Advanced Matrix Theory and Linear Algebra for Engineers

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Lecture No. # 29

Diagonalization – Part 2

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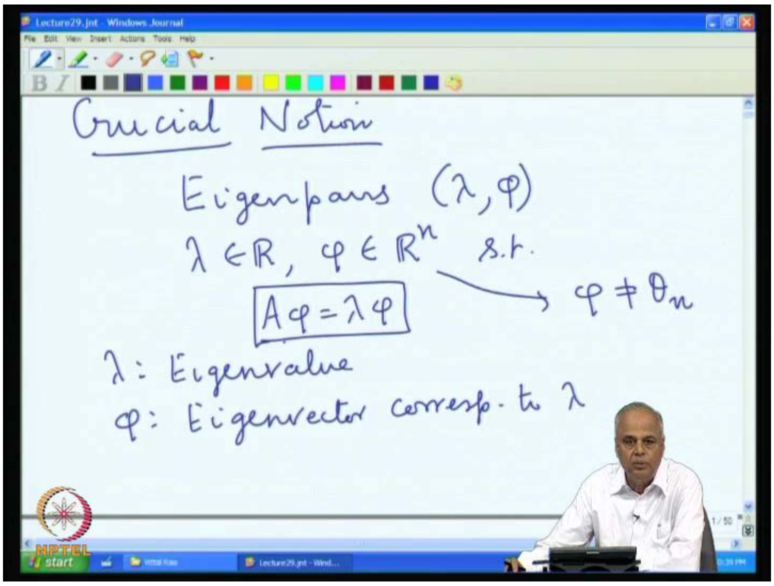

We have been looking at the notion of diagonalizability of a matrix A in R n cross n that is we are looked at a matrix A, which is n by n and all is entities are real, we are looking at the notion of the diagonalizability of that matrix. Become the crucial thing for answering this question is the notion of Eigen pairs. When we say Eigen pair lambda phi, lambda is a real number, and phi is a vector which has n component such that, A phi equal to lambda phi. Then we say lambda phi is an Eigen pair, lambda is called Eigen value, sometimes called characteristic value also, and phi is called Eigen vector corresponding to lambda. And here the phi beyond those want it to be non zero, because otherwise this equation a phi equal to lambda phi will have solution 0 solution for any lambda.

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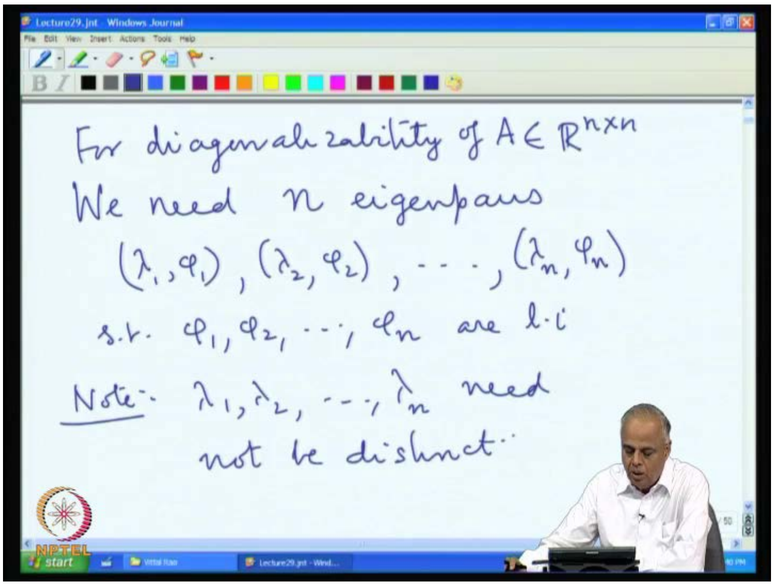

So, we are looking for those lambdas for which there is non zero solution, such a pair will be called an Eigen pair. For diagonalizability of A what we found was that we need n Eigen pairs with A is n by n matrix in order to diagonalize a we need n Eigen pair lambda 1 pi 1, lambda 2 pi 2, and so on lambda n pi n; such that, pi 1, pi, 2 pi n are linearly independent. The Eigen vectors appearing in the Eigen pair combination with linearly independent; note lambda 1, lambda 2, lambda n need not be distinct, what we mean is the sum of them could be repeated.

So far diagonalizability our problem is to find such n Eigen pairs, where the Eigen vectors involved in the Eigen pair or linearly independent.

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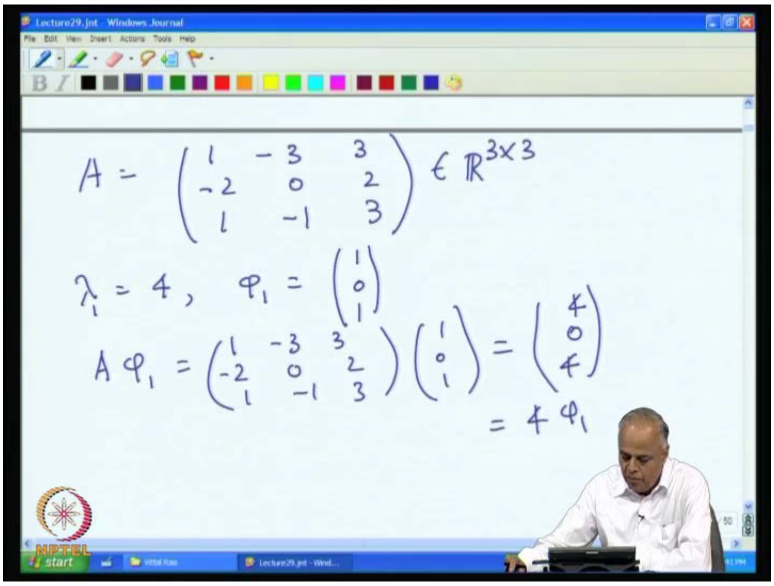

Let us look at some simple examples, let us take the matrix A 1 minus 3 3 minus 2 0 2 1 minus 1 3, which a matrix which is all real entries and it is a 3 by 3 matrix. Let us consider the number lambda 1 equal to 4, and the vector pi 1 to be 1 0 1. We now verify that, A pi 1 is 1 minus 3 3 minus 2 0 2 1 minus 1 3 into 1 0 1, and when multiply we get 4 0 4 which is 4 times pi 1.

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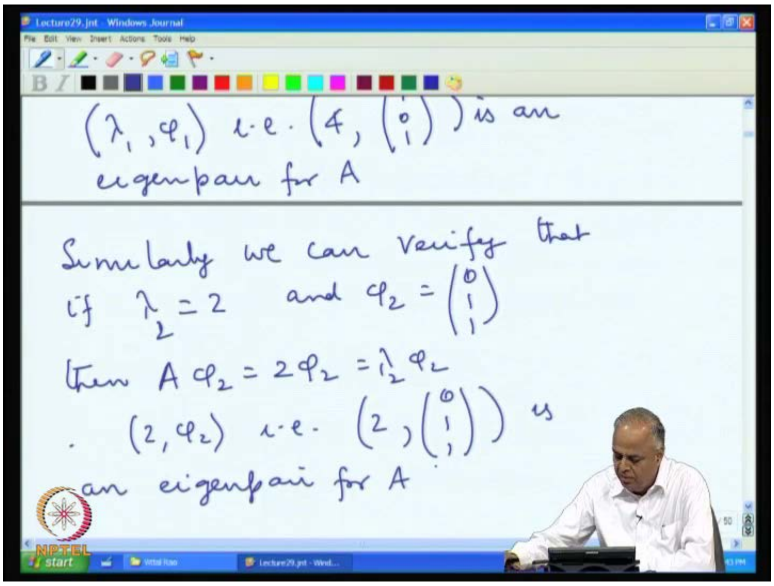

Thus, we find that, pi 1 is an Eigen vector corresponding to Eigen value lambda 1 equal to 4. Since, lambda 1 equal to 4 is an Eigen value, and pi 1 is an Eigen vector corresponding to this, we get lambda 1 pi 1; that is the number 4 and vector 1 01 is an Eigen pair for the matrix A. Similarly, we can verify by simple multiplication that, if lambda 1 equal to lambda 2 equal to 2, and pi 2 equal to 0 1 1, then A pi 2 is 2 pi 2, that lambda 2 pi 2; and therefore, 2 and pi 2, that is 2 under vector 0 1 1 is an Eigen pair for A.

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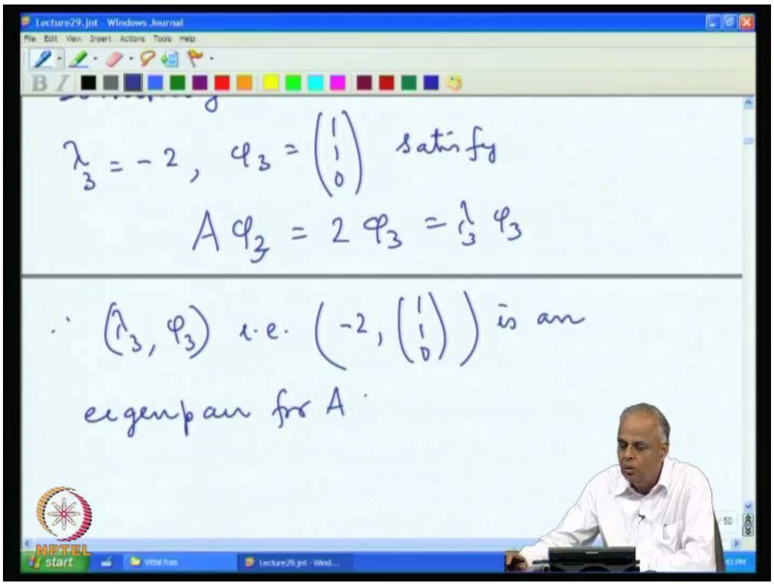

Similarly, you can also verify one more Eigen pair lambda 3 equal minus 2 pi 3 is equal to 1 1 0, satisfy A pi 3 equal to 2 pi 3, that is lambda 3 pi 3; and therefore, lambda 3 pi 3 that is minus 2 1 1 0 is also an Eigen pair for A.

