Advanced Matrix Theory and Linear Algebra for Engineers

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Lecture No. # 33

Hermitian and Symmetric Matrices- Part 2

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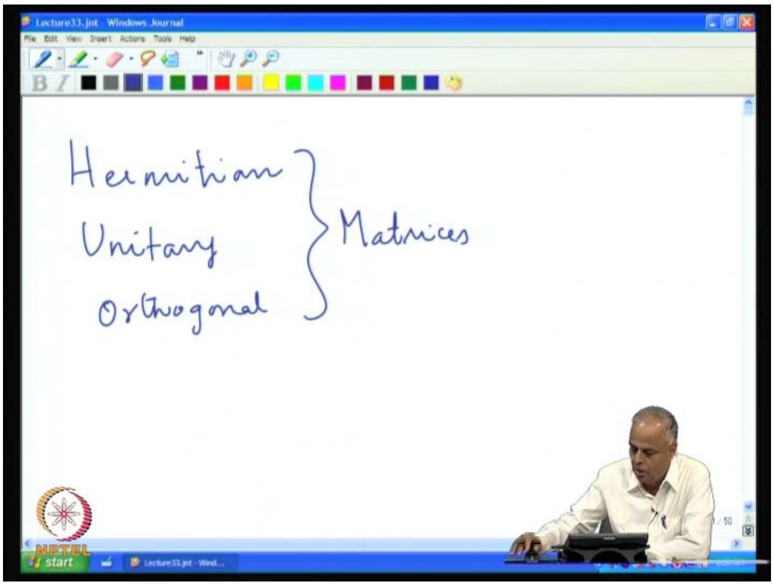

In the last lecture, we introduce the notions of Hermitian matrix, unitary matrix and of orthogonal real orthogonal matrices, this will be three important notions that we introduce let us recollect.

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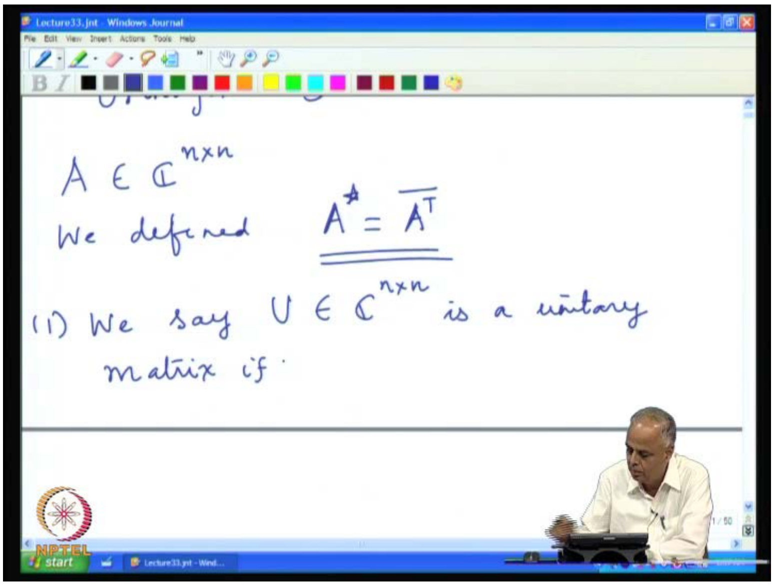

Suppose A is an n by n complex matrix, we define A star the Hermitian conjugate as the transpose conjugate. So, you flip the matrix and conjugate every entry, you get the Hermitian conjugate A star. Then we said we say U belonging to C n n is a unitary matrix unitary matrix.

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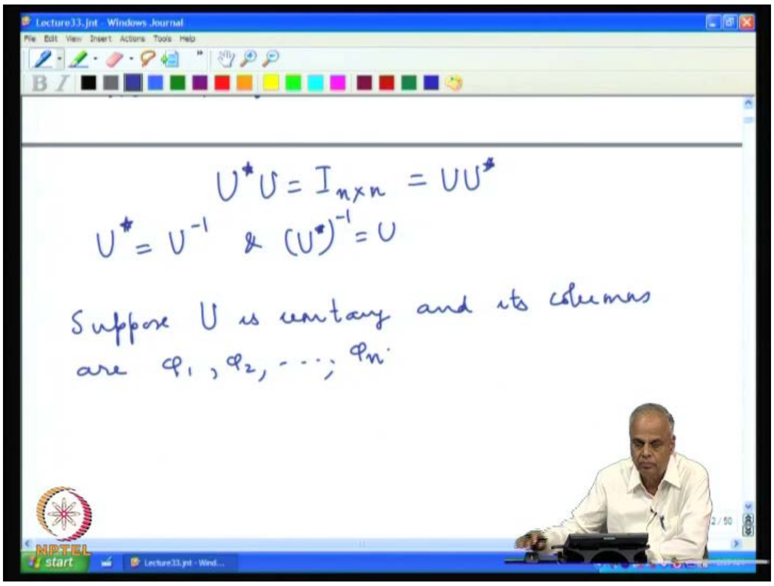

If U star U is the identity matrix and hence U U star will also be identity, this means U star is U inverse and U star inverse is U then we say the matrix is unitary. Now, suppose U is unitary and its columns are phi 1, phi 2, phi n.

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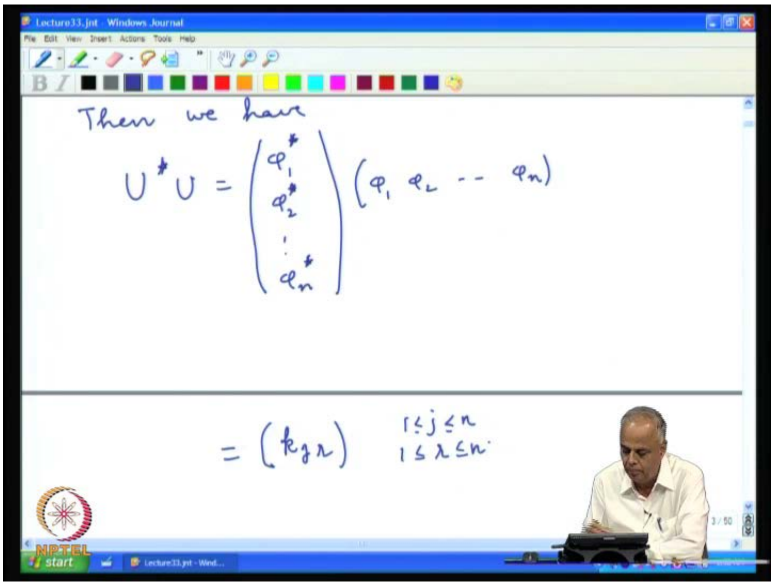

Then we have U star U, U star will be the columns flipped and conjugated and U will be phi 1, phi 2, phi n when we multiply these 2, we get a matrix let us say k j, let us use k j r, a matrix k j r, 1 less than or equal to j less than or equal to n, 1 less than or equal to r less than or equal to n where k j r is simply phi j star phi r.

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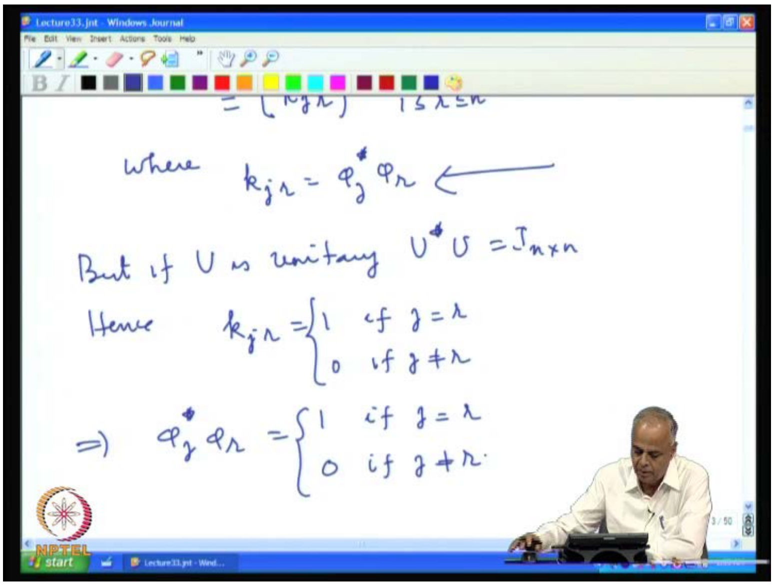

Now, since U is unitary, U star U must be identity, but if U is unitary, U star U must be identity. Hence k j r must be equal to 1, if j equal to r and 0, if j is not equal to r because for an identity matrix, the diagonal entries are all 1 and the half diagonal entries are 0, comparing with this, we get phi j star phi r is equal to 1, if j equal to r; 0, if j is not equal to r.

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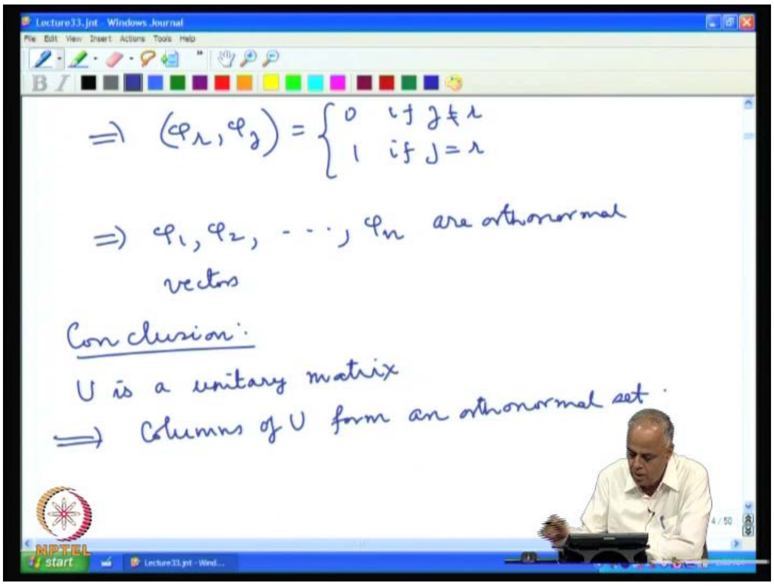

This says the inner product of phi r and phi j is 0, if j equal to r, 1 j not equal to r and 1 if j equal to r that says, the phi 1, phi 2, phi n are orthonormal vectors. Thus, we see that U is unitary then the columns of U form an orthonormal vectors. So, what is the conclusion? The conclusion is U is a unitary matrix imply the columns of U form an orthonormal set.

