Advanced Matrix Theory and Linear Algebra for Engineers

Prof. R. Vittal Rao
Department of Electronics Design and Technology
Indian Institute of Science, Bangalore

Lecture No. # 22

Inner product and Orthogonality – Part 1

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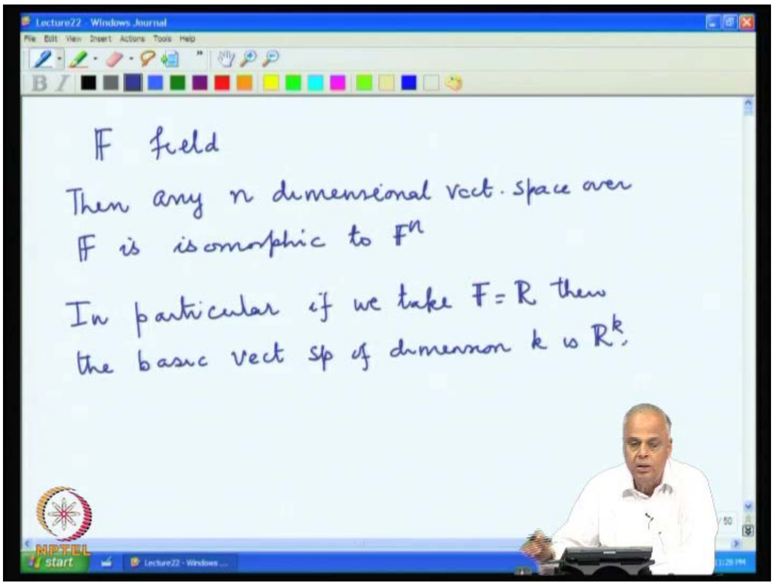

In the last lecture, we found that if F is any field, then any n dimensional vector space over F is isomorphic to F n. And therefore, we concluded that when we are talking about vector spaces over F, the main basic vector spaces are this Fn. Now, in particular if we take F to be the field of real numbers, then the basic vector space of dimension k is R k. And is the because of this reason that we study R k in great detail.

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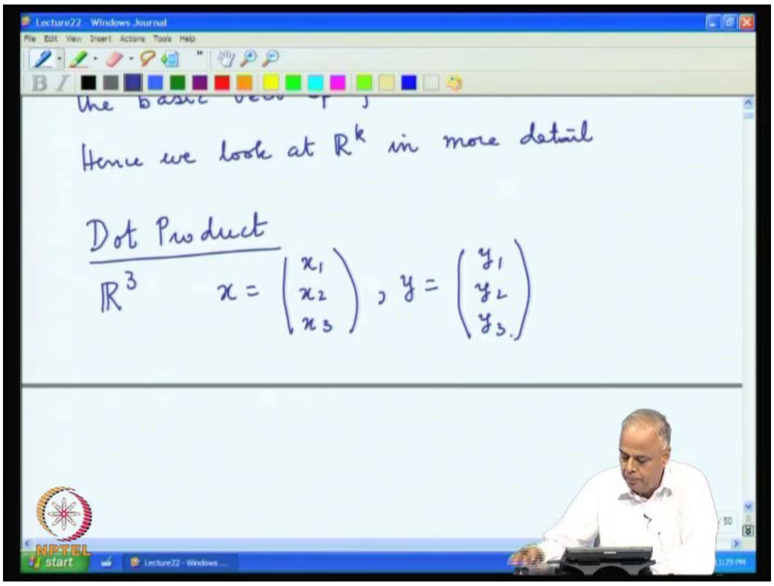

So, hence we look at R k in more detail. The first thing that we observe is R k being a vector space it already has this basic algebraic structure are given by the 2 basic operations of the vector space namely addition and scalar multiplication. Now, we are going to bring in more structure, (( )) more geometric in nature on the space R k. So, we look at the notion of first, the dot product which most of you will be familiar with it. Let us consider the vector space R 3 you may recall that, if we take any vector x whose components are x 1 x 2 x 3 and a vector y whose components are y 1 y 2 y 3.

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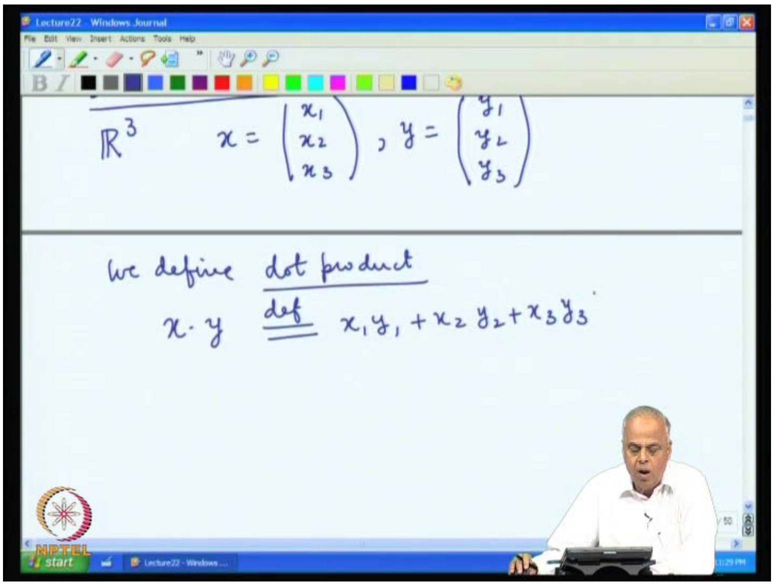

Then we define the dot product, this is in the vector calculus vector algebra per learns known as the dot product x dot y, it is defined to be x 1 y 1 plus x 2 y 2 plus x 3 y 3.

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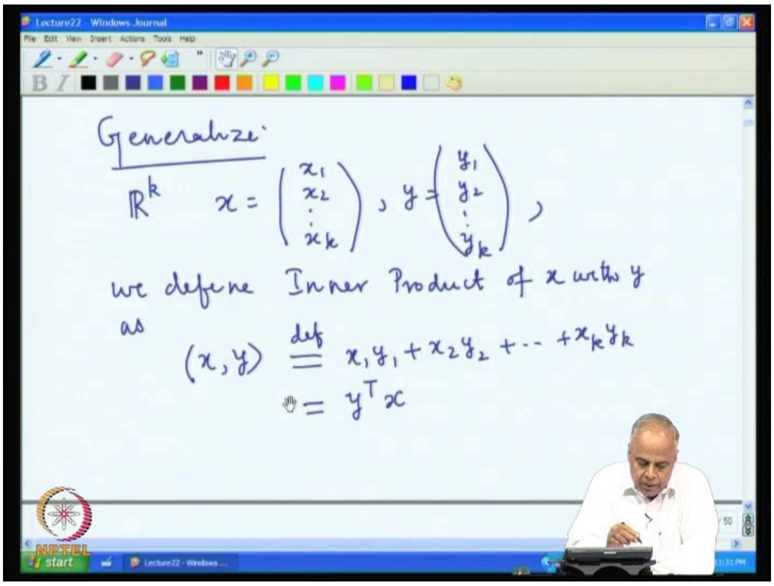

We now, generalize this idea. So, we generalize then we take R k for any k now, not k not necessarily 3, it may be hundred, it may be twenty, it may be thirty fix a k then look at the k dimensional vector space R k and if we take any 2 vectors x 1 x 2 x k and y equal to y 1 y 2 y k analogous to the dot product. We now, define the dot product and R k and from now, on we will refer to this as the inner product in R k and instead of x dot y, we will denote it by x comma y with in a bracket.

So, we define inner product of x with y as it is denoted by x comma y with a bracket and it is defined to be x 1 y 1 plus x 2 y 2 and so on x k y k. We may notice that, this is the same as taking the matrix y and taking it is transpose and multiplying with the matrix x, the column matrix y, it is transpose will be a row matrix. So, we have a row matrix and we multiply this row matrix by this column matrix, the result will be a number and therefore, we observe that the inner product of 2 vectors is a number.