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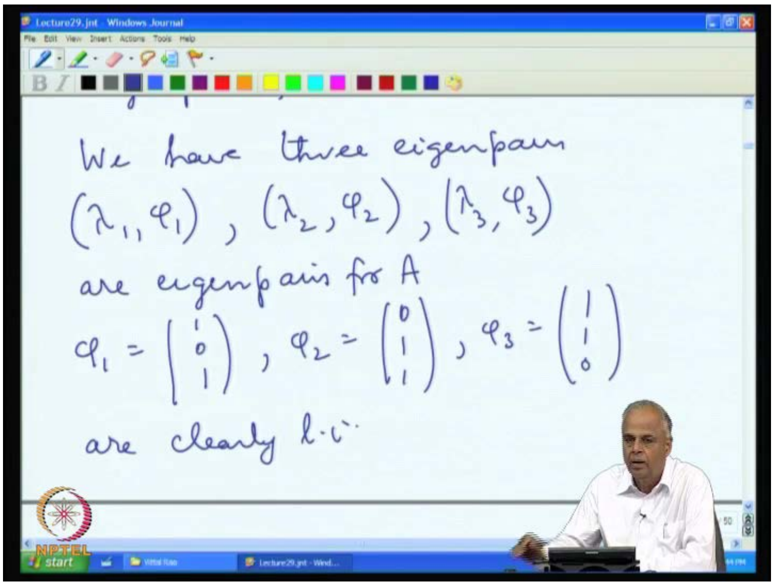

Therefore, we have found now, we have 3 Eigen pairs, what were they lambda 1 pi 1, where lambda 1 was 4 and pi 1 was 1 0 1, lambda 2 pi 2, where lambda 2 was 2 and pi 2 was 0 1 1, lambda 3 pi 3, where lambda 3 was minus 2 and pi 3 was 0 1 1 are Eigen pairs for A, and pi 1 which was 1 0 1, pi 2 which was 0 1 1, pi 3 which was 1 1 0 are clear linearly independent. And therefore, we have 3 Eigen pairs, in which the Eigen vectors involved are all linearly independent, and this was our search here to be diagonalizable.

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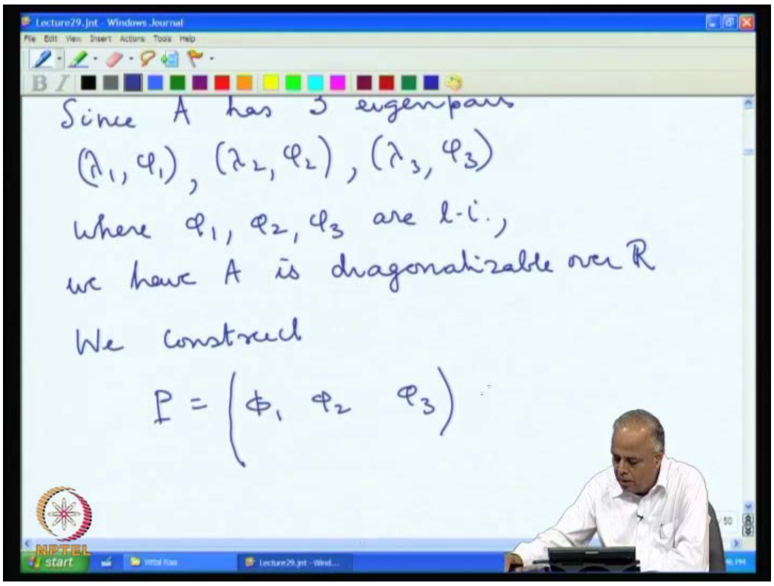

Since, A has 3 Eigen pairs: lambda 1 pi 1, lambda 2 pi 2, lambda 3 pi 3, where the pi 1, pi 2, pi 3 are linearly independent and A is A 3 by 3 matrix and we have got 3 Eigen pair and the Eigen vectors in the Eigen pairs are all linearly independent. We have A is diagonalizable over R; everything involved is over the real numbers and so it is diagonalizable over R, what is the diagonalization process we construct the matrix P has the matrix whose columns are these vectors pi 1, pi 2, pi 3.

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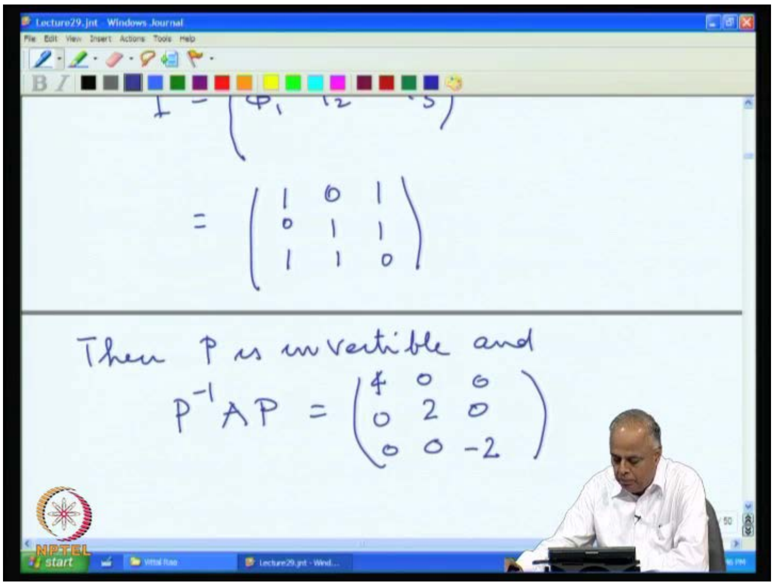

What does that mean? pi 1 is 1 0 1, pi 2 0 1 1 and then pi 3 is 1 1 0; if you take this matrix, then P is invertible, and P inverse A P will be the first column corresponds to the Eigen value 4, the second column corresponds to the Eigen value 2, the third column of P corresponds to the Eigen value minus 2. So, these Eigen values will come in the diagonal and all other entities will be 0 P inverses A P now with this matrix P 1, can easily calculate P inverse, and verify indeed the P inverse A P is this diagonal matrix.

So, if you have an n by n matrix where we can find n linearly independent Eigen vectors and n scalars lambda 1, lambda 2, lambda n and does n Eigen pairs, in which the Eigen vectors involved are all linearly independent. Then we can assert that the matrix A I is diagonalize.

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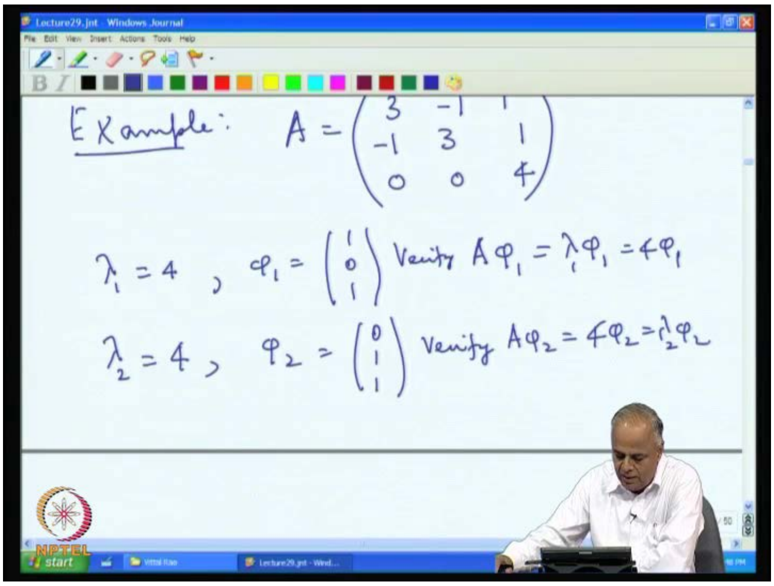

Let us look at another example, we note that, in the previous example in the Eigen pairs that regard the 3 numbers lambda 1, lambda 2, lambda 3, they were all distinct, but this is not necessary this numbers may be repeated. Let as look at another example; A equal to 3 minus 1 1 minus 1 3 1 0 0 4; now, if you take lambda 1 equal to 4, and pi 1 equal to 1 0 1, we can easily verify that A pi 1 is lambda 1 pi 1. Just multiply the matrix a with this vector you will get 4 times pi 1. Similarly, if you take lambda 2 also as 4, and pi 2 as 0 1 1, we can verify A pi 2 is again 4 pi 2, which is now lambda 2, pi 2.

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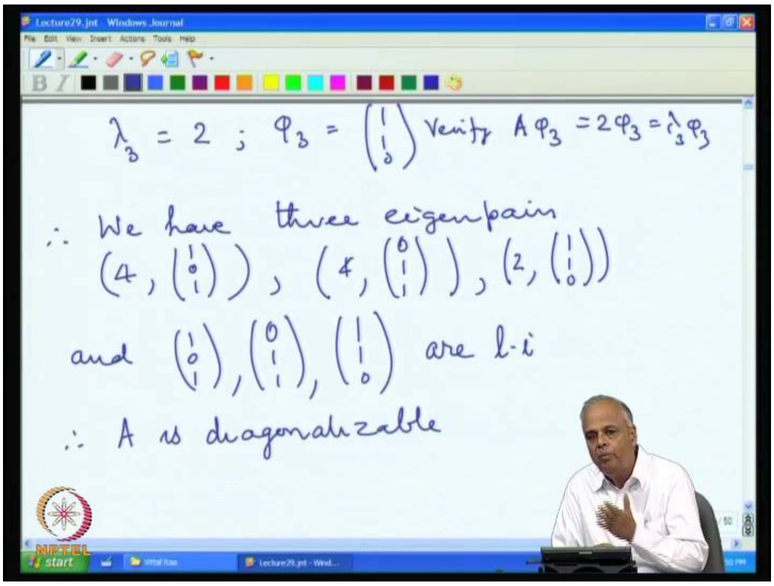

And finally, if you take lambda 3 equal to 2 and pi 3 is equal to 1 1 0, you can verify with that matrix A multiplying pi 3 gives as exactly 2 pi 3, which is lambda 3 pi 3; and therefore, we have 3 Eigen pairs now. The first one is lambda 1 pi 1 which is 4 and 1 0 1, the second one is lambda 2 pi 2 which is 4 0 1 1, and the third one was lambda 3 pi 3 1 1 0. Notice that the number 4 repeats in the first and the second pair, but the corresponding Eigen vectors, we are chosen are different, and 1 0 1 the Eigen vectors involved in this are linearly independent. Therefore, A is diagonalizable, A is diagonalizable, we have got for a 3 by 3 matrix A, we have formed 3 Eigen pairs lambda 1 pi 1, lambda 2 pi 2, lambda 3 pi 3 and pi 1, pi 2, pi 3 are linearly independent, and ends we have A is diagonalizable.

