than equal to c. So, that shows that the solution exists; you have the solution exists; solution for mod t less than equal to c. So, if you have tried to plot these curves, you have two branches; if you have this will be more clear, if you have a point here; this solution curves are something like circles. These are circles, actually.

So, you have solution curves like this. Similarly, you will have solution curves from here. So, these are different solution curves for the initial value problem. So, let me put it that way. So, you will have solution curves curve, coming that depends on each c. So, depending on, you will have a minus root c here. We will find out what is that minus root c, and you will have root c. So, let us look at an initial value problem. If you have the initial value problem IVP, and suppose you have described y at t naught, is equal to y naught.

So, if that is to determine your c here, this will imply; your c will satisfy y naught square plus t naught square. So, this will imply; your solution is y square here, plus t square is equal to y naught square plus t naught square for your information. So, if you try to see this thing, it depends on why y naught is depending on the sin of y naught which branch of the solution, which is the solution defined thing. Suppose, assume that, let me plot the curves in the same page, so that, we will complete this particular problem. Suppose, your y naught is positive; what will happen? If you have this, if you try to plot this curve, then again, come back to the, let me take another color. So, if you plot this, you have your t naught here, somewhere your t naught; it can be t naught, be negative also, and your y naught here. So, if you have y naught, you will have the circle. So, if your y naught is here. So, this is your y naught; your t naught, y naught.

So, you see, the solution exists only, if this is nothing, but square root of y naught square plus t naught square. This will be, this point will be minus of square root of y naught square plus t naught square, you see. So, you have your solutions from existing in the interval mod t, the solution exists, mod t less than equal to plus square root of y naught square plus t naught square; this is an interesting example. On the other hand, if this is the case; y naught is positive. If you have y naught is negative, you will have the other branch. You will see, you will have the other branch where, you are here; this is say, y naught here; y naught. So, that is how, solutions will come like that. So, the solution is defined from this interval. So, this is, of course, you can or have anything; you have trouble at the origin, because of the singularities. So, you will have the solution curve. So, this is an interesting example of a separable equation, one can deal with it.

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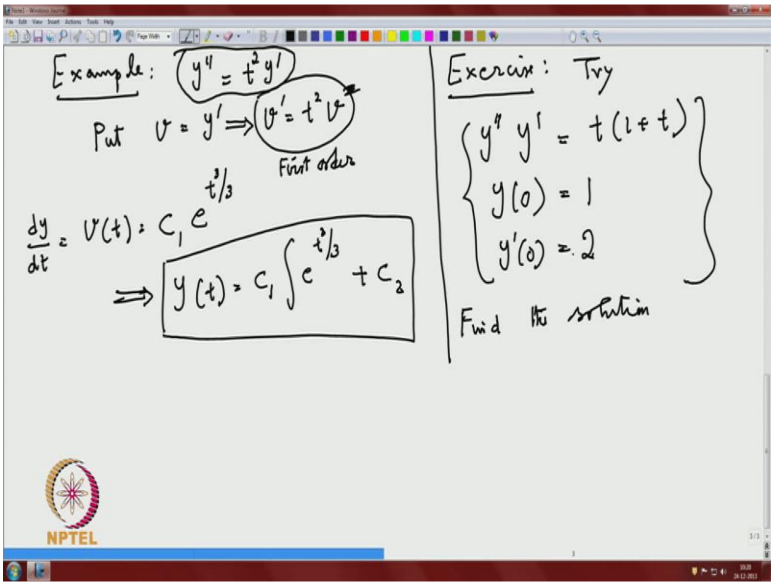

You can also use these equations to solve sometimes, probably; equations can be used to solve a second order equation. So, let me here, present you one more example without details, but a second order equation. Suppose, you have this simple equation of the form y square equal to; we have not studied second order equation, but you do not need to know anything about the second order equation.

Suppose you have this equation. What you do is that you put v is equal to y prime. Then, this equation will become v prime equal to; you have a equation t square v; you see, this is a first order equation, first order and you can solve for vt. So, you can write down your vt by the usual way; some constant c 1 into e power t cubed by 3. So, you can have your solution. Now, what is vt? This is nothing, but dy by dt. So, you have another first order equation, which you can integrate and eventually, you can write your solution yt is equal to c 1, integral of e power t cube by 3, plus some c t; that is it; another then, we will get.

So, such equations are can be solved. So, I will give probably, you can try another exercise for you; exercise, try the equation. A simple equation y double prime, y prime equal to t into 1 plus t; find the solutions of the initial value problem, y at 0 is equal to 1 y prime 0 is equal to 2, here 2; that is the way you can get, find the solution. So, this is because of the special form as you see that, because of this special form, there is no y here; you are able to bring it to the thing. Same thing, you can do it; you can bring it down this; this equation you can bring it down to the first order equation, solve it and then, you will get another first order equation, and then find the solutions to it.

