Ordinary Differential Equations
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Module - 3
Lecture - 10
First Order Linear Equations

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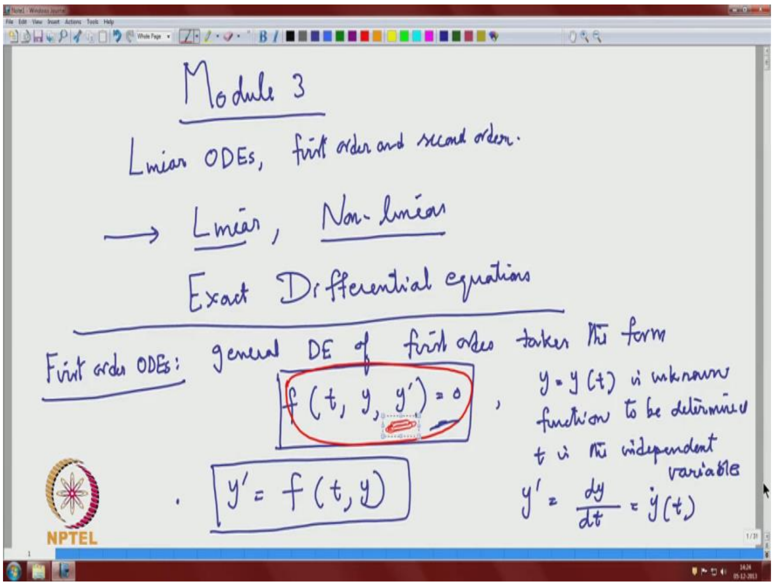

Welcome back again to this lecture. So, we are going to start the module three of this course. In this module, we will be discussing about a special class of differential equations; namely linear, linear O D E's, and we restrict ourselves to the first order and second order. We will also study the first order linear systems, which include the n'th order equations, n'th order linear equations. And what we have seen in module one and module two. Module one you have seen some interesting examples, and module two you have seen the preliminaries required for these course.

So, that two are the kind of elementary discussions, and from here onwards we start our actual study of ordinary differential equations. So, we thought initially, before going to the existence uniqueness theory. we will straightway come to the, some of the easier differential equations, known as linear O D E's, which will explain what are linear O D E's. And we already classified differential equations into order wise; first order, second order, third order etcetera. Now it is another classification of differential equations, what are linear and non-linear. Now you are already familiar a quiet a bit what are linear and what are non-linear, but we specify more about it. In this first, when we discuss this linear O D E's. We also discuss something about call exact differential equations. So, this is the plan differential equations, and this is the plan in this module, maybe we will finish this module in 5 or 6 lectures. So, let me begin with before thing.

So, what is a general first order since the first lecture of essential O D E's, after the preliminaries examples. So, a general differential equation of first order. So, we will first think that first order differential. So, we will start with first order O D E's. So, a differential equation of first order, takes the form f of t y y prime equal to 0, where what is y y equal to y t, is the unknown function to be determine, t is independent variable. Now you what is meaning of independent dependent, all concepts already understood in the preliminaries, and y prime I denote for d y by d t.

So, there are different notations we will see for the derivative y y prime d y by d t this also you may see that y dot of t. So, you will see all these things according to the convenience, comfort level, and the way it is written, you will use various notations. As remarked in the introduction, this itself is probably have difficulties in dealing with that. Probably we will see some of them as we along, but what we will be seeing in most of our thing, a special type from here, but I call where y prime is equal to sums out of f of t y form. These are formed this f is not necessarily this f. This is a general form again. This is more general, because if you are given in the form it is not necessary that from y prime you can solve this equation this. If you think that this is a questioning t y y prime, in general you will not be able to solve it in a nice way y prime to write this one. So, this is a very special class of equations.

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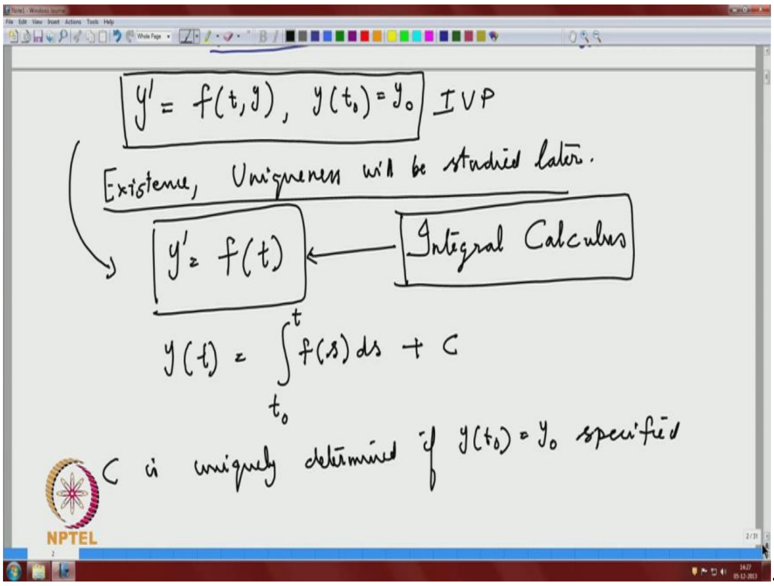

So, how does an initial problem looks like, so you have the initial value problem. So, an initial value problem for this looks like; y prime is equal to f of t y and y at t naught is equal to y naught. So, you see this is the initial value problem. So, you have to study initial value problem. The regarding the existence, uniqueness we will study later, existence uniqueness will be studied, uniqueness will be studied later. We will not, will that. This will see later; that is in another module the regarding existence about the conditions under which you have already introduce in our preliminaries like; the continuity Lipchitz continuity all that, and you will see in fact, different methods to study this equation. And the more easier one again I remarked; when is the y prime is equal to f of t is. So, the function f do not depend on y, and this is an easier problem, which we will discuss soon, even this problem.

