Advanced Matrix Theory and Linear Algebra for Engineers
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Lecture No. # 28
Diagonalization- part 1

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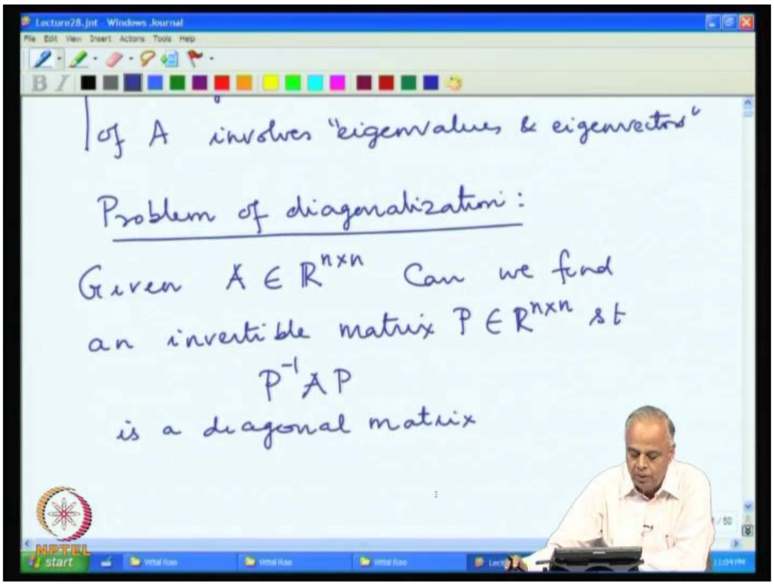

In the last lecture, we observed that finding for bases. The bases for the four subspaces of A involves eigenvalues and eigenvectors. We said that, this is going to involve, the notion of the eigenvalues and eigenvectors. If you want to find suitable bases which will fall our problems. Then, we found that the problem of diagonalization. We said that, this is also going to involve eigenvalues and eigenvectors; this is the main problem, but introduces the notion of the eigenvalues and eigenvectors. And so, we started looking at this problem of the diagonalization.

What is the problem? The problem is given a matrix A, which is real n by n, can we find an invertible matrix P such that, P inverse A P is a diagonal matrix. We found that this problem, such a question arises from the type of change of variables, that you want introduce in a system of equations in order to reduce it a diagonal system or an easy system. So, the question is can we find matrix P, which converts A to a diagonal matrix by this transformation P inverse A P.

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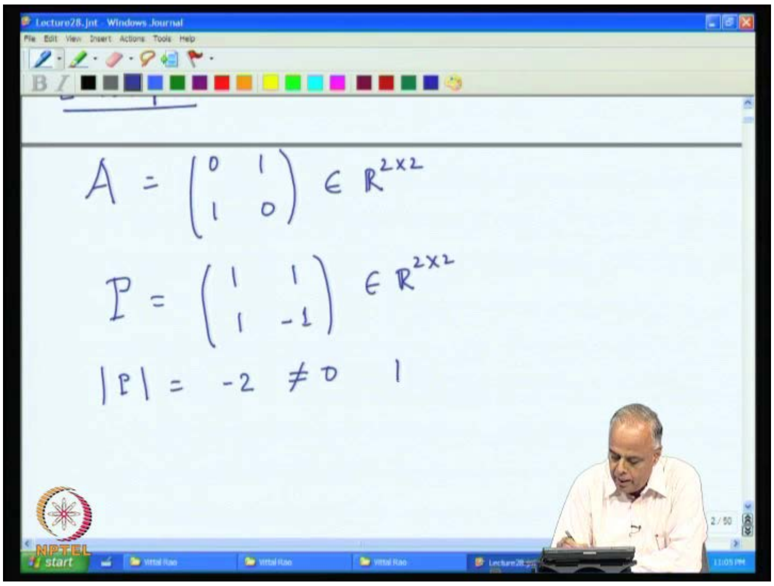

Now let us look at some examples; first example, let us state the 2 by 2 matrix 0 1 1 0, 2 by 2 matrix, so is real 2 by 2 matrix. Now, consider this matrix P, we shall see later how we found this P 1 1 1 minus 1. This is clearly in R 2 2, so this is again a real matrix 2 by 2 and the determinant of p is minus 2 which is not 0 and a learned to earlier classes, that the matrix P is invertible to the determinant is not 0.

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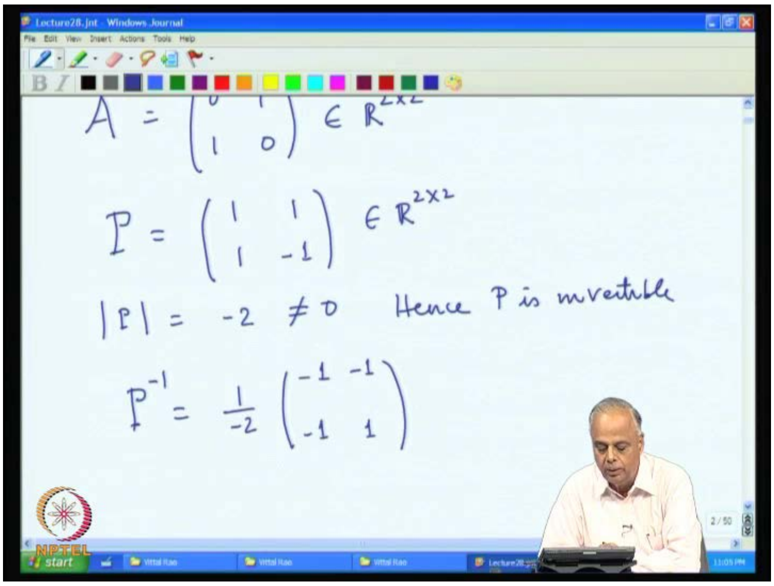

Hence, P is invertible and for 2 by 2 matrix is the inverse is easy to find, the inverse is 1 over the determinant, the interchange the diagonals and then, change the sign of the half diagonals. So, the inverse of the 2 by 2 matrixes is one by the determinant, the diagonals interchange the half diagonals signs changed.

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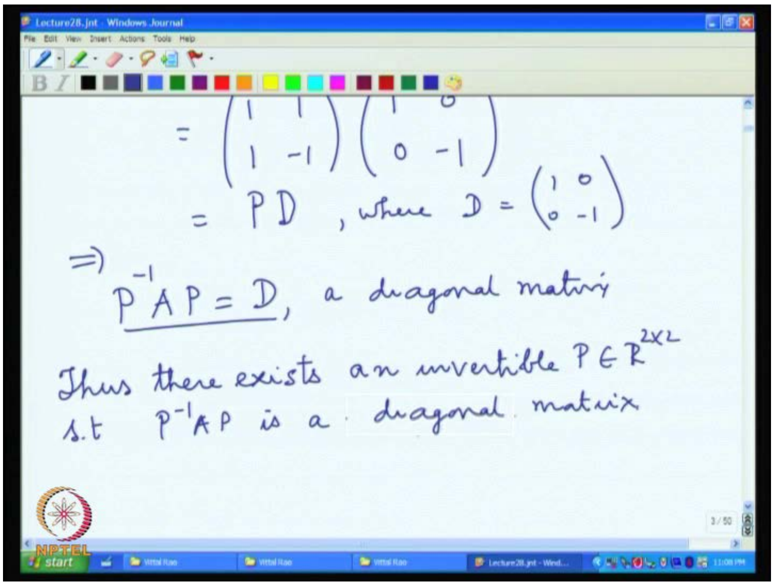

So, this is the matrix P inverse and we see that, let us calculate AP; A is the matrix 0 1 1 0 and P is the matrix 1 1 1 minus 1. When we take the product, we get 1 minus 1 1 1, which we can write as 1 1 1 minus 1 into the diagonal matrix 1 0 0 minus 1. Therefore, this matrix 1 minus 1 1 1 can be return as the product of these 2 matrices is easy to check, when you take the product of these 2 matrices is we get this matrix. But 1 1 1 minus 1 must P and will call this diagonal matrix as D, where D is the diagonal matrix 0 1 minus 1. And since P was invertible, we get P inverse AP taking P to the other side D, a diagonal matrix. Hence, we have an example here, of a 2 by 2 real matrix A for which we are able to find P, which is invertible and P inverse AP is diagonal.

So thus there exists, this is the question we rise, weather we can find such a P; in this example, we are found such P there exist and invertible P, which is in R 2 2, 2 by 2 matrix such that, P inverse AP is the diagonal matrix. So, we get the answer in the affirmative for this matrix. Our question was, can be find a piece of P inverse AP diagonal matrix.

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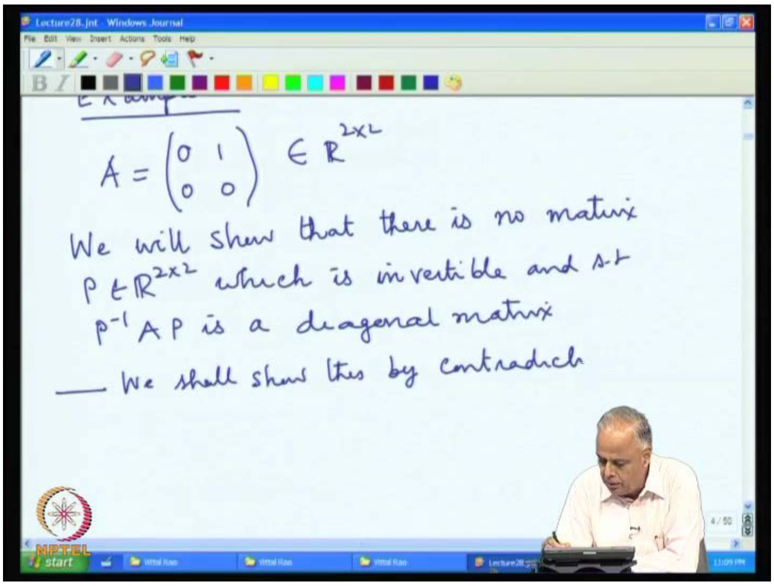

In this example, for this matrix, we are got the answer as s. Now, let us look at another example, example 2; let consider A to be the matrix 0 1 0 0. Now, we will show that there is no such, we will show, this is the matrix real; we will show, that there is no matrix P in R 2 2, which is invertible and such that P inverse AP is a diagonal matrix. And We will show this by contradiction we shall show this by contradiction. What you mean by that is we assume that their exist such a P; I am sure that leads to the contradiction.