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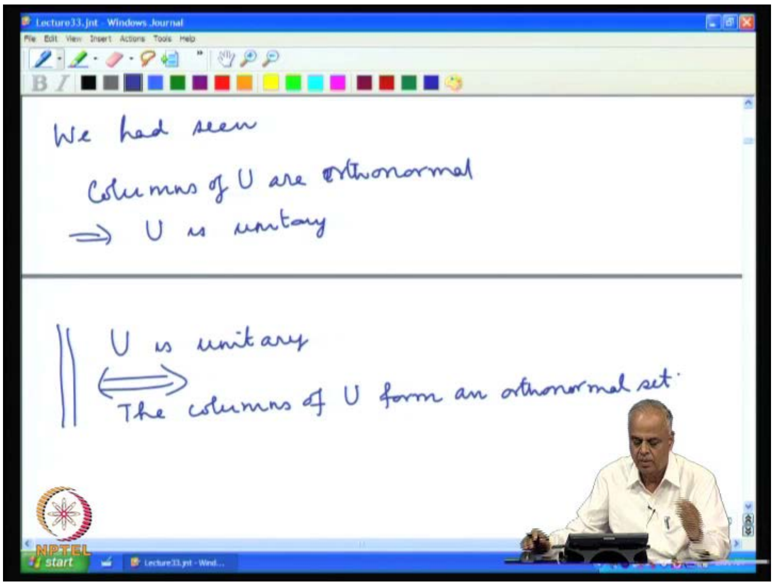

Now, we have verified last time that, if the columns of U are orthonormal then the U is unitary. So, we had seen that columns of U are unitary are orthonormal implies U is unitary, thus combining these 2, we get the important conclusion that U is unitary, if and only if, the columns of U are form an orthonormal set. Thus, we can easily recognize whether a matrix U is unitary or not by looking at its columns and seeing whether they form an orthonormal set and at all.

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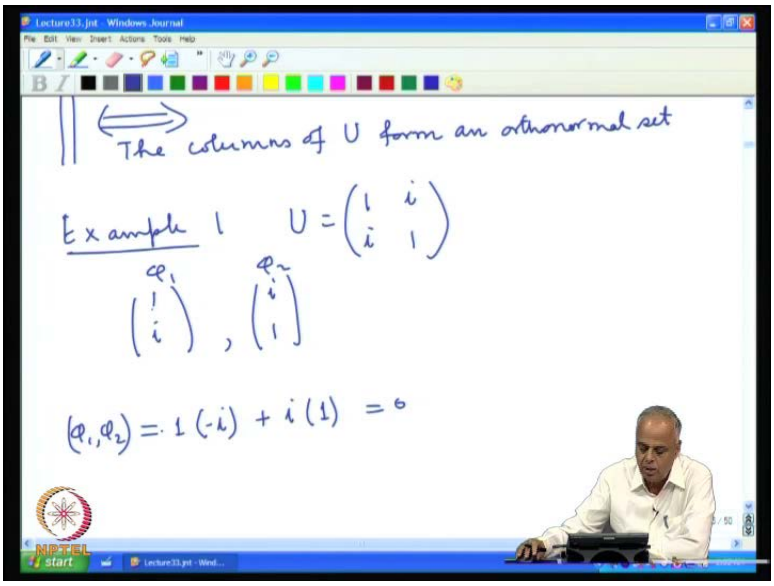

So, let us look at some examples before we see the real version, let us take the matrix U to be 1 i i 1 then the columns are 1 i and i 1, if you look at their dot product so, we have the dot product of 1 with 1 i with i 1. So, the first component into the conjugate of this first component of the second vector plus second component of the first vector times the conjugate and that is equal to 0. So, the column phi 1 is the column phi 2. So, we have phi 1 comma phi 2 is equal to 0.

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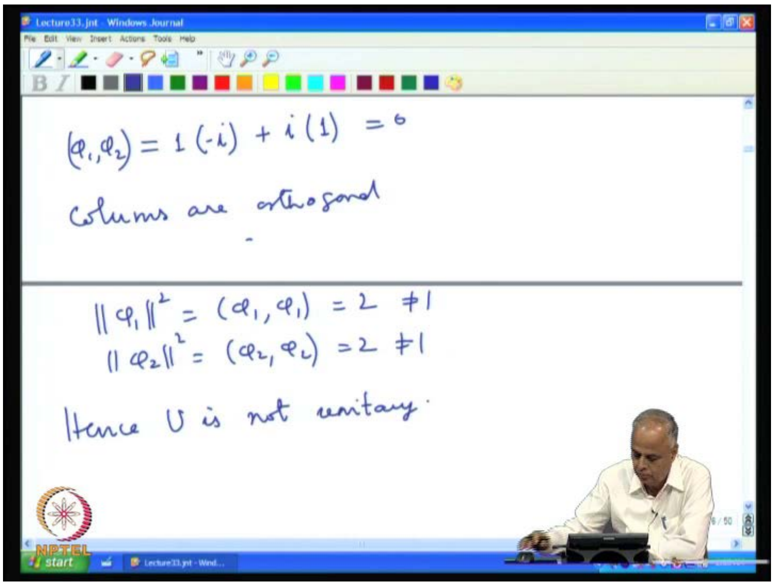

So, the columns are orthogonal the columns are orthogonal, but their lengths the length of phi 1 square that is phi 1 comma phi 1 is 2 not equal to 1. Similarly, phi 2 square which is phi 2 comma phi 2 is equal to 2 not equal to 1. Therefore, even though the columns are orthogonal, they are not of length 1 and hence U is not unitary.

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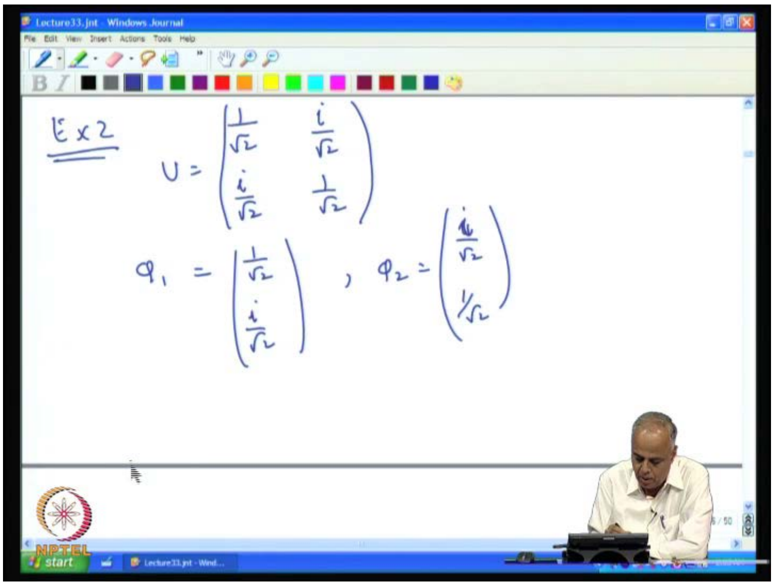

Let us look at another example, U equal to 1 by root 2 i by root 2 i by root 2 1 by root 2. Now, we have the columns are phi 1 equal to 1 by root 2 and i by root 2 and phi 2 equal to 1 by root 2 i by root 2 1 by root 2.

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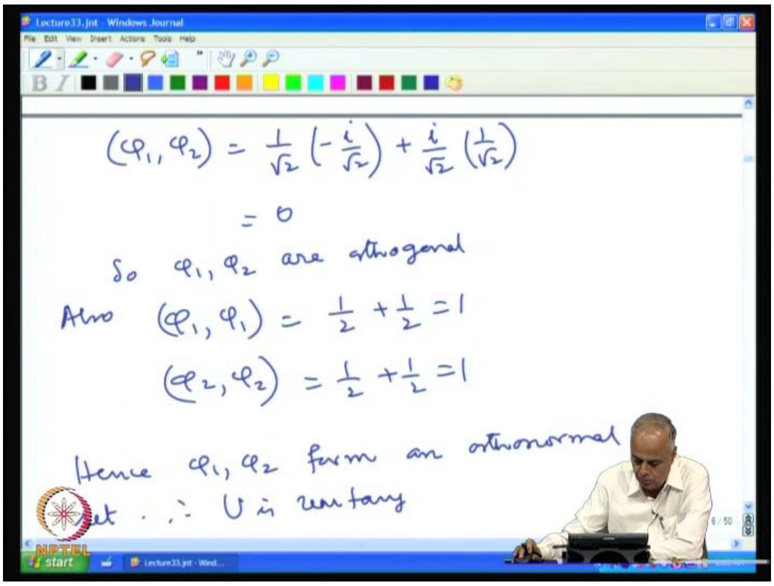

Again, if we take the inner product of phi 1 and phi 2 we get the first component of phi 1 into the conjugate of the first component of the second vector plus the second component of the first vector into the conjugate of the second component which is 0 and hence the vectors are orthogonal. So, phi 1, phi 2 are orthogonal also phi 1 comma phi 1 is 1 by 2 plus 1 by 2, the inner product of phi 1 with itself is 1 by 2 plus 1 by 2 is 1; phi 2 comma phi 2 is also 1 by 2 plus 1 by 2 which is 1; and therefore, they are normalized. Hence, phi 1 and phi 2 form an orthonormal set. So, thus the columns of this given matrix form an orthonormal set and we observed then it must be unitary. So, therefore U U is unitary. We can directly check this fact by calculating U star U, what is U? it is 1 by root 2 i by root 2 i by root 2 1 by root 2.