Solving this equation, this is an essentially d y by d t equal to f t, and this is nothing, but this is I call it integral calculus problem. We will come to that integral calculus. So, the easiest burn on the integral calculus, is developed to solve one of the easiest differential equation. So, that is what you will be doing it in this one, but we will explain to you little more about it. So, when you solve by this differential equation, you know that d y by d t is equal to, and you write your solution to y of t is equal to, in the initial value problem if you (()) integral t naught to t. Let me use another f of s d f plus c. So, it is a constant. And if you want to see is uniquely determine, if y equal to at t naught is equal to y naught specified if you know that. So, this is what you are going to do it here, all right.

As I said in a in addition to this order classification, you have the classification in terms of linear.

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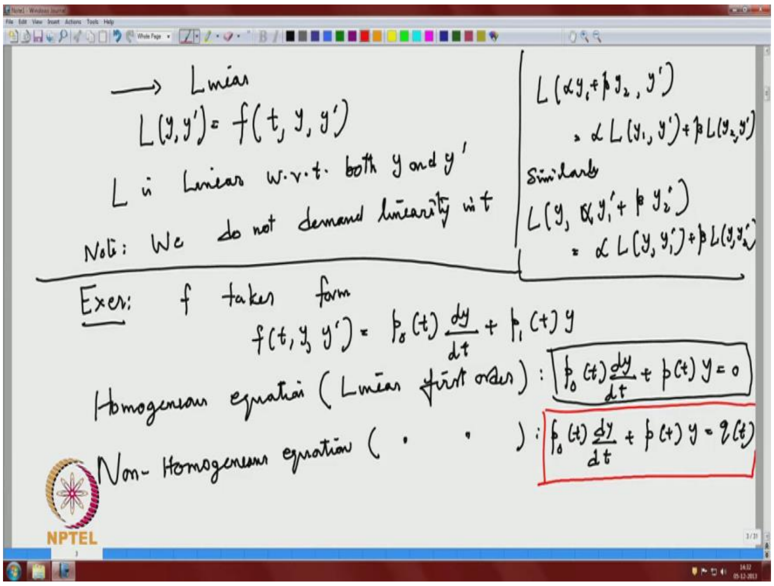

So, what you mean by basically linear. So, you can think that when you given this functional relation between the independent variable y and y prime, (()) t as a parameter. If you view this function y is equal to l of y y prime, you are exactly demanding l is linear. Now know linear from what is a concept of the linearity l is. Linear with respect to both y and y prime. Note we do not demand linearity in t linearity in t. So, what is linearity. Linearity basically tells you if you have l of alpha y 1 plus beta y 2 two functions y prime you live linearity alpha into l of y 1 y prime plus beta l of y 2 y prime. And similarly l of y alpha y 1 prime alpha y prime plus beta y 2 prime is equal to alpha into l of y y 1 prime plus beta l of y y 2 prime. So, you see the linearity and if you look at the…

So, look at the linear algebra preliminaries, which you have studied, and probably I can leave it as an exercise; that f x the form that general form of f can be written once you have this linearity property, you can write f takes the form f of t y y prime of the form p naught of t d y by d t; that is y prime plus p 1 if t into y of t. So, you get a homogeneous equation, this is called homogeneous equation, we call it. We get the homogeneous equation, a general form of homogeneous equation which is linear first order of the form.

This is the homogeneous equation p naught t d y by d t plus p 1 of t y equal to 0, and if you put a non non-homogeneous term on the right.

So, your non-homogeneous term, non-homogeneous equation again linear first order will take the form p naught of t d y by d t plus p 1 of t y equal to, or we use t only here just we take p of t here equal to q t you see. So, this is the first general form from first order homogeneous equation, and this one first order non-homogeneous equation, as you soon see we will not be considering in this generality again, because I said in the previous, a few minutes back heavy coefficients have trouble.

So, we will see one example coefficients. We go to the previous slide, as I said we have described this. We will be considering only this equation. We are not considering coefficients, that troubles comes when that coefficients of the y prime vanishes. If the coefficient of y prime even it is a function of t does not vanish you can take it here by d y it. So, the problem comes when there is a vanishing coefficient in the highest order derivative. So, we consider a highest; say second order deferential equation. And if you have a vanishing coefficients in that highest derivative, then it can cost problems for your differential equation and initial value problem, and such equations are normally categorized in the class of singular equations.

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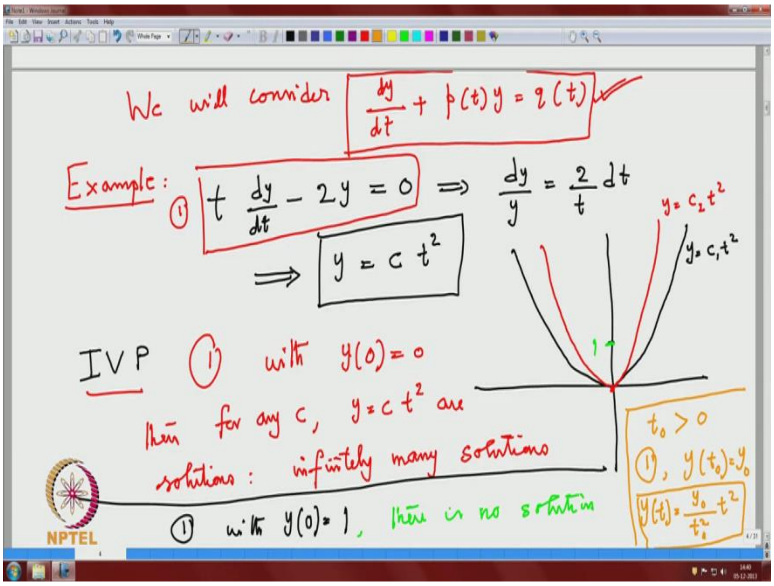

