Ordinary Differential Equations
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Module - 1
Lecture - 1
General Introduction

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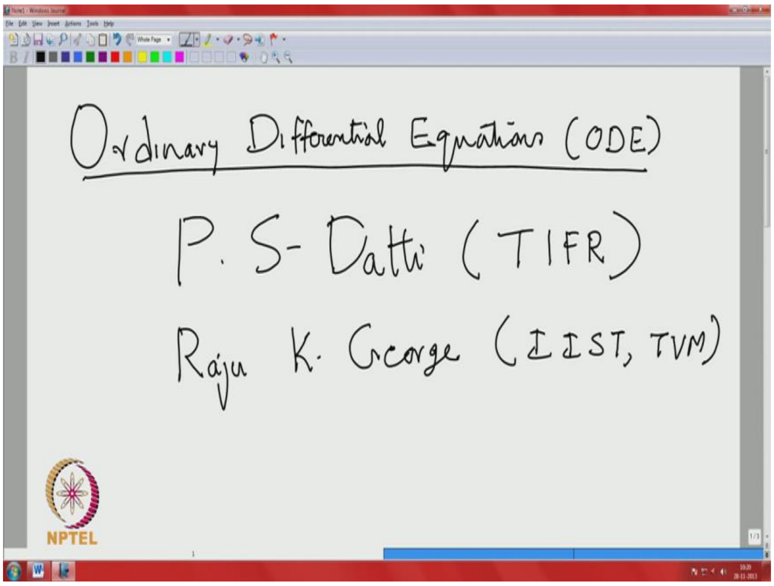

Yeah, good morning to everybody. This is a course on ordinary differential equations popularly known as ODE. So, we will be using this ODE course to represent ordinary differential equations. I am A.K. Nandakumaran from department of mathematics, Indian Institute of Science, Bangalore. In this course, we will be basically covering standard university syllabus. So, all the material covered in most of the Indian Universities will be covered in this course.

But in addition to the normal syllabus, we will also plan to introduce certain notions and especially some topics like: phase plane analysis, phase portraits the, what is called as the qualitative analysis of differential equations. So, it is a little more material then what is usually introduced in the university. Thus this course is going to be very useful not only for the students especially the MSC students of Indian Universities and Institutes, it will be a really beneficial to the faculty or the teachers who teach this course, because they get a little more idea then what is normally covered in our place.

We want to see this course in a entirely different prospective, In this course, we cover approximately in 40 to 45 lectures. And 3 of us are involved in teaching this course, the P.S. Datti from TI Tata Institute of Fundamental Research, Bangalore its basically called TIFR camp center for applied mathematics, and Raju K. George from IIST Trivandrum Indian Institute of Space Science and Technology and of course myself.

In the first lecture, this first lecture, we would like to plan to give an overall view of the entire course and at the end of it. We will explain to a end of this first lecture, we will explain the material covered in this course. Before that, we will give a various issues some aspect of this course why we want to this course and what are the prerequisites required to learn this course we will be introducing all these things. So, this lecture, the first lecture going to be very general and students will get an, or and the teachers will get an overall idea of this entire course.

And that will help them immediately help them immediately to go to a particular module, if they are familiar with certain other module. So, they do not have to if your familiar with certain aspects of differential equation say all your existence theory linear equations everything, if you are familiar. You do not have to spend time and you can straight away go into the nor analysis of linear and non-linear system phase plane, phase portrait which you probably you some of you may not have seen it. On the other hand, if you student for the first time and if they want to learn differential equations they can start including the basics.

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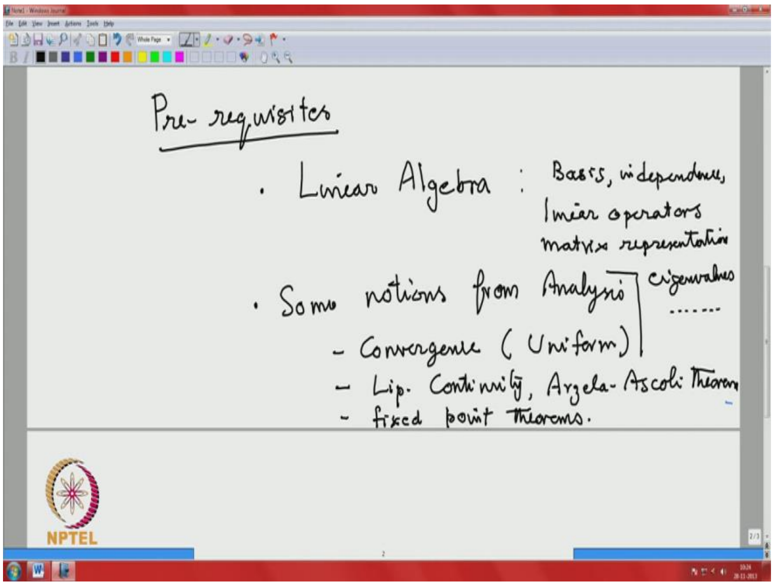

So, what are the. So, let me begin with the prerequisites required for this course, prerequisites we expect the both the students and the teachers are familiar with a first course on linear algebra, we will elaborate little bit there and then the second thing what they need to know about the some notions from analysis, like: convergence, especially the uniform convergence like notions, like: lipschitz continuity something like what are called Arzela Ascoli theorem.

And some theorems like: Banach fixed point theorem and linear algebra the notions like: basis, independence and we also need to know the concepts like: linear operators, matrix representation. The concepts like: Eigenvalues and things like that Eigenvalues etcetera you need to know that. So, this is what a the prerequisites, we expect from this course to fully understand, but to make this course a self stained course, we will be spending few hours probably 5 to 6 hours in recalling this notions.

So, that those who are not fully comfortable with these thing get to know what they exactly want to know, but definitely recall the entire material, basic material we need to know we may not be able to prove each and everything, because it is basically 1 or 2 courses additional courses; which you should be familiar with just to tell you that part you should be familiar with and you can get into that if you are not understood fully from this course about the prerequisites you can go back and study that material to understand this course ok. We want to say before proceeding further, we want I would like to explain to you why we have planned to introduce this course to you.

