Advanced Matrix Theory and Linear Algebra for Engineers

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Module No. # 06

Lecture No. # 21

Linear Transformations- Part 5

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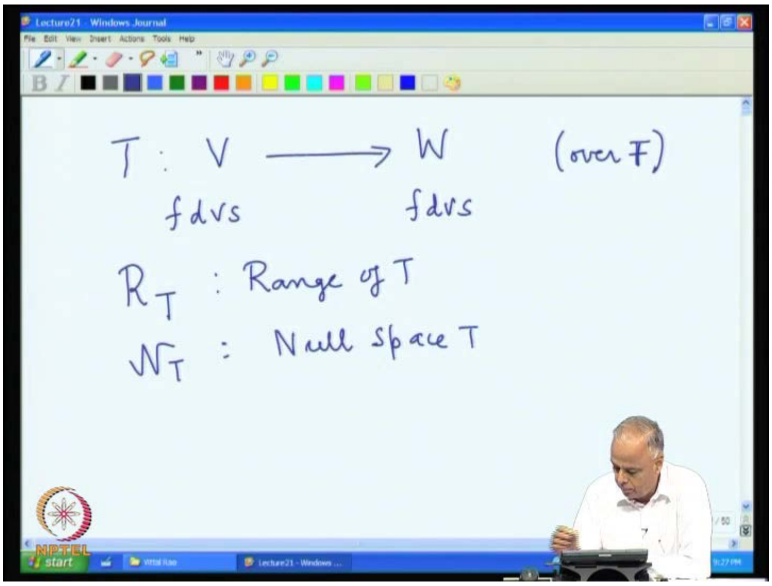

We have been looking at linear transformations T from a finite dimensional vector space V to a finite dimensional vector space. So, let us say V is a finite dimensional vector space, W is a finite dimensional vector space and we have a linear transformation from V to W. We assume that both these vector spaces or vector spaces over a field f. We looked at two fundamental subspaces connected with T. One is R T the range of T and the other was the null space of T which we denoted by N of T.

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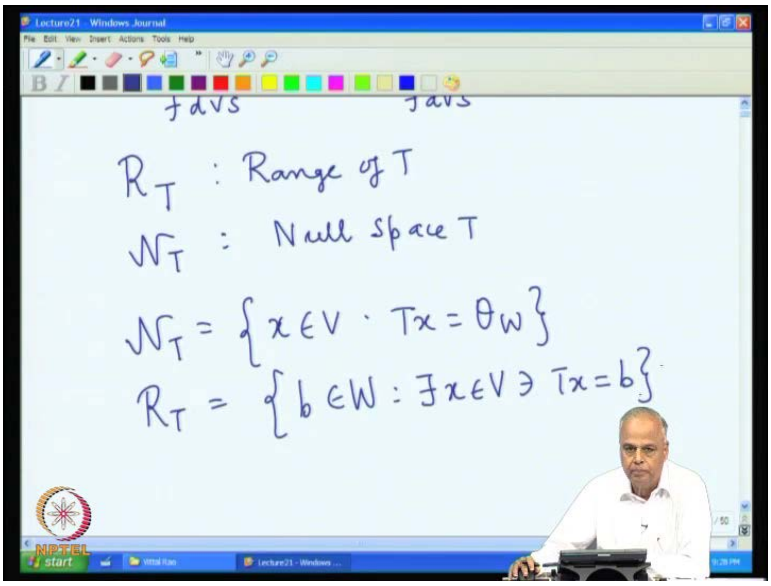

If you recall we defined n of T to be all those vectors in V which get mapped to the 0 vector in W and the range of T was all those vectors in W for which we can find a pre image in V such that T maps the pre image to b.