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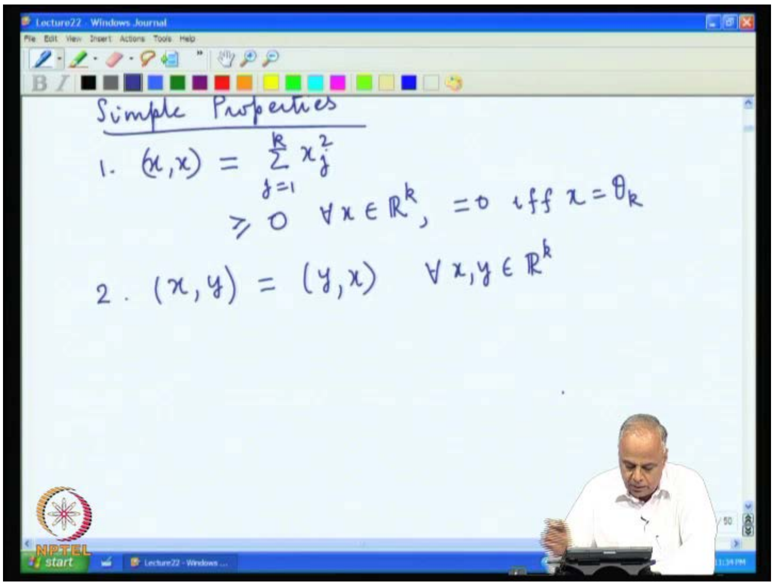

So, we define the inner product of 2 vectors in R k by this definition. We observe for some simple properties of this inner product (no audio from 05:28 to 05:37). From the definition, we observe that x comma x will be x 1 square plus x 2 square plus x 3 square plus x k square. So, we will simply write it as j equal to 1 to k, x j square and therefore, this being the sum of non negative quantities, this will be always greater than or equal to 0 and when will that become 0 when the sum of the squares is 0, but the sum of the squares is 0. Since, all are real numbers if only when each 1 of these entries is 0 that means, x j is 0 for all j, but x j is 0 for all j means x is equal to 0.

So, if and only if, x is the 0 right to the first important property is the inner product of any vector with itself is always non negative and it become 0 only when the vector is the 0 vector. The second important property is, we have a product note that the product of 2 vectors, this product is inner product, the result is not a vector, but is the number is the real number and this real number is always non negative and become 0 only when x is the 0 vector.

Now, we have introduced some new action in the vector space, the moment we introduced some new action, we are interested in what are its implications on the 2 basic operations of the vector space namely addition and scalar multiplication. Before that, we observe that the definition here is symmetric in x and y, if we interchange the positions of x and y, the result is going to be the same, you will get y 1 x 1 y 2 x 2 y n x n, but the result is going to be the same. Therefore, we observe the first, the symmetric property x comma y is the same as y comma x for every x y in R k, that is the inner product of x with y is the same as the inner product of y with x.

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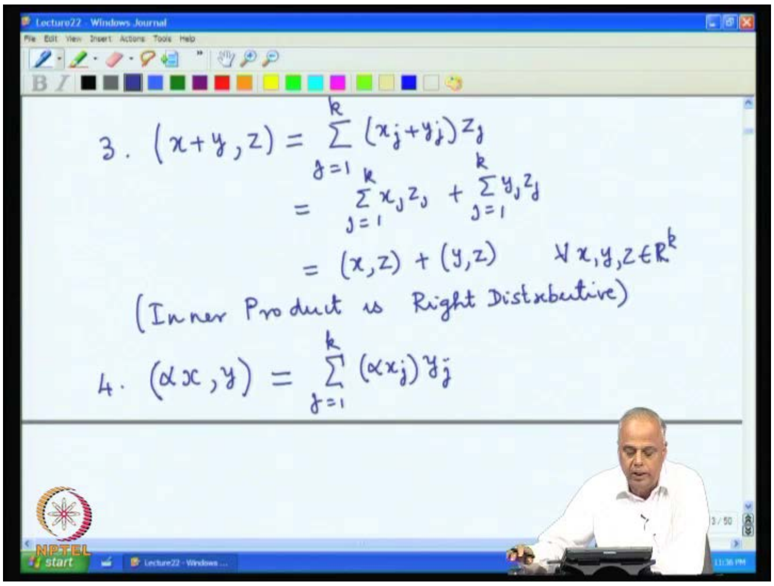

Then we look at the effect of this operation of inner product with the 2 basic algebraic operations in the vector space namely addition and scalar multiplication. So, suppose I take 2 vectors x and y and add them and then take the inner product with z, that is the same as by definition j equal to 1 to k, x plus j has coordinates x j plus y j and z as coordinate is z j which is equal to j equal to 1 to k, x j z j plus j equal to 1 to k, y j z j, but this first sum is nothing but the inner product of x with z plus the second term is the inner product of y itself with simply (( )) to say that this inner product is distributive from the right x plus y comma z is x comma z, y comma z.

So, we will (( )) inner product is right distributive (no audio from 09:24 to 09:30). This is the effect of addition, we we interpreted with respect to the inner product. Next, we will see the effect of this inner product on scalar multiplication; this is true for any vectors x, y, z and R k. Next, we look at this effect of inner product on scalar multiplication. So, we take a vector x and we take a scalar alpha and then take its inner product with a vector y, what is this equal to the components of alpha x or alpha x j and this is to be multiply with the components of y j.

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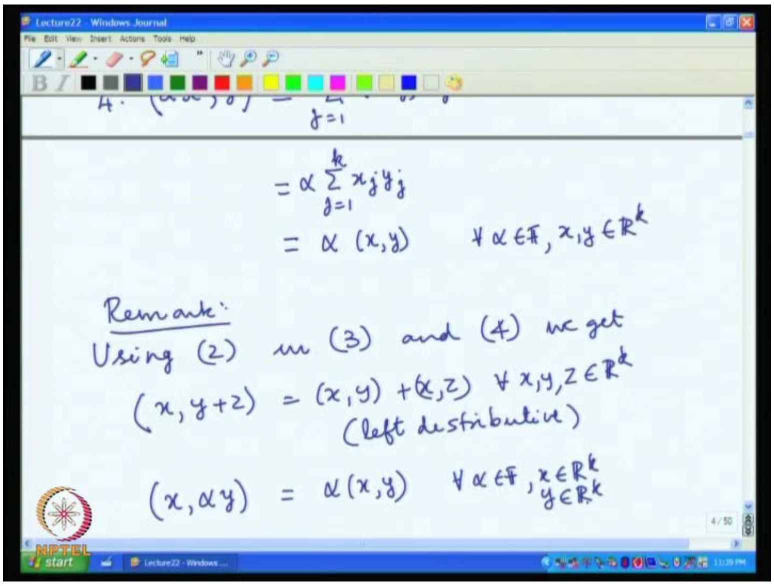

And this is the same as summation j equal to 1 to k, alpha can be pulled out of the summation x j y j and the summation is nothing but x comma y, so it is just alpha x comma y. This simply says that the alpha can be pulled out of the inner product from any 1 of these practice from the first practice remark, since the inner product is symmetric right distributive will also imply left distributive and pulling out alpha from the first term in the inner product will also imply pulling out alpha from the second term in the inner product.

So, using the symmetric property which we have called as true, the symmetric property using 2 in the distributive property and scalar multiplication property, we get x comma y plus z is equal to x comma y plus x comma z for every x, y, z in R k, that is left distributive, the inner product is left distributive and we get x comma alpha y is the same as alpha comma into x comma y for every alpha in F and x in R k. So, we are also, we have to say that for every alpha in F, x, y in R k. So, we have left distributive, right distributive and the constants can be pulled out of the inner product either from the first factor or from the second factor. So, this is a very important generalization of the notion of the dot product that we had in vector algebra, what does this inner product give us?

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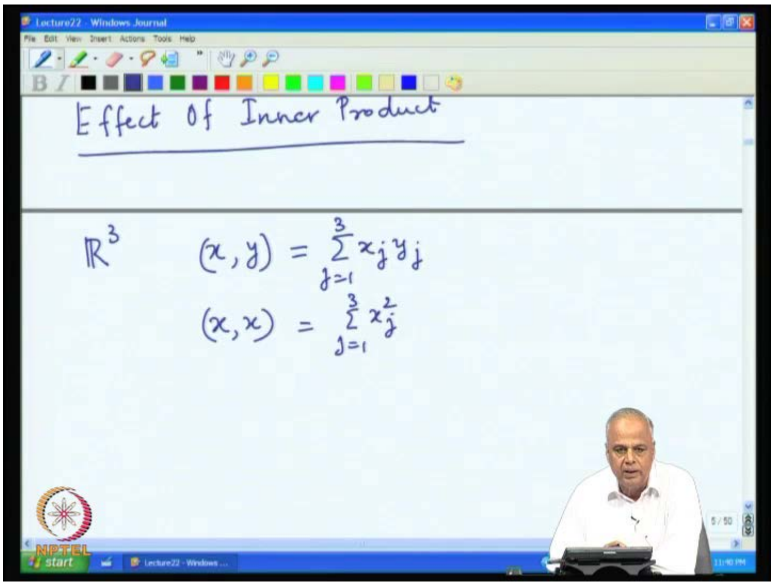

So, the effect of inner product, if you recall again look at R 3. Now, for x comma y in R 3, which is the dot product now, we are going to denote by this notation is summation j equal to 1 to 3, x j y j and we Now, if we Now, take x comma x, we get j equal to 1 to 3, x j square which is (( )) length of the vector in the 3 dimensional space the square. So, the square of the Euclidean length which we normally define as the length, that is the distance from the origin to the point x 1, x 2, x 3.