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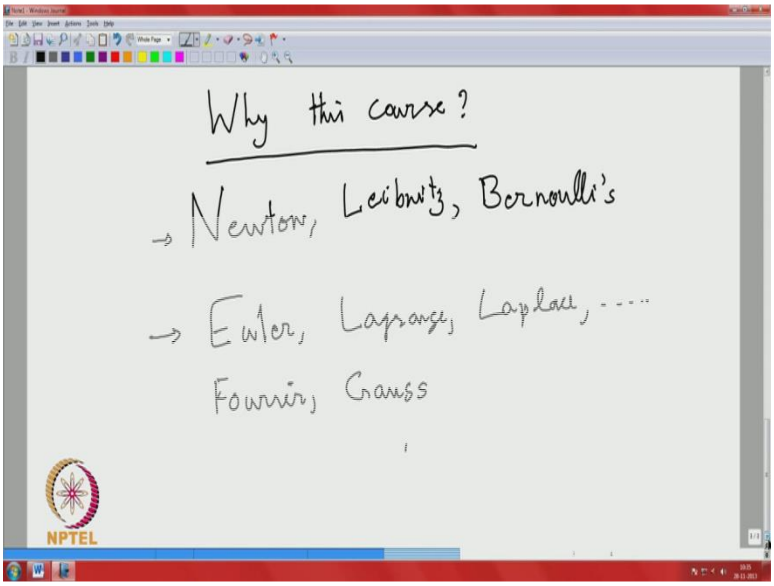

Why this course we want to spend few minutes. Let me to know why this course. As you know very well differential equations lies at a heart of analysis; this is the 1 whether it is ordinary differential equations or is a the partial differential equations you feel that the differential equations really lies at the heart of analysis. And if you look at the history of the development of Science and Mathematics in particular and more precisely the analysis, the root cause of development of all these notions lies at the differential equation.

Because differential equations models the physical systems and to understand the physical systems, you have to learned the differential equations and to develop a systematic theory; which we will tell more about it soon and analysis and other Mathematics had to be developed. And that is what happened in the history of the you… For example, if you see the development of starting with the integral calculus and the basic notions of analysis, and then even for the initial developments of complex analysis these are all happened in probably in the 19^{th} century.

Then the 20^{th} century the very powerful functional, the development of functional analysis, then operator theory related to that and this really exploded the development. And if you look at really all these things, you can really see that differential equations is the root cause for developing all these things, but the 1 of the unfortunate thing is that the analyst themselves have forgotten this fact, the differential equations are the root cause for these thing. And it is not especially when you are developing the when you develop the a curriculum or a syllabus for the differential equations this fact is not taken into account at all.

And in the process, the beauty of differential equations is lost and especially the interplay between differential equations and with other subjects like analysis, even algebraic topology, differentiate geometry and all the beauty is lost. So, if you want to understand differential equations in a beautiful way and it is a impact on other things, you have to understand a certain interplay between the differential equations and the other subjects. So, we will not be able to do it the entire thing, but we want to address this course or teach this course in our period to see the impact of at least the algebra and geometry.

This is a completely different from, what you see in a normal or standard university course, where a set of artificially created methods are introduced to the students and most of the students think that differential equations is a course consisting of a set of methods. And even the set of methods they may not be aware, how this set of methods are coming up, because if you want to introduce this set of methods you have to get into the analysis. And other aspects which is not exhibited in our teaching or in the even in the syllabus it is not exhibited.

So, our main motive in this course is to see some aspects of algebra and analysis, as I mentioned that is why this preliminaries, you have to prerequisites, you have to go through it. But, definitely as a for example, if you if you want to see 1 example to show that in this introduction you have got the Eigenvalues of the matrix, its introduce in a very casual manner, but most of the students do not get an impact of this Eigenvalues. But after studying this course you can see that how this Eigenvalues are and its Jordan decompositions are used.

So, powerfully in analyzing the linear and non-linear systems and especially understanding the stability analysis, phase plain, phase portrait you can see. So, it is a automatically motivates if you understand the linear algebraic motivates not only the differential equations. It is also get good enough motivations, why we study analysis the Eigenvalues of a matrix and more generally in functional analysis, the use of spectral theory. And all these developments in the other subjects of course. In fact, help you to understand differential equations this way. So, it is a give and take policy and which we want to show it to the students take. And that is why we want to see the course in this morning.

So, let me continue little more about that 1, a little more about this introduction to differential equations probably, the beginning of in differential equations started from the attributed to the development attributed to Newton and Leibnitz after the invention of the differential and integral calculus and. So, the early work comes from Newton, Leibnitz and Bernoulli probably starting from the late s 17^{th} century. And there are other people like in addition to the Newton, Leibnitz basically, the main people Newton, Leibnitz Bernoulli.

All these are early yearly part of the century only Bernoulli there are 2-3 Bernoull's. And then there are other many other people contributed to something like euler Lagrange laplace there are many people there are I, will not be able to write all the names like euler Lagranges Laplace ebell there are many people like Poenkare many other people fourier of course, you cannot miss that fourier gauss and many other people. And if you look at in the Yale part of this thing the main issue was to address the physical problems.

So, if you have a particular physical problem you model it and the model turns out to be differential equations, may be ordinary differential equations, it can be partial differential equations or integral equations is anything. So, the issue was to solve that differential equation. You also have to understand this is a preanalysis era the 17^{th}century and early part of it and 18^{th} century are all preanalysis era, where even proper definitions of functions convergence continuity are all not available.

So, the idea was to obtain the solutions to a differential equations, which represents the which represents actually the physical models and derive solutions in whatever, way you obtain functions in the simple form. And you predict to your physical phenomena's. And that is and the early methods in this directions, were integrating factors and separation of variables etcetera. But soon in the people like euler and other mathematician realize that making an attempt to solve a differential equations is futile.

The reason is that, they were trying to develop even though, it is a each differential equations represent different physical phenomena and trying it to solve, it separately they want trying to develop a systematic theory, for the differential equations that that was the beginning of the thing. And the realization came that devising explicit methods to solve differential equations is a futile attempt and that, even stands today the differential equation which you can exactly solvable are very limited and most of the practical problems there are no explicit solutions; that is a where it.