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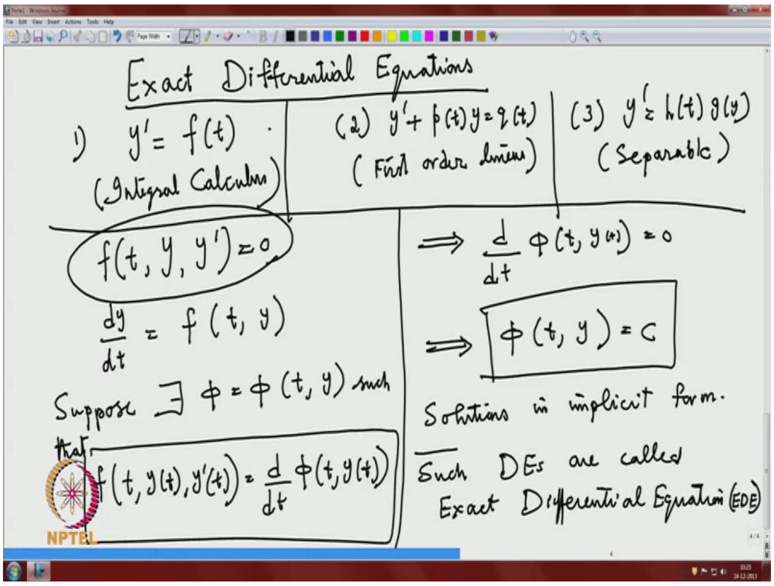

With these two or three examples, now, we will go to what is called the exact differential equations. In this lecture, we will try to explain certain things about the exact differential equations, and then, we will move on to the second order linear equations, and how to study second order linear equations. So, we will go to what is called exact differential equation. We are covering these things even though, these are all important and these are the things, taught in the university system. So, we want to go to that, but we advice the students, those who are taking this course to work out few more examples then, I am explaining here, to get familiarized with these things.

What we have seen? Basically, we have studied three types of equation. One is the integral calculus problem. Suppose, you are given an equation of this form; this is the integral calculus problem. That is what the terminology I am using here; integral calculus. That is the first thing, we have studied. The seconds thing what we have studied is the first order linear equation; y prime plus pty is equal to qt. This is what we have so far completed. This is the first order linear equation and essentially, what we have seen is that by multiplying suitably, if necessary, you can bring that. Everything, you can bring back to the integral calculus problem. The third one what we have studied now, is y prime is of the form; this is non-linear, but separable form; h of t into g of y; separable form. All these equations are in the category of this exact differential equations; that is what we will be (( )). So, we will want to formally, study this example. As I said, the most general form of your differential equation is fty; y prime equal to 0. you can define that terminology of exact differential equations even, for this equation, but since, we are not planning to deal with this more general equation, our equation what we are considering is the standard one; dy, dt is equal to f of ty.

So, let me start with that thing, suppose there is a two variable function. Suppose, there exists a function phi, which is a function of 2 variable where, you; this is very important thing; it is not that when I am defining phi, v is restricted to and we do not treat that y is a dependent variable of t; y is not just yt right now; phi is defined in a domain of the plane ty where, you treat both t and y are independent variables. So, it is a 2 variable function. When I write yt, then you are basically, restricting your yt to the solution curves. At present, it is a function of 2 variables in the ty plane, basically. Since that you can write this equation. So, let me define even, for this thing or this full equation. You can write it such that you can write your f of t, y of t, y prime of t; you see now, you restrict as a total derivative that is what, if you look at the studies of these 3 equations 1, 2, 3 essentially, we have done this thing of d by dt of phi of t yt.

Suppose, there exists such a function satisfying this thing, then that implies d by dt of phi of t yt equal to 0. In other words, this can be integrated now, so that is why, phi. So, the solutions are given in implicit form, is constant. These are the solutions in implicit form. So, such differential equations are called exact differential equations. We want conditions under which, we want some verifying conditions. So, that we want to know whether, a given differential equation is exact or not. If the given differential equation is exact, you can immediately, find the solutions and what we have seen is in the case of 1, 2, 3 whether, in the calculus of first order or separable equations; you can do this quickly, and if not if the differential equation is not exact, can you find out some multiplying factor; can you multiply the differential equation so that multiplied differential equation is exact. So, we want to find with that in mind that multiplication factor is also, in mind; we are going to see something more general.

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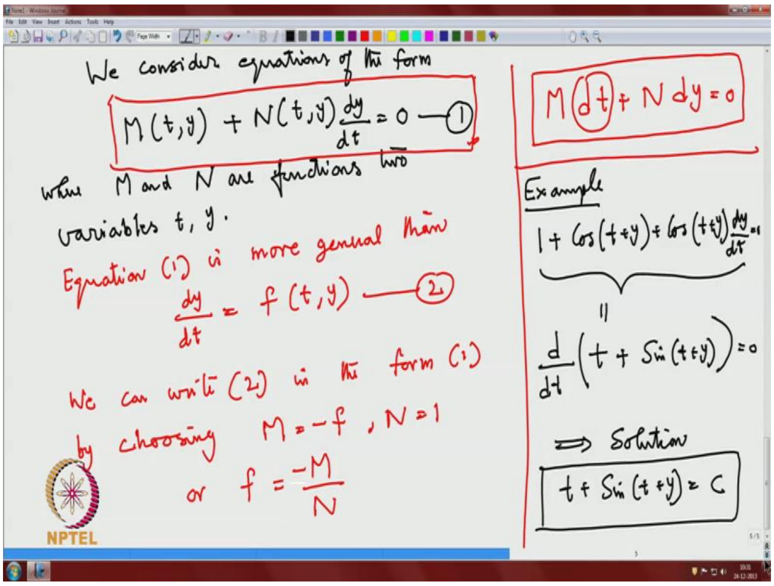