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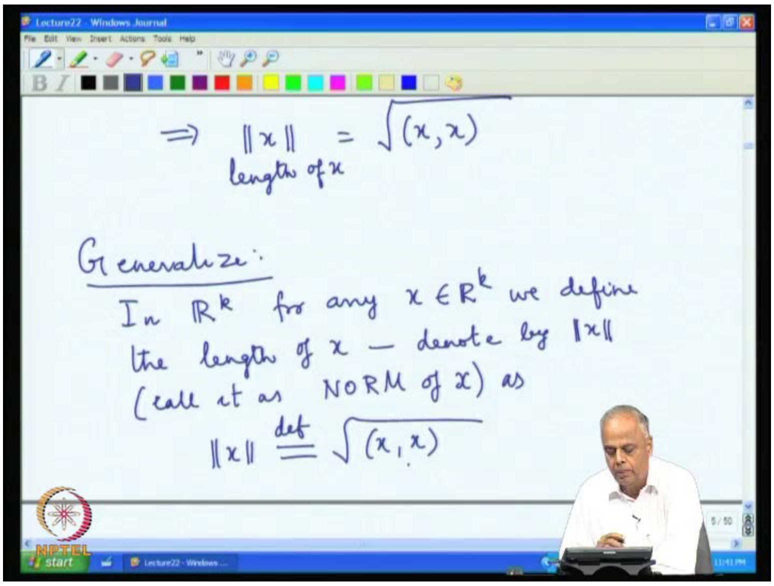

And therefore, we have that the length which we now, denote by this the length of x is square root of x comma x. Now, this is generalize to define the length of a vector from the inner product through this definition. So, we moment we have the notion of the inner product, we can define the notion of the length through this definition, by experience that we gain from looking at R 3. Therefore, generalize in R k for any x belonging to R , we define the length of x which we denote by this symbol and from now, on call it as NORM of x as NORM of x by definition is square root of x comma x.

Notice that there no problem of taking the square root of x comma x because we have already observed that for any vector x comma x is a non negative quantity and therefore, we are taking on square root of a non negative quantity so, you will get only a real quantity. However, we may wonder, there are 2 square roots plus and minus which 1 do I take. Since, we want the length to be non negative; we do not want negative lengths. Therefore, we always take that square root which is not negative.

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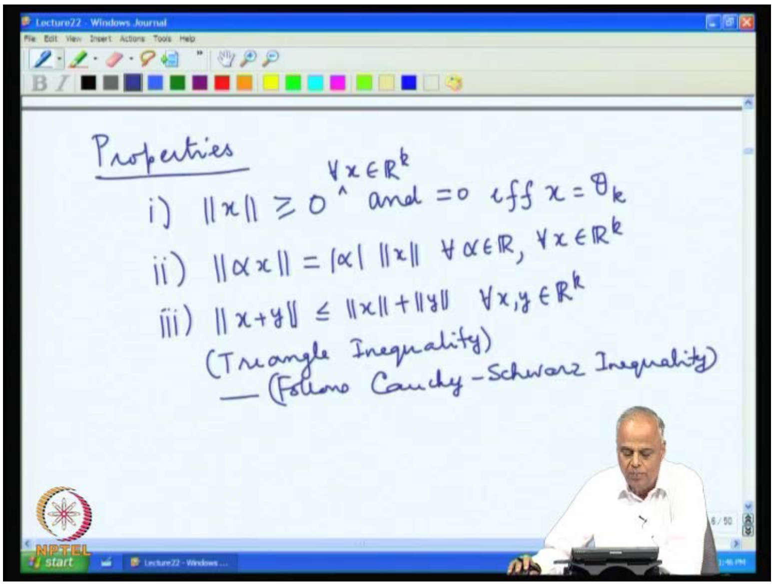

Now, what are properties of this length? This way of defining the length since, we have generalize from R 3, the properties of the length in R 3 are carried over to the properties of the length in R k, what are these properties in R 3, if you take 3 dimensions and if you take any vector, the length is always going to be non negative and only time the length is 0 will be the 0 1 when the vector is 0 vector. Secondly, what is the effect of the length on the 2 basic operations of this vectors, if we add, if we multiply a vector in 3 dimensions by a number, if the number is positive, the length simply gets multiplied by that number, if the number is negative, the length simply gets by the multiplied by the modulus of that number.

So, in any case the length gets multiply by the modulus of the scalar which is multiplying the vector and we have a simple law in geometry that, if we have a triangle any side, it is length must be less than or equal to the sum of the 2 sides. In vector language, this translates into what is known as the triangle inequality, that the length of this sum of a vector is less than or equal to the sum of the lengths of the vector. Let us summarize this, these get generalize in the most general set of R k as well.

The first property is the length of a vector is always non negative because square root of x com x comma x is always non negative and equal to 0, if and only if, this is 0, this square root is 0, if and only if what is inside that x comma x is 0, but the property of the inner product says x comma x is 0 only when x is 0; and therefore, we get this is equal to 0, if and only if x is the 0 vector and when you multiply a vector by a scalar alpha, this is for every x in R k when you multiply vec vector by a scalar then the length gets multiplied by the absolute value which is for every alpha in R and for every x in R k.

And the third is the property of this effect of this notion of length and the basic operations of addition x plus y is less than or equal to x plus y, the length of x plus the NORM of x plus NORM of y for every x y in R k. This is called the triangle inequality, we shall not get involved in a proof of this, this follows from what is known as the Cauchy Schwarz inequality, we should we shall look at it at a later time, but it follows from the notion of a Cauchy Schwarz.

So, in other words we we have on R k, we have the inner product, the inner product in turn induces the notion of a length or the NORM and this obeys all these standard ideas that we have about length, that length should be non negative, it should become 0 only when the vector is 0 and length should get dilated by the absolute value of the scalar multiplying it and the triangle inequality is satisfied.

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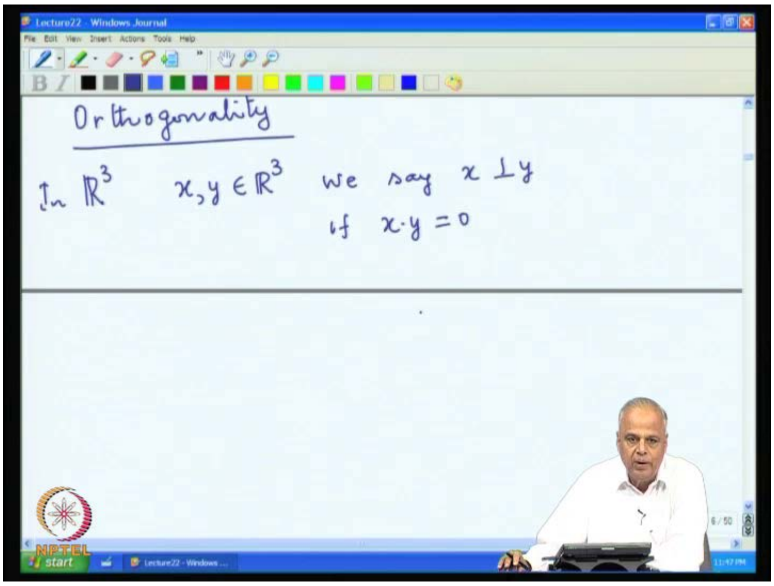

There is 1 more effect of the idea of inner product which is the notion of orthogonality. This is where the inner product wings in the orthogonal geometry. So, we should now, look at the next implication of the inner product. The first implication of the inner product is that induces the notion of a length. The second interpretation or the second influence of the inner product is the notion of the orthogonality. So, what do we mean by it, again look at R 3. So, in R 3 in our vector calculus or vector algebra course we learn that, if x and y are in R 3, we say x is perpendicular to y or we have x is perpendicular to y, if x dot y is equal to 0 which x dot y is what we are generalized x comma y that inner product.