So, you may see one or two examples later. So, we will consider this regular type of equations d y by d t plus p t y is equal to q t. So, this is the equation we are going to completely understand in this lecture; a complete understanding of this equation, we will do it here. So, we will start with an example, where is that trouble. you will see this examples again and again in future, but let me start with an example, very simple example; when there is a vanishing coefficient the singular type equations, we will take t d y by d t minus 2 y equal to. So, you want to solve this equation that initial condition how do you solve it. You follow your normal procedure. When you follow your normal procedure, you write from here. Now we know the meaning of what is that, which we explained in the beginning, the change of variable formula. So, this is nothing, but d y by d t is equal to 2 t d t. You integrate; you do everything that is a simple integration. And I will not do the calculations here.

You can write log mod a and log mod t and all that. At the end of it you will get y is equal to c into t square. So, it is a general solution you will get it, for this equation. Now look at the initial value problem. a simple initial what are these one. These are nothing, but parabola. So, if you plot this parabola, all these are parabolas irrespective of c, it is all passing through their region, because when t equal to 0 y is also equal to 0 . So, you will have. If you fix your t, you will have one parabola. So, this may be something like y equal to c 1 t square. If you choose another one you will have another parabola, is may be y equal to c. it may know look like parabola, (( )) parabolas at the (( )), can anyone imagine parabola, so that is not a problem. So, you will get a different parabola. you see all these trajectories pass through the origin.

So, you put an initial value problem, this problem; this is my initial value problem one together with my initial conditions y at 0 equal to 0, together with my initial conditions equation one with y at 0 is equal to y 0, then for any c y equal to c t square are solutions. So, what does that shows that, is against our well possess criteria basically, that there are infinitely many solutions you see. Even such a linear first order equation, the situation is not that easy to handle infinitely many solutions. So, now, you understand why we are planning to consider only these term. So, this can create even such a simple first order linear equation, and it vanishes only at one point. You have a team, you have infinitely many solutions with p d e of this initial conditions. on the other hand, if I put 1 with y 0 is equal to 0 y 0 equal to 2 or 1. Say I put 1 here, I put 1 here is this fine, y 0 there are no solutions. There are two solutions. So, you have a situation for same problem, h instance of infinitely many solutions, and h instance of no solution.

You can also have a situation, if I take the same problem, if I try to avoid t naught; say greater than 0 for example, and if I consider this p d e this. Let me call it this p d e 1 the p d the O D E 1 naught p d e sorry. This O D E together with initial condition y t naught, is equal to any number, does not matter y naught. Then if I put t equal to t naught here, I get y equal to y naught, I get my c equal to y naught by t naught square y t is equal to y naught by t naught square, t square is the unique solution. So, you have the unique solution if I move. This is for all t greater than or equal to t. So, you have solutions for unique solution for this initial problem. You have a situation, when you are the initial conditions prescribed at that singular point, then you have the infinitely many solution situation you also have a situation, but there are no solutions. So, that is why we do not consider for the time being in this first order case. You may say something later, but we will try to restrict to this situation. So, we will go to understand these first order equations.

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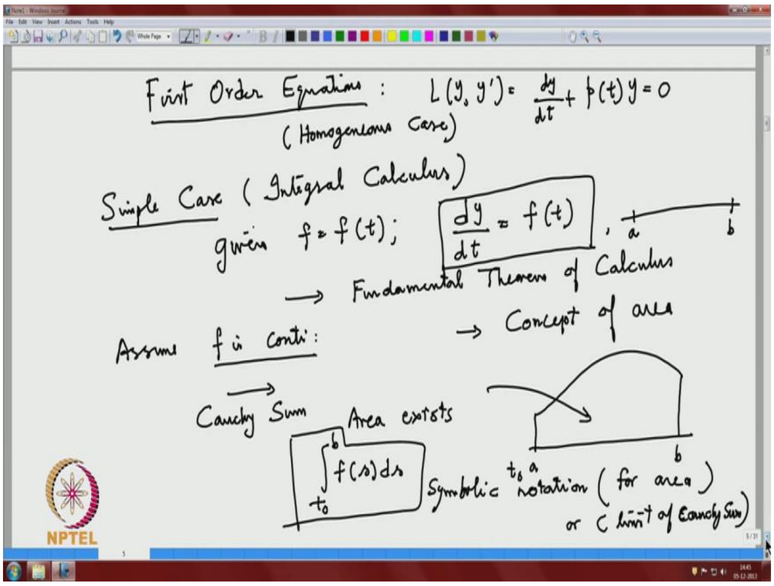

So, we will have first order. You want to solve it now first order equation. We are going to, this equations. So, this is our equation for the simplicity. I write it l of y y prime is equal to d y by d t thus p t y is equal to 0. So, let me start with homogeneous case, then we will come to non-homogeneous case to start with. You will see homogeneous equation, you want to understand that. So, let us take the simple case, what is the simple case I am keep on telling you, is the integral calculus problem. Let me spend 2 minutes, because I want to you to see that integral calculus is developed in the process of solving ordinary differential equation, which quite often ignored, because you study integral calculus in a different way, and O D E is in a different subject. So, you have to, when your integral calculus O D E course is there, we have to begin from the integral calculus problem.