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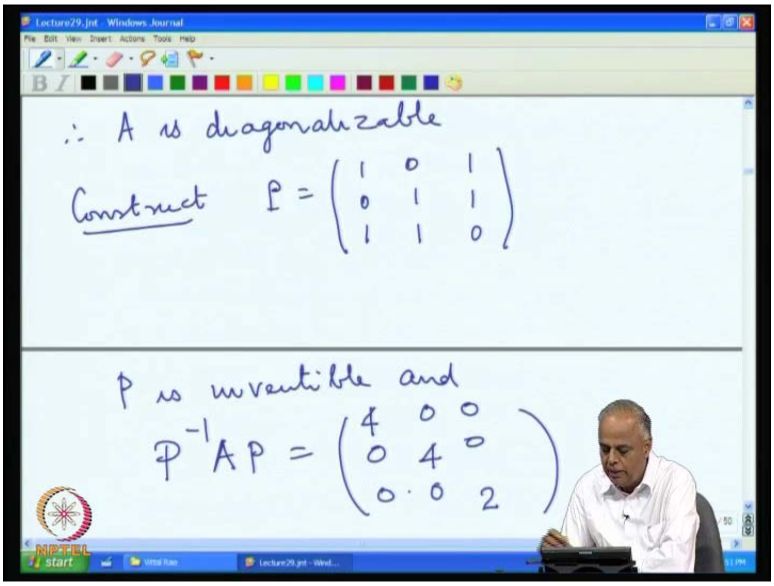

What is a diagonalizing matrix, again we construct P which is the matrix consisting these 3 vectors along the diagonal than as before P is invertible, it is can be easily verified and we can verify that P inverse A P must be a diagonal matrix. Now, what are the diagonal entries, the first column corresponds to the Eigen value 4, the second column also corresponds to the Eigen value 4, and the third column of P corresponds to the Eigen value 2.

So, the diagonal matrix, we get will be 4, 4, 2 along the diagonal and all the rest 0, so thus we see that whenever for an n by n matrix R n cross n matrix, we are able to get n Eigen pairs lambda 1 pi 1, lambda 2 pi 2, lambda n pi n, where lambda 1, lambda 2, lambda n are real numbers not necessarily distinct. As we found in the last example, and vectors pi 1, pi 2, pi n are linearly independent then we can diagonalize the matrix.

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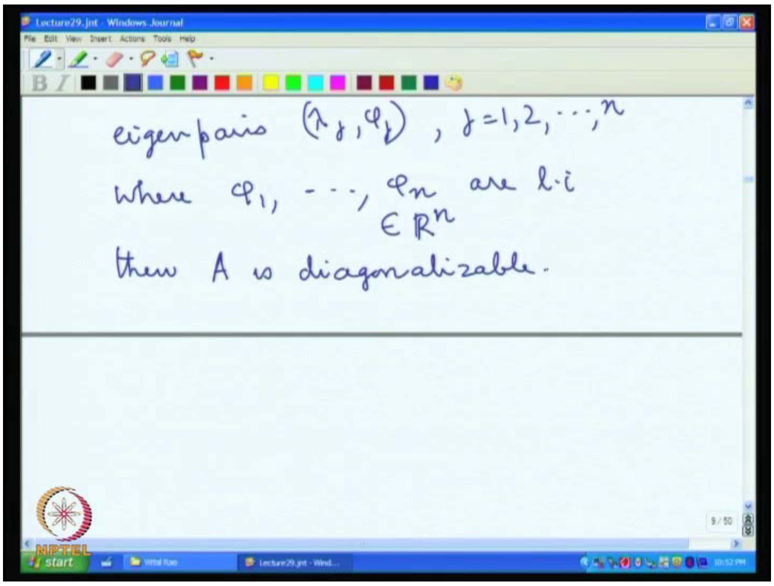

So, again we summarize this, this is the important thing to note is that if A belong to R n, n has n Eigen pairs lambda j, pi j; j running from 1 to n. So, if you tag n Eigen pairs where pi 1, pi 2 are linearly independent, then of course, these vectors pi 1, pi 2 are in R n, they are all real vectors then A is diagonalizable and how do we diagonalize it.

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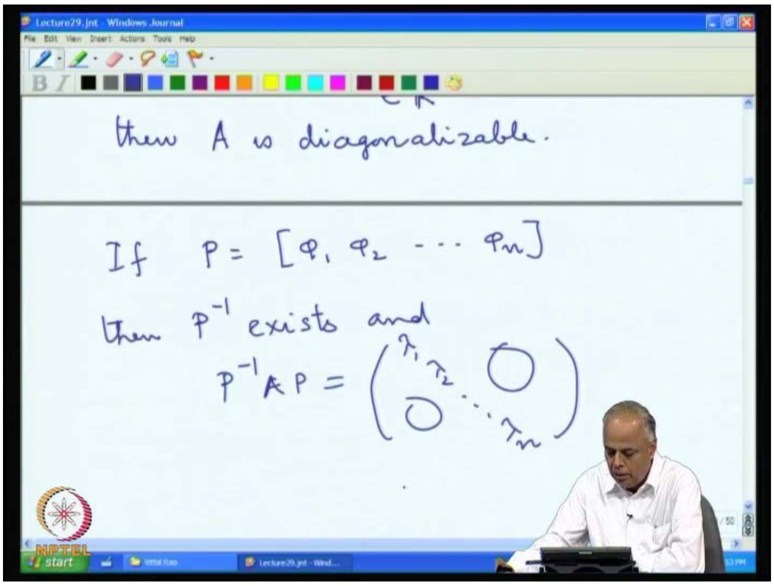

If you said the matrix P to be the matrix his columns are the pi 1, pi 2, pi n, then P inverse exist and P inverse A P will be the diagonal matrix, which will have this lambda

1, lambda 2, lambda n, along with diagonal entries; therefore, the moral of the story is in order you decide or you want to diagonalize A matrix.

Whether you want to decide whether A matrix is diagonalizable or not and when you have decided that, it is diagonalizable how do you diagonalize it, what is the matrix P that makes P inverse A P diagonal all these depend on finding this Eigen pairs, if there are n Eigen pairs we are d, 1 we know it is diagonalizable, we can construct the past x P with the columns as this end n. So, our search therefore, is for this Eigen values and Eigen vectors.

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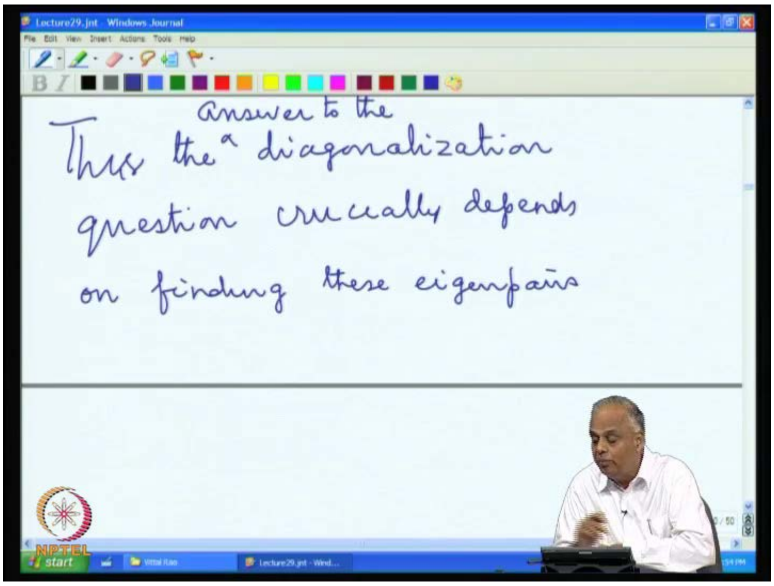

Thus the diagonalization question crucially depends, actually the answer to the diagonalization question crucially depends on finding these Eigen pairs, crucially depend on finding this Eigen pairs. not only we want to find the Eigen pairs, we want to found n of them, we want to find the n of the m, in such a way that the numbers lambda 1, lambda 2, lambda n are a real, and the vectors pi 1, pi 2, pi n are linearly independent.

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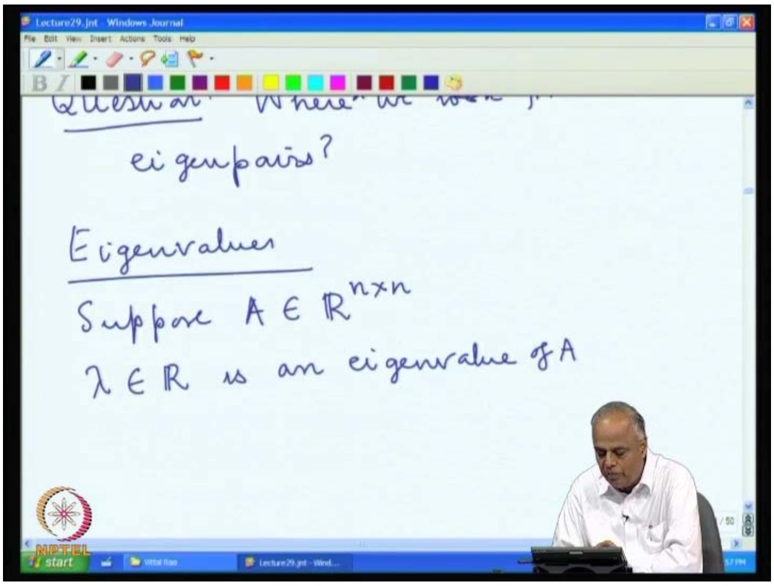

So, the question that arise is where do we look for these Eigen pairs, where do we look for these Eigen pairs, that should be our main work, in practice is going to occupy the main analysis of diagonalizability or otherwise of a given matrix, the search for this Eigen pairs.