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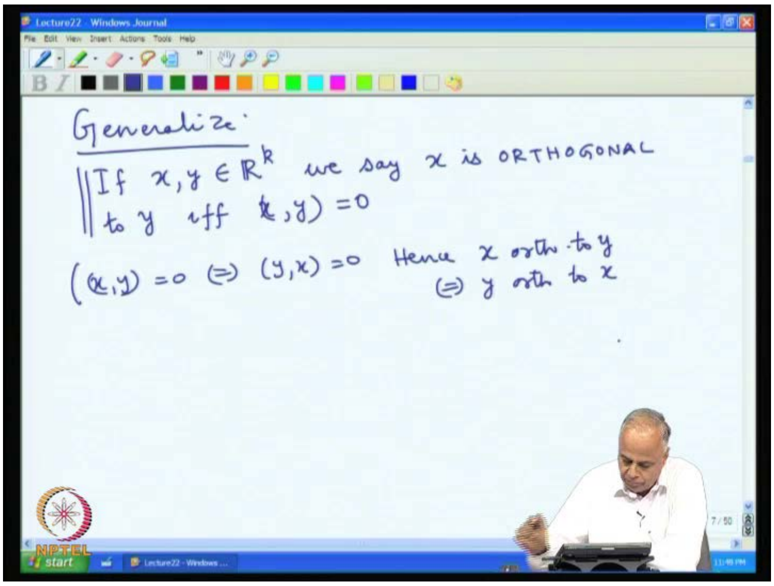

So, we generalize this notion to say that in R k, x is set to be orthogonal to y, if the inner product x with y is 0. So, we generalize this idea to the following, if x y belong to R k, we say x is orthogonal to y, if x comma y equal to if and only if x comma y is equal to 0, recall that the inner product of the 2 vectors a number, that numbers must become 0, the inner product of 2 vector is 0, we says x is orthogonal is to 0. Now, if x y is 0, y x is also 0. So, x is orthogonal to y, y is automatically orthogonal to 0. So, x, y is equal to 0, if and only if, y, x is equal to 0. Hence, x is orthogonal to y, if and only if, y is orthogonal to x and that is why we would not say, x is orthogonal to y, y is orthogonal x, we simply say x and y are orthogonal.

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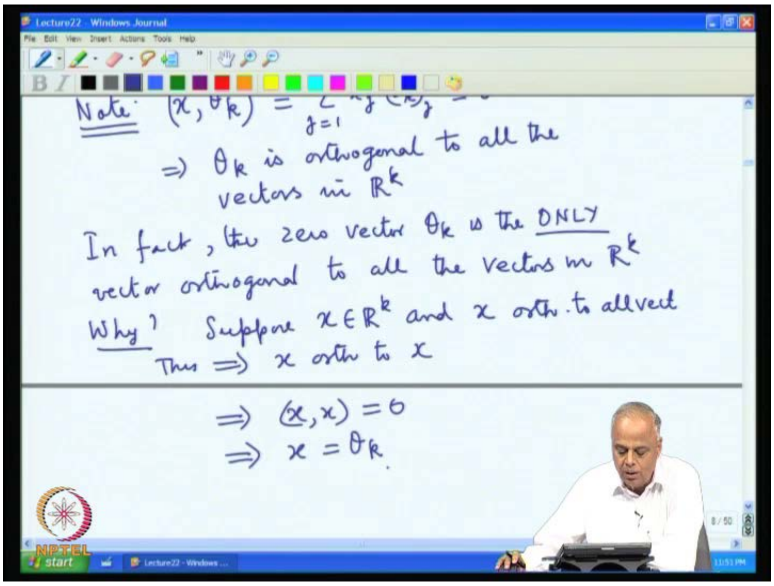

Now, note x comma theta k for any x in R k, what is the x comma theta k, we have to look at j equal to 1 to k, x j and j th component of theta k multiply. Now, but j th component of theta k also 0 and therefore, that is 0. So, that implies theta k is orthogonal to all the vectors in R k. Now, in fact this is the characterization of the 0 vector because this is the only vector which is all orthogonal to all the vectors in fact, theta k the 0 vector theta k is the only vector, orthogonal to all the vectors in R k.

This is only vector no other vector can the orthogonal why suppose x is in R k and x is orthogonal to all vectors, if x is orthogonal to all the vectors this implies x is orthogonal to itself, because it is orthogonal to all the vectors in particular, it must be orthogonal to itself, but if x is orthogonal to itself the inner product of x is with itself must be a 0, but we know that the inner product of the vector with itself is 0 only when x is equal to theta k. Therefore, the only vector which is the orthogonal to all vectors is the 0 vector.

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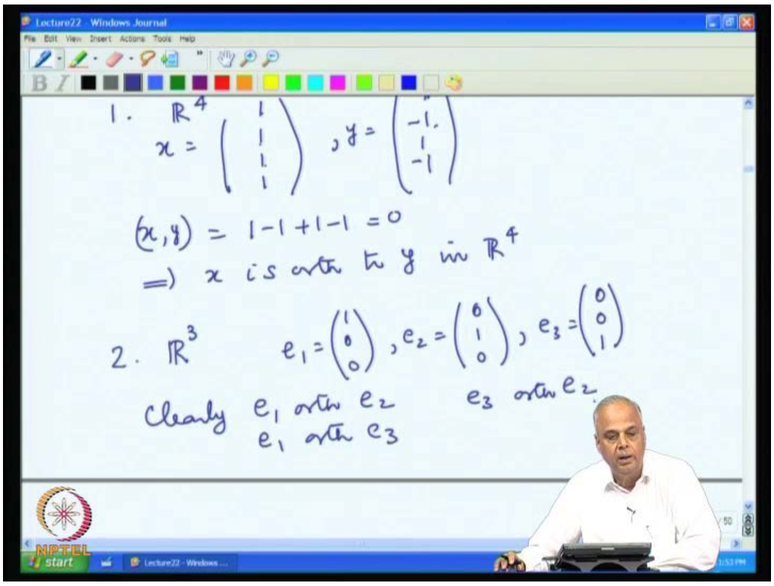

Let us look at some simple examples of this orthogonality format. Let us consider R 4, it say the vector x equal to 1 1 1 1 and y equal to 1 minus 1 1 minus 1 clearly, we have x comma y is x 1 y 1 which is 1, x 2 y 2 which is minus 1, x 3 y 3 which is 1, x 4 minus 4 x 4 y 4 which is minus 1. So, it is going to be 1 minus 1 plus 1 minus 1 which is 0 which implies x is orthogonal to y in the vector space R 4. Let us look at another 1 example. Let us look at R 3 and let us look at x 1 are let use as a standard symbol, let us call it as e 1 which is 1 0 0 and e 2 which is 0 1 0, e 3, 0 0 1. Then clearly e 1 is orthogonal to e 2, e1 is orthogonal to e 3, e 3 is orthogonal to e 2 because all the inner products are 0.

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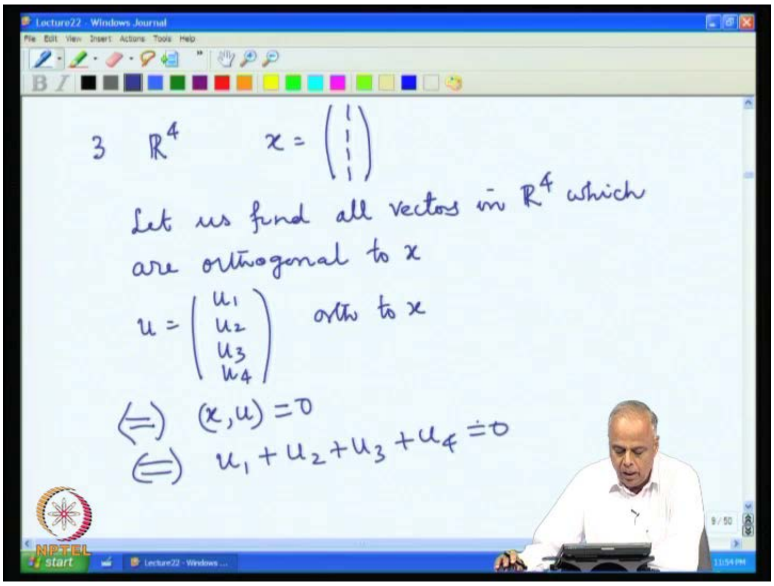

Next, look at another simple example consider R 4, consider the vector x equal to 1 1 1 1. Now, let us find all those vectors which are perpendicular to x. So, let us find all vectors in R 4 which are orthogonal to x. Now, let us take a vector the u which is u 1 u 2 u 3 u 4. Suppose, u is orthogonal to y x, this can happen only when the inner product of x and u is 0, but what is the inner product of x and u, it is u 1 plus u 2 plus u 3 plus u 4 so, this happen if and only if, u 1 plus u 2 plus u 3 plus u 4 equal to 0. So, therefore, a vector to the orthogonal to x, if and only if the sum all of all its components equal to 0 that means, u must be of the form alpha beta gamma since, sum of the 4 components must be a 0, the fourth component must be alpha minus beta and thus there alpha, beta, gamma are real numbers. So, this is the this is the set of all vectors which are orthogonal to x.