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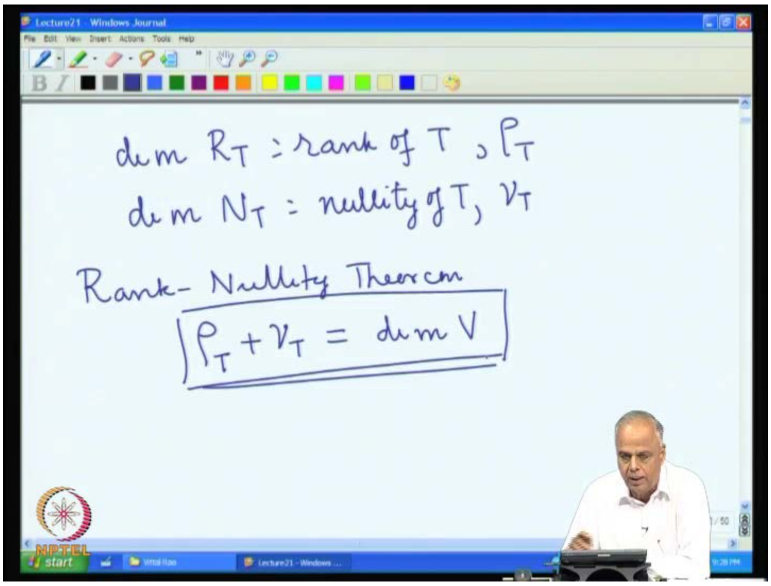

We defined the dimension of the range of T as the rank of T and denoted it by rho T and dimension of the null space of T as the nullity of T and denoted it by V T and we had the fundamental theorem called the rank nullity theorem. The rank nullity theorem gave us that the rank of T plus the null space of T is equal to the dimension of the domain space

V. This is one of the most important theorems for linear transformations and this gives us many information about how to construct basis for the range of T and the null space of t.

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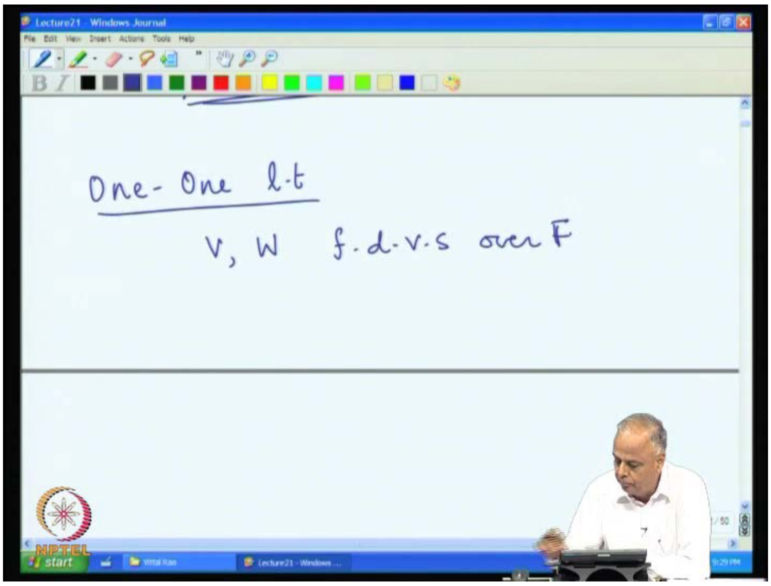

We then introduced the notion of one, one linear transformations. Once again let us consider V W to be finite dimensional vector spaces over a field air.

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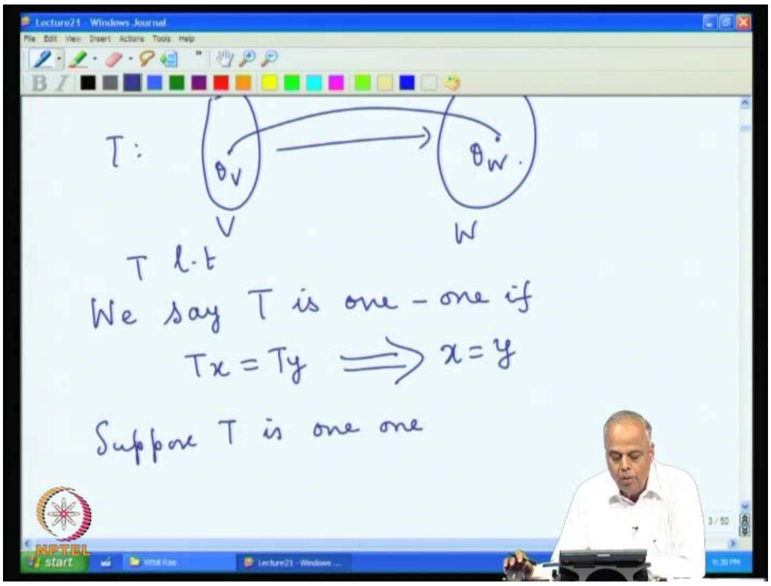

And let us take a linear transformation T from V to W. So, T is a linear transformation. So, we have a linear transformation T from a finite dimensional vector space V to W and we say T is one one if distinct vectors get mapped to distinct vectors. That is if the two images are same then the corresponding pre images must be the same. T x equal to T y if and only if x is equal to y. Then we say T is one one.