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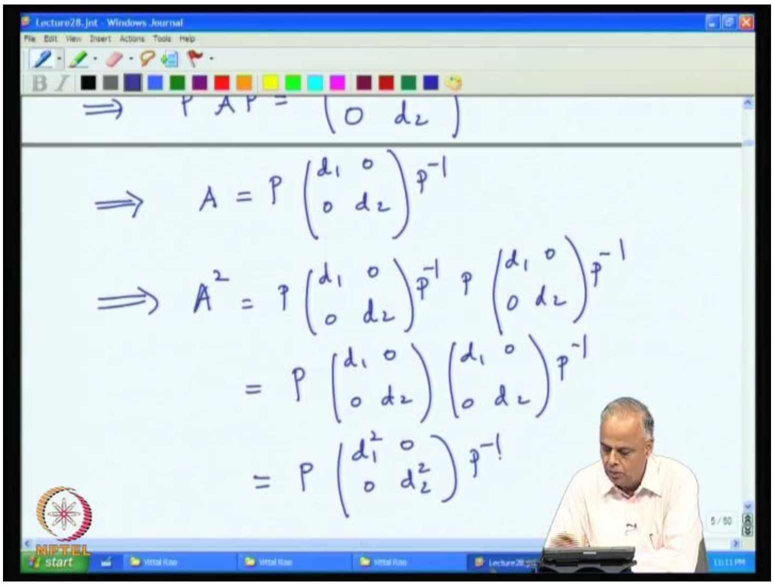

Suppose, there exist an invertible P in R 2 2 such that, P inverse AP is diagonal. Suppose it is possible, we shall show, but least to a contradiction. That is the case, we have P inverse AP is equal to a diagonal matrix, let us call it us d 1 d 2 0 0. Suppose, P inverse AP is a diagonal matrix that says, A is p times the diagonal matrix, times p inverse by pre multiplying by p and post multiplying by p inverse both sides; we get A equal to this matrix. This is show, we get A squared is equal to p into d 1 0 0 d 2 p inverse, that is A has to be multiply by A again, A into A is A squared, which is this. But now, p inverse p is the identity so, this becomes p into d 1 0 0 d 2 into d1 0 0 d 2 into p inverse. This is, because the middle P inverse, P is the identity matrix. But now, is easy to multiply the diagonal matrix is in the middle, to get d 1 squared 0 0 d 2 squared P inverse.

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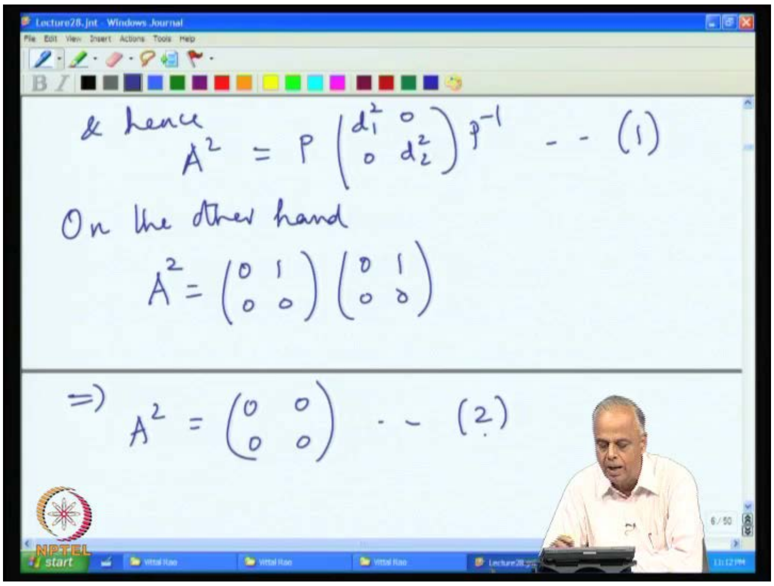

So, therefore we get A squared and hence, A squared is this. So, let us call this 1.On the other hand, we are given the matrix A, let us find what A squared is? A squared is A into A; this word the matrix A, so we are tried to find the A squared. A squared is A into A. When you multiply this 2 matrix is, we get the zero matrix. Therefore, now we compare 1 and 2, we see that the right hand side here must be equal to the 0 matrix.

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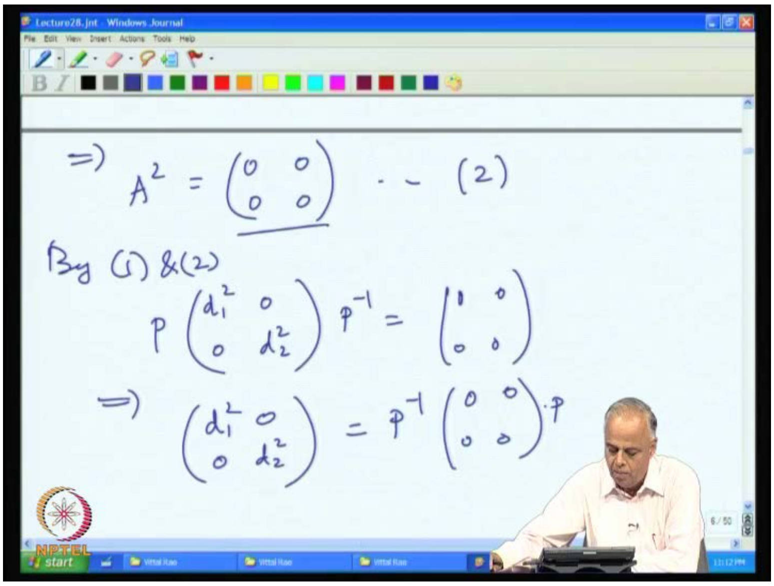

So, we get P into d 1 squared 0 0 1 d 2 squared into P inverse is the 0 matrix. This is by 1 and 2. Now, we again pre multiply by P inverse and post multiply by P. We get d 1 squared 0 0 d 2 squared is equal to p inverse into 0 0 0 0 into p. But 0 multiply any matrix is 0.

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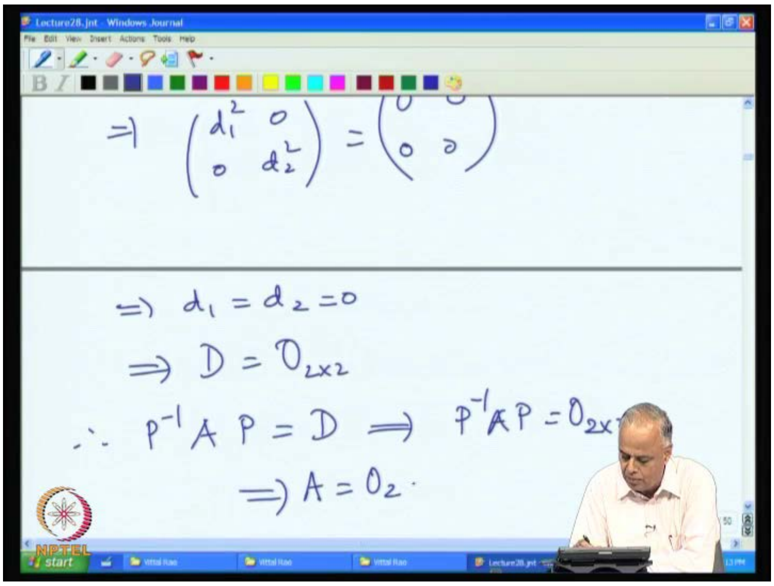

So, we get d 1 squared 0 0 d 2 squared, is 0 0 0 and therefore, it implies d 1 is 0 and d 2 is 0. Which means the matrix D, we got here, we got this matrix D, let us suppose call this matrix as D must be the 0 matrix.

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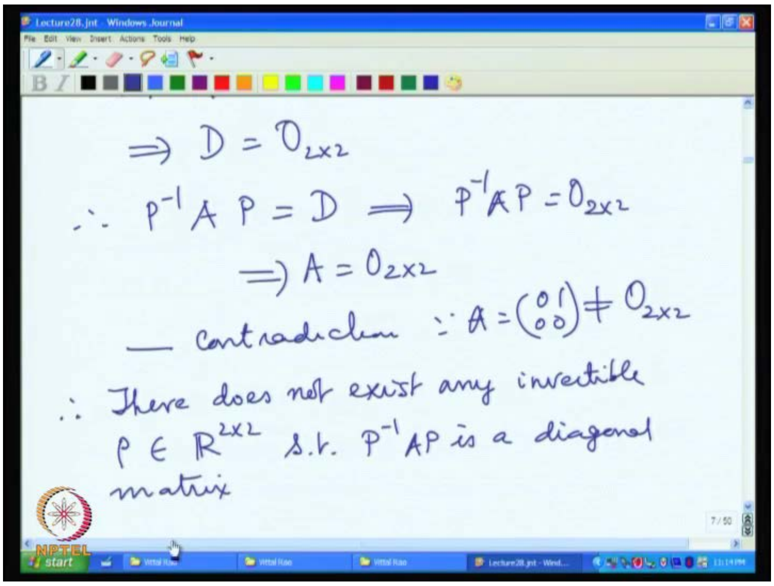

Hence, we get D equal to 0 matrix and therefore, P inverse AP equal to D implies P inverse AP in the 0 matrix. And, one second taking to the P to the other side, we get A is the 0 by 0 matrix. But that, the contradiction because we are given that A is the matrix 0 1 1 0 1 0 0 contradiction, because A equal to 0 1 0 0, which is not the 0 matrix. And therefore, our assumption that supposes there exist such a P, let us we started with the assumption, that suppose there exist and invertible piece of that P inverse AP is diagonal, let us to a contradiction, and hence that assumption must be round.