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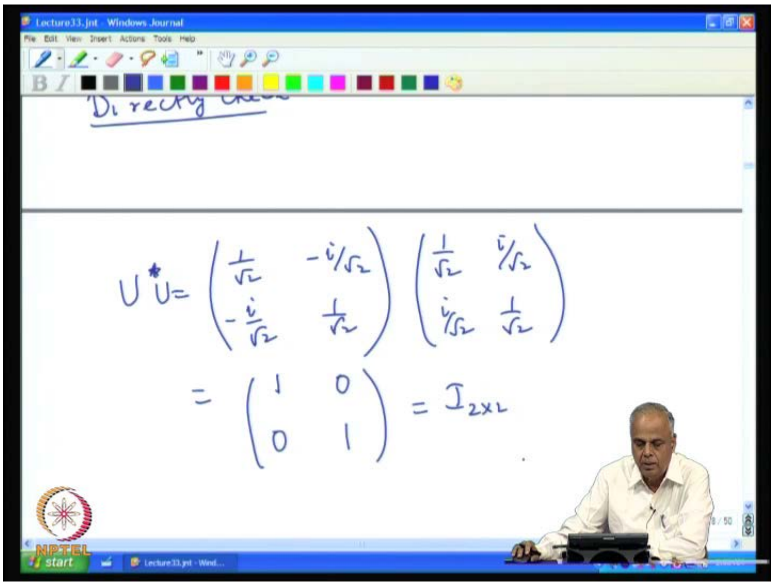

So, U U star will be 1 by root 2 minus i by root 2 minus i by root 2 1 by root 2 which is transpose and then conjugate, times U will be 1 by root 2 i by root 2 i by root 2 1 by root 2 to take the product i get 1 by 2 minus i square by 2 which is plus 1 by 2 which is 1 and then 1 by i by root 2 minus i by root 2 minus i by root 2 plus i by root 2 again we get 1 which is i 2. So, therefore we directly verify that U star U is identity. So, if the columns are orthogonal it is not enough, the columns are normalized alone it is not enough, the columns are orthonormal then the matrix becomes unitary.

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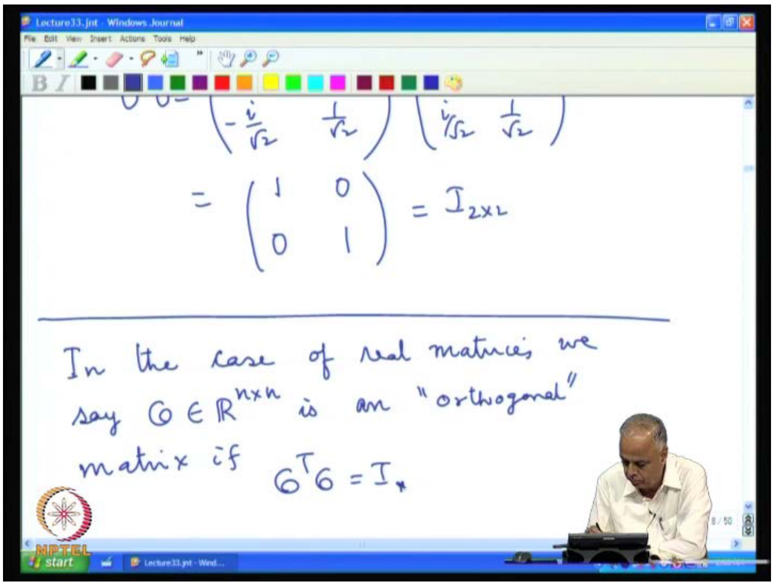

In the real case, in the case of real matrices we say o belonging to R n cross n is an orthogonal matrix, if o transpose o is identity.

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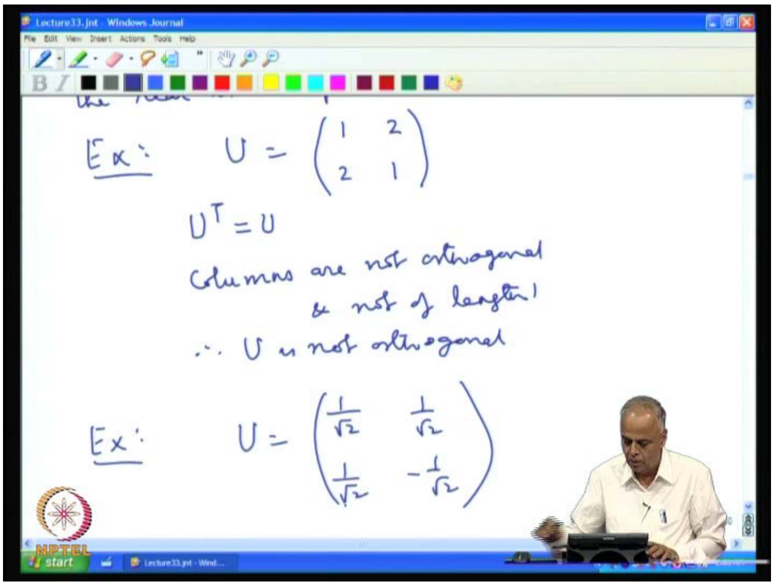

In this case, the columns will be orthogonal orthonormal, but with the real inner product because we are now everything is real so, we let the real inner product in R n. As an example, look at the matrix U equal to 1 2 2 1 we have U transpose is equal to U, but the columns are not orthogonal and not of length 1 and therefore, U is not orthogonal is not a orthogonal matrix. On the other hand, if we look at the matrix U which is 1 over root 2 1 over root 2 1 over root 2 minus 1 over root 2.

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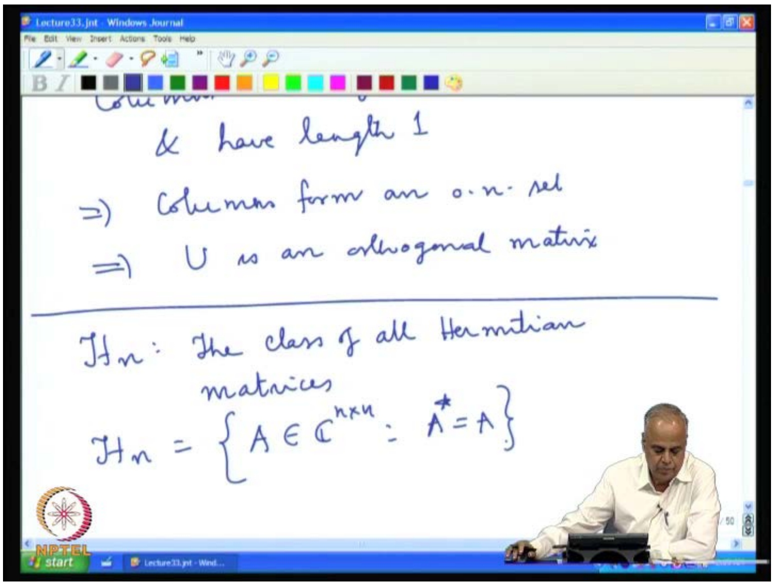

We see that the columns are orthogonal and have length 1and therefore, columns form orthonormal set columns form an orthonormal set and hence U is an orthogonal matrix. So, these were two important notions that we introduced last time namely the unitary matrix and the orthogonal matrix and the most important class of matrices that we introduce was H n, the class of all Hermitian matrices. H n is the collection of all those complex matrices for which A star is equal to A, that is they are self conjugate, the Hermitian conjugate with itself.

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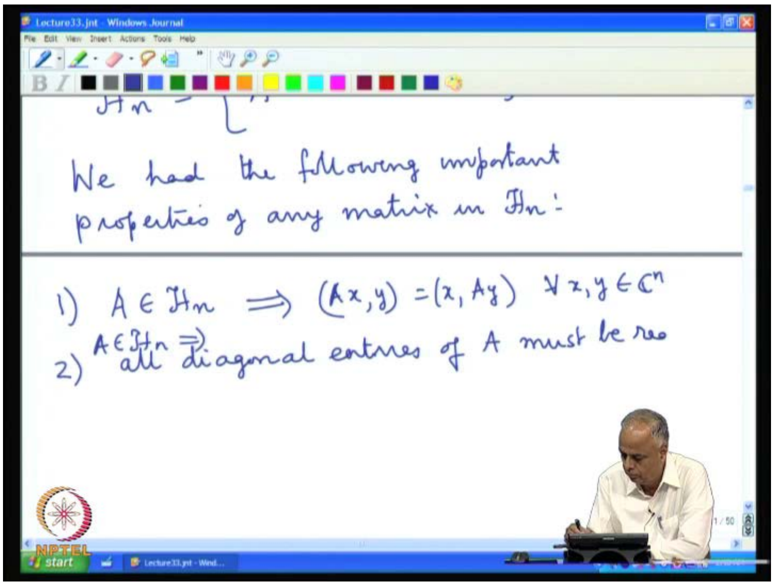

We observed the following important properties following important properties of any matrix in H n that is, any symmetric any Hermitian matrix will have these following properties. The first property we had was that A belongs to H n that is A is a Hermitian matrix implies A x comma y is equal to x comma A y for every x y in C n. Then we observed that all diagonal entries A belongs to H n implies all diagonal entries of A must be real.

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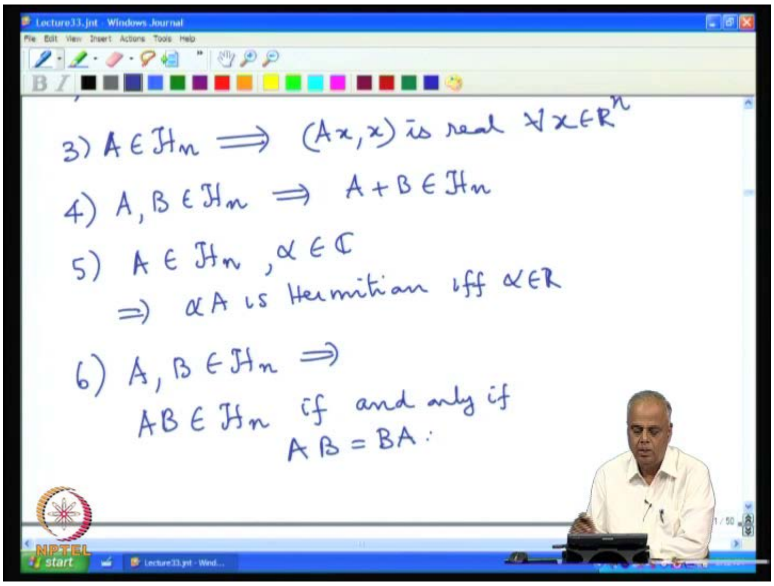

We also had observed that if A belongs to H n that is A is a Hermitian matrix then A x comma x is real for every x in r n, even though the matrix A is complex as long as it is Hermitian and even though the vectors x may be complex A x comma x will always be real. Then we observed that, if A and B are Hermitian then A plus B is also Hermitian then if A is Hermitian alpha is any complex numbers then alpha A is Hermitian, if and only if, alpha is real and finally, we had the property that, if A and B are Hermitian implies A B is also Hermitian if and only if, A and B commute that is A B equal to B A. These are some simple properties of Hermitian matrices.