So, we shall now begin our analysis to find the answer to this question of where do we look for these Eigen pairs, so looking for Eigen pairs involve several things do you want to look for those numbers lambda 1, lambda 2, lambda n, we want this numbers to be real numbers, and then we have to look for those vectors pi 1, pi 2, pi n, and we want them to be linearly independent and a pi, j to be equal to lambda j, pi j. So, these are the ingredients that we are looking for in the numbers, and the scalars that we are hunting for… So, in order to hunt for this numbers and scalars we must know something about them, we should know how they look like, so that we can go and grasp and see are you the 1, we are going to satisfy.

So, what we want to do is let us look at the Eigen values, that is this numbers, the numbers are called Eigen. The numbers appearing in the Eigen pairs are the Eigen values. So, we look at some analysis of the Eigen values to familiarize ourselves as to how an Eigen value look like or where we should go and look for remember, we are looking for the Eigen value as a real number. So, we have to look for this Eigen value in this world of real number, which is infinite. So, we are going to search for n needles in an infinite stock of needles so, it is very difficult to identify the follows.

So, we must have some way of identifying the particular needles, the particular number that we are going to look for therefore, we are going to analyze. So, suppose we have a matrix a which is n cross n real matrix, and lambda real number is an Eigen value of a suppose, so we catch holed of 1 of the known frames of a namely the Eigen values, and see start analyzing m and C how we looks like know, because is an Eigen value, every Eigen value has an Eigen vector when is an Eigen.

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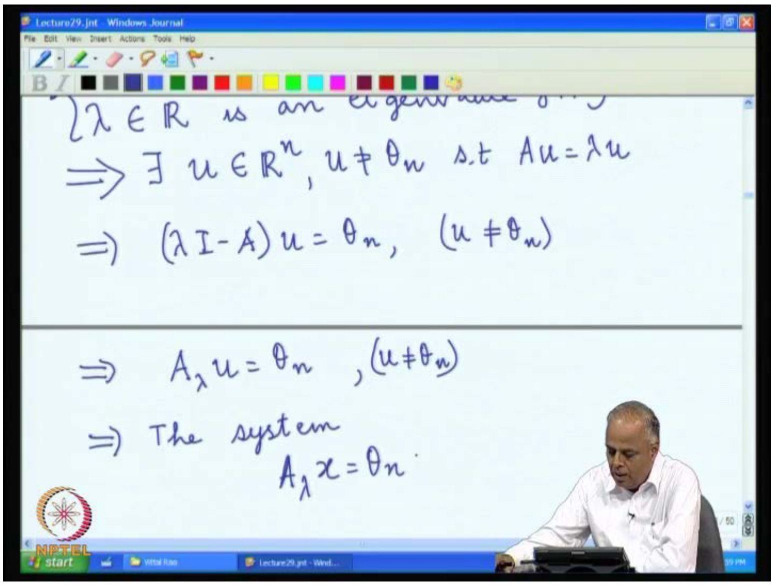

Therefore this means, so suppose A is an n by n real matrix and lambda real number is an Eigen value of A, then this implies there exist A vector u, which is not 0 vector such that, A u equal to lambda n, this is the requirement of Eigen value when Eigen value should always have an Eigen pair are the A vector u. So, that A u equal to lambda n, if that is the case this says lambda I minus A u equal to theta n and remember u is non 0. So, there is a vector u non 0, so that lambda I minus A u equal to theta n by what does that tell us. Now, let us call this matrix lambda I minus A for the time A, as A sub lambda so, there exist if lambda were an Eigen value of a then there may exist an vector u which is different from 0.

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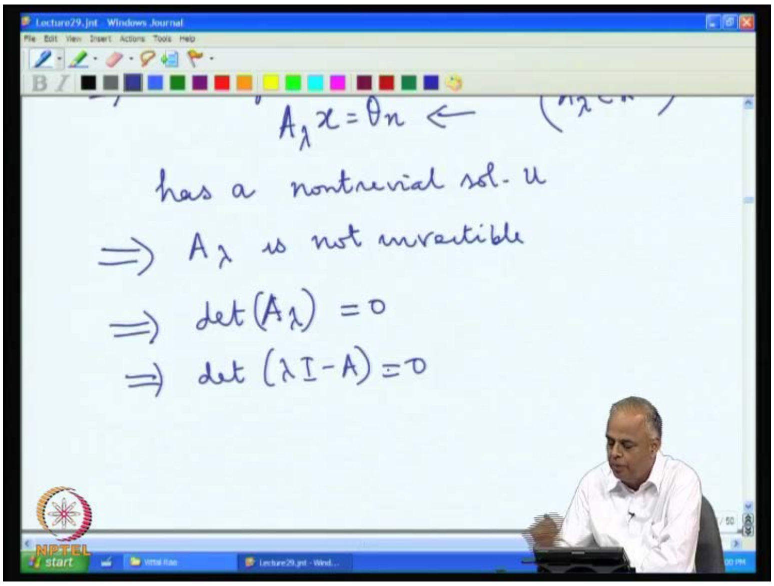

So, that a lambda u equal to theta n, this says the system A lambda x equal to theta n has a non trivial solution, has a non trivial solution u. Now, A lambda is since A is real, lambda is real, I is real A lambda is also a real n by n matrix. So, this is the system, this are homogenous system this homogenous system, whose matrix is a lambda. So, this homogenous system corresponding to the matrix A lambda, as a non trivial solution u; this immediately tells as that A lambda is not invertible, because if A lambda where invertible, this system will have x is equal to A lambda inverse theta which is 0, so x will be 0, but we have a nontrivial solution u.

Therefore, in order that the system has a nontrivial solution A lambda must be not invertible, if it were not invertible; that means, the determinant of A lambda must be equal to 0, because we know that a matrix is invertible, if the determinant is not 0, so not invertible; therefore, the determinant must be equal to 0. So, let us write it a lambda was determinant lambda was A lambda was define to be lambda I minus A.

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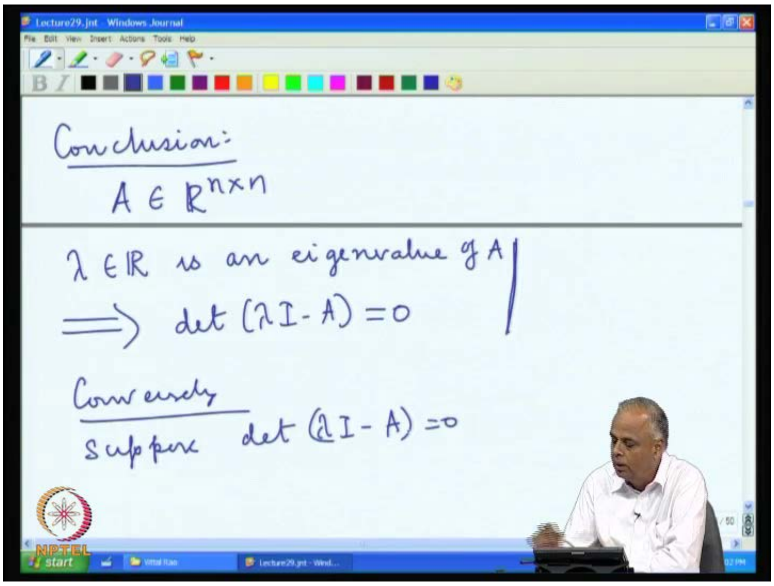

So, what is the conclusion that, we have got the conclusion is that if lambda belonging to R is an Eigen value of A, because A is A n by n matrix and real A is an Eigen value of A lambda is an Eigen value of A implies determinant of lambda I minus A must be equal to 0, this is our conclusion of the above discussion. We just go through this argument again, we said that suppose A is suppose A is A n by n matrix, then if lambda is an Eigen value there is an u such that0 u is not 0, and aim is lambda u, this implies A lambda u equal to theta n, where A lambda with the matrix lambda I minus A. Therefore, the system A lambda x equal to theta n is nontrivial solution u; therefore, the determinant A lambda is not invertible, therefore the determinant 0 and ends with it that, if lambda is an Eigen value of a then determinant of lambda I minus A equal to 0.

So, we know some property of an Eigen value of A, if something has to be an Eigen value of A minimum it should be such that the determinant of lambda I minus A is 0, conversely suppose determinant of lambda I minus A equal to 0 determinant of lambda I minus A equal to 0.

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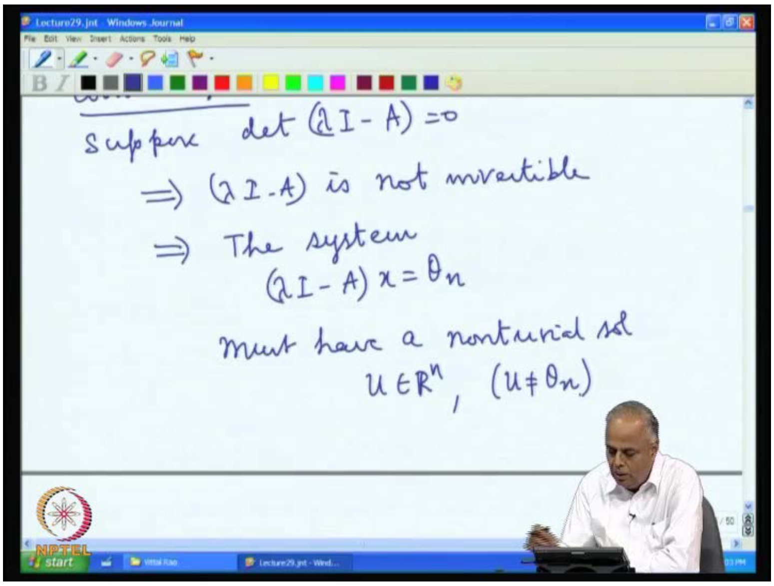

Implies now, the matrix lambda I minus A is not invertible, because the moment the determinant is 0 the matrix cannot be invertible, if this is not invertible that means, the system lambda I minus A x equal to theta n. The homogenous system must have a non trivial solution u belonging to R n, when you say non trivial I mean u naught equal to theta n.