For one one transformation; suppose T is one one then we already have the 0 vector getting mapped to the 0 vector of W. Any linear transformation mapped 0 vector to the 0 vector. So, we already have the 0 vector getting mapped to the 0 vector. Now, because the map is one one; no other vector in we can get mapped to the 0 vector.

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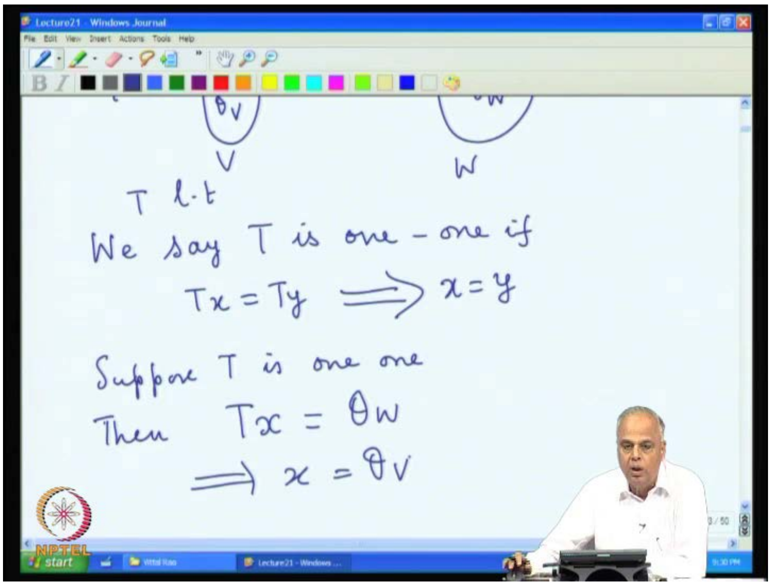

So, thus we have whenever a when a vector gets mapped to 0 vector then x must be equal to theta V because no other vector can get mapped to the 0 vector.

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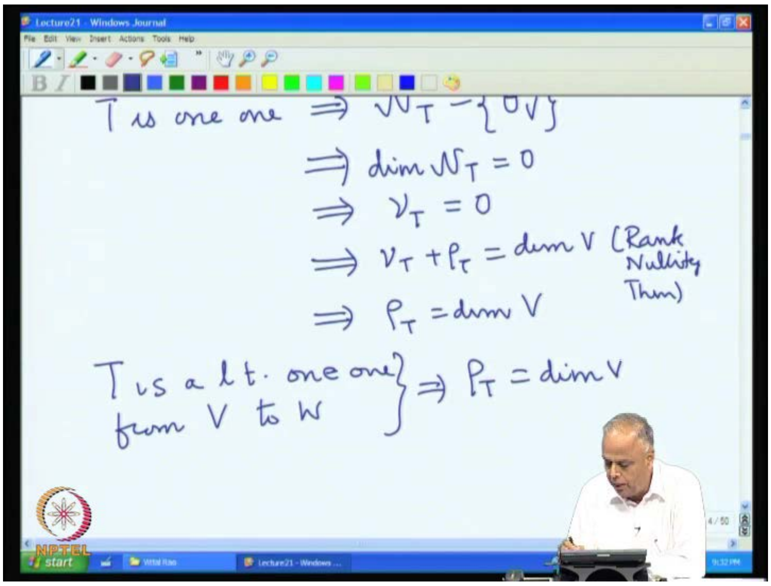

This means that whenever T is one one that implies the null space of T must consists of only the 0 vector because only the 0 vector V gets mapped to the 0 vector of W. This immediately tells us that the dimension of the null space of T must be 0. But, the dimension of the null space of T is what we call as the nullity of T and denoted it by nu T and hence nu T must be equal to 0.