So, what is the integral calculus problem. Given f as a function of t al 1, there is no y. You want to solve, want to find the anti derivative, that all of you know it d y by d t is equal to f of t. you want to find the anti derivative. So, the solution to this, is essentially the, basically the development of integral calculus, and which essentially end with what is the famous theorem called the fundamental theorem of calculus. So, I am not going to write the fundamental theorem of calculus, which you are familiar, but essential what I am stating, if you rewrite it you get the fundamental theorem of calculus. So, I recall some of these things, so when this problem. So, you assume f is continuous, that the minimum assumption; assume f is continuous.

How do you solve this equation, that is what probably you know the development of integration or development of the concept of area. You have to also understand that integral calculus problem. Actually unifies three fundamental problems in the thing, that tangent problem, anti derivative problem, area problem. So, the fundamental problem basically connects these three in the three important problems, like tangent problem, anti derivative, or anti tangent problem and area problem. So, suppose this is given in with say a b. suppose this problem you want to solve it in a b, or any other things. How do you solve this equation. That is concept of a v a which you forgot and which I am not going to give it here, what you do is, that you define the integral via the concept of area which all of you know it. So, you have a of b, you define the area concept, but what Cauchy essentially, than that through Cauchy sum what you call it. I do not introduce it here.

Now you go back to your integral calculus, we integration which you are developed to define the things, what you have done is that this area. If it is a continuous function, the area exists like you prove it by dividing this radian into small pieces. And dividing the Cauchy sum, shows that the Cauchy sums converges and which comes in. and this area is basically you call it integral a to b or t naught to t, if you want it, if this you call it t naught you call it t naught to b. This is basically just like d y by d t is notation. I told you this is a just a symbolic notation for the area, or the limit of Cauchy sums. All this you have to be careful while or you rather you have to understand it that way Cauchy sum. So, that way you define these area concepts. And how do you proceed further, how do you solve these problem, we complete that problem.

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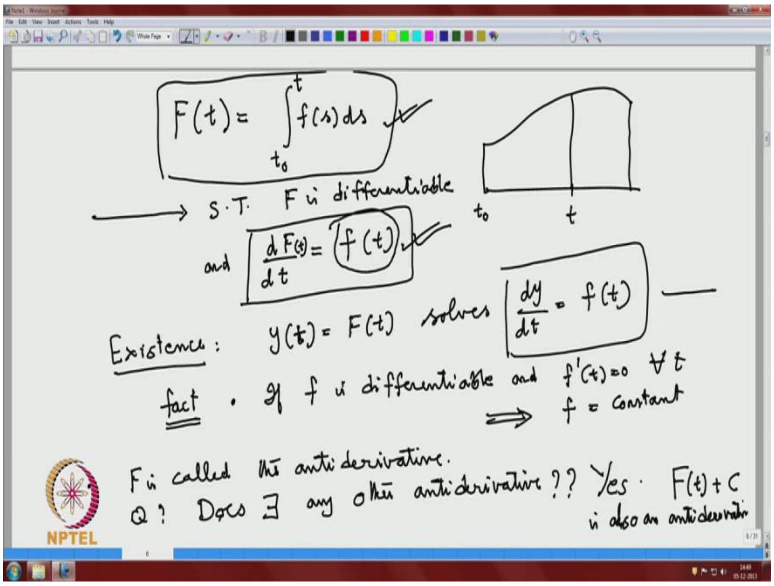

So, you have this area. So, you have your t naught given here, and now you know the area. The function f is continues in this entire interval whatever it is. You put t here, the function is continuous there, the function is continuous here also in t naught to t. So, you can define F of t. after defining the integration via the area with capital end of t, is equal to t naught t f of s d s. You see then the next step implies integral calculus, show that f is differentiable, and compute its derivative d f by d t equal to. If you compute this one, you can see this is nothing but f of t.

So, you basically get to the existence part. So, that gives you the existence part therefore, existence to you get it y t equal to, not for the initial value problem, for the solution to the deferential equation y t equal to f t solves d y by d t equal to f of t, you see you have a solution. So, this is the first part of fundamental theorem of calculus essentially. After that there is one important fact, this is also you need it in fundamental theorem of, or you put it in the form which you know it. This tells you that if f is differentiable, and f prime of t equal to 0, for all t, then this implies. This is not revealing fact actually, one needs to prove it or understand it. So, that is why I am writing down the fundamental theorem calculus, all the development in three parts; one is this, this one, and development of defining.

This appropriately using Cauchy sum, and showing this one. And then the second part is this, is any differentiable function for which f prime of t equal to 0 for all t, then f will be constant the third part comes uniqueness aspect. We can write it when you put an initial value problem, the uniqueness comes into picture, you say the third part of fundamental theorem part calculus, is the crucial one, probably that is the one you will be seeing it. So, this F is called the anti derivative. The question is that; does there exist any other anti derivative, does there exist any other anti derivative, that is the question.

So, you have produced one, if you given small f is continuous, if this function f is continues, then you produced an anti derivative using the area. The next question, what I am asking is there any anti derivative. Yes you can immediately identify some anti derivative, what are the anti derivatives. You have one anti derivative, and you add the any other f to given f t, you add any other is also an anti derivative. So, you have one anti derivative produce to via this 1, whatever the cost free limits and then if you add any constant, because the derivative of that constant is 0 . So, if you take g is equal to d g by d t is also equal to d of by d t and which is equal to f by here. So, the question is that.