So, we consider slightly more general equation than the previous one. So, we are going to consider something more. More means you can also, the multiplying factors are taking in the account in this differential equation; we will see soon that. We consider equations of the form M t y plus N t y into d y by d t is equal to 0, you see where, M and N are functions of two variables; that is what again, functions of two variables, t and y. So, it is defined on some domain in t y plane; t y, we will define that. Of course, this equation, let me call this equation 1, which we are going to use it and also, mark it here. So, you can immediately, this equation 1 is more general than d y by d t is equal to f of t y. Let me call this 2. Why it is more general? You can always, write your equation 2 in the form of 1; we can write 2 in the form 1 by choosing M equal to minus f, N equal to 1 or f is equal to minus M, and f is of the form M by N, you see.

But the advantage of choosing the equation 1 is that you will be able to; even, if you multiply by; this is one choice of M and N, but what we can do is that you can also, multiply by a multiplying factor, the same multiplying factor for M and N, and you get back the same f. So, the advantage is that you can write f in the form M by N, in different ways with different factors, and what we will see that is for certain representations, the equations may not be exact, but if you choose an appropriate multiplying factor, is possible that you get the exactness of the differential equation. This is exactly, we will be doing it. So, you can see that when this is exact. So, this gives you an equation 1, gives you an added advantage of representing this equation. In books, I also want to make sure, the students who are studying the university system, quite often, these equations will be written in this form; m d t plus n d y equal to 0, but I want to retain in this form. So, that we do not have any ambiguity of d t, d y, but basically, this is it, that here.

What we are going to say, for example, let me give you 1 example here and then, we will go the theorem; a quick example, a simple example. Consider this equation; 1 plus cos of t plus y and plus cos of t plus y d y by d t equal to zero. So, you see, but this can be easily seen; it is not that very easy to see that. This is nothing, but you can see that; you do not need much knowledge about it. This is nothing, but t. If you differentiate, you get 1; sine you differentiate, this is 1. Then, you will have the other factor, you see t plus sine of t plus y, you see. So, if you differentiate, you get this 1; this differential product form and you will have that, quickly. You have to differentiate this one also. That is what will give you; if you differentiate this, you will have these two terms. So, this equation will reduce to this form and you can find that this gives you the solution in implicit form, you see; solution t plus sine of t plus y is equal to constant, you see. So, you get many of these things; not that every differential equation is exact.

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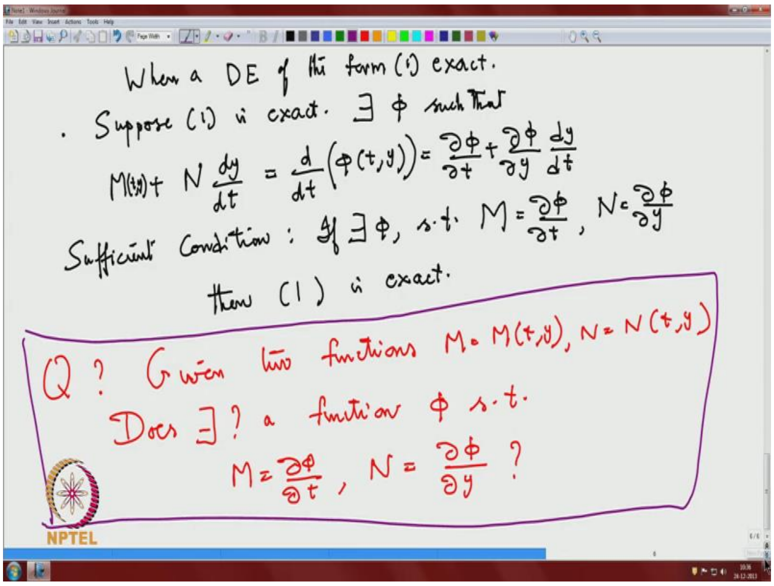

So, we will go to; our interest is to find these equations. So, we want to (find) when a differential equation is exact. Keep that one in mind; for us, 1 is this one, and this is our differential equation 2. So, we want to know when is the differential equation of the form 1, exact; that is our form 1, exact. That is our (( )). Suppose, it is exact; let us look for that is what you are looking. Suppose, we want to derive the thing; suppose, it is exact; suppose, 1 is exact; what does that mean? There exists phi; assume the differentiability of mu; such that M plus N d y by d t equal to 0. I am suppressing the variables t and y. So, if you want, you can write M t y in the beginning. That gives you if this is exact, if there exists a function; this is equal to d by d t of phi t y; now, of course, y is treated as a function of y. So, if you do the total derivative, if you calculate, this will be d by d phi by d t, plus you will have d phi by d y, into d y by d t. This immediately, because of this one, of course, from here, you cannot conclude M is equal to d phi by d t and N equal to d y d, but that gives you a necessary sufficient condition.