But the rank nullity theorem told us that the nullity of T plus the rank of T is the dimension of v. So, we have nu T plus rho T is equal to dimension of V null. This is the rank nullity theorem and since nu T is 0 we have rho T is equal to dimension of V. And therefore, T is a linear transformation one one from V to W. This implies that the rank of T must be equal to dimension of v. So, that the rank of a one one transformation is always equal to the dimension of the domain space. The domain space here is v.

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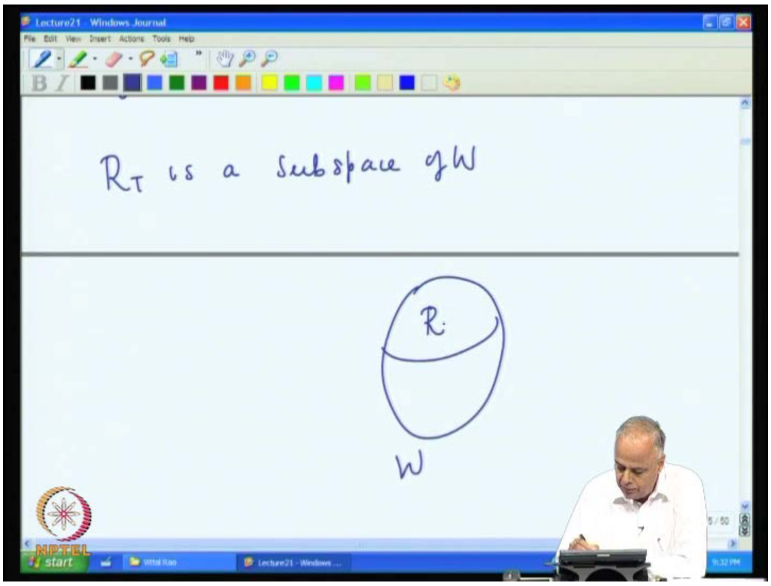

But the range of T is a subspace of w. So, we have W here and the range of T is sitting inside that.

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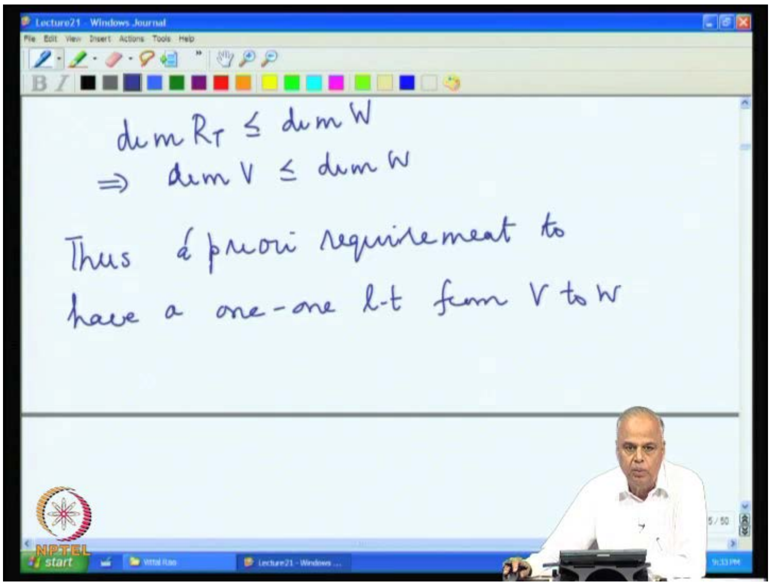

And therefore, the dimension of the range of T must be less than or equal to the dimension of w, but, we have just found that if T is one one; the dimension of range of T is equal to the dimension of v. So, the dimension of V must be less than or equal to dimension of w.

So, therefore, if the dimension of V is more than the dimension of W; we cannot expect to have a one one linear transformation from V to w. So, thus a priori requirement to have a one one linear transformation from V to W. Recall we are assuming all are finite dimensional.