So, the and it is in this scenario you have to see the qualitative analysis like: existence, uniqueness theory, stability analysis, large type behavior all that you want to do it and that is you are the analysis geometry and a linear algebra and other mathematics development is important to understand the differential equations. So, a second phase of differential equations started from the beginning of 19th century, where the most of the analysis also started to develop. So, you will see these things in the coming lectures.

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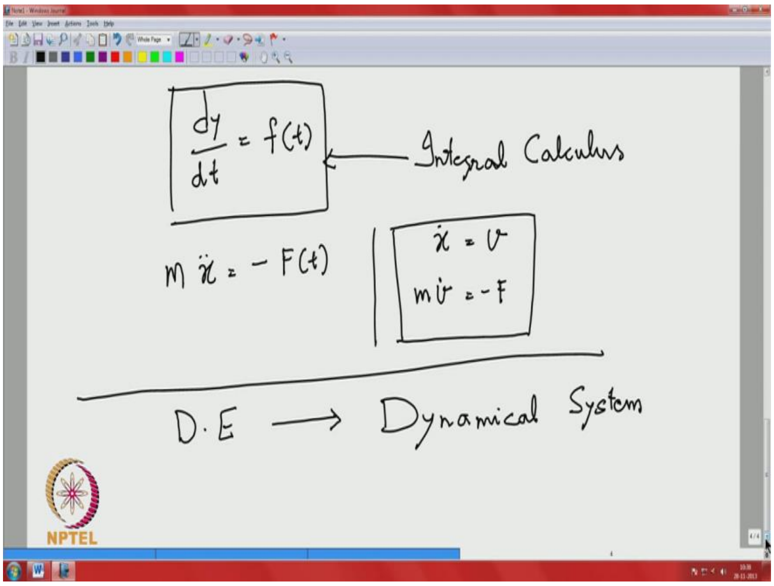

So, let us begin. So, what is basic differential equation, if you look at what the most easiest differential equations probably can be coming from d y by d t is equal to f of t you see and then. So, this problem all of you know after the study of integral calculus of this problem is nothing but your integral calculus problem. This is just to convey to you that integral calculus problem. So, what do you say given a function you want to determine y of t.

So, attempting to solve this precisely your fundamental theorem of calculus and other things will come into play and the beginning of equations is of course, Newton's second law of motion, you will have the all of your familiar with this kind of equations; which represents the second law of motion, of course this can be written as a first order system if you put x dot is equal to v, then you will have m v dot is equal to minus f of t. So, you see. So, you can give these second order equations.

So, here is a 1 point I want to tell, I will tell little more about these things as we go along these introduction and throughout the lectures, you will see the importance of a this equations and another way of the thing you view differential equations as a dynamical system. And this is the view, we would like to perceive throughout this most of the time we will be perceiving this thing dynamical systems; where you can view y t is a trajectory y t trajectory of a or a motion of a particle in some space.

If it is a 3 dimensional, it will be a motion of the particle you can say the motion of the satellite or you can say the motion of the missile or of any planet or anything. So, here the thing, by the way the dynamical system point of view gives a better feeling about your differential equations. And you can see a plenty of you are going to see different examples, different thing, throughout to this course, you will be seeing it. And what do you say then you see that t is your independent variable and y is your dependent variable. So, given time t given independent variable t, you will see your solution y t which is the trajectory of this thing.

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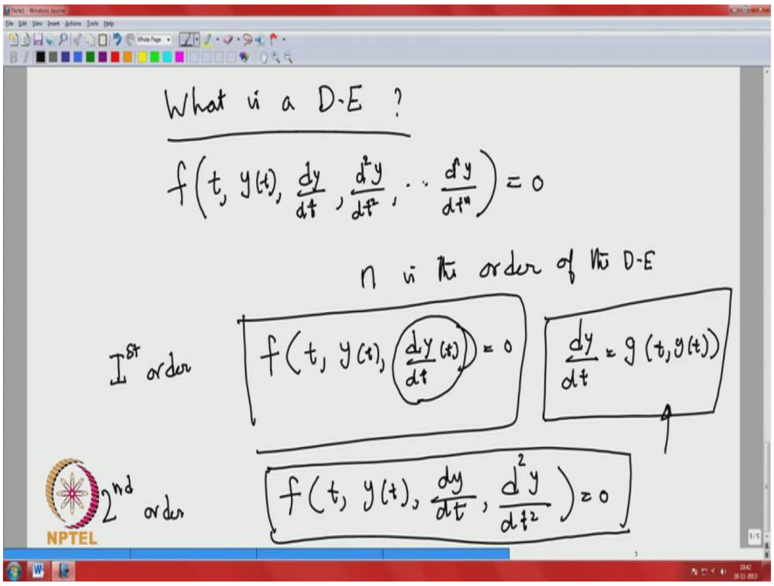

So, when is a just is. So, what is the differential equation basically, what is a differential equation, as you see which you are going to see that formally it is nothing but a relation. So, you will have a independent variable t and you will have an unknown function which a function of v y t. And you will have its derivatives it depending on what type of problem, it can also come d square y by d t square and so on, you can go up to d power n by y by d t power n and you will have a relation connecting these.

So, formally, I can define a differential equation is a relation connecting a independent variable in if it is a motion of a trajectory you can think t is a time variable, but it need not necessarily, the time all the time you will see other types of examples. And y is the unknown variable and there its derivative in relation connecting with that 1. And this n is basically, called the n is the order of the differential equation order of the differential equation.

So, for example, if you have a first order, if you want to understand a first order equation first order equation is nothing but a relation connecting t y t and d y by d t able to understand this equation is not easy. So, most of the time you will be seeing a something a simpler form of this equation, where you will be having d y by d t is equal to g of t y t and this is the differential equation, which we may most of the time. We will be addressing it as I say here, this and this are not. So, same if you want to get you should be able to solve your d y by d t from this relation in terms of t and y and in general that will not be possible.

So, that we will restrict to because this is a much general category. This is a more general category and this 1 is a particular case of this differential equations. And for a second order equation, if you want to see that we will not go further second order you will have a differential equation connecting t y of t and d y by d t can you see d y by d t is equal to sorry you will have d square y also, you will have d square y by d t square is equal to zero. So, this represents a second order equation.