So, you will have immediately, a sufficient condition. What is a sufficient condition? If there exists phi, smooth; whatever, smooth and as you required, such that M is equal to d phi by d t. You are looking for a 2 variable function, c phi, such that a single 2 variable function, phi such that M is its derivative with respect to the first argument; d phi by d t, and N is the derivative with respect to the second argument; not d phi by d t; d phi by d y. That will imply, then 1 is exact, you have. So, you want to know more about it, then 1 is exact. So, the question is that; you can immediately, see that, this thing. So, the question we want to post, the question now, in this form; given 2 functions M and N in 2 variable M and N; M is a function of 2 variable in a domain on t y plane; N equal to n t y; does there exists, the question is that, does there exists f function phi, such that M is equal to d phi by d t; the question we formulate here. So, at least, we get it; N is equal to d phi by d y. So, if you can do that, you can answer this question. You have the, you see the question of controlled, question of x had differentiability; x at differential equation; can be answered, if you can find that one. So, that is the question we want to answer. So, let me put it in the form of a theorem, a simple theorem; it is not a difficult theorem.

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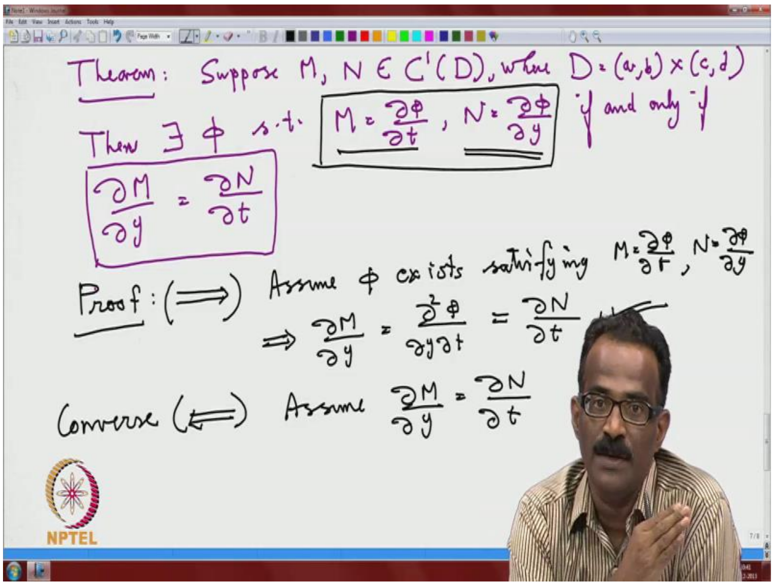

We want to make it, write it, show the theorem. Suppose, M N belongs to this notation probably, you may study; this is c 1 of d; it is nothing, but a continuously differentiable functions. A function is continuous in a domain d where, just the d is a rectangle; you can write it in r t y plane, a b cross c d. So, you see, it is a 2 variable function, defined on a domain in r 2; r 2 is the t y plane, and M and N, you need smoothness. That is, it is continuously differentiable. Then, there exists phi, such that M is equal to d phi by d t, N is equal to d phi by d y; this is if and only if d phi by d y is equal to d phi by; no, sorry; interesting thing is you want conditions, in terms of the M and N, if and only if d M by d y is equal to d N by d t. You see the interesting fact here, is the condition in terms of the given data. M and N are functions given to you in the differential equation. So, you can verify a given differential equation. The moment you put your differential equations of the form M plus N d y by d t, you can check that condition, an easy condition to be checked at, just a differentiation, which anybody can do it easily, to verify that it is a exact differential equation or not.

The condition is not very sufficient, because one way is trivial, because if d M by d y is equal to d N by d t; this is equivalent to saying that d square phi by d y d t equal to d square phi by d t d y, an interchange of differentiation is true. So, immediately, get one part of it and then, proving the other part is also, not difficult. So, we will give you a proof of that; we will give a proof of it, because it is not very difficult; it is easy. Assume one condition. So, we will have the thing, one way of proving it. There exists phi that is why; we will prove this part, first, imply. So, assume, phi exists, satisfying M equal to d phi by d t and N equal to d phi by d y. Then, that imply this is a trivial part, I am telling; d M by d y is equal to d square phi by d y d t. By interchanging, for smooth functions can be interchanged; by interchanging, you get this is equal to d N by d t. So, that is fine. So, that is ok; so converse.

Converse is the one you have to prove. So, you want to prove this one. So, assume d M by d y equal to d N by d t. Now, what we have to prove? We have to prove the existence of phi, satisfying these two conditions. We want to find the existence of phi, satisfying these conditions. If we look at here, if you look at this equation, you want to satisfy these two conditions, you want to do that one. So, what we do is we have to achieve 2 constraints; we have to find 1 v, satisfying two equations; M equal to d phi by d t and we also, have to satisfy the same phi, satisfying this thing. So, what do we do is that whenever, you have difficulty, 2 objectives to be achieved; first, we concentrate, first we keep it aside, one of the constraints and look at only, the other constraints. So, we can see that whether, we can find this thing, and that is what we are doing it. So, look at the constraint 1.