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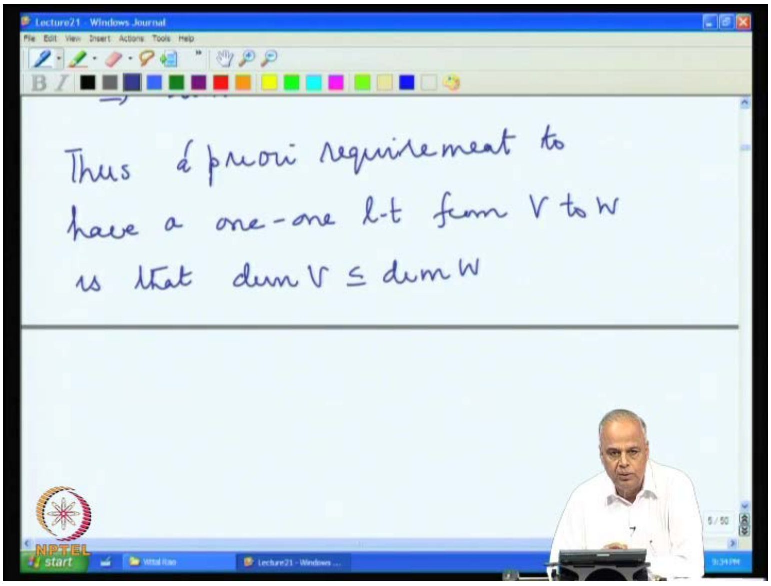

V to W is that the dimension of V be less than or equal to the dimension of W. For any chance dimension of V is greater than dimension of W there is no hope of finding a linear transformation from V to W which is one one.

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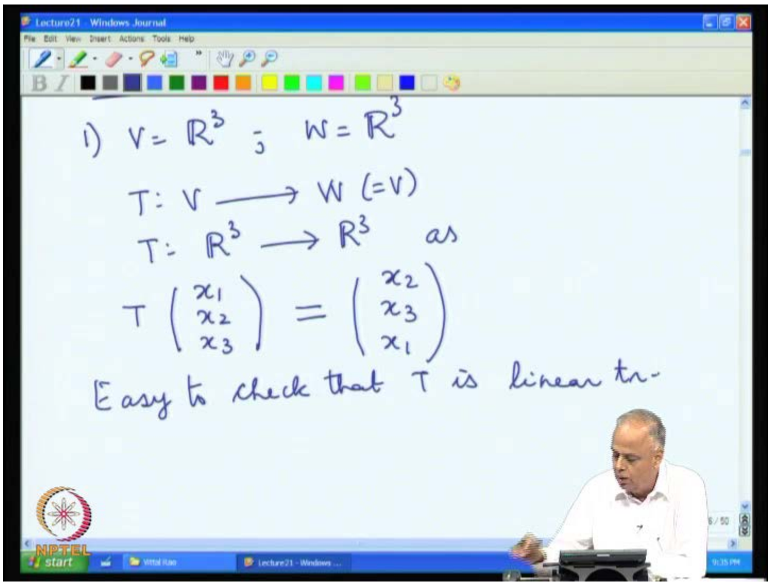

Let us look at some simple examples. Let us take V to be R 3 and W also to be R 3, the usual 3 dimensional vector spaces over the real number. Now, let us define a mapping from V to V now W in this case is equal to V. So, we are looking for a mapping from R 3 to R 3. We now define this T as it takes the vector, any vector in R 3 is of the form x 1 x 2 x 3. So, it takes the vector x 1 x 2 x 3; it should take it to another vector in a R 3. The new vector the, that is x 2 x 3 x 1. So, it sort of a permutation of the coordinates of the vector x.

Now, easy to check that T is linear transformation. So, it is easy to check that T is a linear transformation. We have only to see the T preserves addition that is T of x plus y is T of x plus T of y and T preserves scalar multiplication that is T of alpha x is alpha T x.