We will in addition to that there is a classification called linear differential equation and non-linear differential equation. So, we will have a set of lectures on first order second order linear equations and also an nth order which will be converting to a system of. So, we will have a separate analysis separate study about this equation, and then there will be a general study of a first order equations, as well as the first order system we will do it is a study same thing.

We will explain little more about this thing. So, if you have more than 1 dependent variable instead of just 1 independent 1 independent variable t, and 1 dependent variable y you have an equation, but on the other hand if you have more than 1 unknowns it will lead to a system of differential equations on the other hand if you have more than 1 independent variable say t s etcetera and it will lead to partial differential equations. So, we will not be doing anything regarding the partial differential equation in this course and that is an entirely something different.

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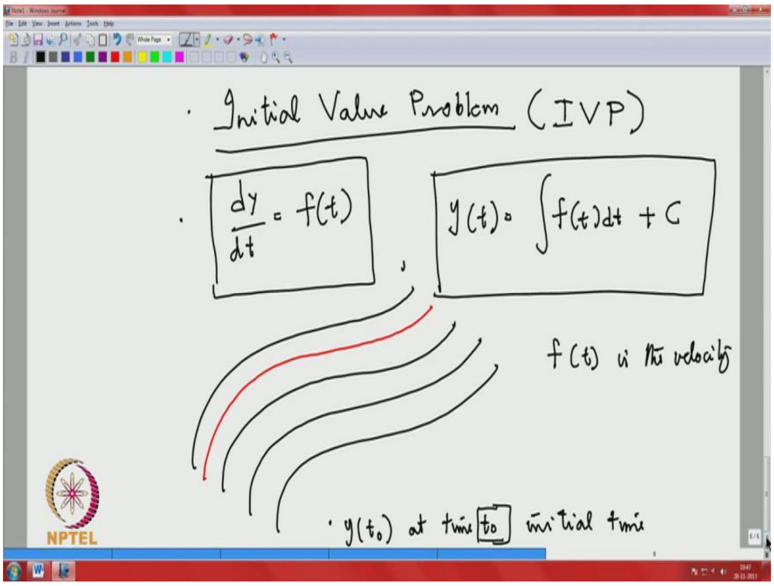

So, now we want to explain little bit about what is called a initial value problem, because what we are going to do with initial value problem. So, let me explain which is normally called IVP. So, whenever you see IVP its nothing but initial value problem. So, let me start with your easiest problem called the integral calculus problem y t is equal to f of t, you have to understand that f of t depends only on t. So, when your given such a relation the integral calculus problem tells you that you want to find given f, you want to solve your y of t.

This is formally you can write it as all of you would have seen this thing, you write this as integral of say, I use the word f of s d s or f of t d t does not matter. So, you will write f of t d t and probably constant. This is very clear because if you take if you have 1 solution to this problem, integral calculus problem which you are familiar at the 12^{th} level then you can add a constant to you will get this that will also be a solution. So, the solution definitely is not unique, if you have 1 solution you will have can always add a constant to get another solution.

But, what you essentially telling, in the entire integral calculus problem what the fundamental theorem essentially tells you, if you really look at the fundamental theorem carefully it tells you that these are all the solutions which you want to do that yeah. So, if you have 1 solutions of this form and the integral calculus tells you that if f is a continuous function, which we will not be covering here, but you can see that such a thing you can be define the integral can be defined using the concept of area that is an another issue and what the finally, tells you that all solutions to this problem are given by this 1.

So, this indicates if you wanted to have a particular solution. So, you can think that, as I said there y t may be representing a trajectory. So, there will be 1 trajectory if you have there will be many trajectories to this solution is possible to that is what if you have 1 trajectory something like that, if you have 1 trajectory then you can add a constant to that trajectory and you can get many solution. So, if you want to find a particular solution going from here if you.

So, that is very natural physical problem you are in a place particular place at time t naught, you will be at this point y t naught. And then this equation tells you that: you will know the velocity or all the time this exactly it gives you an f t. So, you are given a position and then you what we call it a initial time this time t naught is called the initial time and then you have the f t is the velocity f t is the velocity is the velocity and that is a standard physical problem. If you know the initial position and velocity at all the time and your job is to determine that trajectory.

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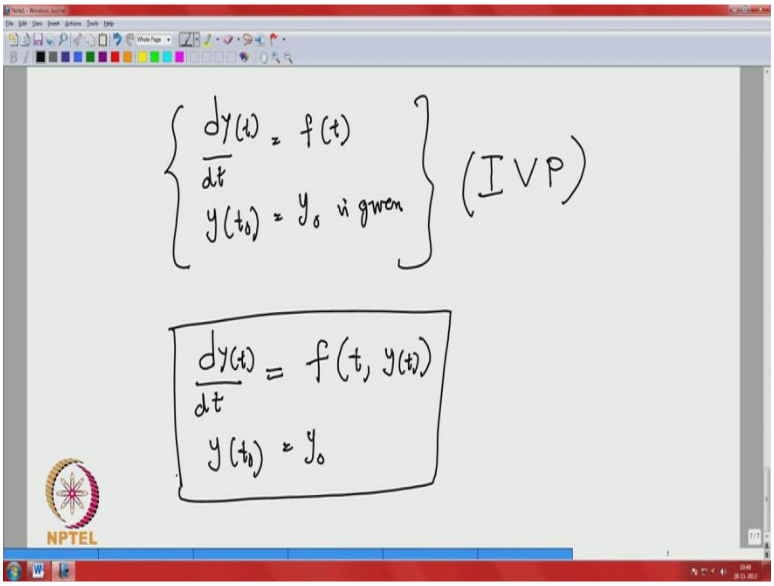

So, this constitutes a reasonable good physical problem d y by d t equal to f of t and then y at t naught your initial position y naught is given to you given. And this together is called the initial value problem and this can be this really motivate and this can be done for any general thing. So, if you have a general differential equation; which we will be addressing in this course d y by d t equal to the f of t it may not only depends on the independent variable t it also depends on the position at that time and y at t naught is y naught is given.