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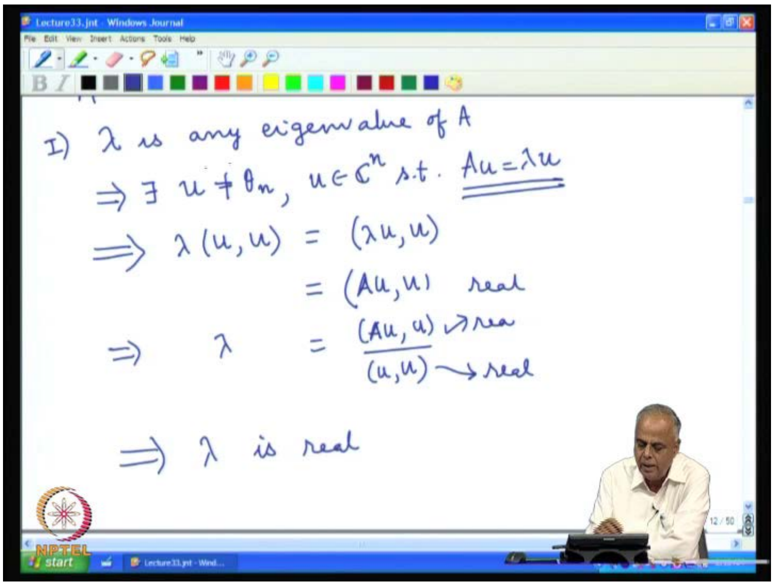

Now, we mention that we are going to study Hermitian matrices in detail because they posses some nice Eigen properties. So, we shall now begin our study of Eigen properties of Hermitian matrices (no audio from 16:37 to 16:47). We begin with the matrix A which is Hermitian and so, we now look at certain property, the first fundamental property that we are going to look at that the following. Suppose, lambda is an Eigen value of A so, suppose we consider an Eigen value of A. Now, lambda is an Eigen value means there must be a corresponding Eigen vector.

So, this implies there exist a vector U not equal to theta n, U belonging to C n such that A U equal to lambda U which is what is meant by saying, that lambda is an Eigen value. This equation A U equal to lambda U, U is a non trivial solution for the system; it always must exist, if lambda has to be an Eigen value. Now, that says lambda times U comma U can be written as lambda U comma U, because we know that, when a complex number is a multiplier of a vector in an inner product in the first factor then it can be pulled out without any change.

So, lambda is a multiplier of U and it is in the first factor so, it comes out as lambda outside. Now, lambda U is A U because we have U is an Eigen vector corresponding to the Eigen value lambda. Now we had the fundamental property of Hermitian matrices that A U U is always real A x x is always real, we observed that, if A is Hermitian A x x is real. So, by property three that is, we get this must be real, so this is real.

So therefore, this implies lambda equal to A U U by U U, I can divide by U comma U because U is a non 0 vector and hence U U will be not 0, it will be a non zero number. Now, if you look at the right hand side, the numerator is real, the denominator is real because U U is always real and greater than or equal to 0, the ratio of two real numbers would be real and therefore, it says lambda is real. So, therefore, we have concluded that any Eigen value of A must be real.

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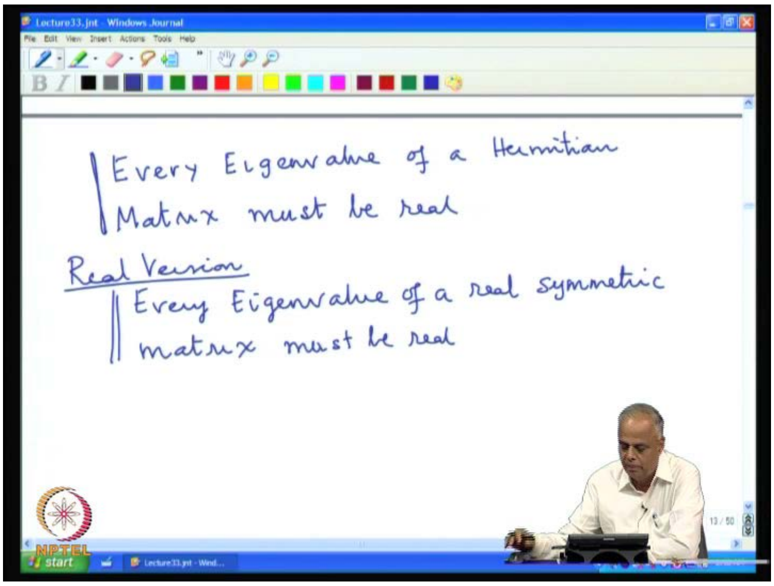

So, the first fundamental property is that every Eigen value of a Hermitian matrix must be real. Now, we observed last time that every real symmetric matrix can be thought of whether complex Hermitian matrix and hence, we get that every real version every Eigen value of a real symmetric matrix (no audio from 20:28 to 20:36) must be real. So, there lots of quantities that they are associated with a Hermitian matrix that are real. We found that, if A is Hermitian matrix all the diagonal entries must be real. We also found that, if A is Hermitian then the A x comma A x is always real for all ma vectors x. Now, we have found that all the Eigen values must also be real. So, every Eigen value of a Hermitian matrix is real and every Eigen value of a real symmetric matrix is also real.

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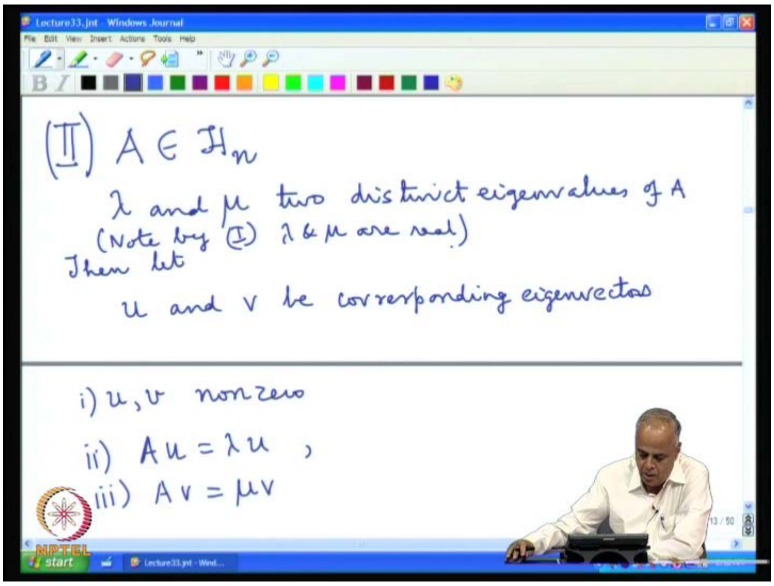

So, these are two, this is the first fundamental property namely the Eigen values being real, it is the fundamental property of Hermitian and real symmetric matrices. We now look at what it means in terms of Eigen functions Eigen vectors. So, again let us look at a Hermitian matrix A is Hermitian and suppose lambda and mu two distinct Eigen values of A (no audio from 21:44 to 21: 50) two distinct Eigen values of A. Now, since lambda and mu are Eigen values then let U and V be corresponding Eigen vectors. So, we have two distinct Eigen values what we mean is lambda and mu are different.

There are two different Eigen values we are considering and we are considering the corresponding Eigen vectors, what does this mean? The fact that u and v are Eigen vectors means u and v are non zero, one and u is an Eigen vector corresponding to lambda and v is an Eigen vector corresponding to v. Now, notice that we have already shown that the all the Eigen values of the Hermitian matrix must be real and therefore, since lambda and mu are considered to the Eigen values of the Hermitian matrix H n lambda A lambda and mu must be real. So, note by one lambda and mu are real. Now, therefore, we have this two distinct Eigen values, lambda and mu and we have the corresponding Eigen vectors U and V, U and V are non zero A u is lambda u and A v is u v.

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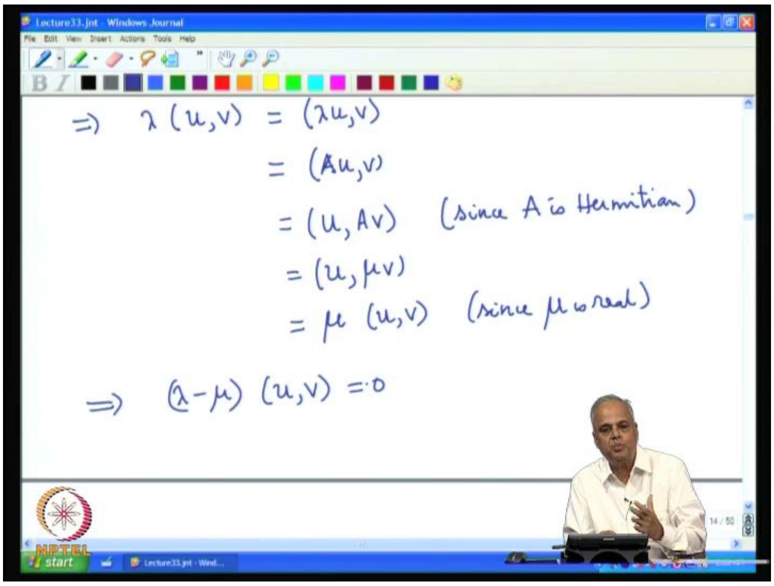

Now, this implies let us look at lambda times u v again, this is the same as lambda u v because the lambda can be pulled out of the first vector. Now, it is does not matter whether in the first factor or in the second factor because it is real, it can be pulled out, so, it is lambda u v. Now, lambda U is A u from this and we know that for a Hermitian matrix A x comma y is y comma A x, the first fundamental property of Hermitian matrices says so, this is u, A v since A is Hermitian.