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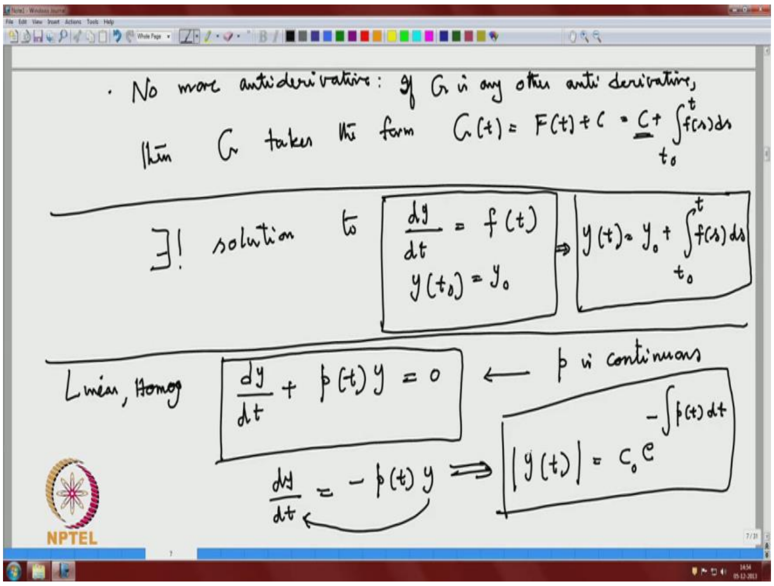

That answer is the third part now no more anti derivatives that. So, this three (( )) no more anti derivative, these three together, whatever I said existence of that anti derivative f prime equal to 0 if implies, and no more anti derivatives, together gives you the familiar theorem of calculus. So, if I formulate it, this statement further, if g is any other anti derivatives, then g takes the form and g takes the form g t is equal to f. We have defined earlier plus c; that means, this has to be of the form say t plus t naught to t f of s d s. So, what you have eventually proved is that, if you compare all that one you have a unique problem, there exist unique solution to d y by d t is equal to d y by d t d y by d t is equal to f of t y t naught is equal to y naught you see. So, the moment you fix your initial condition, the constant is fixed. You got any solution to this form of this case where, and your solution has to be of this form, and this c can be determine if you fixed it. So, what is a solution, your y t is given by y naught c will be y naught in this case integral t nought to t f of s d s. So, that is what the first thing.

Now my idea, I want to go to the next level. So, what is the next level. I want to understand my solution d y by d t linear. So, while going to the next level linear homogenous, and is that plus p t of y is equal to 0. We want to solve this equation. Again I see the whole thing, what I am going to do in this whole lecture. Every first order linear equation in this form can be converted essentially to a problem of the integral calculus, and integral calculus problem has solutions. So, we will do that now, as I said we assume p continues, if p is not continues, p is continues, p will continue, p will do that one. So, how do you go ahead in that, how do you solve this equation. So, I want to solve this. See which all of you know it, let me complete it. So, you have d y by d t is equal to minus p t y. So, if you solve this equation, bring y here, t there minus p t d t, so you know that. And take logarithm that only when you take log. So, let me recall what I have done in the introduction, what you get is only mode t. So, you will get something c naught e power minus integral of p t d t. So, you see. So, how did you remove if we call my introductory lecture. Any way let me state one small that one, that was in the introduction.

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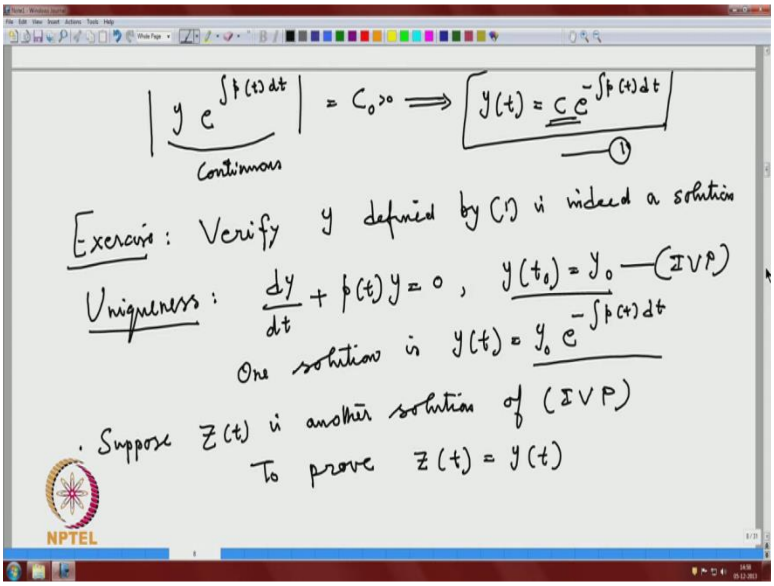

So, state and how do you remove this modulus. To remove this modulus I will bring my exponential term here, integral of p t d t is equal to some constant. Now this is a continuous function, and this is our exercise, which you probably would have done it by now. If you have a continues function which modulus is constant that function, it will be a constant and the case here, because this is modulus, this is positive. So, you will get your solution y t is equal to e power c e power minus integral of p t d t, let this is such solution. You have your general solution, you see you have your general solution in that form. So what you have done is that. So, you have a solution. So, you have your. In fact, you represent. In fact, if you want it as simple exercise I have produced that solution in the later form. you can verify y defined by 1 , this it call it 1 e by 1 is indeed a solution.

So, you see these linear equations, the non-linear equations, you will study existence and uniqueness, and you can apply that general theory to here, get the existence and uniqueness, but I am trying to show you in this case, you do not need to appeal to that general theory, to get a existence uniqueness. So, you have a direct existence here. You can also get your uniqueness which is what I quickly refer now; uniqueness of this equation, this is the equation. Let me recall once again each time, to get familiarise. So, you have p t y equal to 0 with y at a t naught equal to y naught. So, one solution is y t equal to. We put t naught is equal to y naught. From here you get c equal to y naught. You get y naught into e power minus p t d t. I want to solve uniqueness. Very simple, very easy proof. Suppose z t is another solution of I v p. I call this I v p. now consider this is a simple thing, what do I want to prove it. I want to prove z t is equal to y t. So, this is simple idea y t is of this form. I want to prove z t is equal to that one, bring it here, show that product is constant, as simple as z t.