This is your this is your initial value problem, you see you have a very nice way of understanding that the there is a. So, much difference here, because in this case your dynamics what we call it namely: the velocity is given to you a priory, you do not need to know that the. So, the velocity just not depend where you are it depends only on the time. Here the problem is that the velocity not only depends on the time and it also depends on the position making it a the in general this is a very non-linear problem.

It is a highly non-linear problem. And that is and the difficulty what you can anticipate here, even such a simpler problem to solve like a simple integral the beautiful theory of integral calculus is developed. And it was not an easy job. So, that immediately we can have anticipation, if such a simple looking problem is difficult to solve you can really anticipate a much deeper difficulties in these more general problem.

So, you are trying to basically invert a differential operator of these thing and trying to solve that is why, it is a become a really a nontrivial theory is required to develop to understand these kind of problems. So, you can really feel the as you go along the difficulties coming here.

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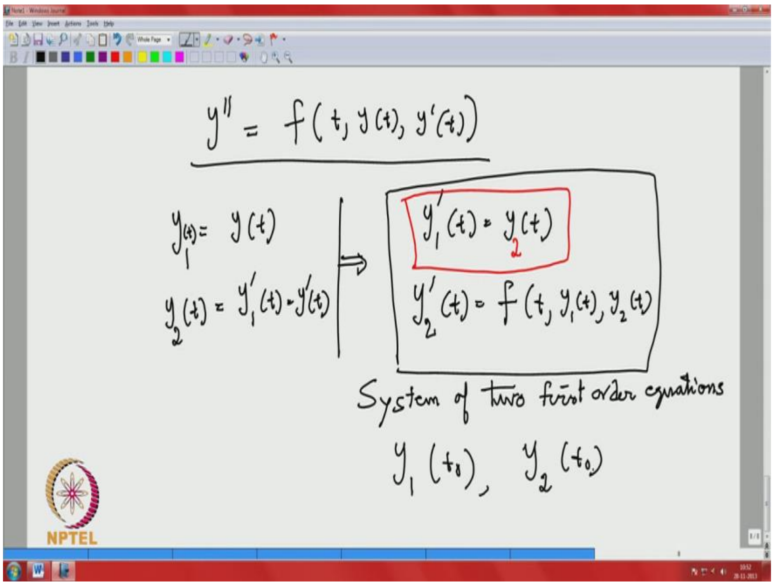

So, if you have a second order equation; A second order equation in a slightly simpler form. Let me write to here y of t y of t and y prime of t that is the a suitable here you may have an options. So, that, but let me tell you one thing. So, I let me convert this differential equation into a system of differential equation; which generally possible. So, I will put y1 of t is equal to y of t and I put a another variable.

So, I introduced 2 dependent variable y1 which is nothing but y and y2 is nothing but y1 prime of t that is equal to y of t. So, with this together we apply this here, what is this 1 y1 prime of t is equal to y of t and then y2 prime of t, what is y2 prime of t is nothing but y1 double prime of t that is nothing but y double prime of t the this is y prime of t. So, y double prime of t that is nothing but f of t y1 of t and y2 of t.

So, you have a system of 2 first order equations system of 2 first order equations first order equations is it and naturally if you look go back and see each first order system. So, if you want to understand the first order system of course, depends on this is y of t. So, this will be y1 prime of t is nothing but this is y2. So, this is y2 of t not y1 of t. So, if you look at this equation this is a first order equation for y1. And hence if you look at the previous page, what I said that: if you have a first order equation you have 1 initial condition basically, because that is a and… So, you require 1 condition for y1 and another condition initial condition for y2. So, if you want to have a it is a kind of nice problem you need an initial condition for y1 at t naught and you need an initial condition for y2 also.

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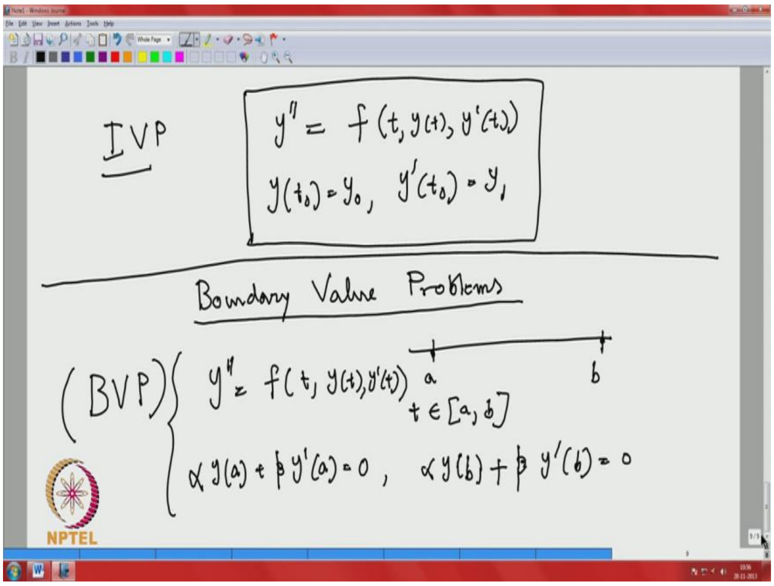

So, 1 of the possible equation. So, an initial value problem for a second order equation looks like this, there may be other possibilities; which you may see as the course proceeds. So, you will have y of t y prime of t together with a 1 condition for y at t naught y naught 1 you need condition for this is for y 1. So, you need a condition for y2 that is nothing but y prime of t naught. So, it is a y 1.

So, this is a standard initial value problem; which again you will be studying this equation and especially you will have a little more detailed study about this equation in it is. So, linear equation we will explain when the stage comes to understand the linear equations as this is an introduction we will not get into the detailed thing. But, as far as second order equations are concerned there is another interesting set of problems called boundary value problems.

Quite often this second order equations may be defined in a just an interval a b. There are many applications especially a set the equations like coming from Sturm Liouville systems representing the mechanical problems and vibration problems. So, you are going to spend anyways some time on this Sturm Liouville and other things where you have a second order equation and the conditions are not defined at the initial values, but the conditions are defined at the boundary value.

So, the boundary value problem in short form we call it b v p and the applications why boundary value problem comes we will also see when we introduce the Sturm Liouville systems as well as a the general boundary value problem and you will see all that 1. So, typically a boundary value problem may look like f of t y of t y prime of t. And this is for t in a b. So, it is a prescribed the interval with you can have a very general boundary value problem.