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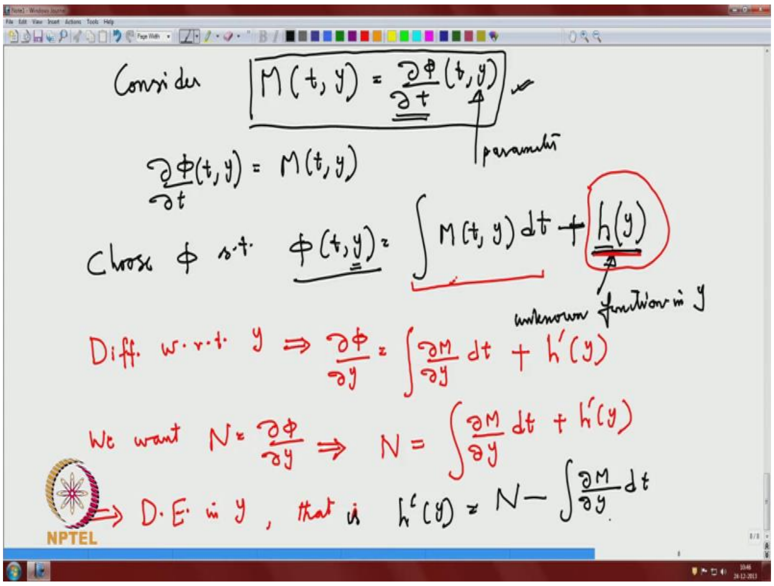

So, consider this equation. You want to see that we do not know we want to know this; does there exists M t y is equal to d phi by d t t y; you want to prove this one. Look at here, this equation; that is the first idea of understanding this equation. When you look at this equation, it is nothing, but a first order ODE in t, and y appears like a parameter; that is what it is a parameter; that is how you have to see that. So, there is no differentiation with respect to y. So, you can see that this is nothing, but a first order equation. We use this d by d t partially, by d t to represent that there is another variable. So, this is an equation in t variable, first order equation in t variable, treating it as, M is given to you; M achieved. So, you have to just integrate.

So, this is an integral calculus problem what we have studied in the last class. This is an integral calculus problem for the differential equation in t. So, if you integrate, choose, the best way is that you choose phi, such that phi t y is just to integrate M; that is a natural way to do integration; f t y and integration is with respect to t, but of course, when you integrate, there will be an integration constant. That is where, the difference comes. When you have an integration constant, and that constant will be depending on this parameter. So, for each fixed parameter y, you will have a constant h. Then, the parameter changes as this constant change. So, here, you have a function of y alone, and this constant is the unknown, now; the constant unknown function in y. So, if you differentiate naturally, if you differentiate d phi by d t, you will if you differentiate phi, d phi by d t, you get this differentiation; you bring it inside, d M by d t, and this will vanish, and your first equation is satisfied. So, that is what you do it.

But now, you have the constraint of the second equation. Now, substitute this in the second equation. So, you differentiate it. So, you want to determine, still you want to determine h. To determine h, you use the second condition often. So, differentiate. So, you have to differentiate, now; differentiate with respect to y. That will imply, you have to do a proper differentiation; you have to differentiate with respect to y. Now, this is an integration with respect to the t. The integral would not get cancelled.

So, you will have the integration, coming here; integral of d M by d y, suppressing the argument and, but d t, somewhere d t here. Then, you have to differentiate h; you have to differentiate h with respect to y. So, that is denoted by h prime of y, because there is only one argument. Now, you can write it, d h by d y, and you want this to be, we want N equal to d phi by d y. So, that implies, you have an equation. So, you have, that implies N is equal to integral of M, d M by d y d t, h prime of y, you see. So, that implies a differential equation in y variable, now; that is, you will get that is; let us come here; that is h prime of y is equal to N minus integral of d M by d y d t. So, you integrate. N and M are fine; you will be able to find this one.

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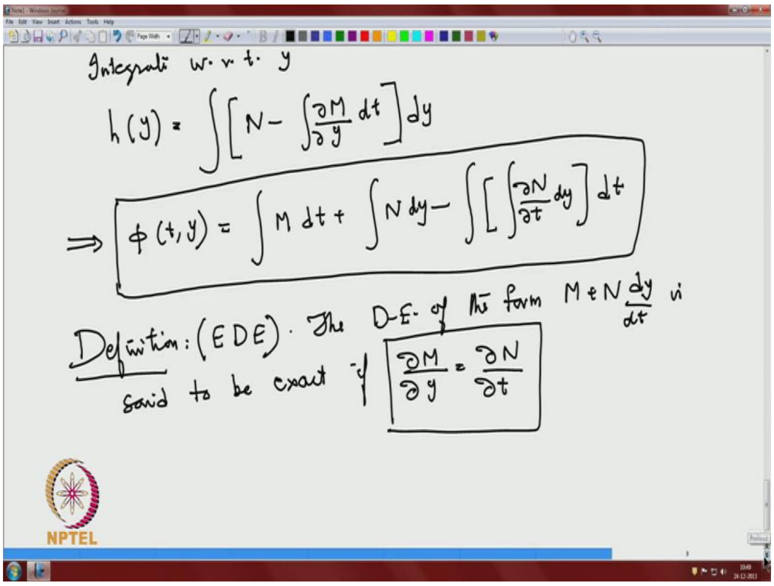