Now, u v A v is mu v by this property because v is the Eigen vector corresponding to the Eigen value mu. Now, mu has to be pulled out is a constant is a number, it has to be pulled out of the second factor, it will come out with the conjugate, but mu is real. So, conjugate does not matter so, it will be just mu into u v, since mu is real. So, therefore lambda times u v is mu times u v and that says lambda minus mu times u v equal to 0.

Now, lambda minus mu is a real number u v is a complex number and the product is 0, the product of two complex numbers is 0, if and only if, one of them is 0, but lambda minus mu cannot be 0 because lambda and mu are distinct and therefore, that says u v equal to 0, since lambda is not equal to mu because we are considered two distinct Eigen values. Therefore, u is orthogonal to v what this therefore says is, if you take two distinct Eigen values, lambda and mu and look at their corresponding Eigen vectors they must be orthogonal they must be orthogonal.

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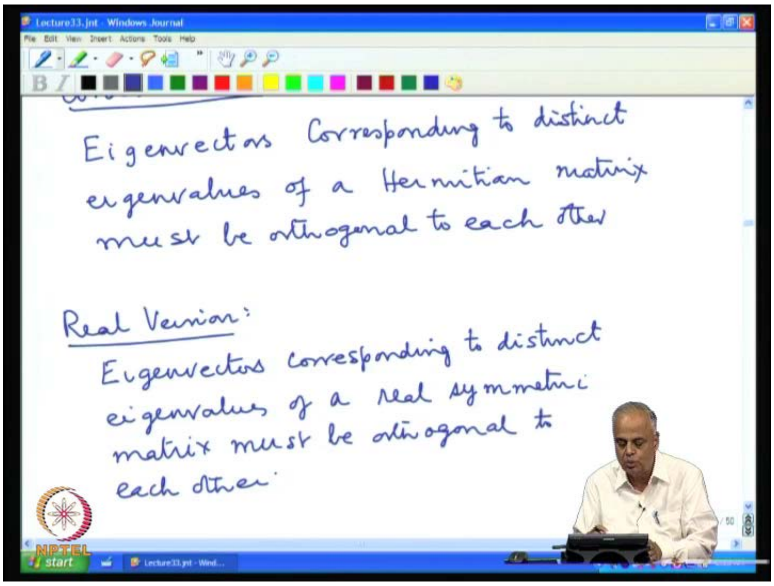

So, conclusion, this is the second fundamental property of the Eigen values and Eigen vectors of Hermitian matrix. So, Eigen vectors corresponding to distinct Eigen values, Eigen vectors corresponding to distinct Eigen values of a Hermitian matrix must be orthogonal to each other. So, they are nicely structured, the Eigen vectors are nicely structured they are orthogonal to each other.

Now, the real version again is that Eigen vectors corresponding to distinct Eigen values of a real symmetric matrix are orthogonal to each other must be orthogonal to each other. Now, when I say orthogonal in the real case, we mean the real inner product that is x comma y is x 1 y 1 plus x 2 y 2 there is no conjugation because everything is real. So, we have now these two fundamental properties all the Eigen values must be real and Eigen vectors corresponding to distinct Eigen values must be orthogonal to each other.

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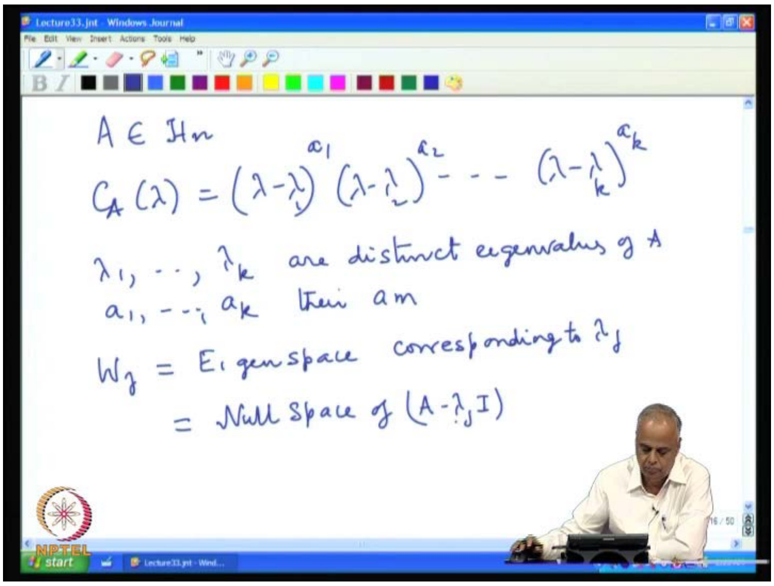

Now, let us see what does this mean? So, therefore, let us look at A Hermitian matrix H n and let us say its characteristic polynomial is lambda minus lambda 1power a 1, lambda minus lambda 2 power a 2, lambda minus lambda k power a k where lambda 1, lambda k are the distinct Eigen values of A a 1, a 2, a k their algebraic multiplicity, this our usual notation, this is how we denote the characteristic polynomial and the distinct Eigen value. Now, correspondingly we have the Eigen space the Eigen space corresponding to Eigen value lambda j, what is that, that is thus the null space of A minus lambda j i.

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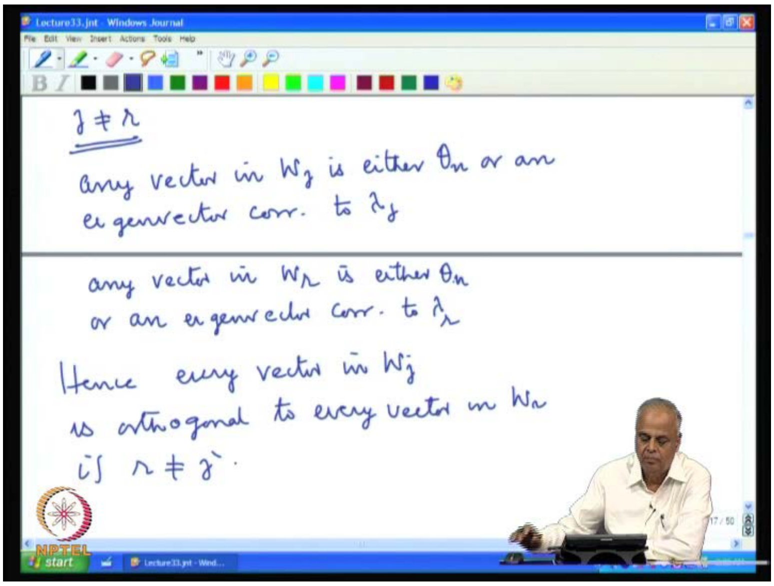

Now, any vectors let us consider j not equal to r, that is we are considering two different Eigen values lambda j, lambda r. Suppose, two indices j not equal to r any vector in W j is either the 0 vector or if it is not a 0 vector, it must be an Eigen vector corresponding to lambda j. Similarly, any vector in W r is either the 0 vector or an Eigen vector corresponding to lambda r. Now, if zero vector, zero vector is orthogonal to all the vector so, 0 vector is orthogonal to all the vectors in W r, if it is not 0, it is an Eigen vector corresponding to lambda j, it will be orthogonal to 0 and it will also be orthogonal to all the Eigen vectors corresponding to lambda r because Eigen vectors corresponding to distinct Eigen values are orthogonal to each other.

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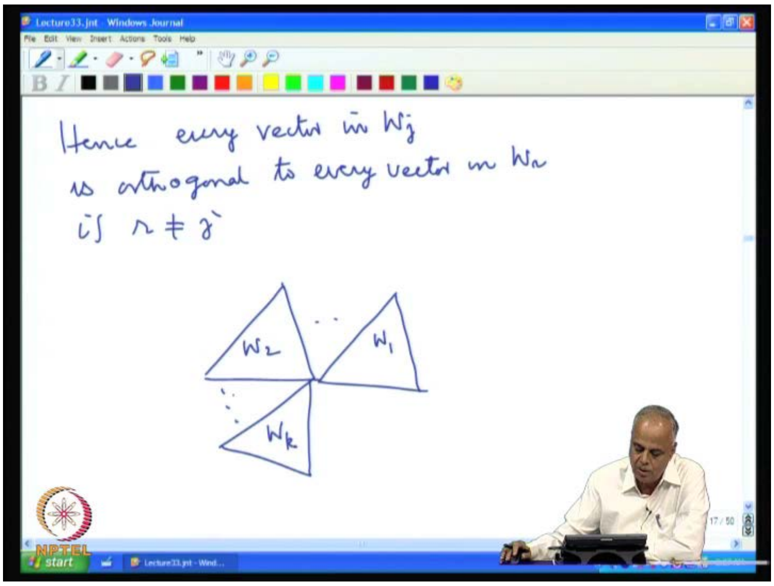

Hence, every vector in W j is orthogonal to every vector in W r, if r is not equal to j. So, now let us see how how this picture looks like, we have this W 1, we have W 2 all of them being orthogonal, the only vector that will be common to two orthogonal things will be the 0 vector. So, we have like that and finally, W k so, we have W 1, W 2 etcetera and we have W k they are so positioned, that if you pick any one of them every vector there is orthogonal to all the other vectors in every other piece. So, w j vectors are orthogonal to every vector in W r, if r is not equal to j. So, what this means is W r is contained in W j th perp and W j is contained in W r perp, if r is not equal to j. Now, what does this lead us to?