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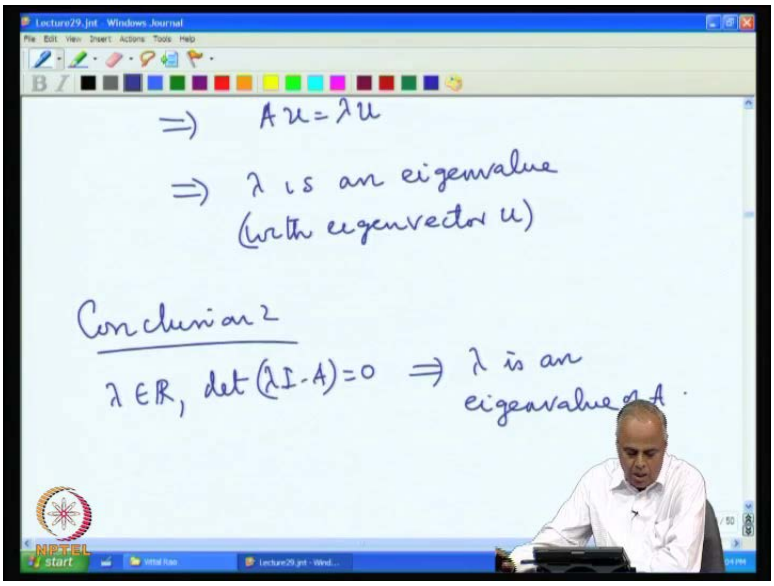

This means, A x equal to lambda A u is means, A u equal to lambda u.

That says, lambda is an Eigen value with Eigen vector u; therefore, what we have is that this is the conclusion 2, we have got is that determinant lambda belonging to R determinant of lambda I minus A equal to 0 implies lambda is an Eigen value. Now, count comparing conclusion 1 and conclusion 2, we see that each of the corners of the other.

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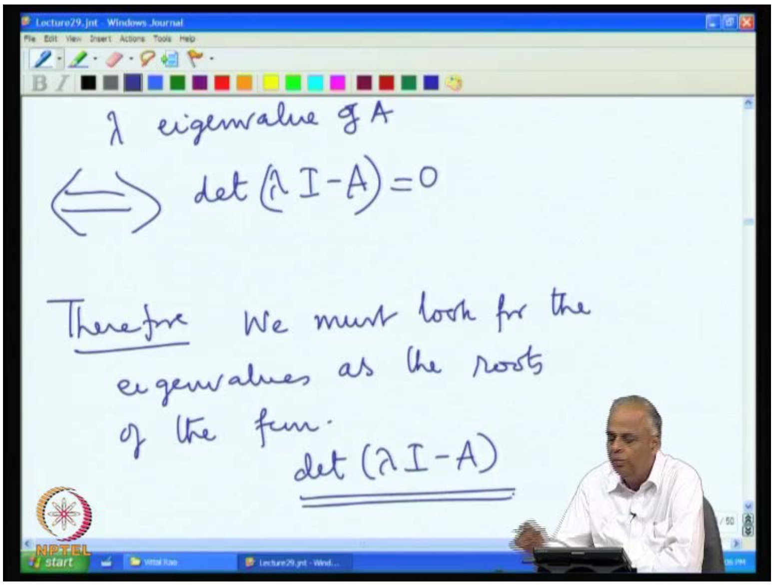

And therefore, combining these 2 we get that important result or theorem whatever, you are going to call it namely A belongs to R n cross n, A is A real n by n matrix lambda belongs to R, then lambda Eigen value of A if I do not, because you have seen both ways it is true determinant of lambda I minus a equal to 0. Now, we have got a characterization of the Eigen values of A, what is the characterization? It has to make the determinant of lambda I minus A equal to 0.

So, we do not have to search all over the place for this Eigen value in this infinite world of real numbers, we have to look for those follows who make the determinant 0, this particular determinant lambda determinant of lambda I minus A 0, Therefore, we must look for the Eigen values as the roots of the function determinant lambda I minus A, we have to find it as the roots of this function. Now, we know given the matrix A how do you go about searching for the Eigen value, first with construct this function which is now A function of will call it D lambda this is the function of lambda whichever point.

At which this function becomes 0 that is going to be an Eigen value, if 4 is a point so, the D 4 that is determinant of 4 I minus A is equal to 0, then automatically 4 must be an Eigen value. So, every root of this function D lambda must be equal to must be an Eigen value of A; therefore, the search now we know is going to depend on 0(s) or the roots of this function D lambda. Now, we have some control of over search for the Eigen values; and therefore, we must analyze this function D lambda in order to understand better about it 0(s), because the 0 S are going to be our Eigen values.

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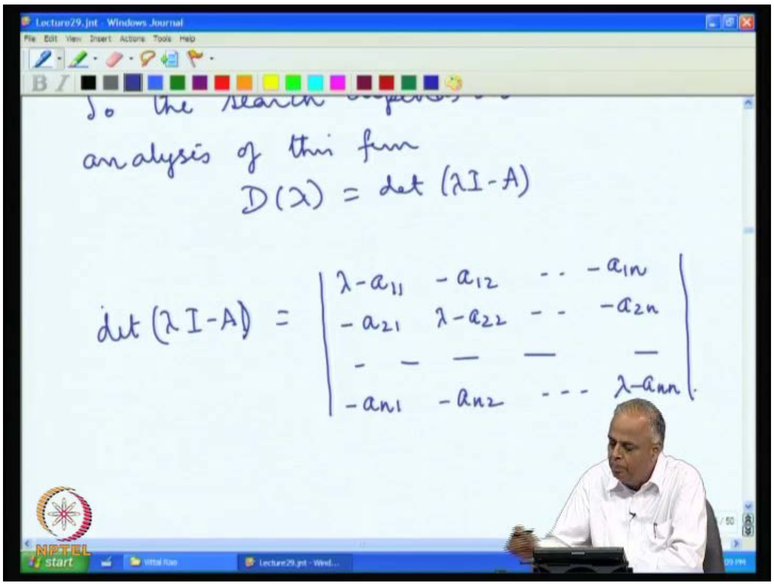

So, let as analyze, so the search depends when I say the search the search for Eigen values, the search for Eigen values depends on our analysis of this function D lambda equal to determinant of lambda I minus A. So, we now look at this function, so now determinant of lambda I minus A, it is like. So, if the lambda I is only the identity matrix with A lambda struck along the diagnose, it is a diagonal matrix with A lambda struck along the diagonals and from that we have to subtract the matrix A. So, we get the determinant as lambda minus A 1 1 minus A 1 2 minus A 1 n; it is simply the matrix minus A except that along with diagonals, we have to stick lambda, so it is minus A 2 1 lambda minus A 2 2 minus A 2 n and so on, the last will be minus A n 1 minus A n 2 and then lambda minus A n.

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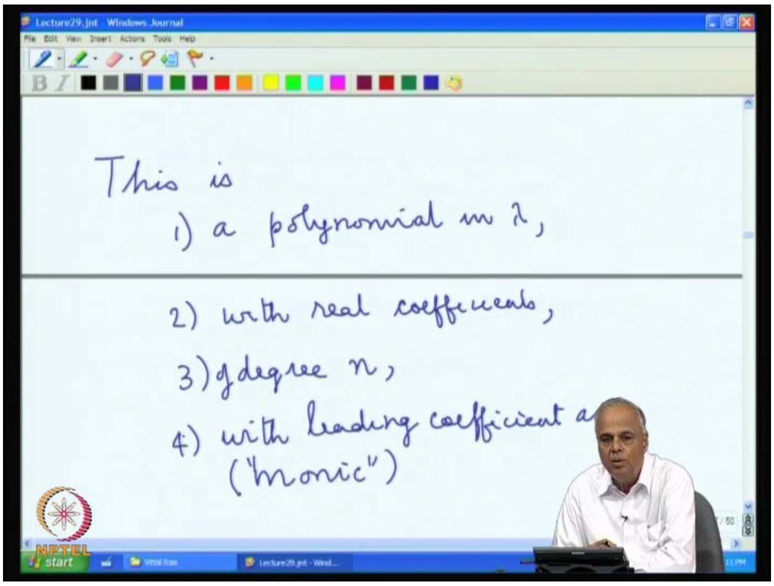

So, this is the n by n determinant, which is determinant of lambda I minus n, when we expand this determinant, do you see that the product of diagonal terms, is one of the terms in the expansion, this determinant we see that the product of the diagonal terms is one of the terms in the expansion we give lambda to the power of n; that is the highest power of lambda become yet, and thus is going to be first that is going to be A lambda to the power of n tern. And since, every entry is either a linear polynomial lambda or A constant and the determinant is simply. The product involve the product of the entries. The determinant is going to be simply product of some of this polynomials, and hence it is going to be a polynomial, and we have seen that the highest degree term is lambda to the power n.

So, this is one here polynomial in lambda 2 with a polynomial with real coefficients, because every entry is real there. So, all the coefficients are going to be real in the polynomial, and so the highest degree term of degree n highest degree term is going to be lambda to the n. So, degree n and the highest degree term lambda to the power of n has coefficient 1, because it involves the product of the lambda, lambda and lambda which is along the diagonal. So, with leading coefficient, when I say leading coefficient I mean the coefficient of the highest degree term the leading coefficient as 1.

So, we have the 4 important properties of the Eigen values of the determinant are lambda I minus A, it is a polynomial. It is a polynomial with real coefficients it is a polynomial with real coefficients of degree n, it is a polynomial of real coefficients with degree n and leading coefficients is 1. Whenever, the leading coefficient is one we call it a monic polynomial. So, thus this D lambda is a monic polynomial of degree n with real coefficients, this is called the characteristic polynomial of the matrix A.