Therefore, there does not exist any invertible p in 2 by 2 real matrix is such that, P inverse AP is a diagonal matrix. So thus, we are two examples; in the example one, we are able to find A matrix piece of the P inverse AP is diagonal. In example two, we to should that cannot be any such matrix. And therefore, the possibilities are more. There are R cases where, we can find the P that the P inverse AP is the diagonal; there are cases were P cannot be found. Therefore, it become necessarily for us to classify those matrix is for, which P inverse AP is diagonal and those matrix is for, which we cannot find P inverse AP is diagonal.

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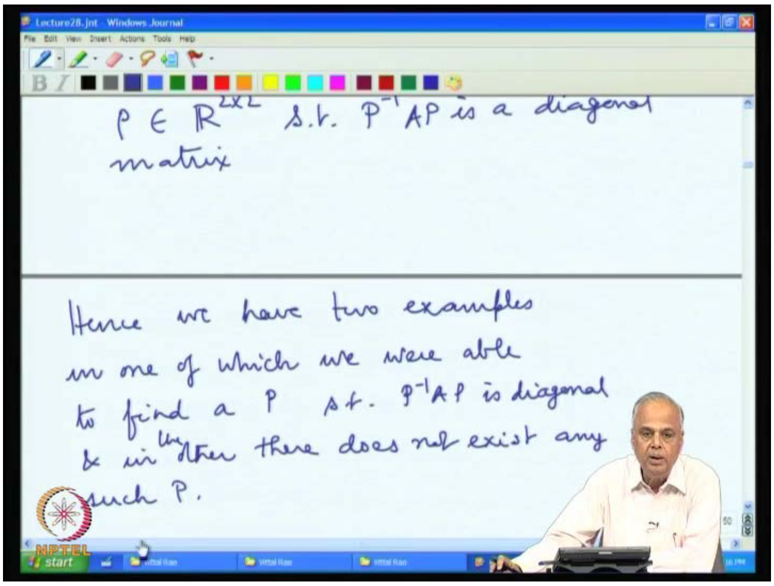

Hence we have two examples, in one of which, we were able to find a P, such that P inverse AP is the diagonal and in the other there is does not exist any such P. Therefore, become necessary for us, to bring it some test, which would decide weather, we will able to find P or we will not be able to find the P.

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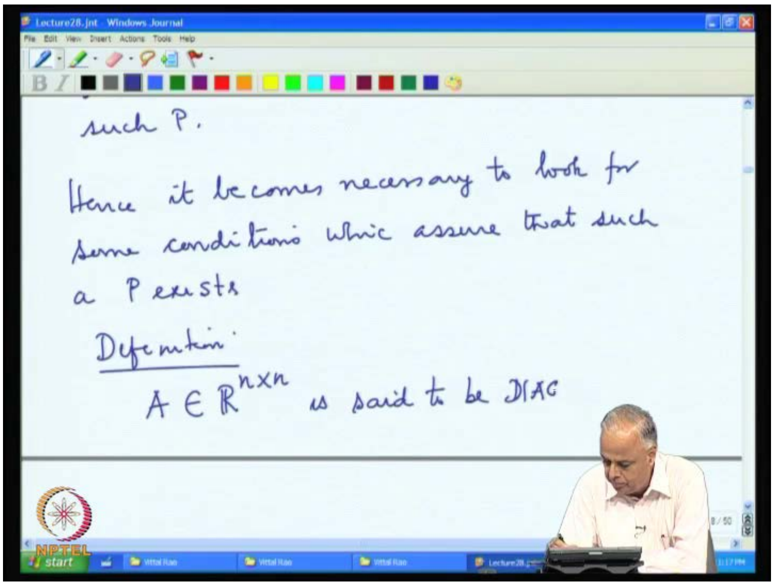

Hence, it becomes necessary to look for some conditions, which guarantee; which assure, that such a P exists. So, we show look for such conditions, we will use terminology now.

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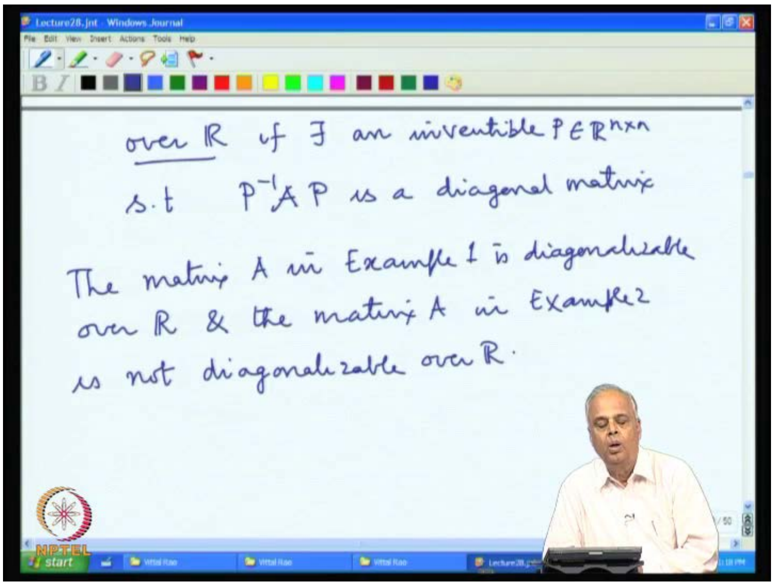

So, will give the definition; A real matrix n by n is said to be diagonalizable over R, if there exist an invertible P If there exist invertible an P in n by n real matrix, that is what is meant by over R. That the P, that we are looking for use a real matrix, such that P inverse AP is a diagonal matrix. So, what it means is that the example one, the matrix A in example 1 is diagonalizable over R, because we are found such AP, and the matrix A in example 2 is not diagonalizable over R, because we are able to not find such P is not diagonalizable. We said that cannot excitable and therefore, it is not diagonalizable. So, inch avoiding the there exists piece, that P inverse AP etcetera.

We simply said diagonalizable over R. The matrix A is diagonalizable over R, if there exist the P that, p inverse AP is the diagonal matrix, as observe the matrix of example one is diagonalizable over R, the matrix of example of two is not diagonalizable over R. It therefore, that become necessary for us, to drives and conditions and which we can say, if A satisfy this condition, we are guarantee that, A is diagonalizable over R. If A does not satisfy this condition, then A is not diagonalizable over R. So, this should be some kind of lit must test for diagonalizability over R of a matrix.

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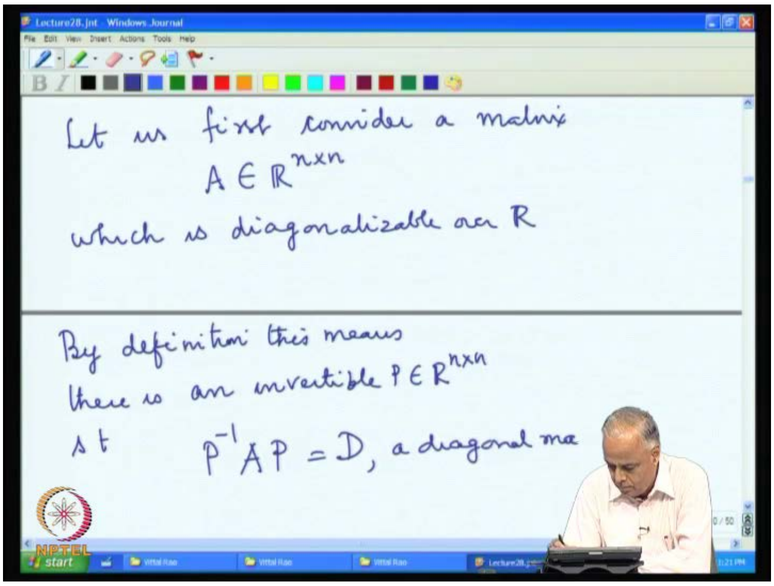

We shall start looking; searching; hunting for such matrix P first such condition. Now, the first thing to do is look at a matrix, which we most diagonalizable; see how it is looks what it is made up are; what makes the diagonalizable; and then from that learn what makes things where, as for as diagonailzability is constant. So let us start, let us first consider a matrix A, which is real n by n, which is diagonalizable over R. Now, what does this mean by definition? Then by definition this means, there is an invertible p, which is real n cross n such that, P inverse AP is equal to D, a diagonal matrix.

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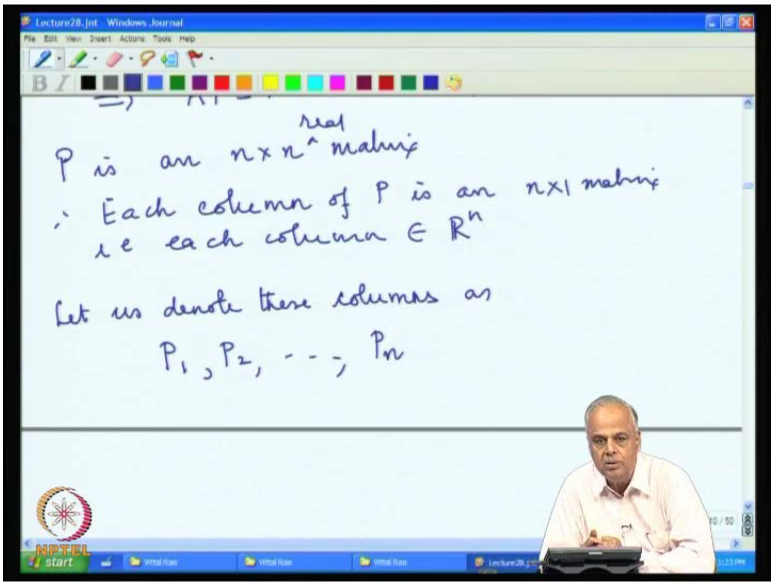

Now, let us write this in a different form. This implies AP is PD multiply the both sides on the left by P, we get PP inverse AP which is AP, and right side become PD. Now, let us look at each one of these sides left side and right side carefully. Now, P is the matrix is a n by n matrix, and therefore it is a n column; each column has n rows. So, it is an each column is n by one vector; each column therefore, belongs to R n. So, P is an n by n matrix, it is real.