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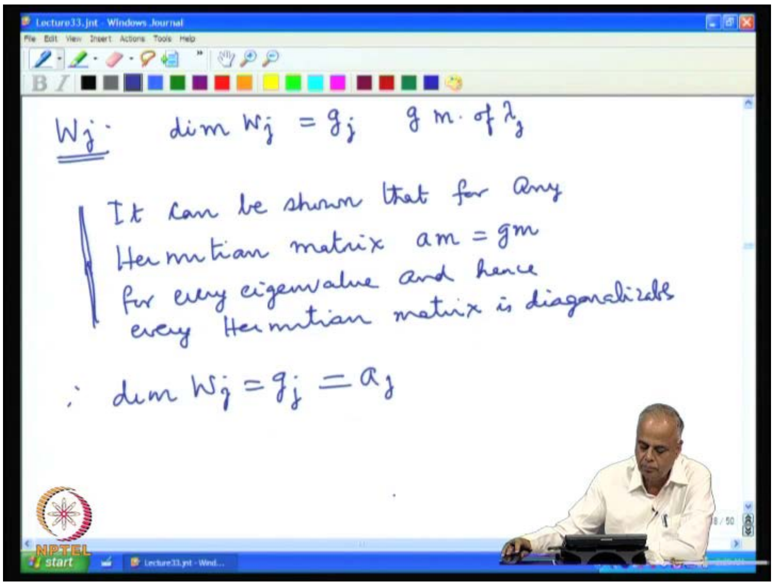

Now, suppose we find let us look at W j, the dimension of W j is what we called as g j, the geometric multiplicity of the lam Eigen value lambda j. Now, we shall at the moment (( )) this theorem, we will simply say it can be shown that for any Hermitian matrix, the algebraic multiplicity is equal to geometric multiplicity for every Eigen value and hence every Hermitian matrix is diagonalizable (no audio from 33:13 to 33:18). So, what does this among to therefore, we say the dimension of W j which is g j the algebraic multiplicity must be equal to a j.

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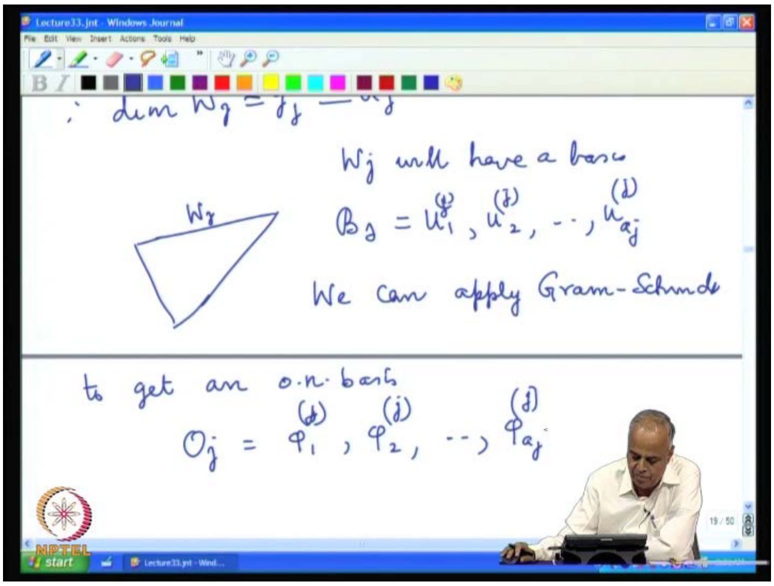

Therefore, if I look at this piece W j, this is the W j its dimension is g j therefore, W j will have a basis let us call it as B j, how many vectors it will contain in the dimension is a j it will contain a j vectors, let me call it as u 1 1, u j 1, u j 2, u j a j the superscript j says that you are looking at the j th Eigen space and the subscript gives the numbering ordering of the basis vector. So, any basis of W j will have a j vectors and now, we can apply Gram- schmdtz to this to get an orthonormal basis for W j.

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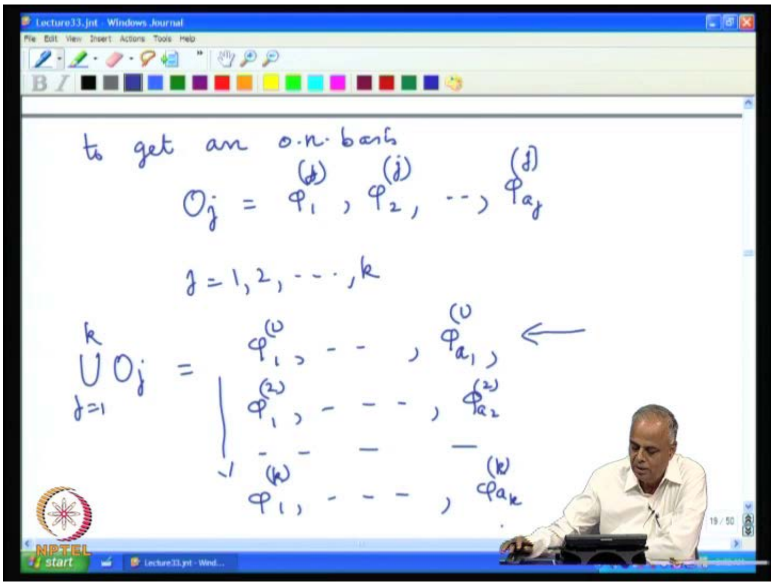

So, we can apply Gram- schmdtz to get an orthonormal basis let us now call it as O j has phi 1 1 I am sorry it is phi j 1 because now, we are looking at the j th space phi j 1, phi j 2 and phi j a j. So, we can get an orthonormal basis for W j consisting of a j vectors we can do this for j equal to 1, j equal to 2 and j equal to k each one of this Eigen spaces, we can construct an orthonormal basis and the number of vectors in this orthonormal basis, it exactly equal to the algebraic multiplicity of the Eigen value. Now, if you put all these O j's together we have the union of this O j's. Now, if you look at let us write this down first phi 1 1, this will be the 1 that correspond to the first Eigen value then i will get those corresponding to this second Eigen value and so on and finally, those corresponding to the k th Eigen value.

So, this will consist of a 1 plus a 2 plus a k which is n so, this will consist of n vectors. If you look at the vectors here, they are all orthonormal, because that is a orthonormal for W 1, but across they are all orthogonal to each other because Eigen vectors corresponding to distinct Eigen values must be orthogonal to each other and therefore, this entire set is an orthonormal set is an orthonormal set and since it contains n vectors, it is an orthonormal basis for C n. So, therefore we can construct an orthonormal basis consisting of only Eigen vectors of A.

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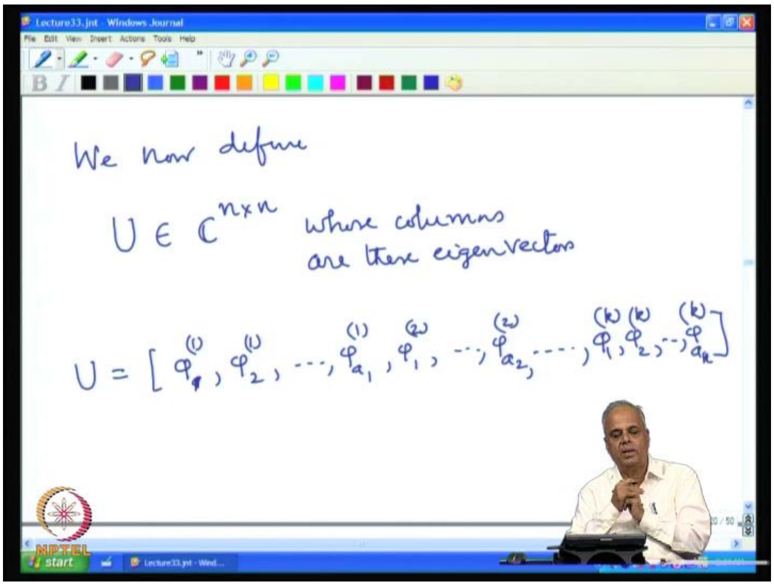

Hence, we have a basis for C n consisting of only Eigen vectors of A. So, what is the consequence of this, we now define the matrix U its n by n whose columns are these Eigen vectors, what do I mean, we construct the matrix U the the first a 1 columns are the Eigen vectors orthonormal Eigen vectors corresponding to the Eigen value lambda 1, there are a 1 Eigen vectors we have applied Gram- schmdtz and we got a1 orthonormal Eigen vectors corresponding to the Eigen value lambda 1.

And that we put at the first a 1 columns then we put the Eigen vectors corresponding to this second Eigen value, the second Eigen value multiplicity is a 2 and it has a 2 Eigen vectors corresponding to it, these orthonormal Eigen vectors are put at the next a 2 columns and we proceed like this and the last a k columns are the Eigen vectors corresponding to the Eigen value lambda k. So, this U matrix is made up of Eigen vectors, it is made up of Eigen vectors of A, it is made up of the orthonormal Eigen vectors of A, a 1 of them which occupy the first a 1 columns or from the Eigen value lambda 1, a 2 of them which occupy the next a 2 columns are the Eigen vectors of lambda 2 and so on and so forth and the last a k of them are the Eigen vectors corresponding to lambda k.

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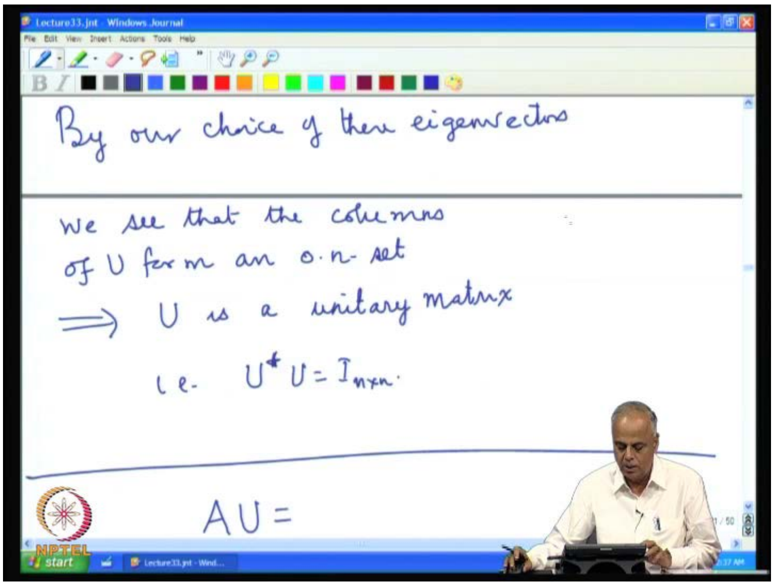

Now, since these Eigen vectors forms an orthonormal set, we see that by our construction the columns of U form an orthonormal set by our choice of these Eigen vectors, we see that the columns of U form an orthonormal set. Now, why do the columns form an orthonormal set, because these are all orthonormal basis for W 1, the next a 2 are orthonormal basis for W 2 and these are all orthonormal basis for W k and these vectors and these vectors are orthonormal because when r is not equal to j, W j vectors are orthogonal to W r vectors.