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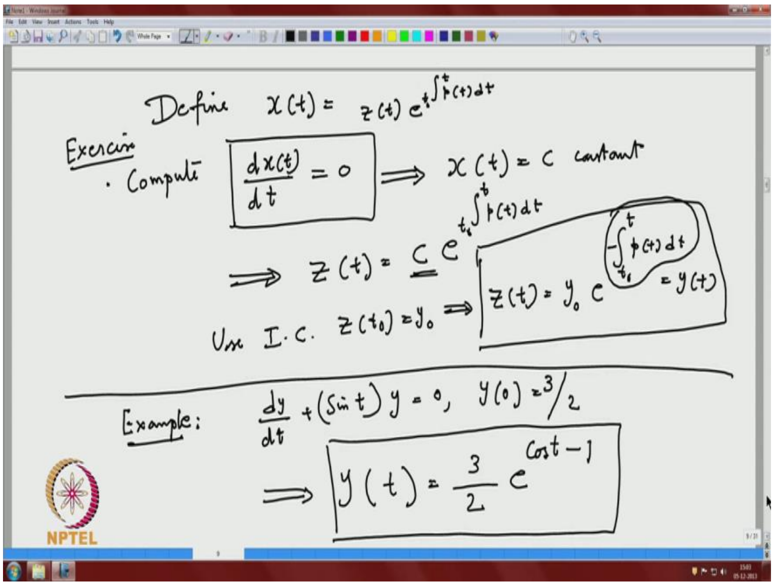

So, define. I want to do it here, because it goes back to the one of the compound of the integral theorem of the calculus. So, define x of t is equal to. The idea of considering is not the thing z t into. You want to show z t is equal to y t, y t contains y naught e power minus t; that minus you bring it here. So, you have a e of t d t. So, this is of course, you can define from, may be let me write it in a x t is equal to z t e power integral t naught to t of s of t is the same variable, does not matter t of d t. Now compute this is a small exercise which you can do it. Compute d x d y by d t is a small exercise you can do, that is not difficult. all exercise if you do it, that time its easy, if you do not do it, the coming exercise will be more difficult. So, you can do d z by d t, keep it in mind that z t is also a solution to a differential equation with a same initial conditions, z t is naught is equal to t naught. So, when you differentiate you get that 1. You differentiate this; you know what to get it. You can actually show that this is equal to 0. And this, what I said, this is one of components of here, fundamental theorem of calculus in the integral calculus.

The moment you have a function exchange, which is move the continues, here in fact differentiable, and if that derivatives is 0, this will imply x t is a constant, x t is equal to c that constant. This implies if x t is a constant, you get z t is equal to, this constant into e power t naught to t p of d t. Now, use initial conditions, z t naught is equal to y naught. So, that implies your z t, if you use this one, you get c is y naught, you get z t is equal to y naught e power integral t naught to t p of t d t. You see this nothing, but your y t. So, you can very simple a thing you can get your existence and uniqueness directly from this equation.

So, you can solve all these equation said integration problem. So, the homogeneous first order linearly equation, is essentially solving and integral calculus problem, to see one example, if we want it, many examples can be given, but you should start working with an examples and exercise. say, suppose I have a d y by d t plus a sin t y sin t y to y is equal to 0, say y 0 you put some number does not matter just to show, but any function, if you write down the solution, the only thing you have to see that, you have the computation of the d t; that is all. If you want to write down new solution, if that function p t is solvable, there is minus sign here, which I missed here, because you are taking this to here. So, there is a minus sign. So, finding the solution of the homogeneous linear equation is essentially you want to integrate that one. So, you want to write here the integral of y t, and you can immediately find the solution, your solution y t and substitute your initial condition. So, I hope the answer is correct, but I am sure it will correct, because it is nothing you're integrate. You can keep plenty of equations like that, to show you, one equation where you may not be able to integrate.

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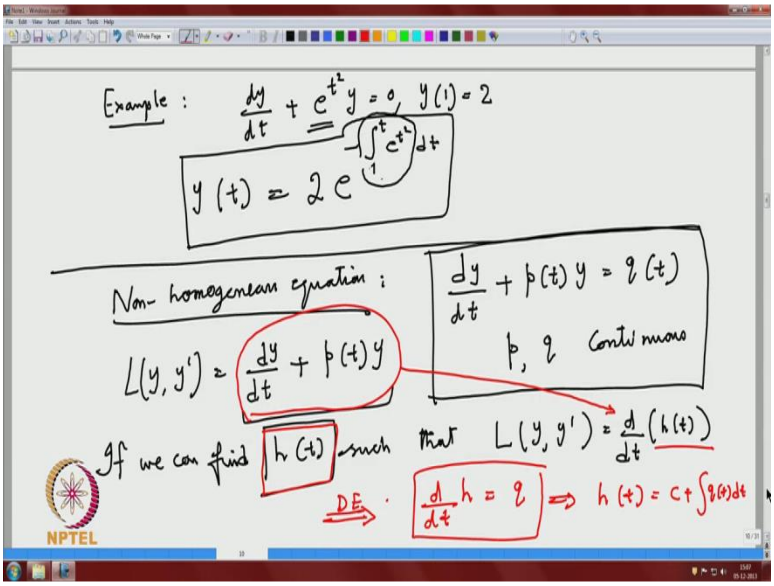

Suppose I write the equation; example, we will also have a difficulty. Suppose I have any equation d y by d t plus e power t square d t y equal to 0. So, all of you probably know, that you cannot integrate this expressly. You can write that integration access, by there is not definition formula to write that one. So, you can only write your solution, y t is equal to the initial value will come; 2 into e power minus integral t naught to t t square d t. So, we cannot do more than that, because there is no way of writing numerically, yes of course, we can compute. You cannot, it is not possible to write this integral explicitly. So, with that let me go to the non homogeneous equation. Say its life is very easy in all this things, because it is nothing more to do then integral calculus problem. So, what is your non homogeneous equation, you have d y by d t plus p t y equal to q t, and then we assume of course, p q continues, p is always several assumption continues, and you have your initial value problem, so we will write that one .