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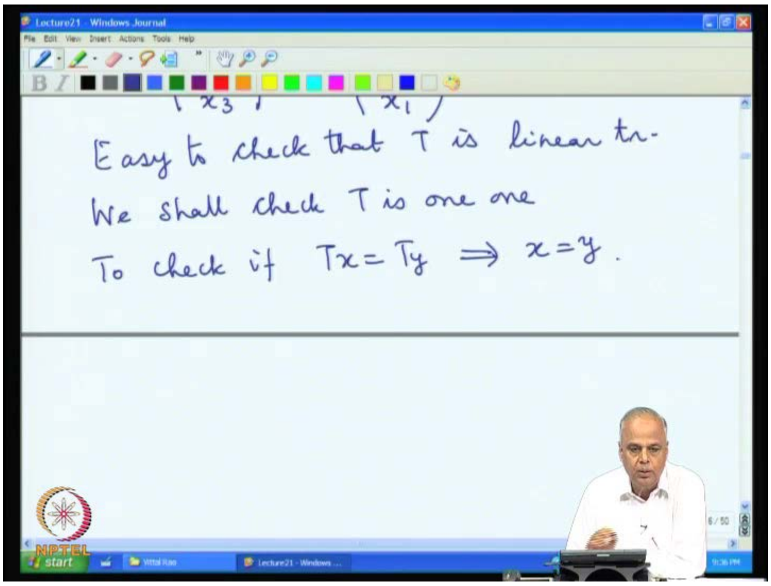

Now, we shall check that T is one one. We shall check T is one one. What do we have to check? We have to check if T x equal to T y implies x is equal to y. This is what we have to check. So, let us check that.

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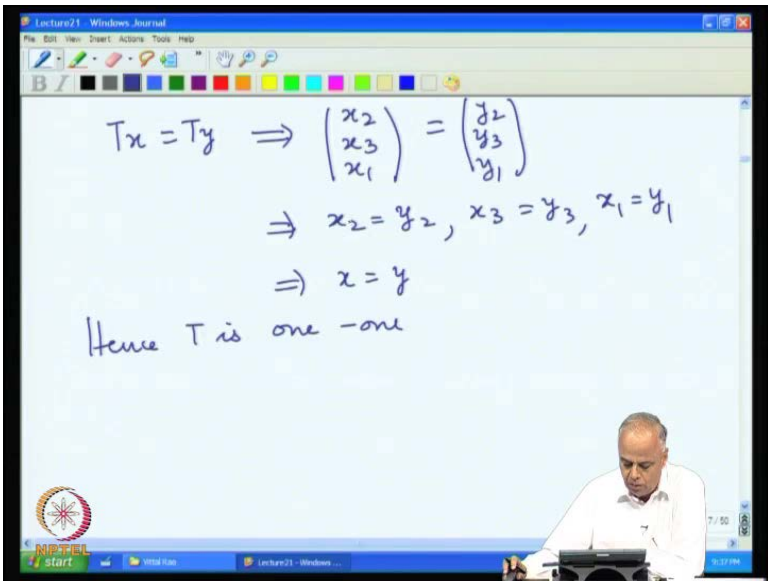

Now, what is T x equal to T y mean? T x means the vector x 1 x x 2 x 3 is taken to the vector x 2 x 3 x 1, T y: the y 1 y 2 y 3 vector is taken to y 2 y 3 y 1. Now, comparing these we get x 2 is equal to y 2 x 3 is equal to y 3 and x 1 equal to y 1 which implies the vector x is equal to y. Hence T is one one.

So, this is an example of a one one transformation. Notice here that the dimension of V is 3 and the dimension of W is also 3. Our requirement we said that a priori requirement for having a one one linear transformation is that the dimension of V must be less than or equal to dimension of W. Hear is a case where we have dimension of V is equal to w.

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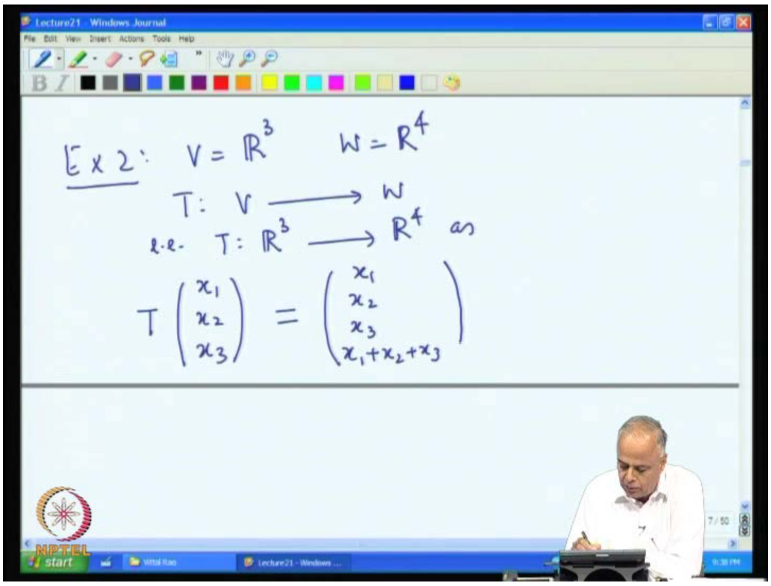