So, they form an orthonormal set, if the columns form an orthonormal set, we know that the matrix must be unitary that implies, U is a unitary matrix, that is what we saw at the beginning of the lecture. So, U is a unitary matrix that is U star U is equal to identity right. So, therefore, starting from the Eigen vectors of A, we have constructed a unitary matrix, what does this unitary matrix do to us, so that is what we are going to look.

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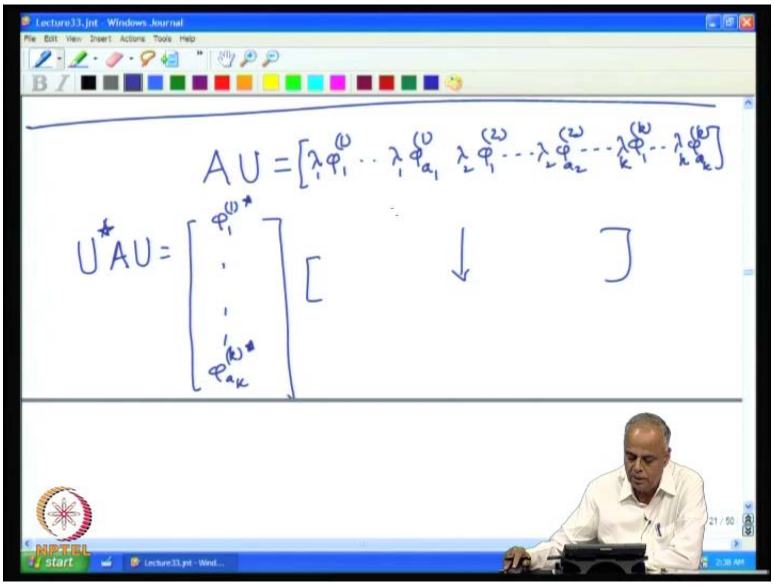

Now, what is A times U? Now, U is this matrix A U will be multiplying each column by A, but when I multiply these first a 1 columns by a because they are Eigen vectors they would be simply multiplied by number lambda 1. Similarly, the next a 2 columns will be multiplied by the number lambda 2 and so on. So, we will get the first column will be just lambda 1 phi 1 1 and then the a 1 th column will be this then we will start with lambda 2 and we will have phi 2 1 lambda 2 phi 2 a 2 and this goes on and we get lambda k phi k 1 etcetera lambda k phi k a k because of the fact that every column is an Eigen vector corresponding to some Eigen value.

So, it just gets multiplied by those vectors. So, therefore, we have U star A U is we will start with phi 1 1 star and we end up with phi k a k star and this matrix here and when we multiply, we get phi 1 star phi 1 is 1 because of orthonormality and therefore, we get lambda 1, but phi 1 star phi 2 will be 0, phi 1 star phi 3 will be 0 because of orthonormality all the others would be 0.

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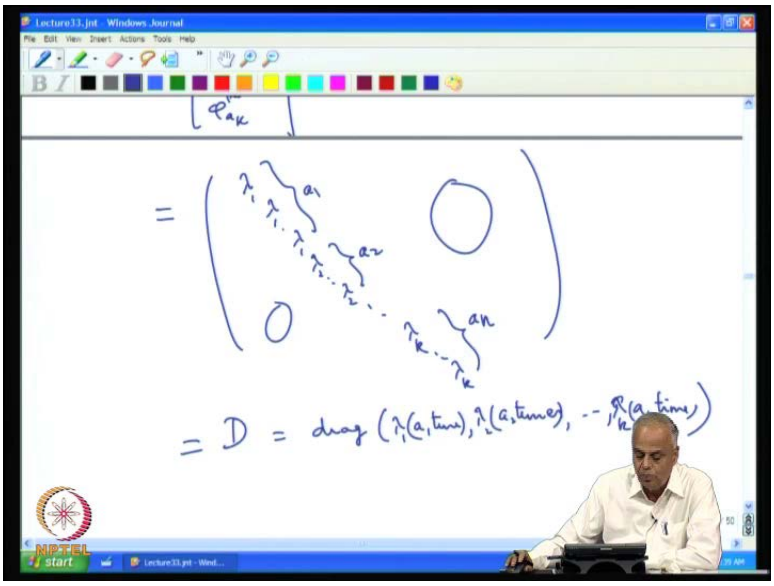

So, we get the diagonal matrix lambda 1 lambda 1 that occurs a 1 times then lambda 2 lambda 2 occurs a 2 times, lambda 1 occurs a 1 times and then lambda k lambda k will occur a k times, this big diagonal matrix n by n we get a 1 plus a 2 plus a n is equal to. We will simply write it as D which is diagonal lambda 1 a 1 times, lambda 2 a 2 times and so on lambda k a k times had diagonal matrix we will simply denote it by D. So, we see that the matrix A is not only diagonalizable, but we have used the unitary matrix for diagonalization.

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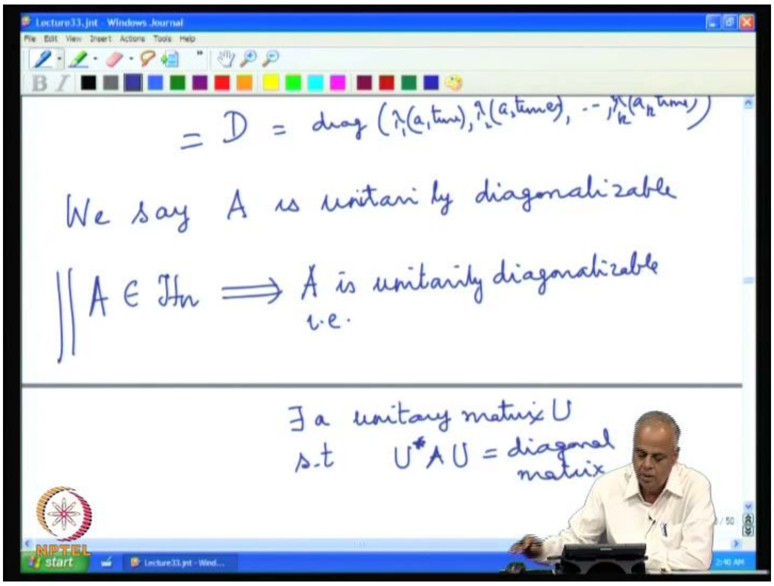

Therefore, we say A is unitarily diagonalizable unitarily diagonalizable. So, thus every Hermitian matrix is unitarily diagonalizable. Therefore, A belongs to H n implies that is A is Hermitian matrix means A is unitarily diagonalizable, that is there exists a unitary matrix U such that U star A U equal to a diagonal matrix.

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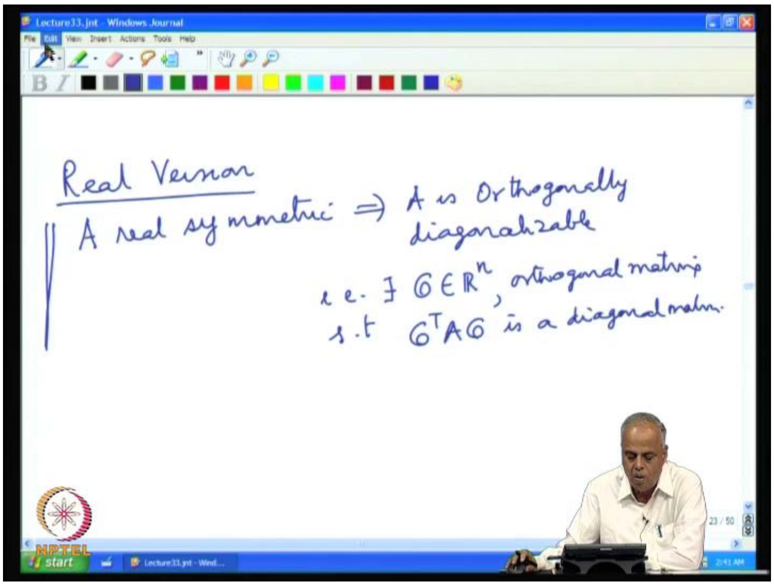

The real version of this is the role of unitary is taken over by orthogonal matrix. So, A real symmetric implies A is orthogonally diagonalizable is orthogonally diagonalizable that is there exists o belonging to R n orthogonal matrix. Now, there is no star so, o transpose A o is a diagonal matrix. So, real symmetric matrices are orthogonally diagonalizable and complex Hermitian matrices are unitarily diagonalizable.

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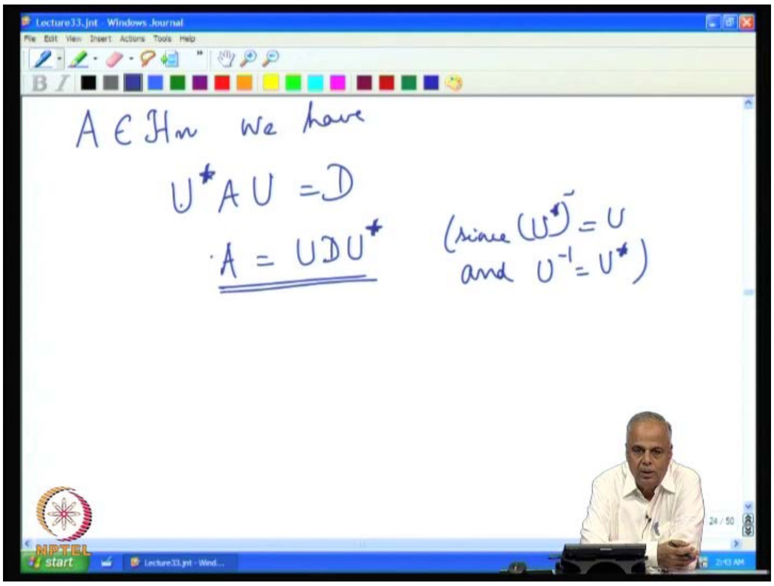

So, therefore, if A is H n we have U star A U is the diagonal matrix D as seen above, so unitarily diagonalize, what does that say, if since U star is inverse of U, U star U is identity so, U star is invertible and its inverse is U. So, if you take the U star to the right hand side, it will go as U star inverse which is U and similarly, U will go to the right hand side as U inverse which is U star. So, this will be equal to U D U star. Since U star inverse U star inverse is U and U inverse is U star. So, this is a very nice representation of a Hermitian matrix.