So, integrate; again, it is an integral calculus problem. Integrate, but integrate with respect to y; you will get h of y is equal to integral of; you have to do that integration; n minus integral of d M by d y, integrated with respect to t, you see, and then, this integration is with respect to y. So, you have that one. So, what is your phi t y? That implies, your phi t y; look at that; you have your phi t y, is here; where is your phi t y, is here; you have an integral M t y h, here. So, you will have your phi t y is equal to integral of M d t, plus integral of N; this is N d y and then, minus integral of d N by d t, d y d t. So, it is here.

We have started with the equation first, for M and then, we used for N. You can also do with the other way; you can start with N first, and then, you can start with M, and you will have a different form. That does not matter; that I will leave it as an exercise. So, you see, instead of M first starting, you start with N and then, proceed. So, you will have instead of h of y, you will have some h of t, and you proceed that way, and you can write down this equation. All this can be integrated out. So, you will have a phi and you will have the phi t y, equal to constant as the solutions to this differential equation.

With this definition, because there is a necessary and sufficient condition; I can redefine my definition now. A definition, I can do the redefine, that differentially; because I would prefer to give conditions in terms of the given data. The given data is M and N. So, I can define the exact differentiation. The earlier definition say, if there exists a phi, satisfying the conditions like M equal to d phi by d t and N equal to d phi by d y, but then, a phi is coming into definition in the definition. But we have a definition of the exact differential equation. The differential equation of the form, M plus N d y by d t is said to be exact, if d M by d y is equal to d N by d t, you see. So, it is just an easy verification. So, if you want to determine a differential equation is exact or not, you have to just compute this differential equation.

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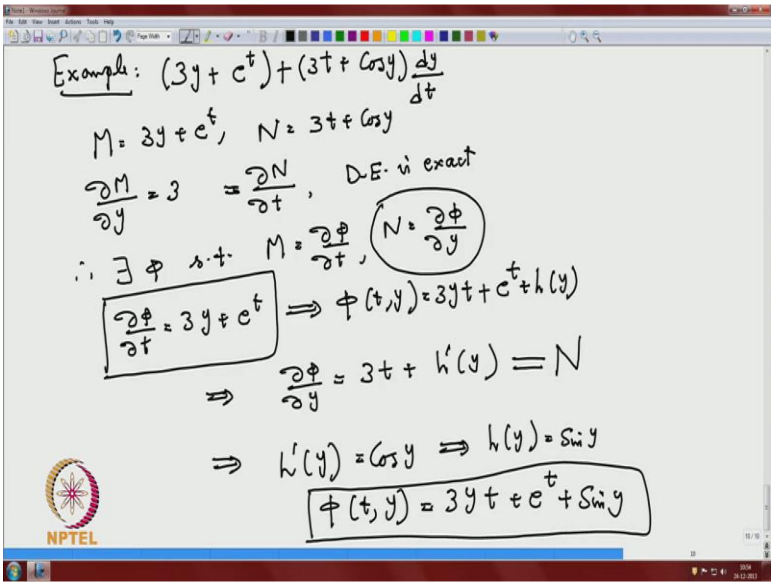

For example, if you want to have an example again, an example like a quick example, you can have plenty of examples, but let me it is an artificially cooked up example, but in equation, you can do it; 3 y plus e power t. You have to put the differential equations. If a general problem is given, you have to put it in a suitable way; cos of y into d y by d t; this is a cooked up example. So, that is not a problem. What is M here; 3 y plus e power t, N is equal to 2 t plus cos y. So, you can compute your; what is your d M by d y; its cooked to compute, is nothing, but 3, and what is your d N by d t, is yes, this would not be exact so you have to say, 3 t; yes, d N by d t. So, this is not two. So, you have 3 t. So, you have three. So, d n by d t is also, equal to 3, you see; this differential equation is exact.

How do you proceed to find phi? You do that one; step by step, you do it. Therefore, there exists phi, such that you do not have to remember the formula. You can proceed to solve the problem. There exists phi, such that M is equal to d phi by d t, and N is equal to d phi by d y. You can integrate it out, one by one. So, if you have what is d phi by d t, you have to find out. So, you want, your d phi by d t is equal to M; you have to solve that; d phi by d t is equal to M. M is equal to 3 y plus e power t, you see, which is, if you treat y as a variable, this y as a parameter and t as a variable, which will give you; you just integrate it out; and you can write down phi t y is equal to 3 y t. I have done the computation, but I do not want to spend time here, in computing, but you can compute it, you see.

Now, you differentiate. That will imply d phi by d y equal to; you compute d phi by d y; if I do that computation, this will be 3 t plus h prime of y; this is a quick. So, that will imply and that has to be, you want this to be d phi by d y to be N. That is how you want that to be N. So, you will have h prime of y is equal to N; N is given to you already here, and you do proceed that one. You have a 3 t; 3 t will get cancelled. So, it is nothing but sine, you see, you got that. So, that will imply h of y is of the form cos y, you see, I am just integrating M or minus cos y.