Let us look at another example. Let us take V to be R 3 and W to be equal to R 4. Notice here that the dimension of V is 3 and the dimension of W is 4. In this case the dimension of V is strictly less than dimension of W and it still satisfies the a priori requirement that dimension of V must be less than or equal to the dimension of w.

Now, let us define a linear transformation from V to W. That is T from R 3 to R 4 as follows. What should T do? T should take a vector in R 3. So, any vector in R 3 is of the form x 1 x 2 x 3; it should map it here vector in R4. Suppose, if it maps it to the vector the first three components are the same as the first three components of the vector x. Now it has the first component say which just some sub the first three components.

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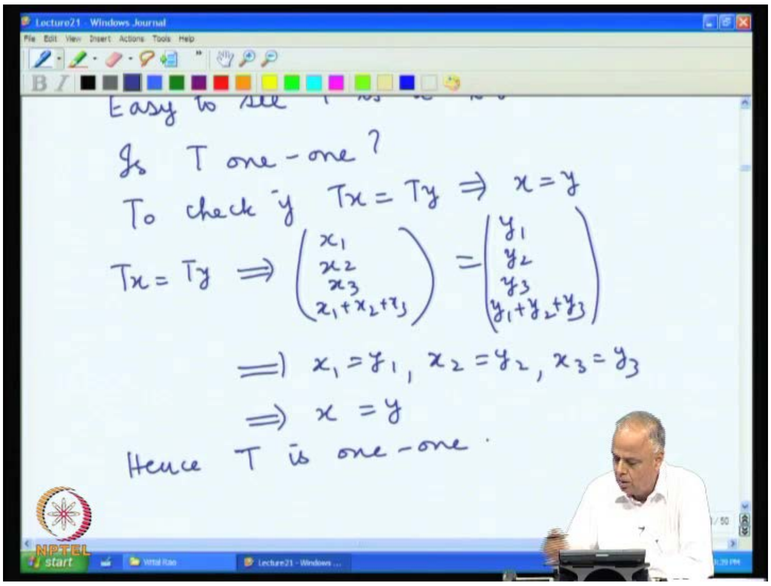

Then once again it is easy to check T is a linear transformation we shall now check whether T is one one. So, is T one one? What do we have to check? Again we have to check if T x equal to T y implies x is equal to y.

So, let us say T x equal to T y. That implies by the definition of T by the definition of T that we have here we have x 1 x 2 x 3 x 1 plus x 2 plus x 3 must be equal to y 1 y 2 y 3 y 1 plus y 2 plus y 3. Then obviously, from this we get x 1 equal to y 1 x 2 equal to x 2 x 3 equal to y 3 which means x is equal to y and hence T is 1. So, thus we can have the possibility exist for a one one linear transformation from V to W if the dimension of V is less than or equal to the dimension of w.

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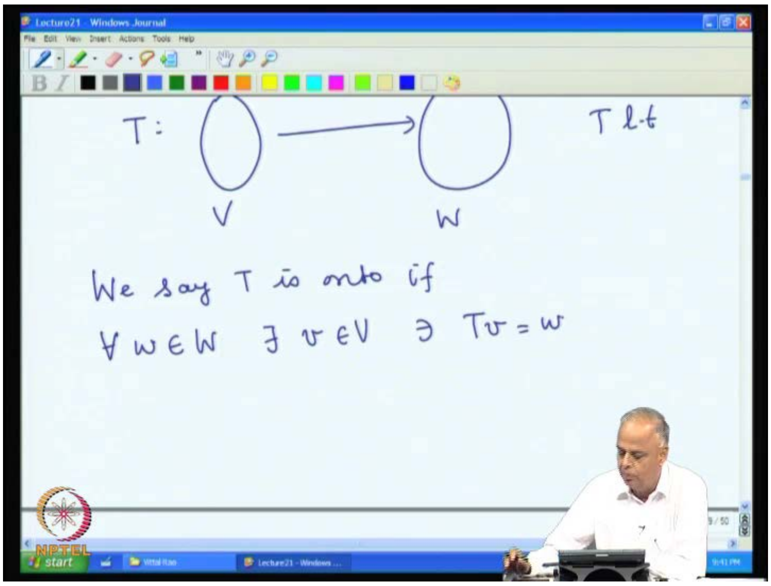