This says a Hermitian matrix can be decompose as the product of three matrices the two extreme matrices are unitary matrices and therefore, easily invertible and the middle matrices is diagonalizable is a diagonal matrix and hence can be treated easily. In other words, we have decompose the matrix A into a product of three simple matrices, the middle one being diagonal is easy to handle, the remaining two being unitary or easily invertible.

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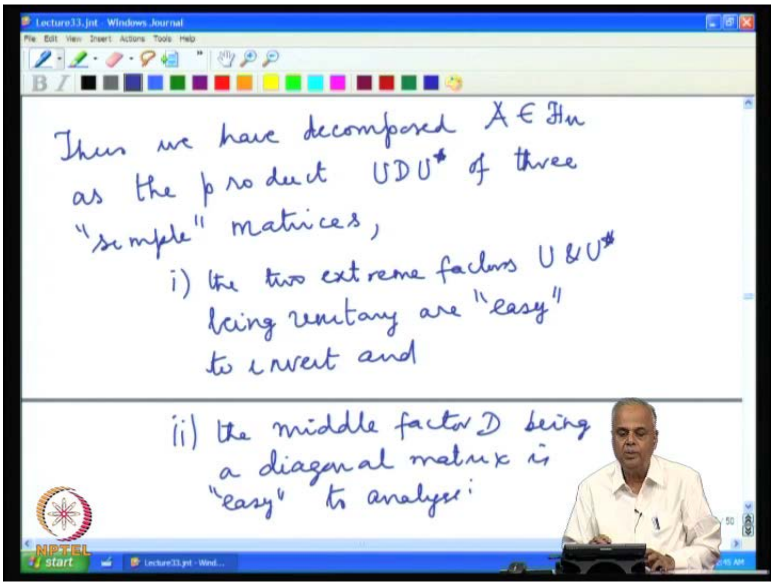

So, thus we have decomposed A belonging to H n as the product U D U star of three simple matrices. The two extreme factors U and U star being unitary are easy to invert and the middle factor D the middle factor D being diagonal diagonal matrix is easy to analyze. So, the nice splitting of the matrix into simple factors, this is a factorization theorem or a product decomposition of a Hermitian matrix into simple matrices A is equal to V D U star U is Hermitian, D is diagonal, again U star U is unitary, D is diagonal and U star is again unitary.

So, this is the very simple decomposition of a Hermitian matrix, a more general version of this sort of decomposition is what we will see as the singular value decomposition, but for Hermitian matrices we have a straight forward decomposition induced by the Eigen values and the Eigen vectors, the Eigen vectors constituting the (( )) matrix U and the Eigen values constituting the diagonal matrix D. You must notice here, that in the decomposition we require U D, U star is known once, U is known, U and D are the two required matters, D is known through its Eigen values, because the diagonal entry D is the diagonal matrix whose entries are always Eigen values, the diagonal entries are all Eigen values, you can notice that here all the entries along the diagonal are Eigen values.

Therefore, to construct this U D U, D requires only the Eigen values and the matrix U you may recall we have constructed using the Eigen vectors. So, the two ingredients required to make this decomposition of the given matrices into product of three simple matrices are precisely the Eigen values and the Eigen vectors and that is why the Eigen values and the Eigen vectors play an important role in the analysis of a matrix.

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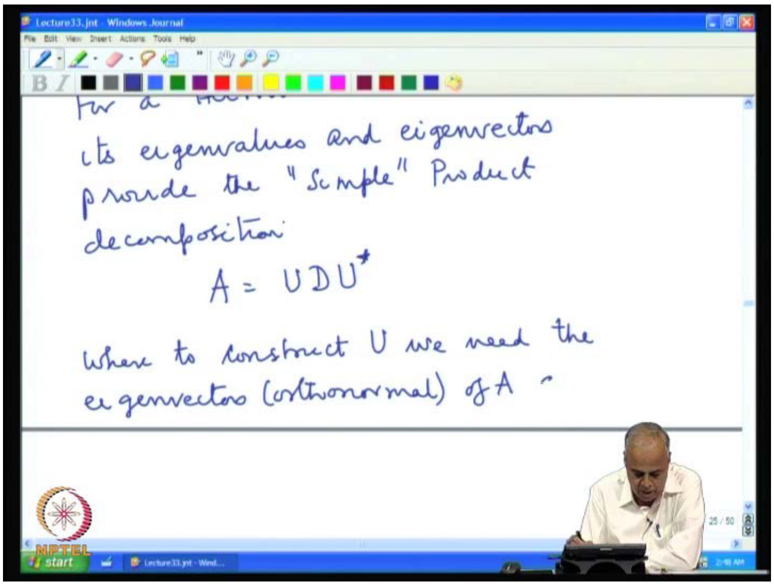

So, in the case of Hermitian matrices, we can conclude that thus for a Hermitian matrix (no audio from 51:40 to 51:47) A, its Eigen values and Eigen vectors provide the de-product decomposition we have call it, we provide the simple product decomposition A equal to U D U star where to construct U we need the Eigen vectors orthonormal Eigen vectors. So, we need the orthonormal Eigen vectors of A and to construct D, we need the Eigen values and their algebraic multiplicity because I have to put diagonal lambda 1 a 1 times and therefore, I need multiplicity algebraic multiplicity a 1, a 2, a k etcetera.

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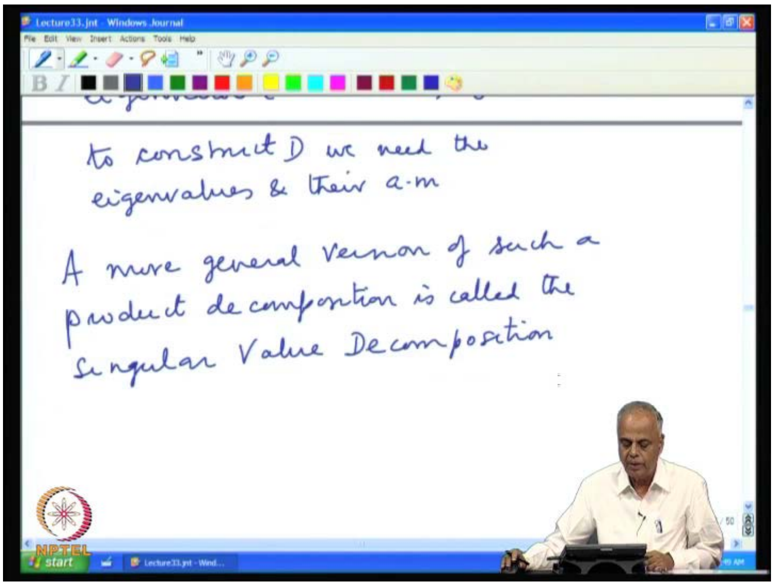

So, a more general version of such a product decomposition is called the singular valued decomposition which is our ultimate goal, we will eventually get to that more general version of this singular value of this type of product decomposition. So, thus we have a nice decomposition of a matrix A into simple matrices unitary diagonal unitary unitary diagonal unitary, the diagonal entries are Eigen values, since A is Hermitian all the Eigen values are real.

So, the diagonal part is a nice simple real diagonal matrix and the other two are unitary, anytime we can invert them by just flipping and conjugating them. So, that is the simple decomposition, we can also view this decomposition as the sum decomposition, we have viewed this whole thing as a product decomposition, we can also viewed this as a sum decomposition which is essentially what is known as the spectral theorem for Hermitian matrices.

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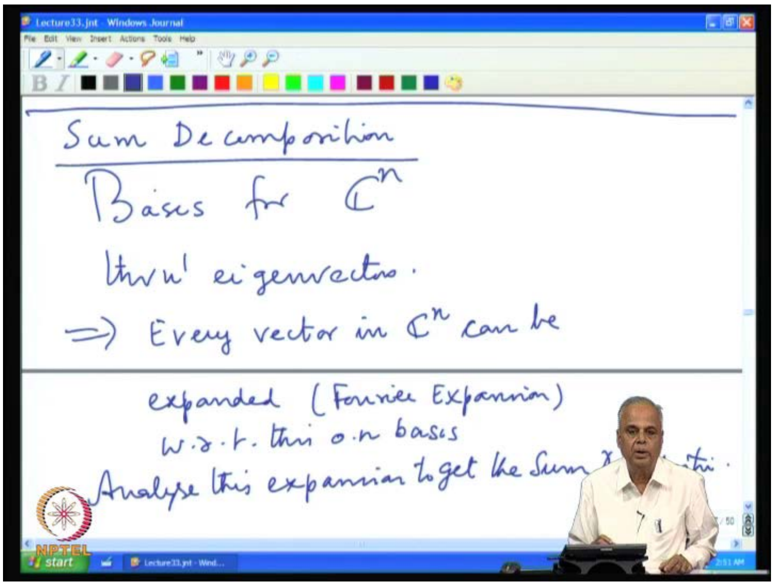

Now, what we actually do for that is, we have the basics for C n. So, we will in the next lecture, we look at the details of these calculations; we look at what is known as a sum decomposition. We have the basis for C n through Eigen vectors and this implies every vector in C n can be expanded remember this so called fourier fourier expansion with respect to this orthonormal basis with respect to this orthonormal basis. We analyze this this expansion carefully to get the sum decomposition and this is what we will look in the next lecture.