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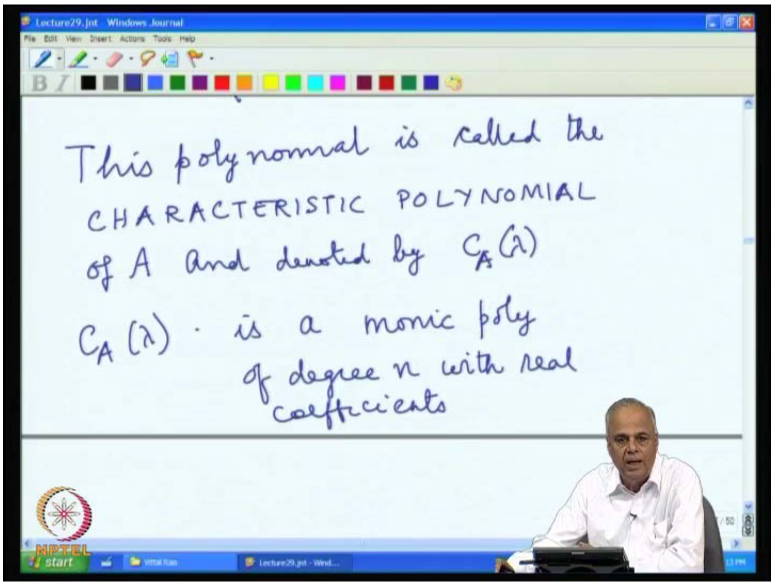

So, this polynomial is called the characteristic polynomial of the matrix A and denoted by… So, let us say C A lambda. So, the C A lambda is by over observations above is a monic polynomial of degree n with real matrix when A is a real matrix, we are considering real matrix when there A is real matrix the characteristic polynomial of A is a monic polynomial of degree n with real coefficients. So, as we are interested in this 0(s) or the roots of this polynomial, because we have seen, but the 0(s) or the roots of this function D lambda or the Eigen values. Now, D lambda is what now our CA lambda is it is a polynomial.

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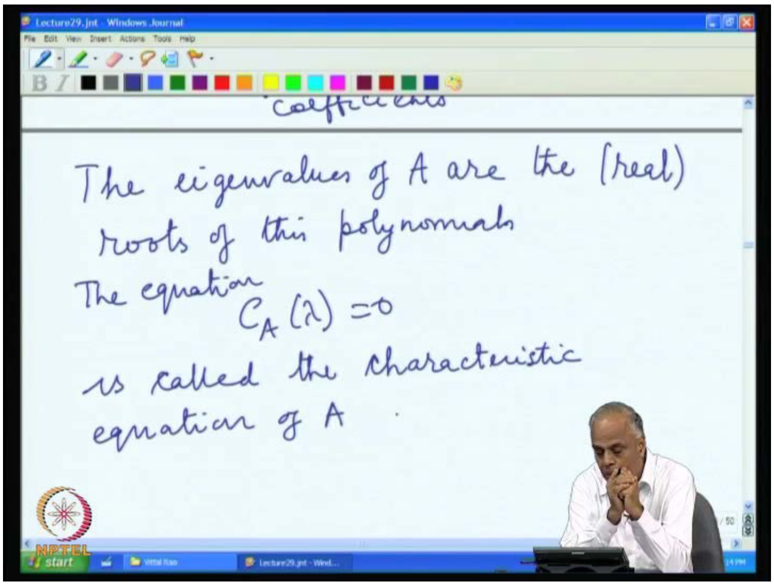

And therefore, the Eigen values of A are the roots of this polynomial. So, when we are looking for Eigen values of A when we are working with real number, we are looking for real Eigen values, and therefore we are looking for the real roots whenever, we are working with real numbers we have to work only for with real roots. So, when we have working of diagonalization problem with real numbers, then we have looking for real Eigen values; and therefore, we are looking at the real roots of this polynomial.

So, we are trying to solve this equation, and find the values solutions of this, so this equation is called the characteristic equation of A; the equation C A lambda is called the characteristic equation of A when we are interested in the roots or the solution of the characteristic equation, because these are the Eigen values of the matrix A. So, now we know exactly, since our search for the Eigen values is now come down to finding the roots of a polynomial.

We do not have to go around searching all over the real numbers, given matrix A we construct this determinant lambda I minus A, when we expand it we get a polynomial, and this polynomial C A lambda is polynomial of degree n, it has real coefficients monic. So, we have a polynomial of degree n, the moment we find the roots of this polynomial of search for the Eigen values is over, because they are lying there; every Eigen values is in the roots and every root is an Eigen value. So, our search for the Eigen value therefore, simply boils down to finding the real roots of the polynomial C A lambda.

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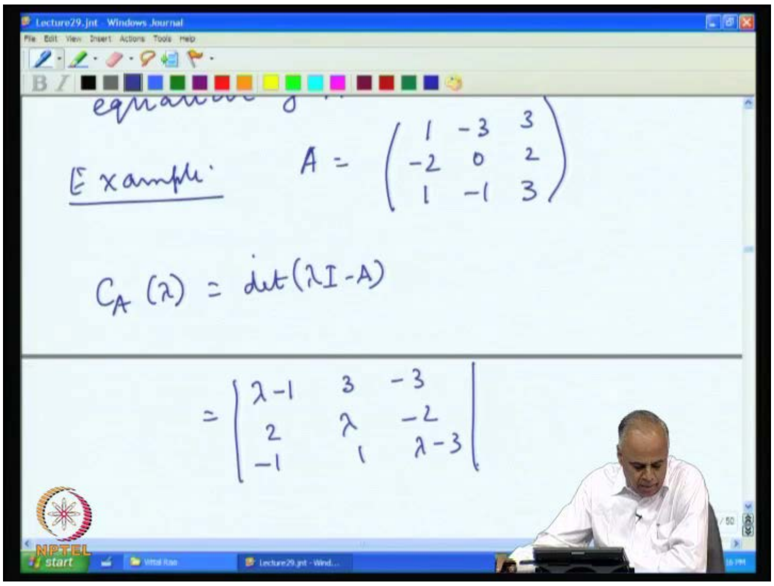

Now, if you look at a simple example one of your earlier examples, if you take the matrix A 1 minus 3 3 minus 2 0 2 1 minus 1 3, then what is C A lambda, it is the determinant of lambda I minus A. So, let what is the determinant it is lambda minus 1 3 we have to negate A, and stick lambda along the diagonal negate A and stick lambda along this determinant. And then expand this determinant, this determinant terms have to be lambda minus 4 into lambda minus 2 into lambda plus 2.

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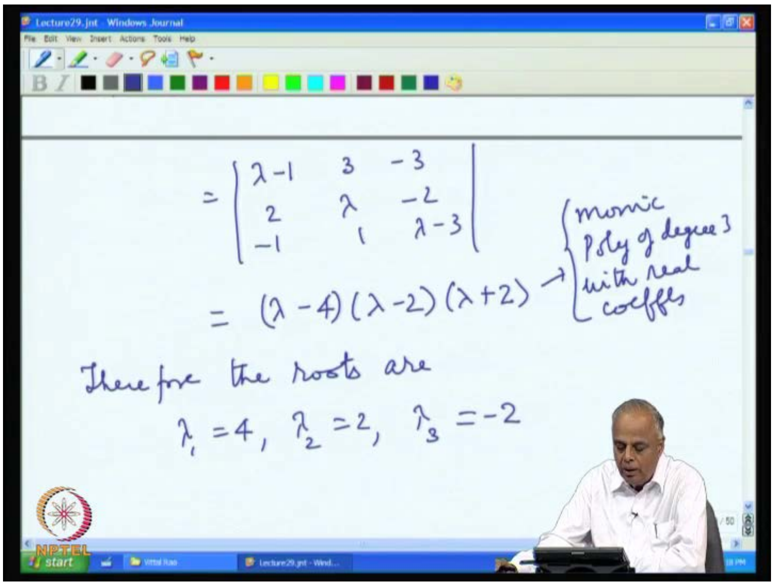

Therefore, the roots are lambda 1 equal to 4, lambda 2 equal 2, and lambda 3 equal to minus 2. You may recall the earlier example, we say when we looked at examples of Eigen pairs - the first example these were the 3 numbers, that appeared in the 3 Eigen pairs. The first Eigen pair at 4, the second Eigen pair at 2, and a last Eigen pair at minus 2.

So, this Eigen values will obviously occur in this Eigen pair combination, so here is A matrix A, when we write the characteristic polynomial, we get remember when you expand this you will get lambda cube, it is a first term. So, it is a polynomial of degree 3 and since, the highest term is the lambda cube; it is a monic polynomial of degree 3, when everything involved is real number.

So, it real coefficient so, the characteristic polynomial is a monic polynomial of degree 3 with real coefficient the degree 3, because we are dealing with A 3 by 3 matrix, we deal with an n by n matrix, our degree will be n; therefore, the roots of this characteristic polynomial namely 4, 2 and minus 2 are the Eigen values of this matrix.

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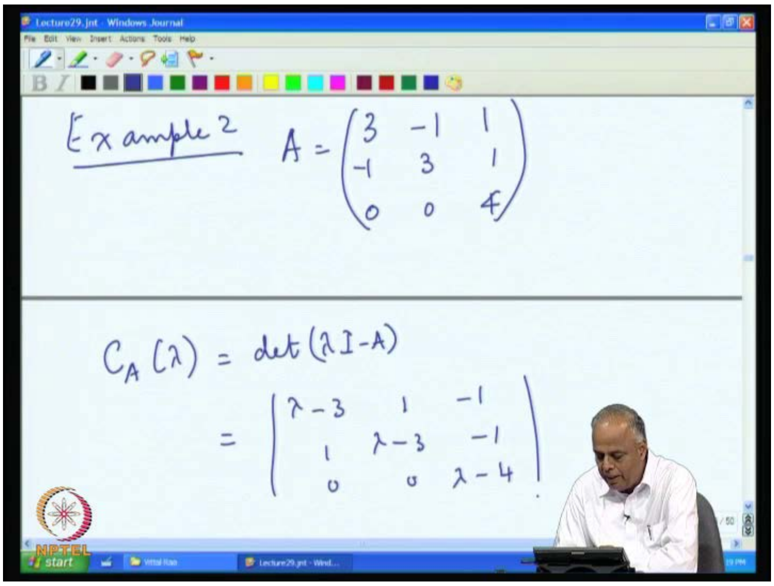

Let us look at another example, this again the same matrix which we consider earlier to give example for Eigen pairs. Suppose, we consider this matrix. What is the characteristic polynomial C A lambda, that is determinant of lambda I minus A, where if I write lambda I minus A, I have to negate A, and then stick A lambda along the diagonal; that is the determinant write A negative of A and stick at lambda along with that.