So, what is your l of y here? So, one of y y prime if you want to do that one l of y y prime; the homogeneous part d by d t. this is the some important concept I want a eventually develop for me in next lecture. You have p t of y, you have linearity here. So, you have this equation, this operator. The idea if you look at the integral calculus problem. If you have d by d t of something, is equal to f of t, it will become an integral calculus problem. So, if we can write or if we can find h t such that one of y y prime is equal to d by d t of h t you can do, we call it basically these are called exact differentially.

We will give a precise definition of exact differential equations later, but the whole thing is that, is it possible to find such an s g, I am not claiming its possible there such an h t exist, but if you are able to do that h t in such a way, this deferential operator can be written as a derivative, a single derivative of a function in this case, only this case. If we can find h t your differential equation, reduces to d by d t of h is equal to q. Once you do this 1, your problem is solved, because it is an integral calculus problem that we give you immediately your h t is equal to some constant plus integral of q t d t. So, you have that one. So, the question is that, can you find that one. In general that may not be possible always, but the form suggest you that.

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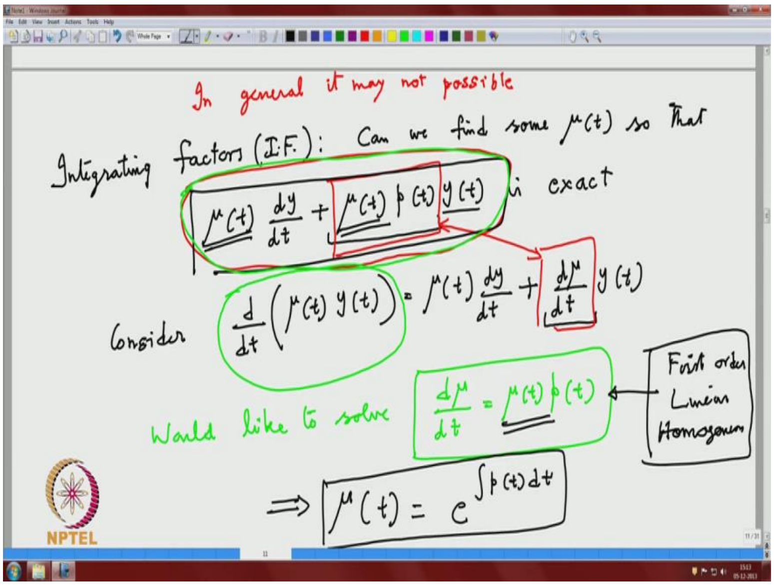

So, in general may not be possible, because we do not know. In general, it may not be possible. So, that is where the concept of integrating factors coming into picture. Normally, written in books, in teachers, in colleges use this notation always integrating factor. the idea that can we multiply, can we find the given deferential equation may not be exact, but the question is, can we find some mu t so that mu t if after multiplying mu t the differential equation, all the operator plus mu t into p t y t can we find, so that this differential operator is exact, can you do that one, because when there is, it is looks like a kind of product form. So, you see this is mu t into d y by d t and y into d y by d t and y into why this intuition behind the reasoning inductivity idea, behind this, (( )) there is something in a product form. So, you have d y by d t here y t here. So, if you want to satisfy d by d t of some product.

So, that gives us a hope that if it can find some sort of, this as the derivative, this gives you a hope to consider. You have to see, it is not that blindly feeling that to consider d by d t of mu t into y t. So, if you consider this one, it will immediately tells you that the first term you get it d y by d t plus you want a d mu by d t into y t, but what do we want, we want in such a way that if we can write this differential equation, is this equation. So, if it is possible to find mu in such a way, that this d mu by d t co insides this one. If you can find d mu by d t and this one equal, then this entire thing. Let me use another colour the entire thing, going to be equal to this one, and we get an exact differential equation. So, by multiplying, if the given differential equation is not exact, it may be possible to make it exact by multiplying that equation. So, what is the advantage of making a exact.

The moment you make that differential equation exact, it is going to be an integral calculus problem. So, you want to solve it. So, would like to solve d mu by d t equal to mu t p t. Now, you recognise this 1, what is this problem, this problem if you look at it, it is a homogeneous linear first order equation, which is what we were discussing so far. So, our non homogeneous linear first order equation, eventually reduced to a solvability of the homogeneous linear equation for mu t, and once you solve this non homogeneous linear equation mu t ,you can use that mu t as a thing, and you know that this solution access. So, this is nothing, but the mu t is a first order linear homogeneous equation. So, you are reducing to the case. So, the non homogeneous case eventually reduced to you first order linear homogeneous equation, and then you find the m u t and replay that one.