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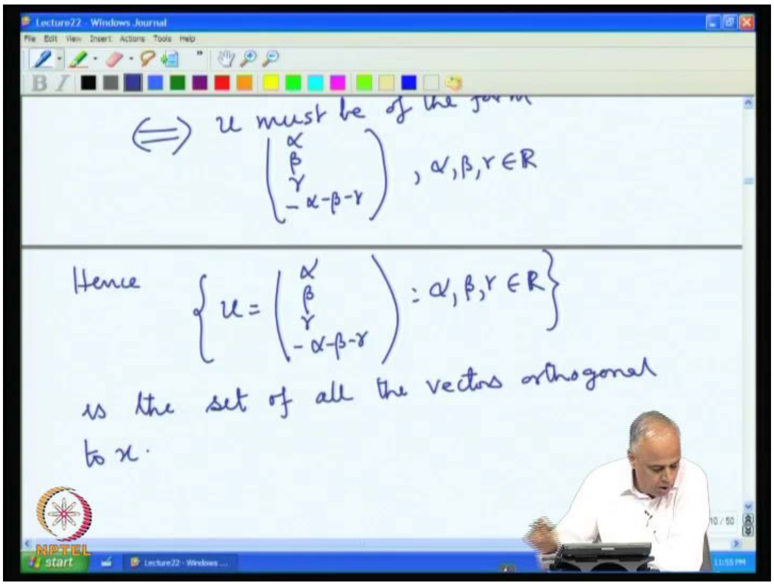

Hence, the set of vectors of the form null quality use in (( )) called u, u equal to alpha beta gamma, minus alpha minus beta minus gamma were alpha beta gamma are real numbers. This collection of vectors is the set of all the vectors orthogonal to x. Let us now, pursue this idea not dose to finding the vector which all are perpendicular to given vector x, but let us try to find a set of all vectors perpendicular to given set of vectors, instead of just 1 vector.

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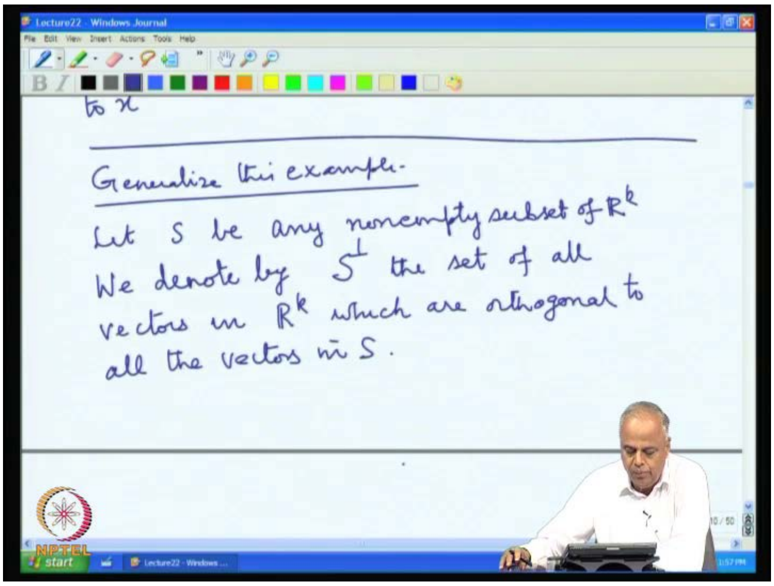

So, what we now do is generalize this example, how do you generalize this example. The way we generalize example is let S be any non empty subset of R k. In the above example, we have taken the set S to be this single vector x, we have just taken the S to be the single vector. Now, what we are going to do is taken arbitrary non empty subset of R k and then we denote by S perp with the super script perp, the set of all vectors in R k which are orthogonal to all the vectors in S.

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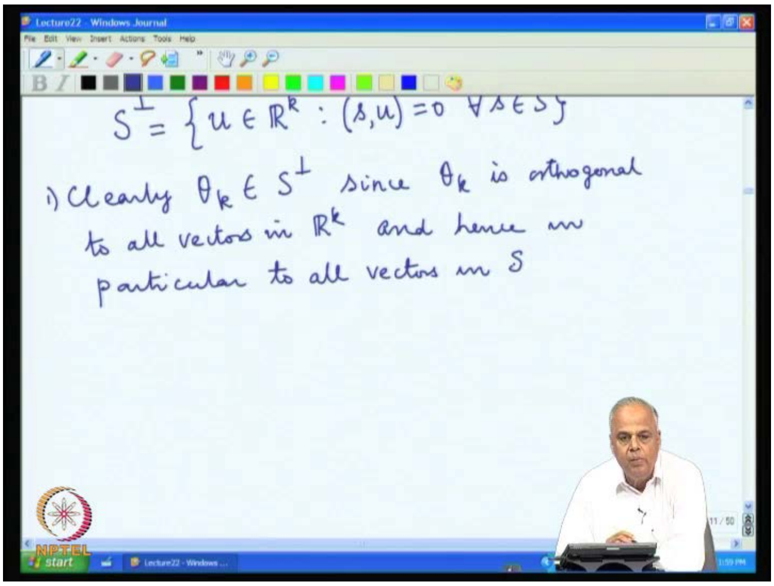

Therefore, in symbolically we can write S perp is the set of all the vectors u in R k, such that u must be orthogonal to every vector S. So, we must have S comma u must be equal to 0 because you must be orthogonal to s and this must happen for every S because it must be a orthogonal to all the vectors. So, it is the set of all u in R k such that, S comma u is 0 for every S in S. This is the collection of all the vectors which all to perpendicular to every 1 of the vectors in S. Now, is this an empty collection or non empty collection will there be any vector at all which is perpendicular to all the vectors in S.

We know that the 0 vector is perpendicular to all the vectors in R k and hence in particular the 0 vector is perpendicular to all the vectors in S, clearly theta k belongs to S perp since, theta k is orthogonal to all vectors in R k and hence in particular to all vectors in S. So, the 0 vector belongs to S perp. Since, S perp is now an empty sub set of R k, the moment we have non empty sub set of R k, we wonder whether it will be a sub space.

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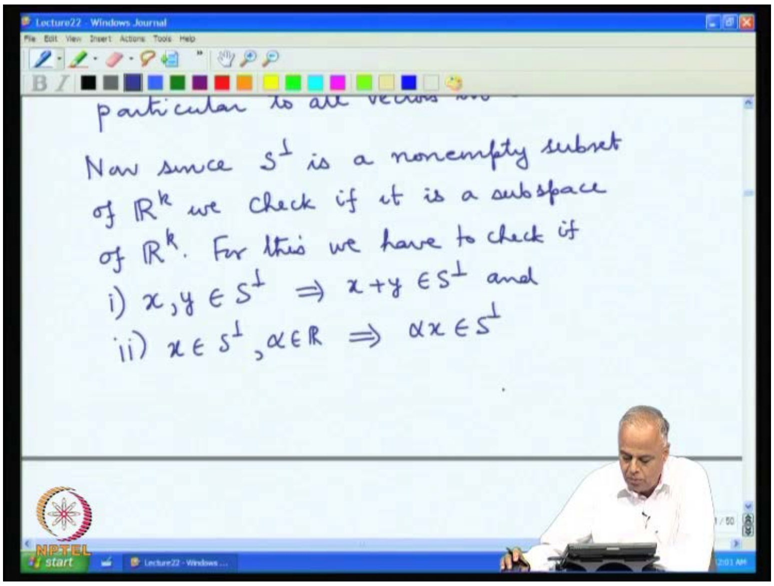

Now, since S perp is a non empty sub set of R k, we check, if it is a sub space of R k. Now, it will be a sub space of R k, remember any sub set of R k will be a sub set sub space of a R k, if it is non empty which is given then it is close with the respect to the 2 basic operations of an addition and scalar multiplication. So, for this we have to check, if x and y belong to S perp implies x plus y belong to S perp and that is closure with respect to addition; 2, if x belongs to S perp, alpha is any real number by the this implies alpha x belongs to S perp. So, we have to check whether S perp is close with respect to addition and close with respect to scalar multiplication.