Therefore, each column of P is an n by 1 matrix; that is P each column is a vector n component. Now, let us denote these columns as P 1, P 2, P n. So, P 1 wills therefore, the first column it will have n entries; P 2 is the second column, it will have an entry P n. It will have n column entry at the column vector.

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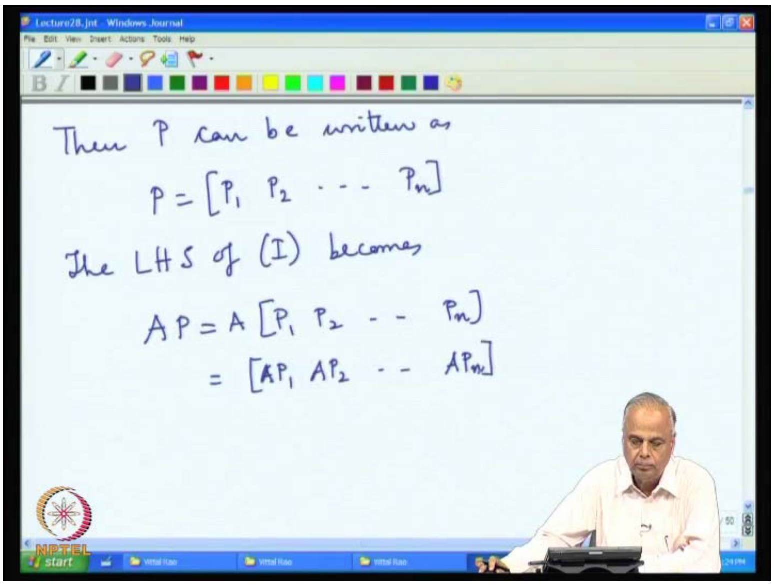

Now, then P can be written as in column notation p, it first column is p 1, second column is 2 and the n column is p n. Therefore, this notation let us go back to this equation one. Which says AP equal to PD? So therefore, left hand side of one becomes AP which is now, equal to A into P 1 P 2 P n. Now, when you multiply the matrix is just becomes AP 1 the first column of these product is obtained by multiplying the matrix A by the first column of P, then the second column is obtained by multiplying the matrix by the second column of P and AP n. This is the left hand side of the equation.

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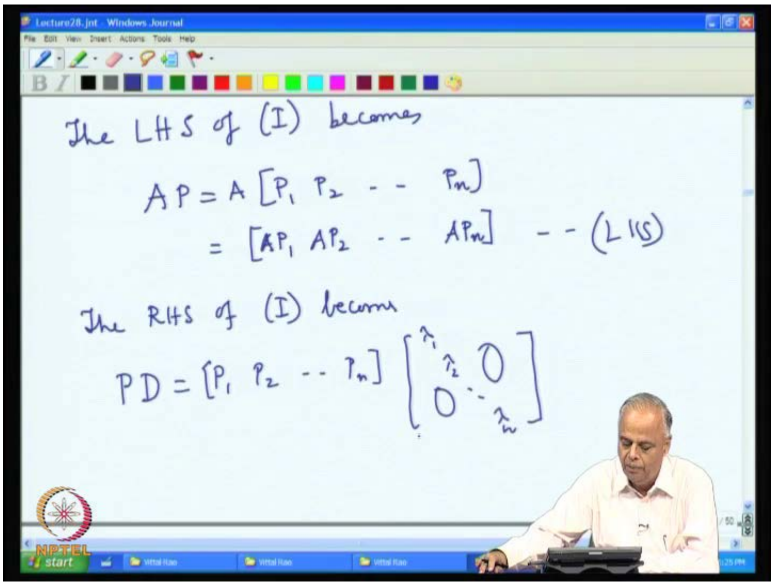

Similarly, the R H S the right hand side of one becomes, let us call is this us R H S, this is the L H S. Let us, call this the left hand side.

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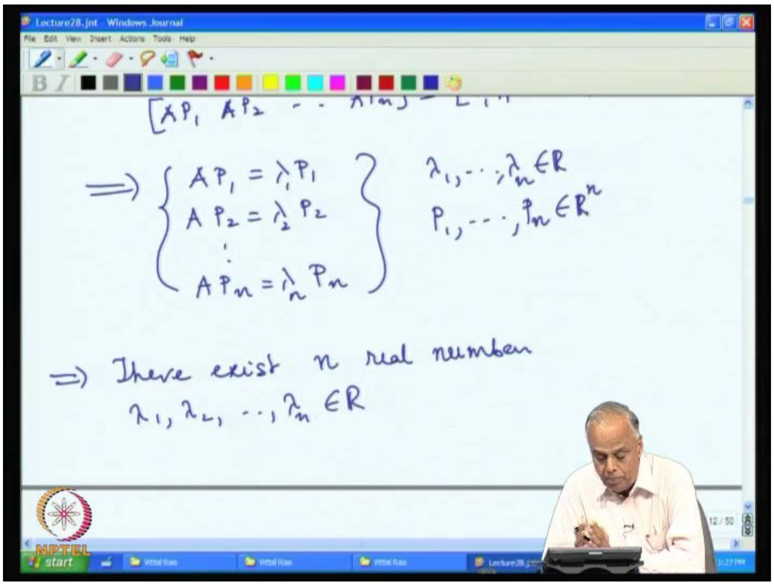

Now, let us look at the R H S. The R H S is P D. Now, P is again return in column notations and D, let the diagonal matrix the lambda 1, lambda 2, lambda n 0. If null multiply this in block notation, this becomes lambda 1 P 1 lambda 2 P 2 and so on lambda n P n. This is the R H S.

And therefore, equating the L H S to R H S. So therefore, L H S equal to R H S into implies this matrix AP 1 AP 2 AP n must be equal to the matrix, lambda 1 P 1 lambda n p; that means, the first column on the left hand side is AP 1 and the first column of right hand side is lambda 1 P 1. Since, the two matrix both sides are equal, the corresponding column must be equal. And therefore, we get AP 1 equal to lambda 1 P 1, AP 2 lambda 2 P 2 and so on AP n is lambda n P n. Now, let us o back this lambda 1, lambda 2, lambda n at the entries in the real diagonal matrix D and therefore, lambda 1 lambda 2 lambda n are scalar. So, lambda 1 lambda 2 lambda n real numbers, recall that the columns are all in R n.

There are column vectors and then have n components that are we get AP1 equal to lambda 1 P 1 AP 2 equal to lambda 2 P 2 AP n equal to lambda n. Now, therefore we have found n scalars lambda 1 lambda 2 lambda n and the n vectors P 1 P 2 P n, that AP 1 is lambda 1 P n and so on so forth. Not only that these vectors P 1 P 2 P n must be linearly independent, because they form the columns for invertible matrix. Therefore, the rank must be n. Hence, we get that implies there, exist n real numbers lambda 1, lambda 2, lambda n. Now, notice that these are only entries the diagonal matrix, there is no compulsion that the same entry need should not repeat or anything like that.

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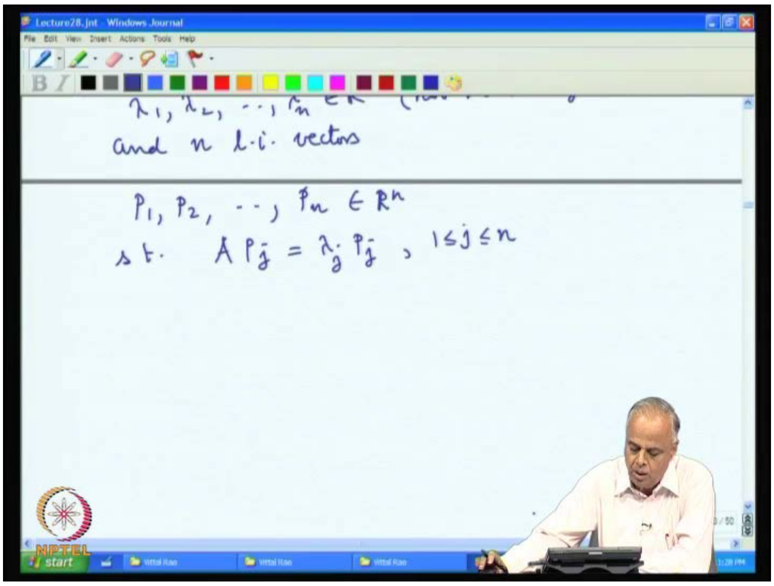

So, these need not to the necessarily district, they may be different numbers or some numbers may be repeating it does not matter. They are real numbers and n linearly independent vectors P 1, P 2, P n in R n, such that AP j equal to lambda j P j 1 less than or equal to j less than or equal to n.

What is our conclusion? Therefore, if use that; if you assumed; or if you know, that A is the diagonalizable matrix. If A was we started of with A as a diagonalizable matrix and their able to conclude that, this would force, we existence of n numbers lambda 1 lambda 2 lambda n, n real numbers lambda 1 lambda 2 lambda n and n linearly independent and vectors P 1 P 2 P n that they exist.