It will reduce to an integral calculus problem; say an earlier the first order reduces to an integral calculus problem. Say the earlier, the first order reduced the essential to an integral calculus problem here. We have do it into two steps. So, how do I write it. I can write immediately this already, you have written mu t. So, I can write a mu t with a constant. Constant does not matter, because I am interested only in integrating factor. Get one integrating factor you can always get another integrating factor by multiplying by a constant. So, immediately, this is going to be integral of. Let me write down immediately is going to be mu t is equal to integral. This we already know how to solve an integral of p t d t. So, you have an integrating factor by solving the homogeneous equation. Let us understand this steps, I am writing it here, one of the very familiar thing which we usually try to write it blindly.

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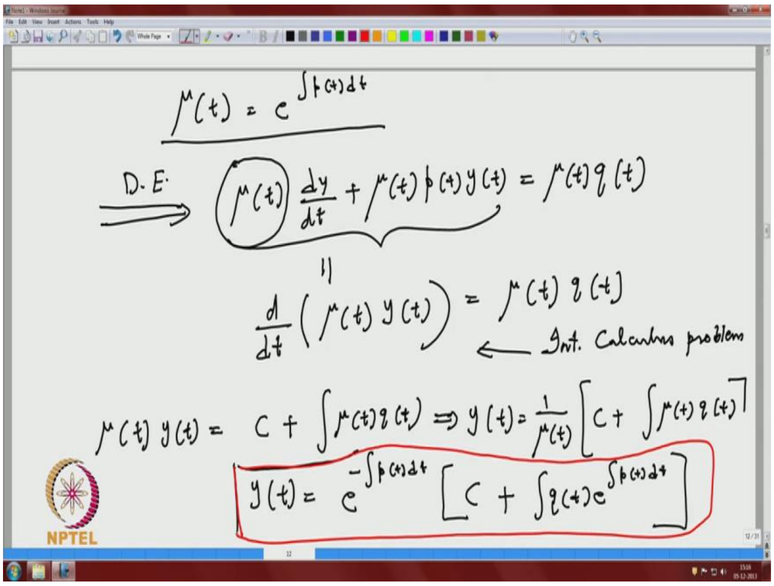

You want to say that there is nothing blindly writing, it is reducing your equation. So, you have your mu t. let me write mu t once again e power integral of p t d t, and then our differential equation, reduce us to. The first thing you are multiplying mu t into d y by d t. let me mu t into p t y t, you have to multiply the right hand side also, you should forget it mu t into q t , and this is what we have done now. With this choice of mu t this will become d by d t. you cannot take arbitrary mu t, you have to have this 1, this form you are not familiar probably, but then you see mu t into y t. So, you have the integral; that is your h t mu t into q t. So, this is an integral calculus problem. And you can immediately write down your mu t is y t is equal to a constant if you want it plus integral of mu t q t.

And then you take mu t mu t is an exponential thing and mu t is greater than equal to 0, it can never vanish. So, that imply your y t is equal to 1 over mu t, some constant into integral of mu t q t. now I am right. yeah is that is the formation. So, you have to. If you substitute mu t you get your solution, constant can be determine by looking at a mu t is this one 1 by mu t is this one. So, 1 by mu t is e power minus integral of p t d t c plus integral of q t e power integral of p t. the interesting thing is that you do not have to remember any of this things. Anyway it is one step calculation. We have done this calculation to show you that, all this integrating factors and everything is coming very naturally to you. So, you do not have to worry about that 1, and probably in the next class I will recall this once again and may be show you one or two examples in the next class, but to conclude this lecture in the example you have to (( )).

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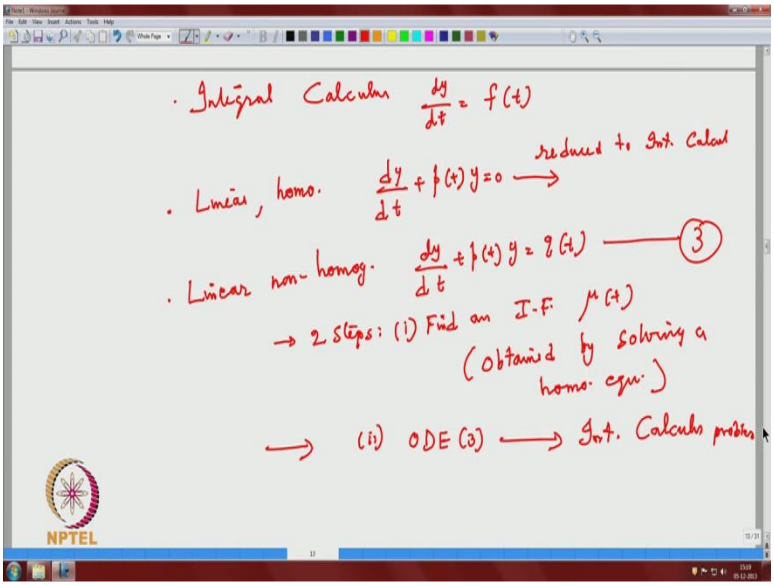

So, let me conclude; first we have solved the integral calculus problem. Solving nothing like d y by d t equal to f of t, and second step linear homogeneous equation; that is equal to d y by d t plus p t y is equal to 0. So, this reduced to a integral calculus problem that is what you have essentially done. If you do this one what you have done log mod y d y by d t by y does not matter. So, you have solved it, so you have a solution. And then linear non-homogeneous you have this equation d y by d t plus p t y is equal to q t. This you have done in two steps; one finds an integrating factor mu t, and this is obtained by solving a homogeneous equation. With the back steps, this is a step 1. In the second step our O D E, this equation this equation, O D E 3 reduces to an integral calculus problem, and this is what we have done, and thus the complete linear first order non-homogeneous problem which takes care of a everything, is completed. So, what I will do in the next class. We will introduce the concept of exact differential equations, and we will understand that in the next lecture.

Thank you very much.