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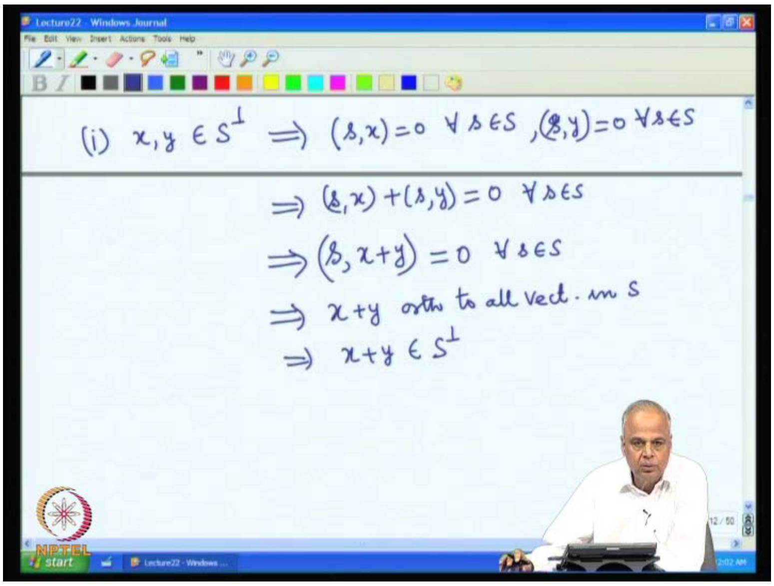

So, let us look at 1, we are given x, y in S perp what does that mean, x is in S perp which means, x is orthogonal to all the vectors in S that means s, x is equal to 0 for every s in S. Similarly, y is in S perp so, y s, y is equal to 0 for every s in S. So, s, x must be equal to 0 for every s in S, s y must be equal to 0 for every s in S. Now, we add, we get s, x plus s y is 0 plus 0 so, it is 0 for every s in S. Now, we know that the inner product is right distributive and left distributive. So, we can write this as s of x plus y is 0 because s of x plus y, s comma x plus y, s comma x plus s comma y, so this is for every. This says, the vector x plus y is orthogonal to all vectors in S that is exactly mean the meaning of the fact that x plus y belongs to S perp. Therefore, we have seen that, whenever x and y belong to S perp, x plus y also belongs to the S perp. This shows that S perp is closed under addition.

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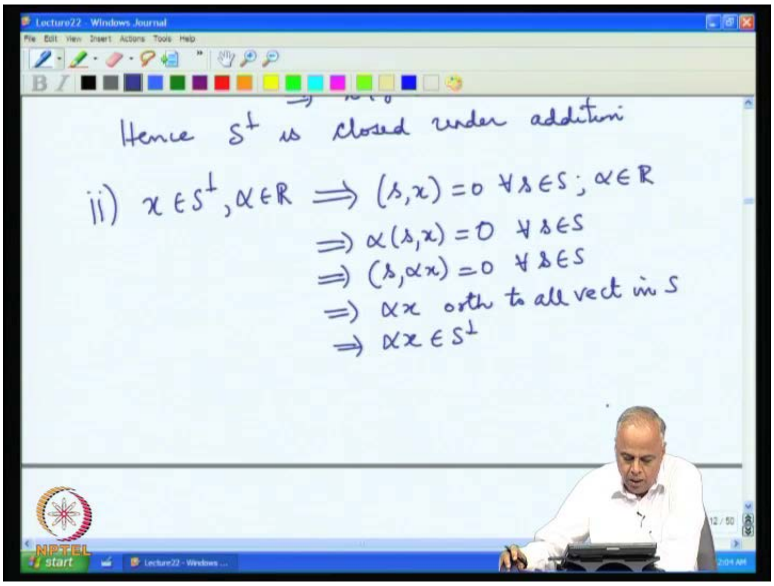

So, hence S perp is closed under addition. 2, we have to now check whether it is closed under scalar multiplication. So, we are given x in S perp, alpha a real number, what is that implies, x in S perp again says x must be a orthogonal all the vectors in s and alpha is in R. Now, s comma x is a real number, alpha is a real numbers. So, we can multiply and on the right hand side, we get alpha into 0 0. Now, we see we know that a constant can be pulled in and out of the inner product from any 1 of the factors. So, this is the same as s of alpha x is equal to 0 for every s in S. This is the same thing as saying that alpha x is orthogonal to all vectors in S that the same thing as saying alpha x belongs to S perp and hence S perp is closed under scalar multiplication.

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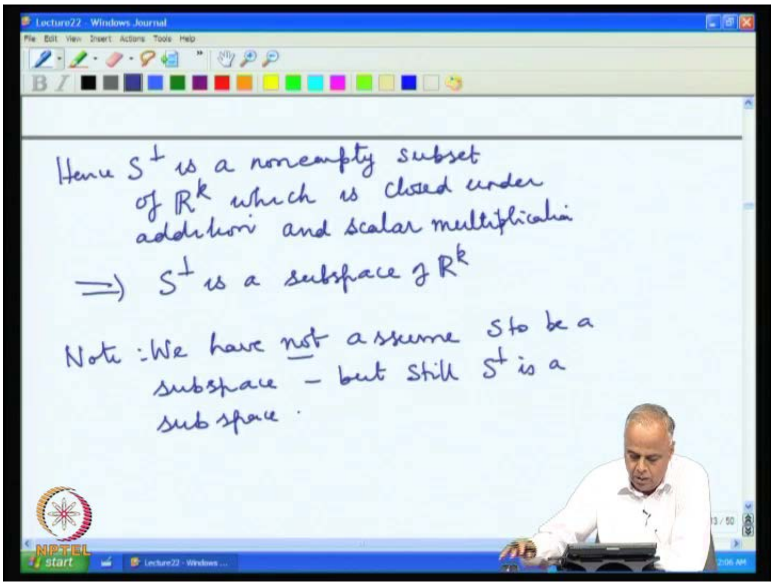

So, what we have seen is that, the S perp is a non empty subset non empty subset of R k which is closed under addition and scalar multiplication; this means the S perp is a subspace of R k. So, what we are done is, we have started it an arbitrary non empty subset S and we concluded S perp, the collection of all vectors which are perpendicular to all the vectors in S must be a sub space. Note that, we are not assume that the original set S was a sub space, irrespective of whether the original set is the sub space sub space or not, the perp S perp will always be a sub space. Note, we have not assume we have not assume S to be a sub space, but still S perp is a sub space.

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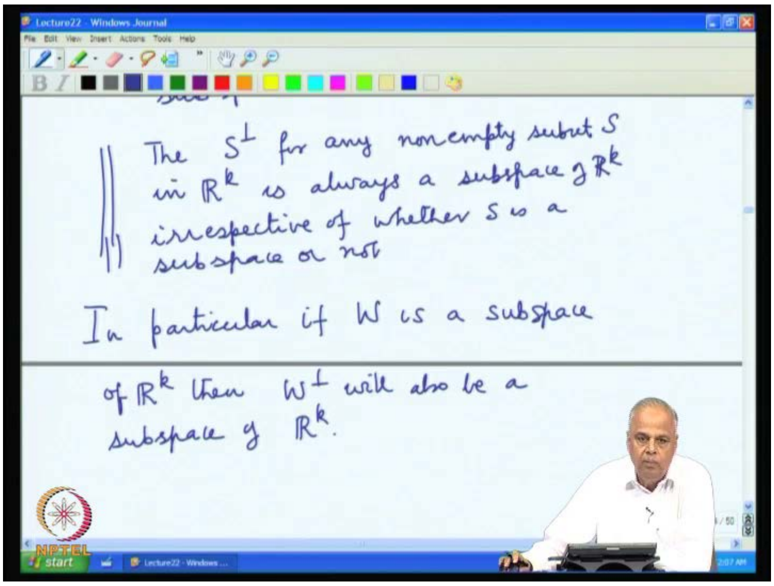

so, the conclusion is the S perp for any non empty subset S in R k is always a sub space of R k irrespective of whether S is a sub space or not. So, whether is a sub space or not, S perp is always a sub space of course, in particular, if W is a sub space of R k then W perp will also be a sub space. We shall be looking at such perps of sub spaces, when we analysis a matrix.