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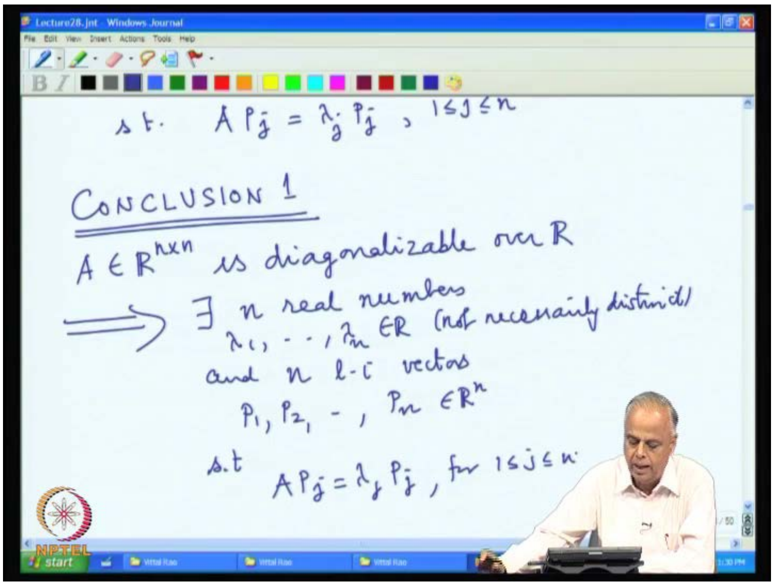

What is our conclusion? Let call this as conclusion 1. A R n cross n n by n matrix real is diagonalizable over R implies their exists n real numbers lambda 1, lambda 2, lambda n not necessarily district, such that and n linearly independent vectors P 1 P 2 P n in R n such that AP j equal to lambda j P j for 1 less than j less than equal to n.

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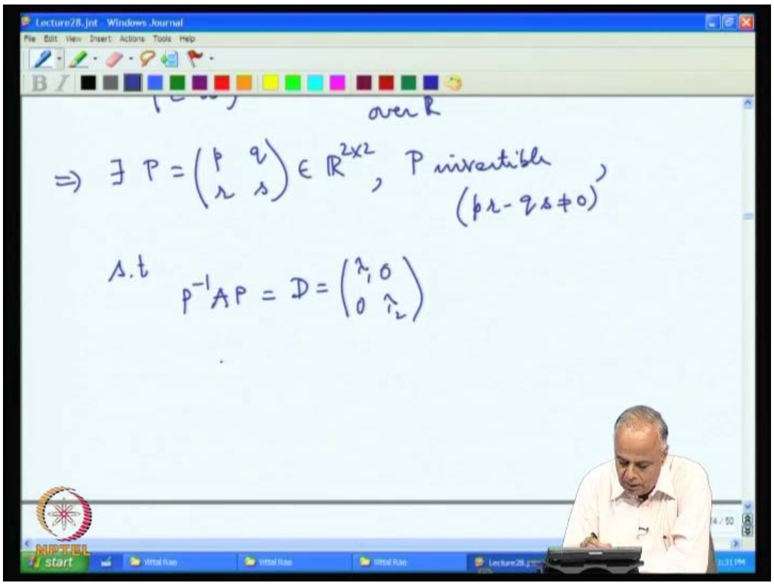

Let us they illustrate they above calculation. So, illustration for a 2 by 2 matrix, just see what is the calculation mean. So, suppose I have the 2 by 2 matrix a b c d belong into R cross 2 and A is diagonalizable over R. What is that mean this implies their exists p. Let call it us p q r s belong to r 2 2 P invertible what does P invertible means, the determinant is not zero, pr minus qs is not equal to 0.

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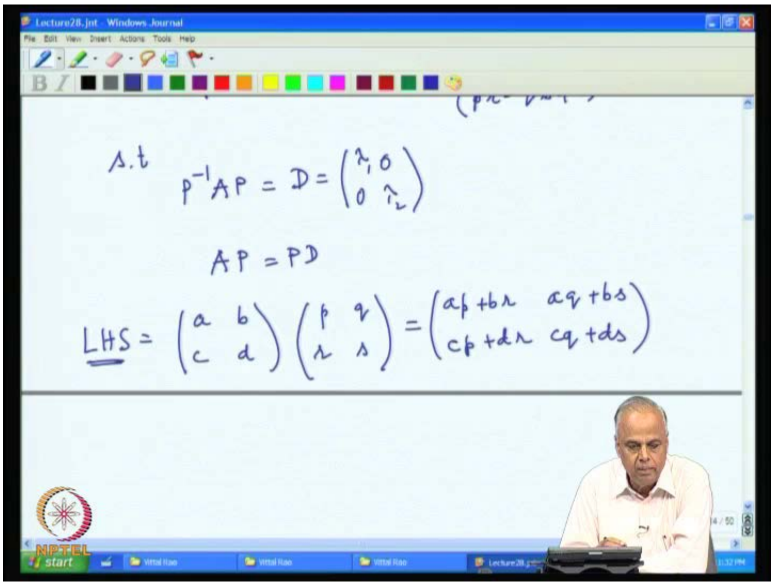

Such that p inverse AP equal to diagonal matrix. Let us, call that us lambda 1 lambda 2. What does this mean? We says that, AP equal to PD. Now what is AP now left hand side A is a b c d, P is p q r s which is equal to ap plus br aq plus bs cp plus dr cq plus ds.

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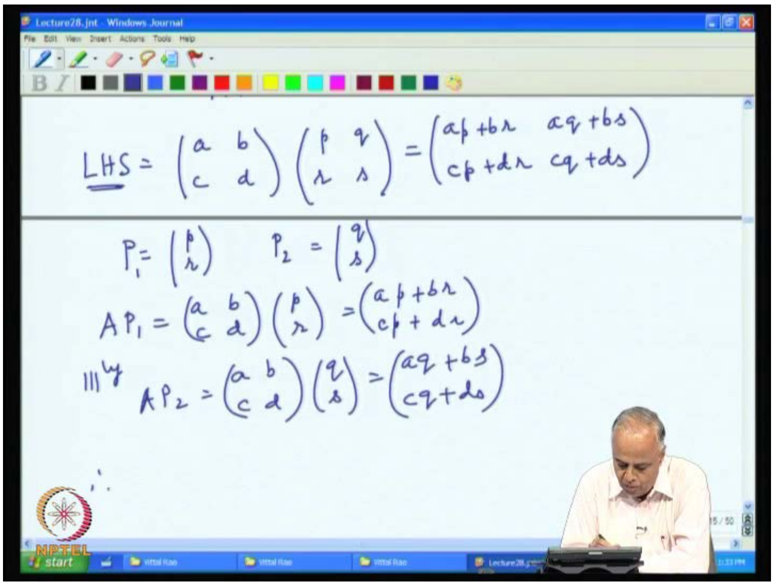

Now, let us look at our column notation we are P 1 the first column of P 1 is pr, P 2 is qs. So, what is AP 1? AP 1 is a b c d into pr, which is ap plus br cp plus dr. This is the first column of the product. This is what we made by seeing the first column of the product will be AP 1. This is what AP 1 is. Similarly, AP 2 is a b c d into qs which is aq plus bs cq plus ds, which is the second column of L H S.

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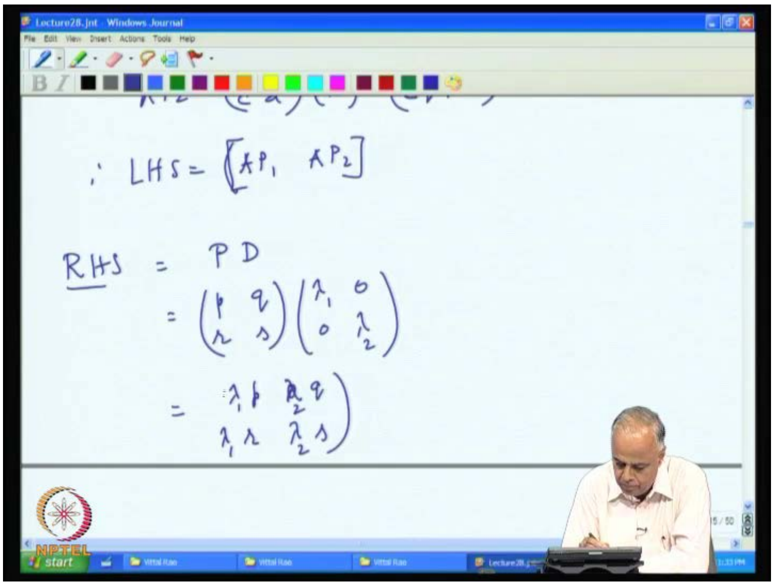

Therefore, L H S is the matrix AP 1 AP 2. The first column of AP 1; the second column is AP 2. What is the R H S? R H S is PD; what is p q r s and d is lambda 1 0 0 lambda 2 and the product is going to be lambda 1 P lambda 2 q lambda 1 r lambda 2 s.

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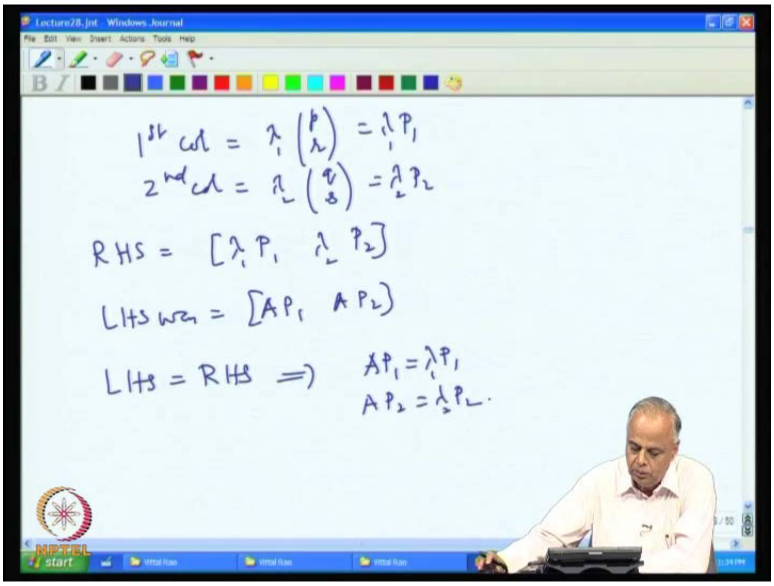

Now, what is the first column of this, the first column is lambda 1 into Pr which is just lambda 1 P 1. The second column is lambda 2 into qs which is just lambda 2 into P 2. Therefore, R H S is nothing but the matrix lambda 1 P 1 lambda 2 P 2. Now, L H S was found AP 1 AP 2 and therefore, equating the L H S. The L H S equal to R H S gives as the corresponding columns are equal for AP 1 equal to lambda 1 P 1 AP 2 equal to lambda 2 P 2. Now, this is what we looked at the general case in the n by n matrix case it is not just 1 lambda 1 and lambda 2 and P 1 and P 2, we get lambda 1 lambda 2 and lambda n real numbers, P 1 P 2 P n vectors such that, AP j equal to lambda j P j. Let us look at even in specific example. Look at to the very specific example.