So, it I can put it something like alpha into y t a plus beta into y prime at a is equal to something like 0 if you want. And 1 more condition something like alpha or some other parameter you can pause it when you will have a solution you will see y at b plus beta y prime at b, it is a general for example, if you take y prime of a equal to 0 or b beta equal to 0 alpha equal to 1. So, you have a y a equal to 0 y b equal to 0 that will be the general that will be a particular case of that.

So, you will be studying. So, basically you have an equation prescribed in certain interval and you have the boundary conditions. And why we study just like, we will have a motivation for the initial value problem, it is along the trajectory you will have the motivation when we introduce because I do not want to spend time now on that boundary value problem, in the introductory lecture, but you will get to know about it soon.

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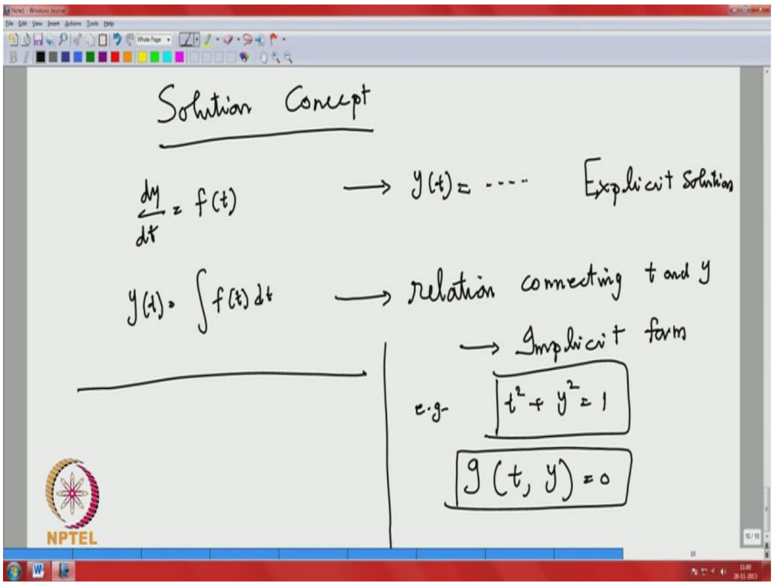

So, with that we will spend little more time on 1 more issue. Let us see, whether we can complete everything what is something like a solution concept quickly, solution concept. There are various things, this is a actually a bigger section and we will not get into the more complicated concepts of solution concept which are indeed useful in applications.

But we want to just tell you something about, the concept which I want to introduce the normal procedure when you have a differential equation which is started 300 years back definitely differential equation associated with a physical problem you want to get solutions in the form what you like it in the in terms of simple functions or whatever it is, but as you see even for the integral calculus problem d y by d t equal to f of t you already know that there are certain f t you may get a representation y t in terms of integral of f of t d t. But integrating the functions in a very closed form may not be possible you see.

So, if you can get a solution y of t, if you can write something y of t equal to something this is called an explicit solution you can write explicit solution as why I am what I am trying to say that writing down your y of t quite often in an this is more or less explicit you may not be able to calculate the integral and write in terms of that. But, still this is explicit, but when you go to more general equations of the form d y by d t equal to f of t y t getting an expression in the y of t may not be possible; what you may get is that a relation is possible that you may get a relation connecting t and y possibly t and y even this may not be possible that is why I said solution concepts are different you can get it here.

This is what you most of the time the methods quite often restrict to this 1 not beyond that. So, connecting t and y and you will see examples, as we go along and when we present examples in our study you will see equations where you get relations coming and this is called what are called implicit form, in the implicitly form. For example, if you get a solution of the form y square plus t square equal to 1 example, if you get suppose your solution representing these form t square plus y square equal to 1 is still possible to solve y uniquely which all of you know it.

So, you will have relations connecting t you may have a relation connecting t and y. And we call this is also a solution to this problem even though you are not able to solve the y in terms of t. So, that is a implicit relation what I want to do here 1 step further even getting this in the either in the implicit explicit form or in the implicit form may not be possible and that is where your analysis will come into play; that does not mean that this equation the differential equation has no solution.

If you are unable to get an explicit or implicit form, it is not possible to conclude the exactly that this does not have an explicit form. It does not have a solution, you may have a solution which you may not be able to represent and that is where the theoretical study of existence and things into come into picture. And the numerical computation of this differential equations comes into play, because the engineers and scientists other science people who working scientist who do the problem they want solutions.

If you are unable to provide solutions in the explicit or implicit form what do you do you cannot stop there, you have to tell them that you can do something else and that is 1 of the major aim of this problem. And the university syllabus essentially restricts to this 2 aspects; when our aim of this course is go beyond this. So, I will quickly tell you little more thing.

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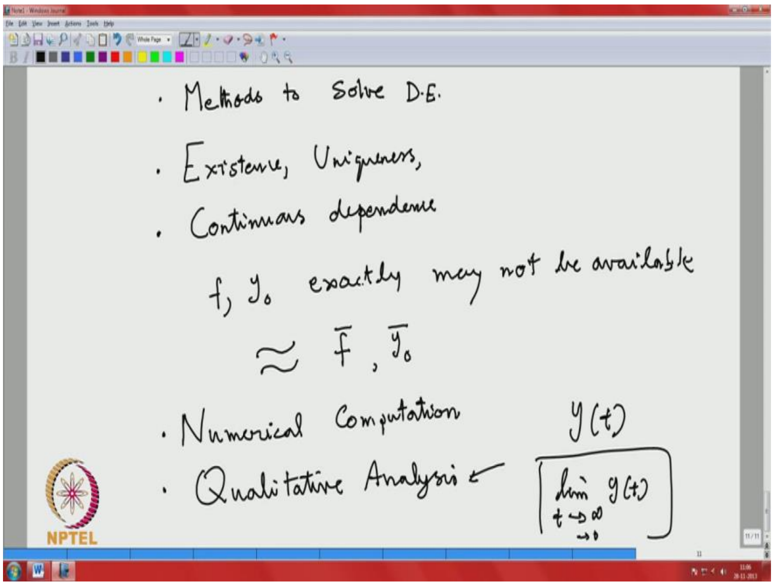

So, what are the things; what are the issues we would like to address through the course of course, we have to address the issue which you are familiar methods to solve methods to solve differential equations as we, but we do these thing, but our concentration is not just this 1. How do you get that methods; what is ideas behind that methods that is what we will be concentrating. And then the second part I told you will not have an implicit or explicit relation some of the ideas which we want to do that 1 existence uniqueness; why existence is required unless you tell them that there is a solution even to proceed for numerical computation, how do you proceed.