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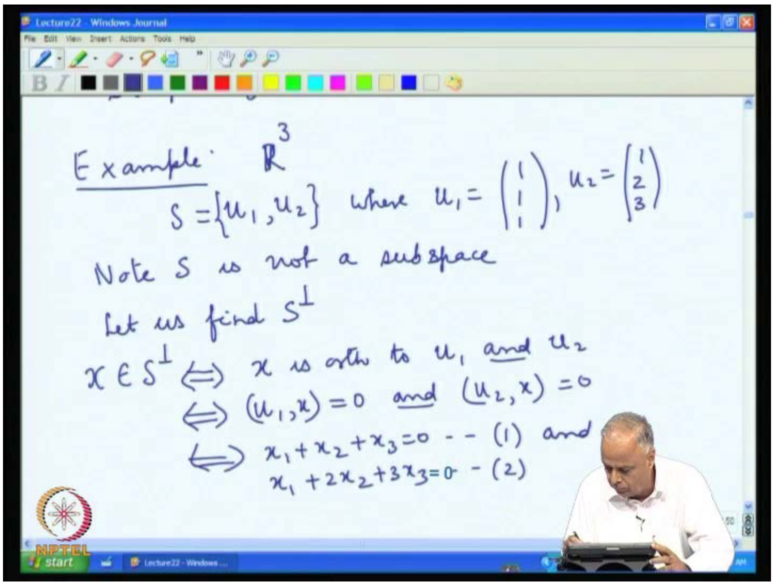

Let us, look at (( )) simple examples. Let us consider v to be R 3. So, in other words, we look at the sub space the vector space R 3. We look at the vector space R 3 and let us take the set S consisting of 2 vectors where u 1 is 1 1 1 and u 2 is equal to 1 2 3. Notice that, S is not a sub space because the vector 1 1 1 is there, but multiples of this vectors are not there. So, a note S is not a sub space s is not a sub space. Now, let us find S perp in this case. Now, what is the S perp, S perp is the collection of all the vectors which are perpendicular to all the vectors in S.

So, suppose a vector x belongs to S perp that implies and implied by, this can happen if and only if, x is orthogonal to u 1 and u 2 because a vector gets qualified to be a S perp only when it is orthogonal to every 1 of the vectors in S that is if and only if, its inner product with u 1 must be 0 and its inner product with u 2 must be 0. Now, x is x 1 x 2 x 3, the inner product with u 1 will give me x 1 plus x 2 plus x 3 as a 0 and its inner product with u 2 will give x 1 plus 2 x 2 plus 2 3 x 3. Let us now, call this equation as 1 and call this equation as 2. So, x 1, x 2, x 3 must be such that both this equations are satisfied.

Therefore, a vector x qualifies to the in S perp if and only if, its components x 1, x 2 x 3 satisfy this condition.

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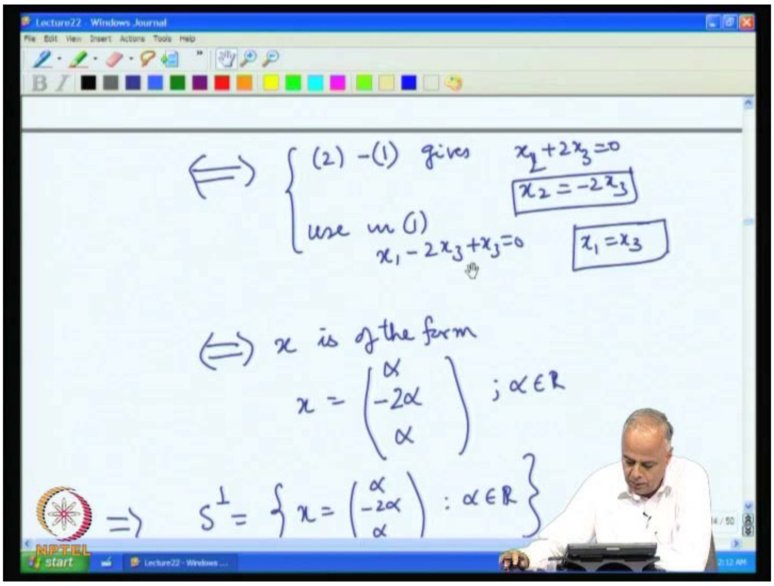

Now, there are 2 equations for these 3 coordinate and therefore, we can eliminate 2 of these variables. So, what does this tell, if we now subtract the first equation from the second equation gives x 2 plus 2 x 3 x 2 plus 2 x 3 is 0 or x 2 is equal to minus 2 x 3, if we now use this in 1, we get x 1 minus 2 x 3 plus x 3 as 0 which gives x 1 equal to x 3. So, we have to have x 1 to be equal to x 3, x 2 be to minus 2 x 3 and x 3 can be chosen an arbitrary.

So, if and only if, x is of the form, x is equal to if we choose x 3 as alpha, x 1 as to be chosen as a alpha and x 2 as to be chosen as alpha minus 2 alpha and therefore, we get S perp consists of all these vectors which are of this form the alpha and this is clearly a subspace and that can be easily verify. So, for this collection of vectors S which which for which had here, this set of vectors the corresponding S perp is given by this subspace of vectors, the S perp is always a subspace.

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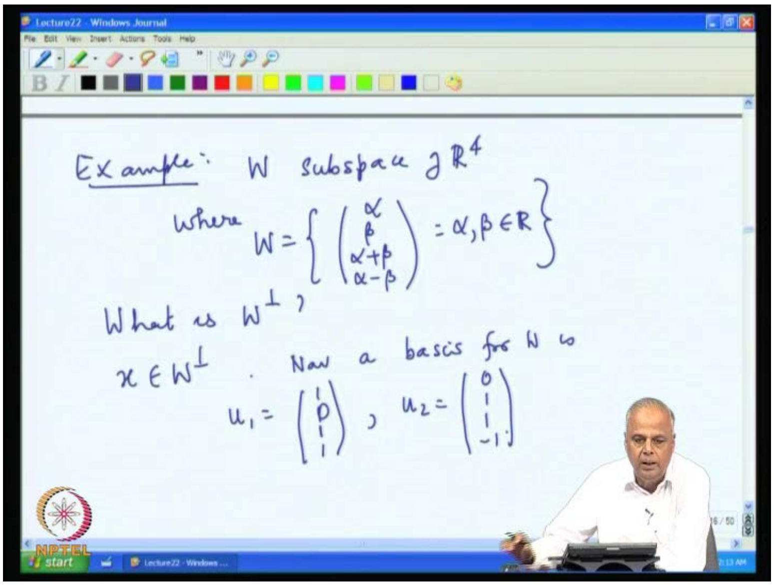

Now, let us consider a sub space W in R 3, W sub space of R 4 where W is defined to be the collection of all vectors of the form alpha, beta, alpha plus beta, alpha minus beta, alpha beta belongs to R. Now, what is the W perp in that in this case? Now, let us say that, we want to find x belonging to W perp. Now, what does it mean to say, that x is perpendicular to all the vectors in W. Now, suppose x is perpendicular to the basis vectors in W then it will be automatically perpendicular to all the vectors in W, lets verified this. Now, a basis for W is u 1 equal to 1 1 1 1 0 1 1 this is taken by, this is got by taking alpha equal to 1 and beta equal to 0 and now, taking alpha equal to 0; beta equal to 1, we get these 2 vectors are linear independent R and W and any vector is the form of alpha u 1 plus beta u 2. So, therefore, this forms a basis.

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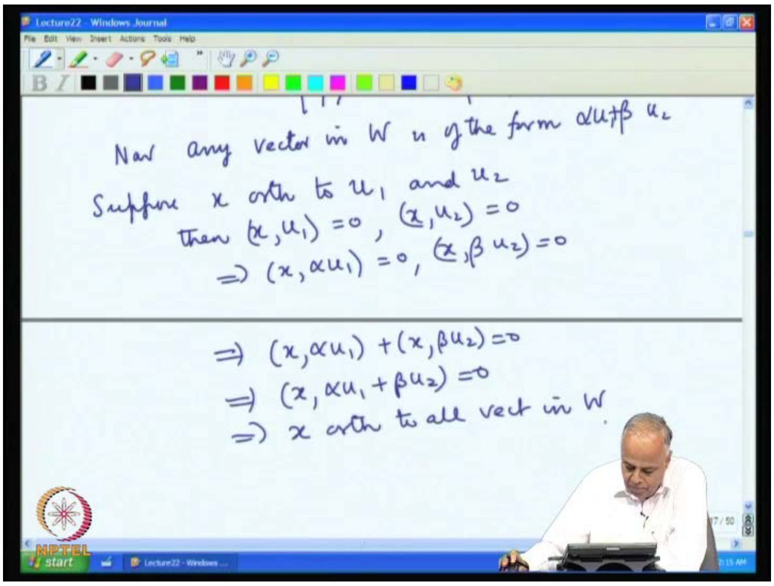

Now, any vector in W is of the form alpha u 1 plus alpha u 2, there alpha u 1 plus beta u 2. Now, suppose x is perpendicular to u 1 that is suppose x is orthogonal to u 1 and u 2 then x comma u 1 equal to 0; x comma u 2 equal to 0 and that implies x comma alpha u 1 is 0; x comma alpha u 2 is 0, because we can pull out a constants in and out, we can take a different constant here, beta if you want and then that will say if we add both of them, x of alpha u 1 plus x of beta u 2 equal to 0 and using the distributivity we get alpha u 1 plus beta u 2 equal to 0 which means x is orthogonal to all vectors in W and therefore, to check whether a vector is orthogonal to all the vectors in W, it is enough if it is ortho check whether it is orthogonal to the basis vectors.