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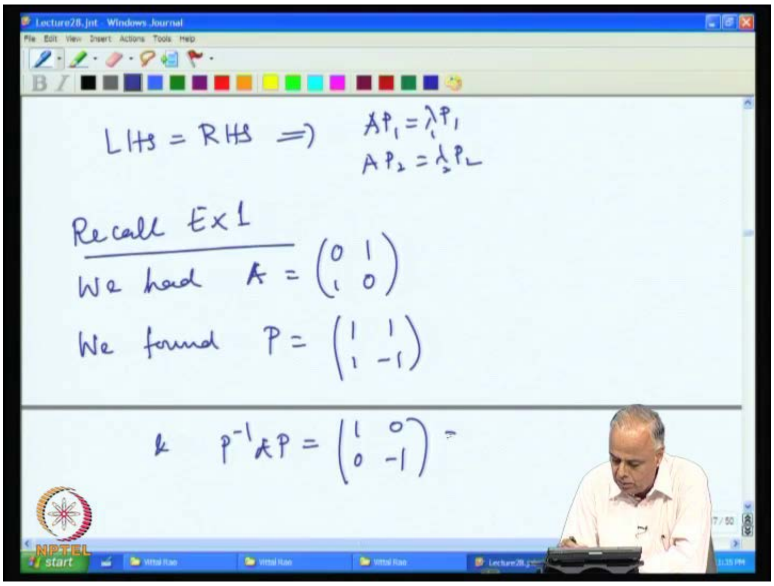

So recall example 1, we had A was 0 1 1 0, we found P equal to 1 1 1 minus 1 and P inverse AP is equal to 1 0 0 minus 1. This is my diagonal matrix.

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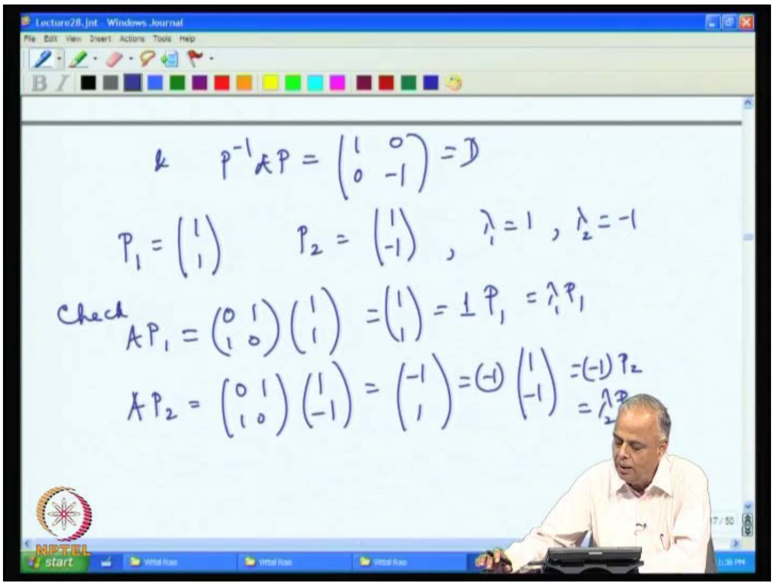

So, in this case by p 1 is the first column of p, p 2 is the second column of D by lambda 1 is the first entry in the diagonal matrix, lambda 2 is second entry in the diagonal matrix. Check, AP 1 is equal to 0 1 1 0 into 1 1 which is 1 1, which is lambda 1 times P 1. which is lambda 1 times P 1 AP 2 is 0 1 1 0 P 2 is 1 minus 1, it will be minus 1 1 which is minus 1 times 1 minus 1, which is minus 1 times P 2, which is lambda 2 times P 2. So, AP 1 is lambda 1 P 1, AP 2 is lambda 2 P 2. When even ever you have diagonalizability, you are assured of the n numbers lambda 1 lambda 2 lambda n and the n linearly independent vectors P 1 P 2 P 3 such that, this AP j is equal to lambda. This is the important thing. If A is diagonalizable over R, then there exists n scalar real numbers lambda 1 lambda 2 lambda 3 lambda n and n linearly independent eigenvectors P 1 P 2 P n such that AP j is equal to lambda j P j.

Conversely now, look at this example by the way, that we consider this specific example of this here, you see that this P 1 and P 2 we got linearly independent. Now, let us look at converse; so now, we are found the movement I have a diagonalizable matrix, this numbers and scalars, this vectors will follow apart. They must exciting side the matrix they encoded; we should find we will able to find this numbers in the vectors from the matrix 0. Conversely will now see, if I am already provided this numbers and the vectors then the matrix A must necessarily be diagonalizable.

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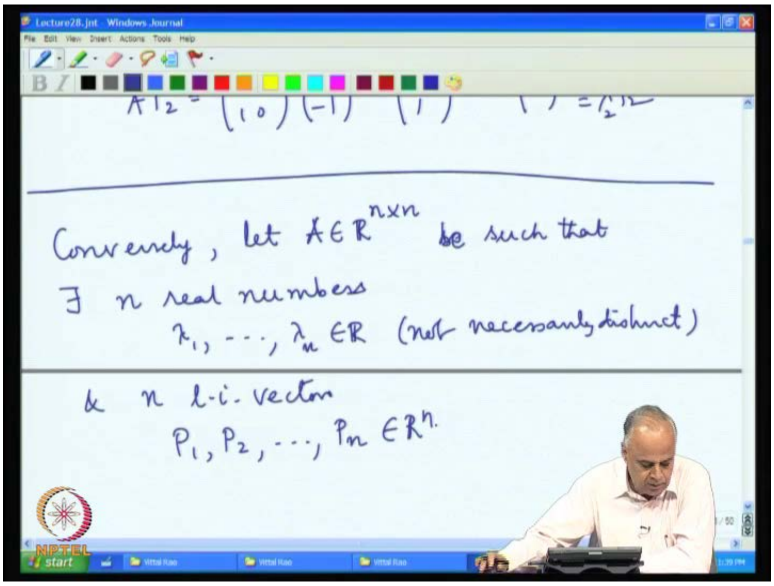

Conversely, let A belong to the real n by n matrix is such that let us b such that their exist n real numbers. Previously, we assumed diagonalizability and prove the existence of these numbers. Now, we are assuming existence of numbers lambda 1, lambda 2, lambda n not necessarily district and n linearly independent vectors P 1 and linearly independent vectors P 1 P 2 P n in R n.

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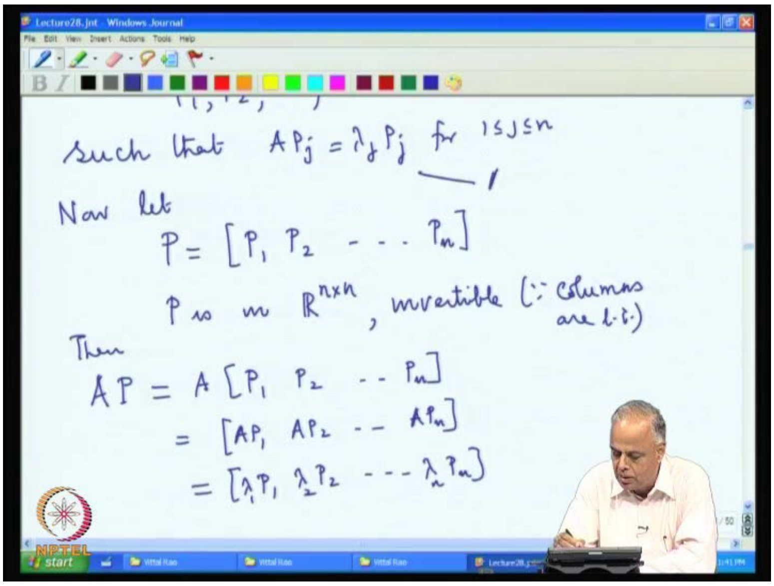

Such that, AP j equal to lambda j P j for 1 less than j less than n. Now, the previous case we assumed diagonazability and the arrived the exists of numbers and this scalar. Now we are assuming the exists of numbers and vectors and we are going to prove that A is diagonalizable. This implies now, let p be the matrix who is the first column is p 1, second column is p 2 and the n th column is p 1. Now, we construct this matrix p 1s we know that n th vector are there we are given their exists n linearly independent vectors. We construct this matrix P and since these are all linearly independent p is in r n by n invertible, because columns are linearly independent.

So, we have A invertible matrix then, what is AP? AP is equal to the AP 1 P 2 P n as observed earlier this product will be AP 1 AP 2 AP n, that we are given that AP 1 is lambda 1 P 1 AP 2 is lambda 2 p 2 lambda n P n that is given to us.

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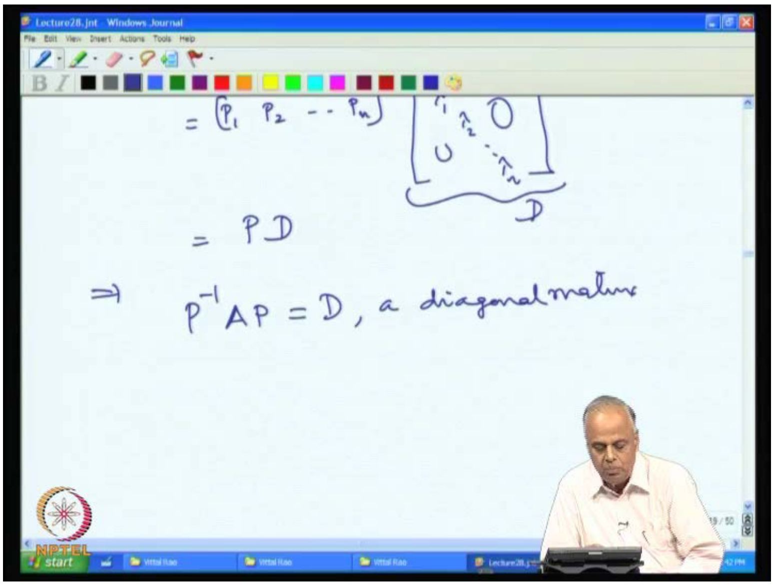

So, this says star by star and this product as before can be return us P 1, P 2, P n into the diagonal matrix lambda 1 lambda 2 lambda n and which is P times the diagonal matrix this matrix what I call D, AP equal to PD which implies P inverse AP is equal to D, a diagonal matrix.