If you do not anything about the equation and you may try to develop numerical schemes you end up with false results, but mathematically if you can tell, that you have the existence and uniqueness the you can do the numerical computations and this is where your play in the analysis will come into play. And there is another interesting fact; which probably you may not have understood or may not you heard in your university system what are called the continuous dependence 1 of the very crucial notions together, with existence unique and continuous.

This is for practical purposes, because if you formulate, if you do a modeling for your differential equation, it will be an approximation for example, the dynamics have or the initial conditions y naught may not be available to you may it may be coming through some data. In fact, even the explicit form of f may not be available, what you may be available is a fine set of data for f using the finite set of data using the approximation schemes you may be able to approximate f.

So, you may not have f and y naught exactly may not be available, what you may get is an approximate value of f bar and y naught bar you see. So, you all will be solving the equation you do not have f and y naught, but you are theoretically studying the equation with using f and y naught, but actually what you may be available may be f bar and y naught bar. So, you are actually may be solving your differential equation with an approximate data. But what is the guarantee that solution you obtain using the approximate data is an approximation to actual solution you see that 1 that may not be true.

You will see examples again in this course even if you use approximate data, the solution build. So, you have to need condition you have to get tell them; under these conditions you may be able to get a an approximate solution. And that is what we will be discussing in the continuous data. And then after continuous dependence of course, you have numerical t you already told you why you need to understand numerical computation because you may not have a solution. But engineers and scientists wants that, they want values.

The last and important part, which we will be covering in these thing what is called the qualitative analysis; you see this is also there are multipurpose 1 of the reasons is that in many situations you may not be interested in actual solution for example, if y t is the solution your interest is you do not want to know what is y t at each time, your interest is the what happens to y t s time progress ok. You may not be interested in the whole path, you may be interested in either limit t tends to 0 you may be interested in limit t tends to infinity or limit t tends to 0.

Whatever, it is or y t and such type of asymptotic behavior or you want to know more about the geometric picture about it. And this you will be seeing in the various things like this is involves in qualitative analysis and you will see, the phase plane analysis, phase portrait and whether your trajectory is stable, whether your trajectory is unstable and all that will be seen. And our ultimate aim in this course, if you are successful is to see some sought of periodic solutions which, is the 1 of the famous theorem called Pokhara Bendixson Theorem.

That we may not we are not sure we will reach that 1, because giving a proof of Pokhara Bendixson Theorem involves much deeper understanding. With that let me now, quickly in 5 minutes tell you about our module, which is already available to you, yeah.

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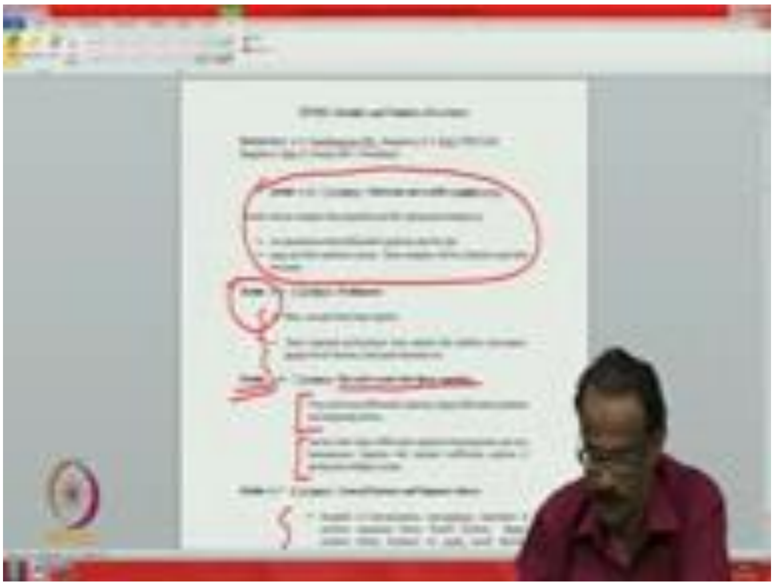

So, this is our whole course going to be here. So, we will start with to get you a feeling we will start with some motivation that is what I had given in this lecture. I hope it is motivated you to learn differential equations, if it not motivated I am sure over the period of time you will get motivated and my colleagues will motivate you more than me as we go along.

So, we may spend some probably 4 lectures, in module 1; you see where after this lecture I may spend around 3 or may be 3 lectures I am not sure we will go along, we will give you some real life examples especially like: dash the thing, population growth and some non-linear systems, may be a satellite example either I will give it or you will see later. And we are giving these examples and we recall these examples as and when required in our course.

So, there are many examples which we cannot give you can give examples of ordinary differential equation, in a every field of science and engineering. But, we have chosen some classical example, this is nothing new most of them are classical examples available in most of the books, but we will do it and we use that examples to do that 1; that is what in the thing. Then as I said the next step is our module 2. We will give you the basics which I said here. We will do, we will whatever the preliminaries, which I explained.

Then to begin with before going to the serious general theory our next module is module 3, in which we will restrict these is the easier equation which you can understand. So, we will spend some time on your first order linear, this is a very special class of differential equation and the interesting thing is that you can convert that problem eventually to an integral calculus problem. And main aim is to motivate you the some of the notions like integrating factors. And a concept of exact differential equations then immediately, you will go to the second order linear differential equation.

You can see that things are not even for the linear differential second order equation, the life is not that easy that you could convert it back, but there is a very nice interesting things here. And it is not easy to solve differential equation only first order linear differential equation, you have a reasonable good theory second order also you have the theory, but solving. So, you need some methods of solving and also if possible, but we will explain to that in the coming lecture. Then we will see the some part of the theory namely the general existence uniqueness theory.