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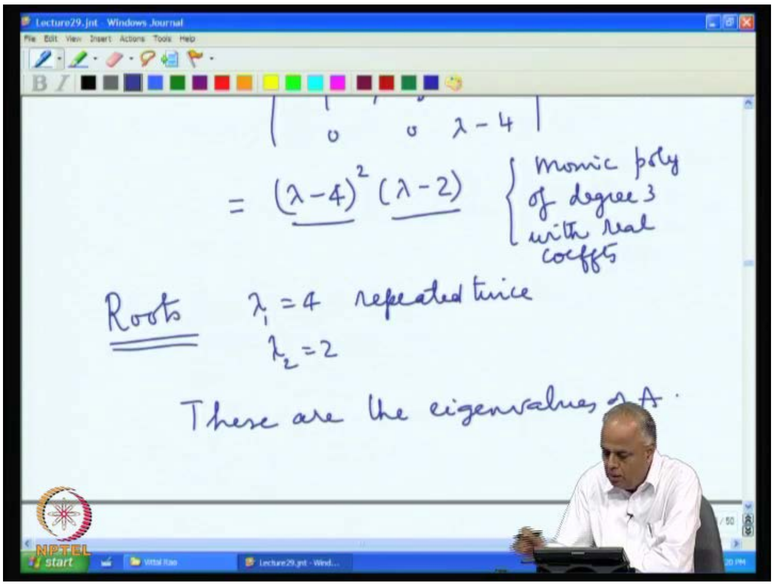

If you expand this determinant this turns out to be lambda minus 4 whole squares into lambda minus 2. So, this is again is a monic polynomial of degree 3, the degree is 3, because A 3 by 3 matrix and when you expand this there will be lambda square term, here in this first and lambda term. In the second the product will be lambda cube so, this monic the cube there must coefficient 1. So, it is a monic polynomial of degree 3 and since, everything involve coefficients are real is with real coefficients. So, 3 by 3 matrix monic polynomial degree 3 with real coefficient, what are the roots this case there is there are only 2 roots actually, one of them is repeated.

So, lambda 1 equal to 4 repeated twice and lambda 2 equal to 2 so, really speaking there are only 2 Eigen values 4 and 2, but 1 of them is repeated twice. So, these are the Eigen values, these are the Eigen values of A; therefore, when you remember if we are dealing with real matrix and we are dealing with real diagonalizability, then we are looking for real Eigen values.

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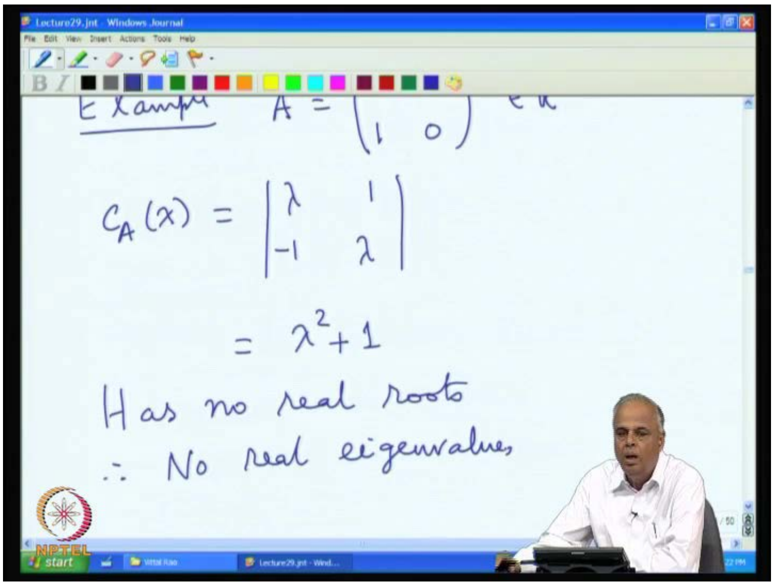

Let us look at another simple example, let us consider matrix A which is 0 minus 1, 1, 0, this again A real matrix 2 by 2. So, it is characteristic polynomial now, must be a monic polynomial of degree two, because set 2 by 2 matrix. So, let us have the characteristic polynomial which is negate the matrix, and then you have to stick A lambda along the diagonal.

So, it is going to be lambda along the diagonal and when you expand this, you get lambda squared plus 1. Now, what are the Eigen values, if you are working in R we are looking for real roots of C A lambda, but lambda squared plus 1 equal to 0 has no real root. So, has no real roots; therefore, no real Eigen values, the moment you do not have any Eigen values; therefore no Eigen pairs, the moment you do not have Eigen pair there is no diagonalizability over R, because we are looking at R.

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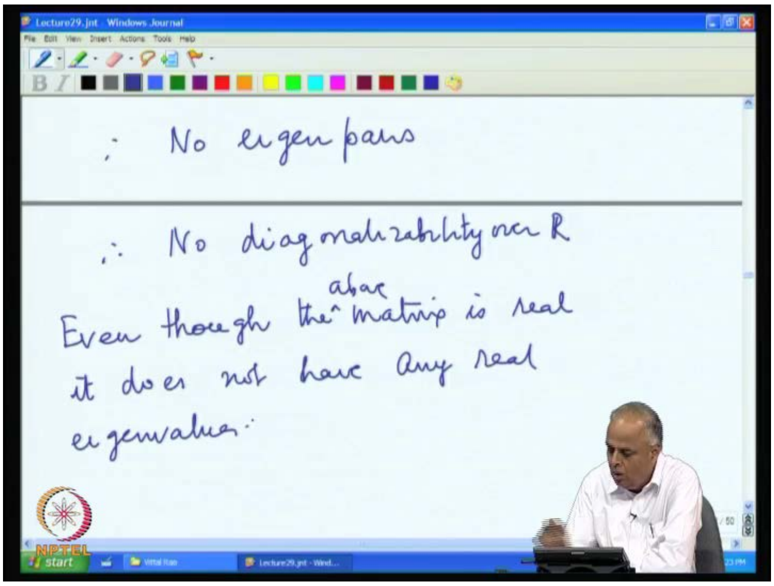

So, this example is a sticking example in the sense that before dealing with real matrices, and if you want to do real diagonalizability it is not always possible, because anyone matrix which is real, it is simple 2 by 2, and it has no Eigen values; and therefore there is no possibility of real diagonalizable. Why did this happen?

Therefore, even though the matrix the above matrix is real, it does not have any real Eigen values. So, what is the problem? Why did this happen? This happens, because we are looking for the Eigen values or the roots of the characteristic polynomial. The characteristic polynomial is the polynomial of degree n with real coefficient, but in general A polynomial of degree n or with real coefficients may not have real roots, and if it has real roots all of them may not be real, some of them may be real, some of them may be complex, and there is who appear as complex.

You place appear complex conjugate pairs therefore, solving polynomial equations with real coefficients over the real's, we may not have any roots or we may not have enough number of roots that is one of the algebraic deficiencies of the real numbers. In fact it is in order to eliminate this algebraic deficiency of real numbers the complex numbers have been created.

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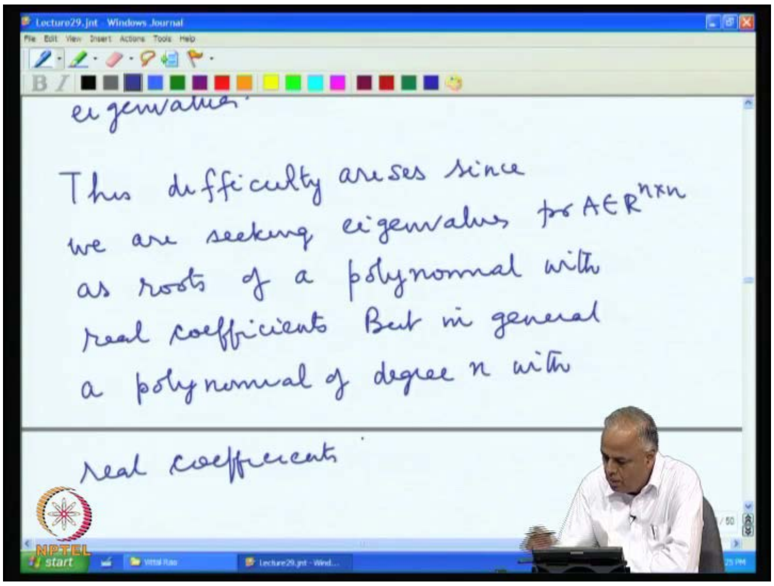

So, let us start note that this problem arises this difficulty arises, since we are seeking Eigen values for A belonging to R n cross n, as roots of A real poly or say A polynomial with real coefficients A polynomial with real coefficients, but in general A polynomial of degree n with real coefficients.

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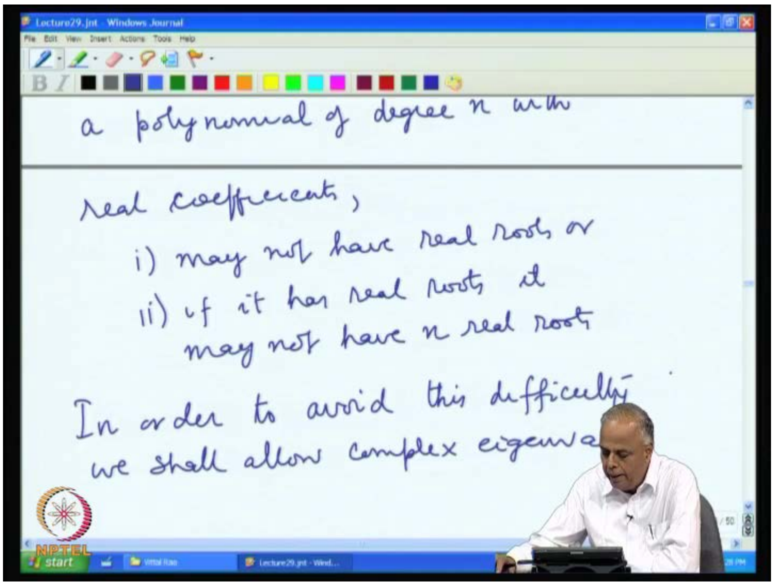

One may not have real roots or if it has real roots, it may not have n real roots therefore, there already a big stumbling block in our search for Eigen values. If you have working I the real m of real numbers this is, because of the algebraic deficiency of the real numbers, what is said as the real numbers are not algebraically closed, that is polynomials with real coefficients cannot be factorize completely over the real numbers that is, they may not have real roots.