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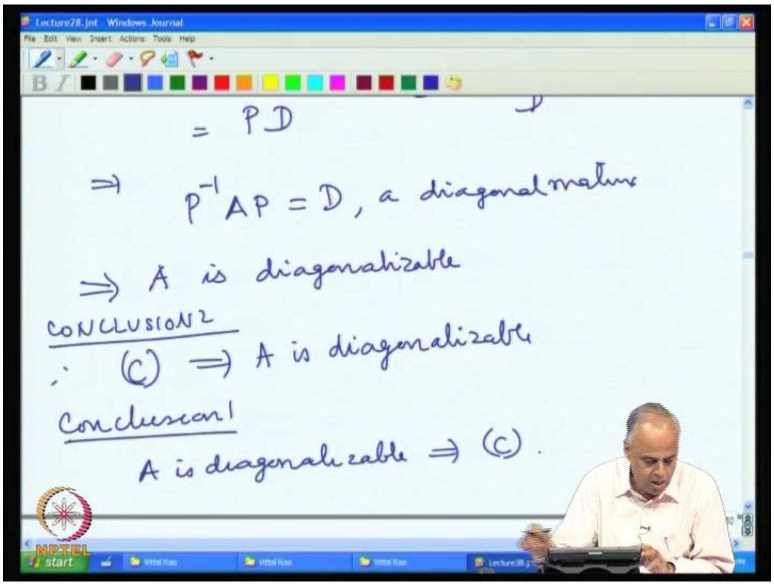

Therefore, the movement we scalar and vectors are given where able to construct of real matrix P that the P inverse AP diagonal which means A is diagonalizable. We now, call this condition as C this whole set of condition as C. what are the whole set of condition?

There are n real numbers n linearly independent vectors such that AP j equal to lambda j P g. What we are seen? Now, is C implies A is diagonalizable.

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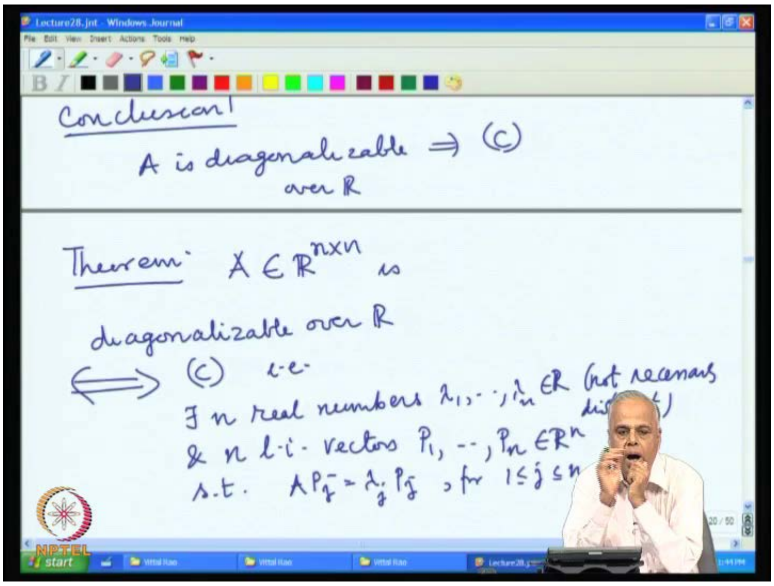

And what we saw earlier, this is conclusion 2. What we saw the conclusion one was nothing, but A is diagonalizable implies condition C and therefore combining two we get the theorem, A belong into R n n. A is the real n by n matrix is diagonalizable over R is diagonalizable over R is diagonalizable. If only see course that is there exists I repeat like this, n real numbers lambda 1, lambda 2, lambda n in R not necessarily distinct, and n linearly independent vectors P 1, P 2, P n belong on to R n such that, AP j P equal to lambda j P j for one less than j less than n. Therefore, we now know how a criterion for a matrix A real matrix A to be diagonalizable over R.

The only problem therefore is, how do if find this number lambda 1, lambda 2, lambda n. How do we find weather we exists these numbers, even if I find this number how do find weather the exists vectors P 1 P 2 P n and even if I prove that there exists P 1 P 2 P n. How do I find this vectors; how do I compare; where there look for the this scalar lambda 1, lambda 2, lambda n and where do I look for this vectors P 1, vectors P 2, P n. Now this state as now to the formal definition of eigenvalues, and eigenvectors. If you look at what we are looking for are vectors P j of the type P j, which has set that when they acted upon by the matrix A that is A times P j; they are very form the remaining the P j direction only thing they may be scaling factor lambda j. In other word the directions remain in variant under A, P 1 P 2 P j tell us n directions which remain unchanged when acted up on A, remain the same direction.

But the vector may be change in the length they the magnitude are the scaling factor lambda j may come into the picture. Therefore, be a looking for vectors which are in variant in direction under the action of A. This gives us to the notion of the eigenvalues and eigenvectors.

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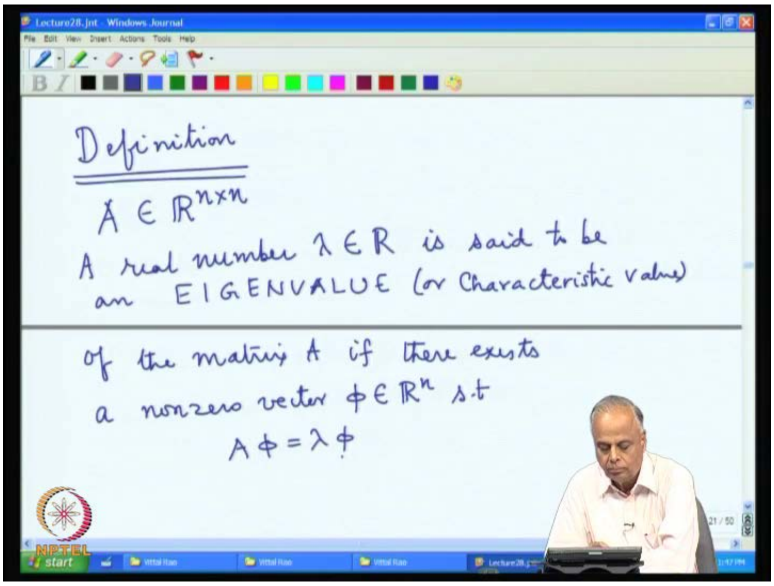

So, definition A is n by n matrix. A real number lambda is said to be an eigenvalue for some times also refer to us characteristic value characteristic value. So, either the Eigen major you want to use it a real number lambda is said to be an eigenvalue of the matrix A, if there exists a non zero vector pi in R n, such that A pi equal to lambda pi; that is the in variant, this is what we are looking at pi remains in the same direction us pi.

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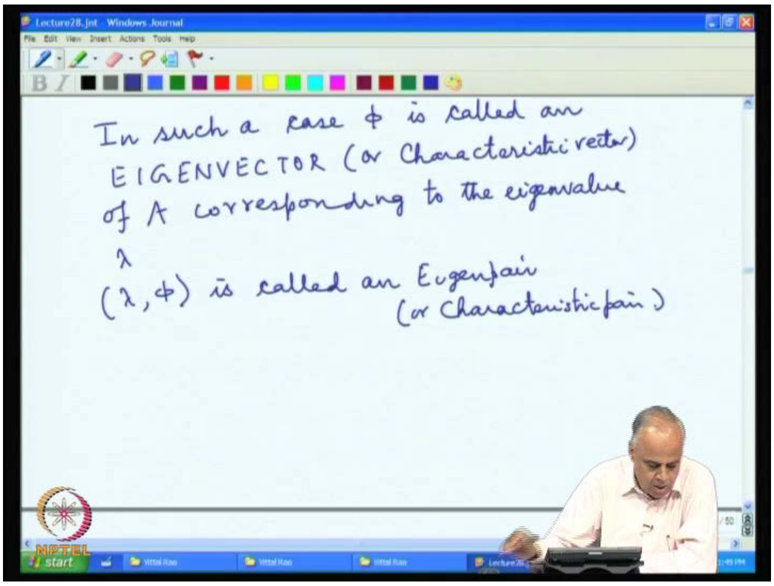

So non zero vector pi in R n, A pi equal to lambda pi, in such a case pi is called an eigenvector, there again characteristic vector of A, the matrix A corresponding it is invariant the scaling factor is lambda, so corresponding to the eigenvalue lambda.

And this pair lambda pi is called an Eigen pair for a characteristic or characteristic pair. Now, if you look that to our criteria for the diagonalizablity. We are looking for n vectors P 1 P 2 P n such that AP j equal to lambda j P j. Therefore, we are looking this numbers lambda 1, lambda 2, lambda n must be Eigen values P 1 P 2 P n must be eigenvectors not only that, since P 1, P 2, P n P wanted them to be linearly independent eigenvectors.