So, basically we will give you different methods to prove the existence and uniqueness, continuous, dependence; we will start with the first order equations. But, then we will also go to the next thing and there are different methods will be there, we will also introduce some methods of solving after that yeah.

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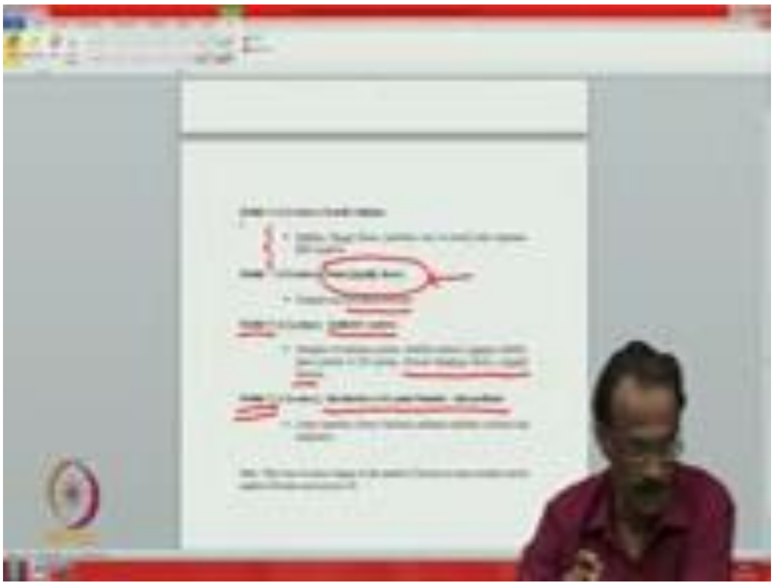

We will go to second linear system here, the powerful use of linear algebra you will be seeing it. So, we will represent all the linear algebra and you see some stability theory for the linear systems will be seeing here. And some periodic solutions and stability will be introduced here. And this is a class of boundary, where you will see a special we will spend not too much time we can spend more time on 1, but we will see some isolation theorems and examples and you will see the Bessel's function, Hermante, Legendres equation.

All that interesting example and we will see something a comparison theorems from 1 differential equation to other equations. And this part our 1 of the important part of our module, which are not really covered not all covered in the university syllabus is the qualitative analysis; already we are done something in the linear system, you will see the various stability analysis Lyapunov stability. We will especially the geometry and phase portrait in these 2 d systems.

This is our ambition to do something on Pokhara Bendixson and hope we will do. We will also give few lectures, we do not spend too much time, because it will go beyond our number of lectures. We plan for a 1 semester course it is a difficult, but we will try to give little bit on the introduction to 2 point boundary value problems and that is what we are planning to do in this entire course.

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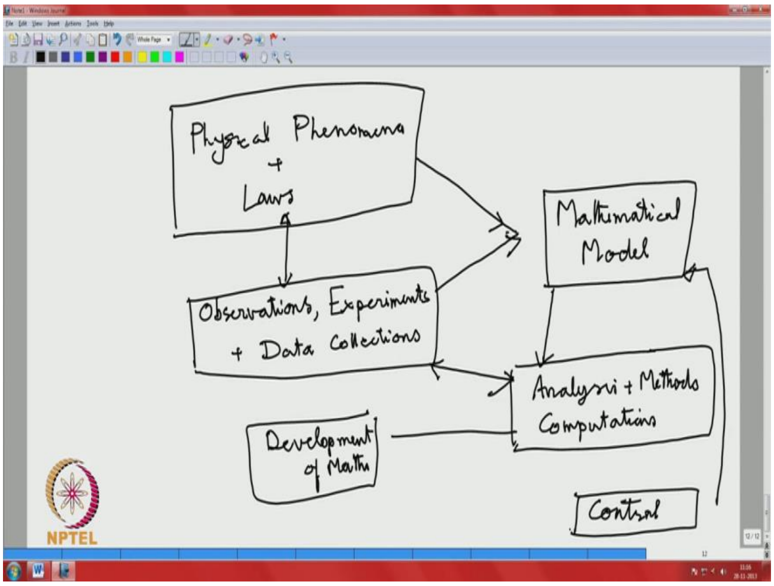

So, let me complete this thing, I have few more minutes in this just I have 2 to 3 to 4 minutes. So, let me conclude the entire thing, with drawing a schematic diagram. So, you have basically you have the physical phenomena, you will have the physical laws here. And that is what your started with your all everything the whole science is that right physical phenomena and physical science.

This to understand the earliest things were observations initially, that is done if you look at our strong everything is observation and that started with observation in the later part by Galileo and other people who have started experiments this is not just experiment create labs and do the experiments and then of course, data collection etcetera on to understand that physical phenomena you have done that. So, here you will have that you see. Using the mathematics available the physical laws and these thing together you do the mathematical modeling, you have the mathematical model here, you see correct mathematical model.

Here is what if you want to understand this 1 here, is the place you have to do the analysis you see this is where you time to do if you have the mathematical model. If you can solve if you can explicitly understand everything no problem, use the mathematical model using these thing interpret your physical phenomena understand the physics and engineering behind it using that 1. For that you need the analysis methods to solve it and of course, you will have the computations.

And the aim of the course is in this box, here only here nothing else you are not here you are only here. So, we are restricting to this box and after doing the analysis and mathematics here is where your development of mathematics coming development of mathematics Maths. And once you do the analysis and methods of computations your model may not be very perfect. This will also help you this is a 2 way. So, after doing that, if the model does not predict you using the analysis properly here use the analysis and mathematics here to improve upon here do further experiments may be for the data collection further of thing you improve your mathematical model.

So, you will come here further again. So, there is a circle between these thing you understand this 1. On this mathematical model may be this is a 1 simpler thing, but you will have other issues like in the mathematical model, where you will have control and other things like this is important now, because you have a mars mission you see you apply control to grade your trajectory. If the trajectories are not taking birth you have to correct it what the ISRO is doing currently.

So, it should mention that 1. So, that leads to another branch of mathematics entirely called control theory here, our aim is that not that 1. So, if you want to spend time yeah you can do that with this is we will finish this introduction. And we this introductory lecture, we will start now, next with some of the examples, real life examples.

Thank you.