Hence, there are re reducible polynomials with real coefficients of degree more than 1; therefore, we are struck here, now as a first stop instead of looking at the problem as to what is the alternatives that available. We will overcome this difficulty by saying that, we will allow complex roots also, because in the above examples suggested, that is going to be problem. If we had allowed complex Eigen values also which means, we allow the Eigen pairs to be complex pair, where the number lambda can be complex.

In the vector pi can be complex even though the matrix is real, then for example lambda square plus 1 will have R roots plus or minus I. Therefore, will have to Eigen values hence, as a first corrective step, we will say even in the matrix is real. We will allow complex roots of the characteristic polynomial, and we will allow complex Eigen vectors which simply means that we now expand our realm, and say it with all matrices as complex matrix a real matrix can also be treated as a complex matrix. And therefore, we will think of everything as over.

C n m, and if you are lucky that we get all the roots was real, then we can work within the real m of the real numbers itself. Therefore, in order to avoid this difficulty. In order to avoid this difficulty, we shall allow complex Eigen value, and hence complex Eigen vectors if necessary.

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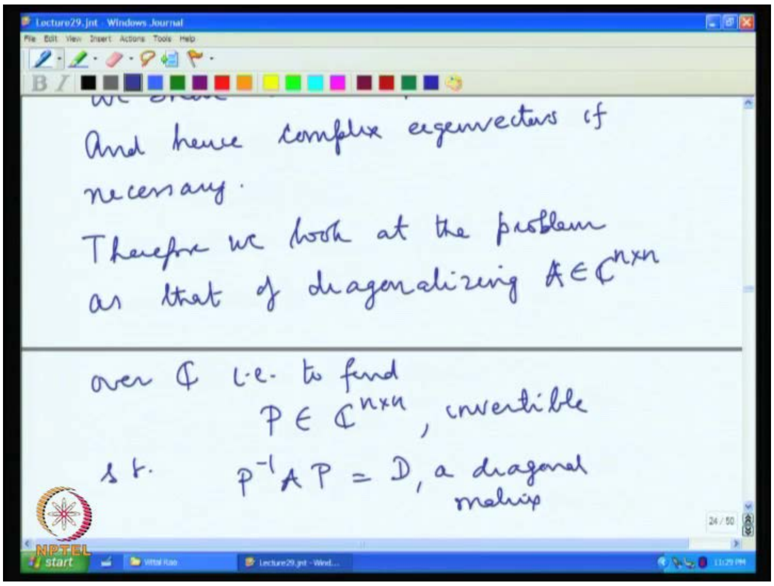

Therefore, we look at the problem as that of diagonalizing A, A complex matrix over the complex number; that is to find P which can be in C n cross n invertible; such that, P inverse A B P is D, a diagonal matrix in C n n. So, therefore we expand everything to the domain of complex numbers, why do we do that.

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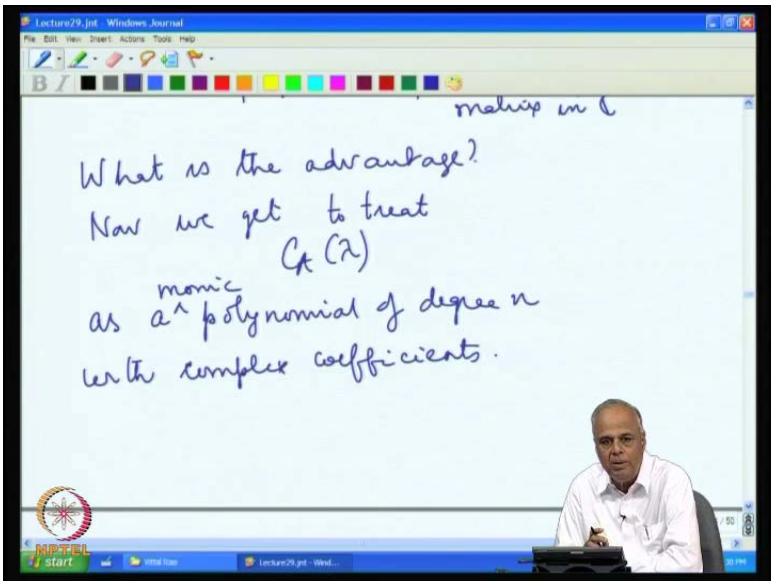

What is the advantage? Now, we get C A the characteristic polynomial as get to treat, we can treat this as a polynomial monic of degree n with complex coefficients, the monic polynomial of degree n with complex coefficients, and the fundamental theorem of algebra says that if you have a complex polynomial with degree n, it will have n roots may be some of the roots are repeated, but if you count the repetition the total number of roots will be n, which mean the complex polynomials can be completely factored into linear polynomials by fundamental theorem of algebra C A lambda will have n roots.

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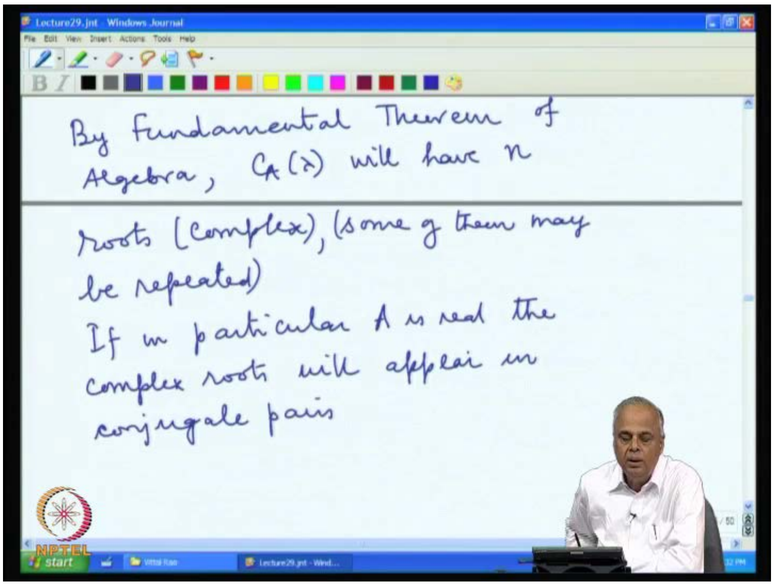

The roots may be complex some of them may be repeated, if in particular A is real is A real matrix, then the polynomial will be real polynomial. And therefore, the complex roots must appear in conjugate the complex roots will appear in conjugate pairs. So, what this says is that as far as, the search for the Eigen values is concern; there is no problem in finding them enough number of them. In fact n of them is the roots of the characteristic polynomial provided, we allow complex roots.

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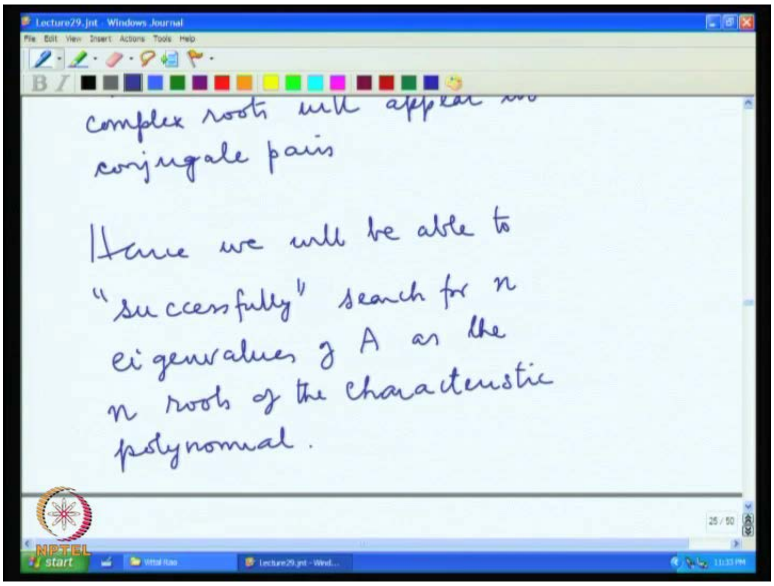

Hence, we will be able to successfully, we will see what is mean by successfully said for n Eigen values of A as the n roots of the characteristic polynomial, and I assert and I repeat that this, n roots will be complex also. If A is complex, the n roots can be complex and if A is real still, the n roots can be complex, but whenever A complex roots appears as A root it is conjugate pairs must also appear as a conjugate.

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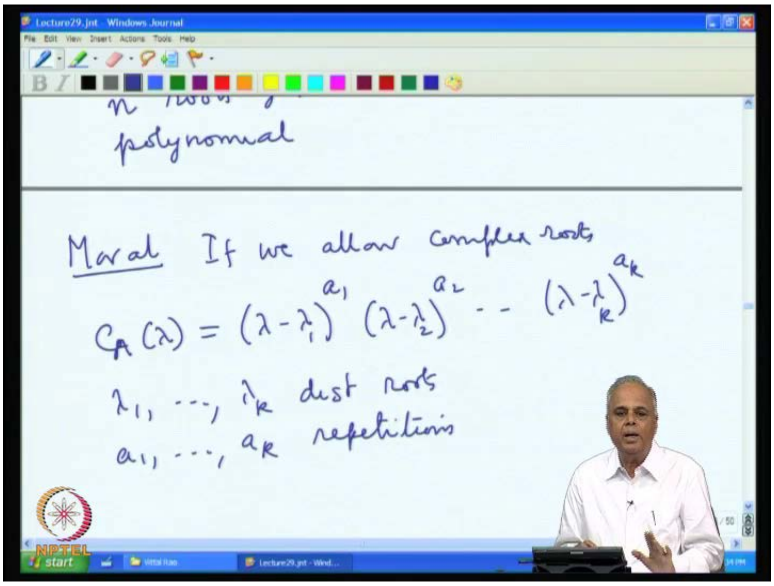

So, what is the moral? The moral is, if we allow complex roots C A lambda can be factored, say lambda 1 is A root may be it repeats A 1 times, lambda 2 is root, may be it repeats A 2 times, lambda k is A root may be it repeats A k times; lambda 1, lambda 2, lambda k distinct roots a 1, a 2, a k the reputations. Thus, our search for Eigen values, we have a clear idea now as to where we should look for them, given the matrix A construct the characteristic polynomial look for it is roots. Now, knowing about Eigen values we shall now in the next lecture look at how we go and search for the Eigen vectors.