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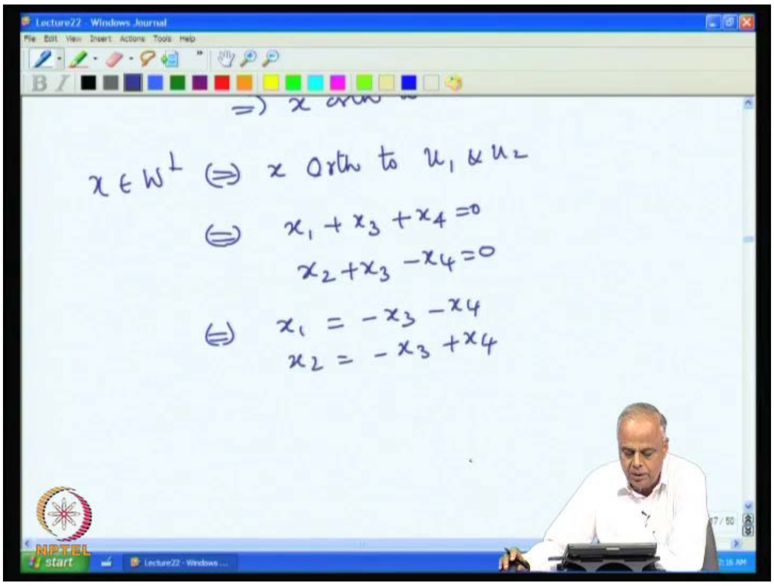

So, x is belongs to the W perp if and only if, x belongs x is orthogonal to u 1 and u 2, if and only if the orthogonality with u 1 will means x 1 x 1 plus 0 times x 2 plus x 3 plus x 4 is equal to 0. So, therefore, if and only if x 1 plus x 3 plus x 4 is equal to 0. Now, orthogonality with x 2 will give me x 2 plus x 3 minus x 4 is equal to 0. So, x 2 plus x 3 minus x 4 is equal to 0. Now, the first equation gives me x 1 equal to minus x 3 minus x 4, second equation gives me x 2 equal to minus x 3 plus x 4. So, it says x 3 and x 4 can be chosen arbitrarily then x 1, x 2 has to be chosen like this.

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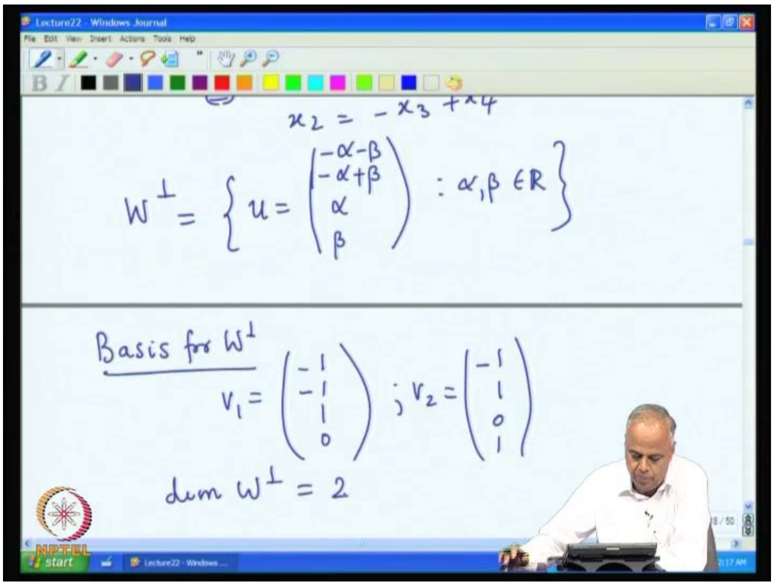

So, we get W perp is equal to the set of all vectors of the form u equal to, if we choose if we choose x 3 as alpha, x 4 is beta then x 2 as to be chosen as minus alpha plus beta and x 3 as minus alpha minus beta there alpha beta belong to R. And now, it is easy to verify that this is a sub space of R k. Now, words are basis, W perp being a sub space a basis for W perp is given by V 1 equal to, if we take alpha equal to 1 and beta equal to 0, we get minus 1 1 1 0 and V 2 by taking alpha equal to 0 and beta equal to 1. So, this forms the basis for w perp. So, the dimension of W perp in this case is 2.

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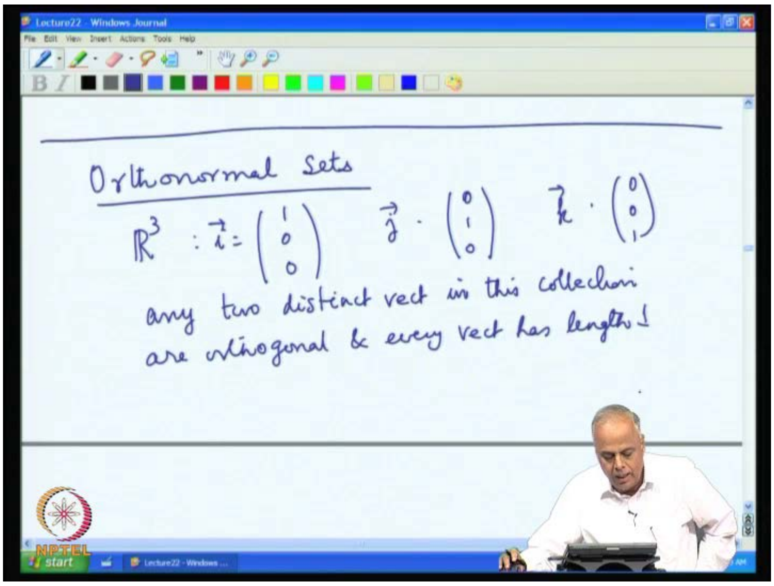

So, we have this notion of the perp of any arbitrary set and the perp of any arbitrary set is automatically a sub space of R k and in particular, the perp of a sub space of is always a sub space. Now, we would like to generalize the notion of i j k vectors which leads as to the notion of orthonormal sets. So, if we now look at the R 3 space then take the vectors i which be vector space we denote by i whose components are 1 0 0, in the vector j which we denote by 0 1 0 and the vector k which be denote by 0 0 1. These vectors have the property any two distinct vectors in this collection are orthogonal and every vector has length 1, when we generalize this notion we get the notion of and orthonormal set in a R k.

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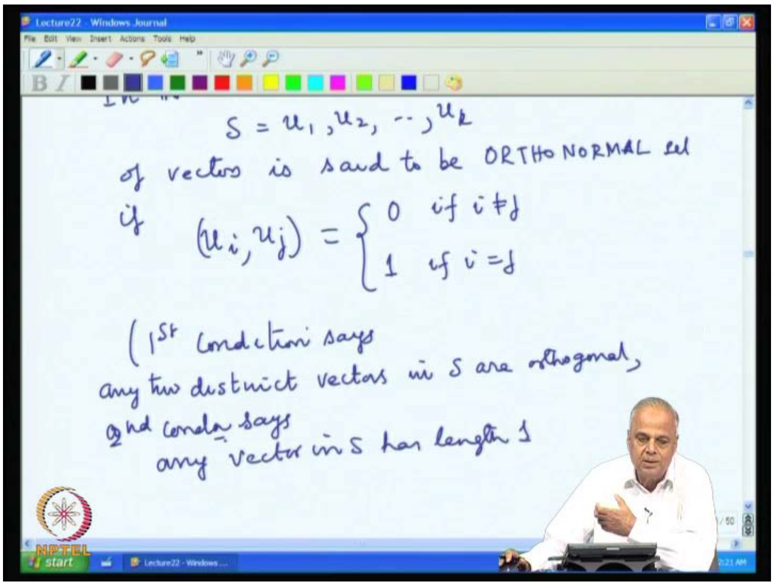

So, we have in R k, a set s equal to u 1, u 2, u k of vectors. So, if we take a set of vectors in R k is set to be orthonormal set, if if you take u i comma u j, the inner product between any two vectors, this must be equal to 0, if i not equal to j, 1 if i equal to j. Now, what does the first condition say, the first condition says i not equal to j that mean the distinct vectors then the inner product is 0 means there orthogonal. So, distinct vectors any 2 distinct vectors in S are orthogonal.

The second condition say if we take a vector and inner product with itself we get the square of the NORM, the NORM is 1 any, the second condition says that any vector in S has length 1. Our idea is to use this generalization of i, j, k vectors to similarly, generate a orthonormal basis analogous to the i, j, k basis for the R 3 to a general R k and use this construction of orthonormal basis to analysis our problem on the matrices. And the next lecture, we will look at the important notion of an orthonormal basis.