So, you will have your phi t y is equal to 3 y t, plus e power t, plus sine y; sine y or cos y; h of y is, h prime of y is cos y, right, h, sorry, h prime of y is equal to cos y here. So, h of y is equal to sine y here. You can work with other things also; that is not a problem, you see. So, this equal to constant, will give you the solution. So, you can work out plenty of things like that; you can do that. So, the easier part is that you have a very easy condition, in terms of d M by d y is equal to d N by d t; that is the advantage; the verifying condition to be that. But if the differential equation is not exact, that is going to be difficult, and that is what we quickly, in the next 10 minutes, we are going to discuss with you; a quick discussion and then that will be, then we will complete this example.

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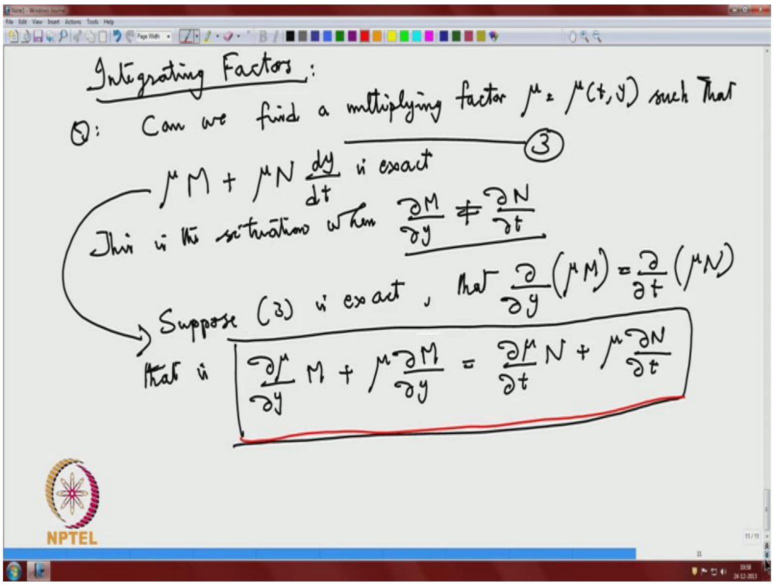

That again, leads to what are called integrating factors; integrating factors, in general differential equation. So, the question is again, can we find M factor, a multiplying factor, which you have already seen in the linear first order differential equation. So, multiplying mu is a function of mu t and y, such that mu t y; mu into m plus mu into N into d y by d t is equal to 0, is exact. You are interested in finding this exact, you see. Of course, this is, if M plus N d y by d t equal is exact, you do not have to do that. The question is coming, when M plus N d y by d t is not exact, that means you do not have. So, this is the situation, when d M by d y not equal to d N by d t, you see. If d M by d y is equal to d N by d t, then that equation is going to be exact, and you do not have to proceed further.

Now, let us suppose, this is exact. Suppose, this equation, let me call it 3. Suppose 3 is exact, what does that mean, again? Suppose, 3 is exact; that is, you have to satisfy this condition, when M is replaced by mu; M and N is replaced by mu n. That is d by d y of mu M is equal to d by d t of mu N. That is d mu by d y M, plus mu d M by d y is equal to d mu by d t N, plus mu d N by d t. Of course, this is like, if you want to determine this is like a partial differential equation, and in general, finding a solution to this equation is difficult. May be, by trial and error, you may be able to cook up a thing. So, in general, determining a solution to this differential equation is difficult, and it is very rare. So, that is where the most general case ends. For a more general situation, you may not be able to find in general, a solution to this differential. By some trial and error, if you are successful in finding the solution to the differential equation, then you can do that one.

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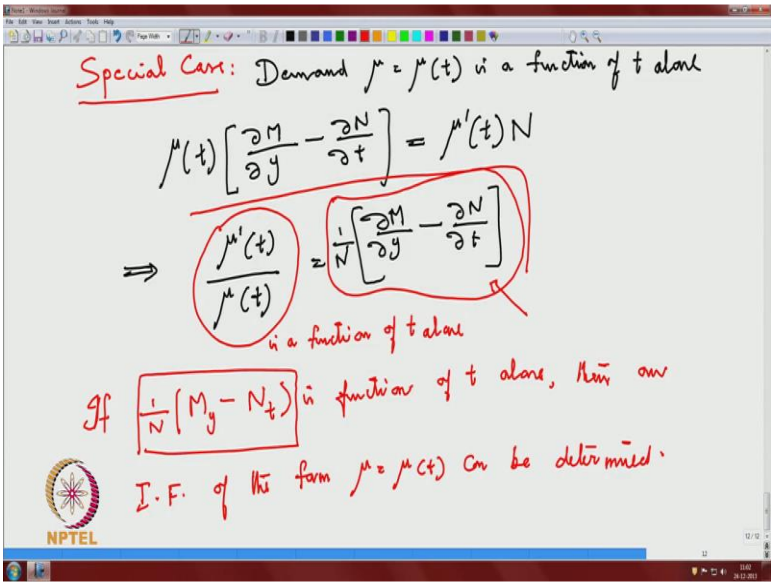

But then, there is a very interesting special case, is still, it may work; special case. The special case is actually, interesting in the sense that sometimes, when you demand little, you may not get anything. That is what the general thing, but on the other hand, if you demand little more, you may get something better, and that is what is happening here. So, you are going to demand something more. We are going to demand mu is equal to mu of t, is a function of t alone. Of course, you can demand mu equal to mu of y, is a function of y alone, and a similar analysis can be done; function of t alone.