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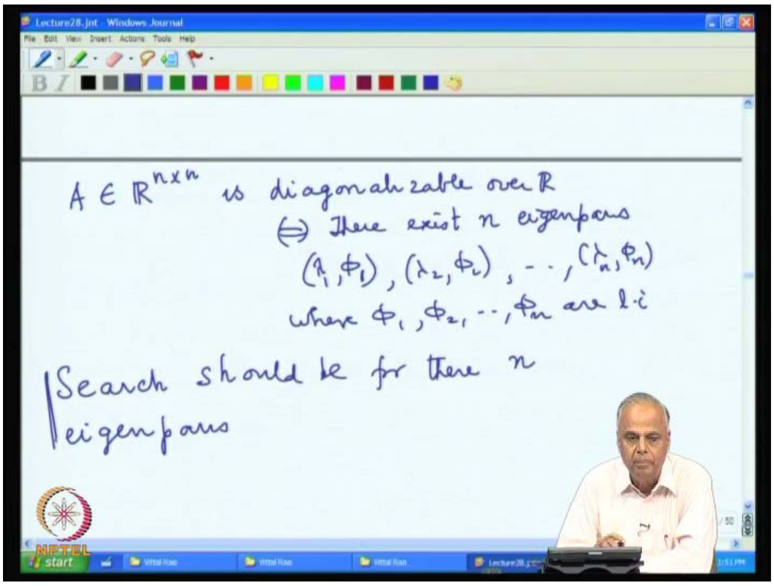

So, the theorem now can be written us, hence by our theorem we get the theorem can be re written in this languages as follows; A belong into R n; that is A is the real n by n matrix is diagonalizable over R; if and only if there exists n eigenpairs lambda 1 pi 1 gives as the same notation is four lambda 1 pi 1 lambda 2 pi 2 etcetera lambda n pi n. Where, pi 1 pi 2 pi n are linearly independent instead of P 1 P 2 P n. I am calling them pi 1 pi 2 Pi n. Therefore, diagonalibility demands that you find n eigenpairs. If you want diagonalizable matrix A, you find n Eigen pairs in such way, where the eigenvector coming out into the eigenpair pi 1 pi 2 pi n are all linearly independent. So, search should be for these n eigenpairs; that is going to be our main search in diagonalizability. Can be find these pairs. Where do I search?

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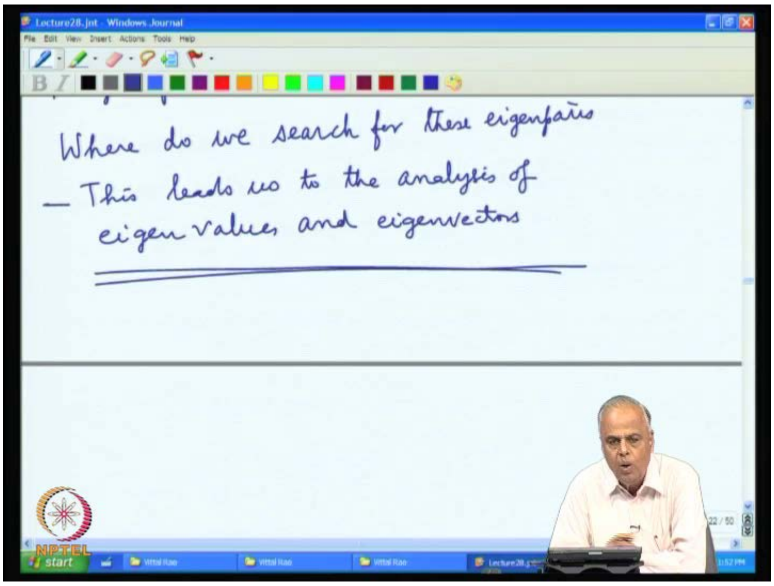

So, where do we search for these eigenpairs and that is the main chapter of eigenvalues and eigenvectors. This leads as to the analysis of eigenpairs, eigenvalues and eigenvectors; this is one of the most important chapters in linear algebra. This notion of eigenvalues and eigenvector and in fact the notion of eigenvalues, eigenvectors respect any transformation and going to play, very crucial rule in the stature analysis of these transformation in particular with respect to matrices.

Now, our analysis should be with respect to this eigenvalues and eigenvectors. What should be our status; how should be going to our strategic; we have to find the pears lambda and pi in pair to find pears lambda and pi. We have to find those numbers lambda and then corresponding those vectors find.

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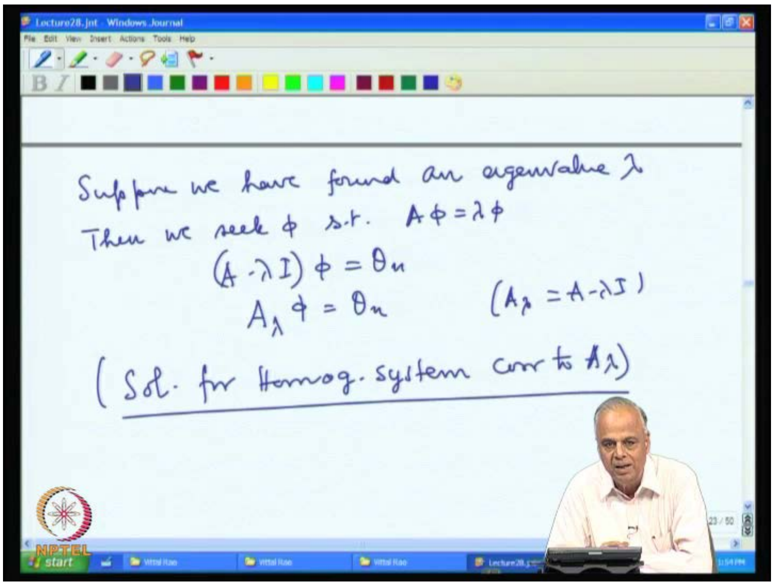

Now suppose, we have found the numbers lambda an eigenvalue lambda. Then, we see pi such that A pi equal to lambda pi. And this pi also looking at A minus lambda I pi equal to theta n or if we call that matrix as A lambda A minus lambda I. Then to boils down to finding the over the solution for homogenous. So therefore, we are looking for solution for homogenous system corresponding to A lambda. Therefore, that we that the L H S; we have seen how to treat homogenous system. So, if we know the eigenvalue, there scene to be some finding n pair eigenvector for that, so therefore, our there is hope for us to find at least the fives in some sense the movement we know the eigenvalue there is the chance, because to the solve homogenous system.

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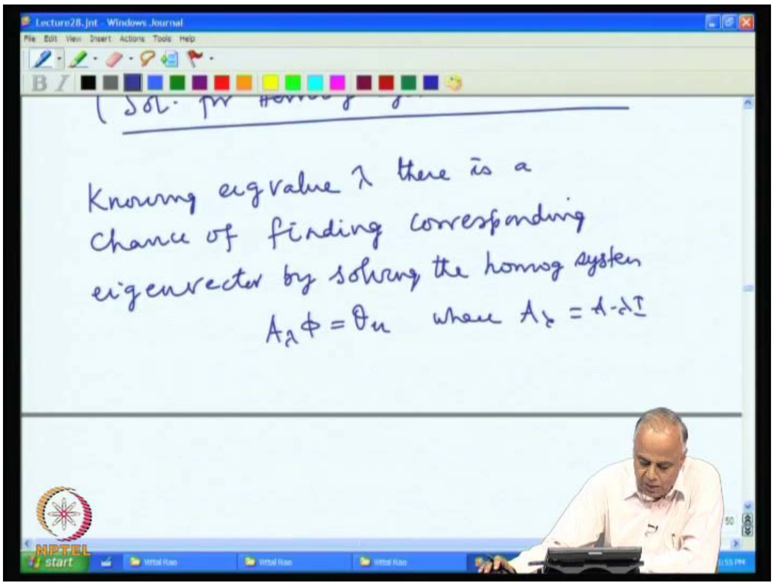

So knowing lambda knowing eigenvalue lambda, there is a chance, of finding corresponding eigenvector, by solving the homogenous system A lambda pi equal to theta n. Where A lambda equal to n minus lambda n. So therefore, our primary set should be further eigenvalues.

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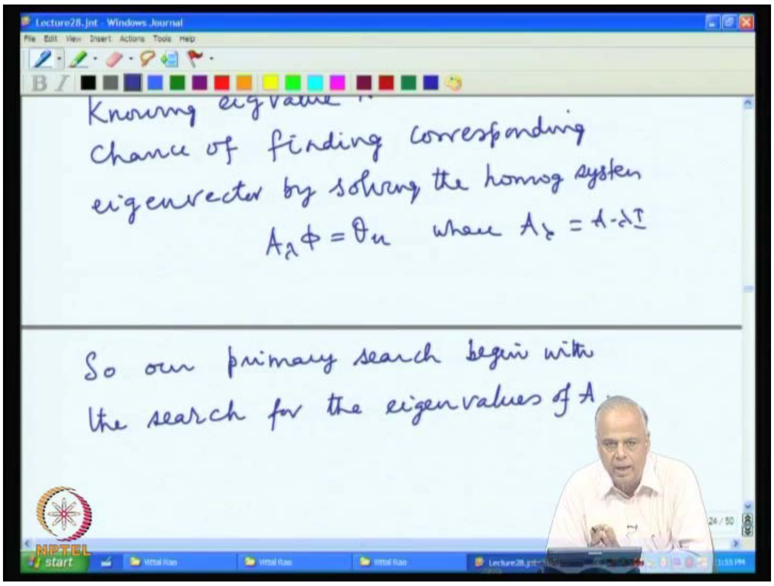

So, our primary search begins with the search for the eigenvalues of A. Therefore, our next study will be given a matrix A, where do I look for eigenvalues. I know that the eignvalues must be real numbers. I must be looking for the eigenvalues among the real numbers, but there are in (( )) component numbers of real number, where do I go on search for that. So I was, I must narrow down by search, but if I my search is infinite set, is a hopeless task cannot go on searching every 1 of those follows are you in Eigen value are you are eigenvalue you cannot go and checking a every real number and check weather is an eigenvalue.

Even if you do not check in real number, if you narrow down the search to the infinite sub set the real numbers. Then again, the problem is that we cannot search one by one, if the infinite set of real numbers, so again the hopeless task. Therefore, some over the other, we have to reduce and focus our search using the matrix A, to a finite search. Some moving look among, because after all looking for the finite number, this was must some was scatter.

So some over look finite set of numbers, and look for the eigenvalues which in that finite set. Our focus will be to reduce our search of the eigenvalue to a search in a finite set of numbers. And so that will be primary consent in the next chapter, to look for the eigenvalues of a given n by n matrix.