If that is the case, look at this equation; if mu is a function of t alone, then this term vanishes. Then, you will have an equation of first order for t. That is what the advantage. So, we are removing by demanding either, mu is a function of t alone, or mu is a function of y alone; we are removing the equation, and the equation becomes a first order equation for ODE, and which we can solve it. That is what we will do that one. In that case, what you will get is that you will get an equation of the form, mu t into; that will imply, if you write that equation; mu t into d M by d y minus d N by d t is equal. We are not sure still, when even, this is solvable and a condition will be given soon, now.

So, this will be now, we can write mu prime, because mu is a function of y alone, into N, you see. So, that implies mu prime of t by mu t is equal to d M by d y, minus d N by d t into 1 over N, you see. Can this equation be solved for mu, but then, look at the interesting fact; this portion is a function of t, now alone, function of t alone, but this is a function of both t and y, because M and N are functions of t alone. If this happen to be the thing, with the right hand side of this equation, is a function of t alone, then this is a first order equation and it can be integrated. So, assume if rather than, assume if 1 over N; let me write M of y for the simplicity; minus N of t is the function of t alone; that is possible. Even though, M and N are functions of both t and y, the combined factor 1 over N, m y minus n t, can be a function of t alone. Then, you see, an integrating factor of the form mu equal to mu t, can be determined, you see; you can do that thing.

So, you see, in that case, this can be done even, with when m mu is a function of y alone, you will get something else here; the mu will come; these things, the roles of d M by d y and d N by d t will be interchanged; 1 by N will become 1 by M. If that happens to be, you have to verify that, and this is valid, because if equation is such that it is going to be 0; d M by d y minus d N. This is the situation and d M by d y minus d N by d t not equal to 0, and in that case, we should calculate that one, and determine by 1 by N or 1 by M, and see that which one is independent; it is a only function of t alone. In the other case, you see that it is a function of y alone. In that case, either mu of t can be determined or mu of y can be determined, and you can multiply your differential equation with that mu t or mu of y to get the solution. This is more or less, the thing we want to tell you in this class, but then, I can give two exercises for you to workout.

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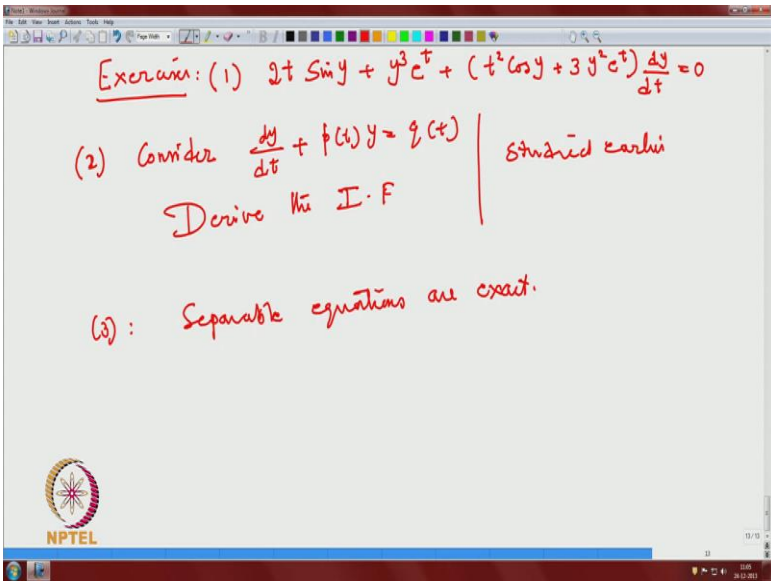

Few one or two exercises, these simple exercises, I want you to work out and one equation, I want, I will write here; 2 t, you determine, whether it is exact or what you can do it with that. I am not even stating the complete problem. I will write only the equation; t plus; these are all again cooked up; cos y and plus 3 y square e power t d y by d t equal to 0, yes. So, this equation can be solved. I want you to solve this equation. You can see that this equation is exact.

The second exercise is what you have already seen. I consider this linear first order equation; consider d y by d t plus p t y is equal to q t; derive the integrating factor, which you will see, using derive the integrating factor. You have seen the integrating factor already, in the first order equation, studied earlier and the third one, which I am going to stop it, you see that the separable equations are exact. So, with this, we are going to finish this class. We have seen that the first order equation can be; it will be an exact differential equation by finding a suitable integrating factor, which you have already seen, and separable equations are always, exact. In the next lecture, we will get into the second order linear differential equation.

Thank you.