We next introduce the notion of an onto linear transformation. What do we mean by an onto linear transformation? Suppose, we have 2 vector spaces; V and W finite dimensional vector spaces over a field F and then we have a linear transformation which maps V to W T linear transformation.

We say, T is onto if the image fills up the whole space W what does that mean if we take the entire W then every part of W is the image of some vector in V. What do we mean therefore, what we mean is for every W in W; there exists a V in V such that T of V is W.

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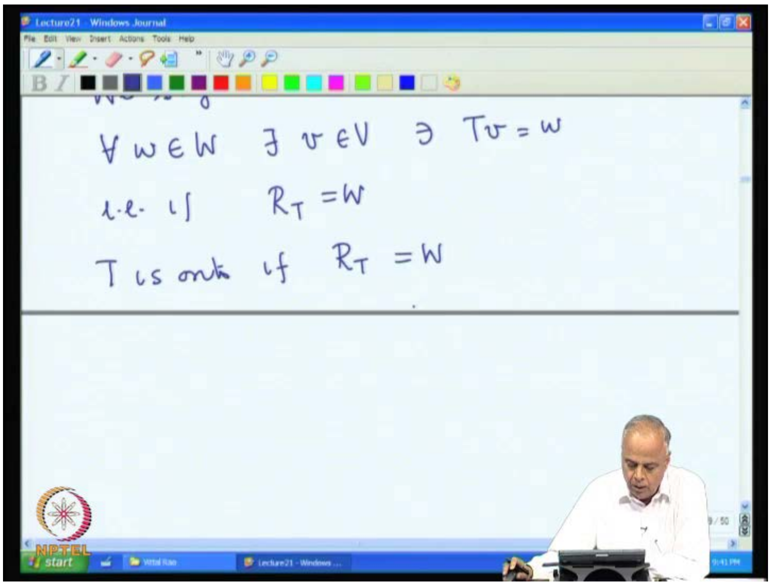

This is the same thing as saying the range of T is all of W. Every vector in W is the image of some vector in v. So, the definition therefore, is T is onto if the range of T is equal to W.

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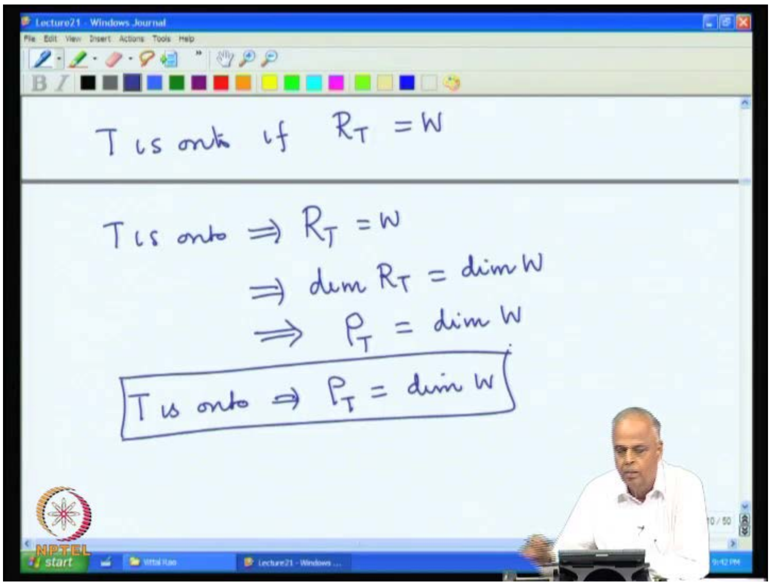

What does this mean? So, therefore, T is onto means range of T is equal to W, range of T is equal to W. Therefore, the dimension of the range of T must be the same as the dimension of W. But the dimension of the range of T is what we called as the rank of T. And therefore, the rank of T must be equal to dimension of W.

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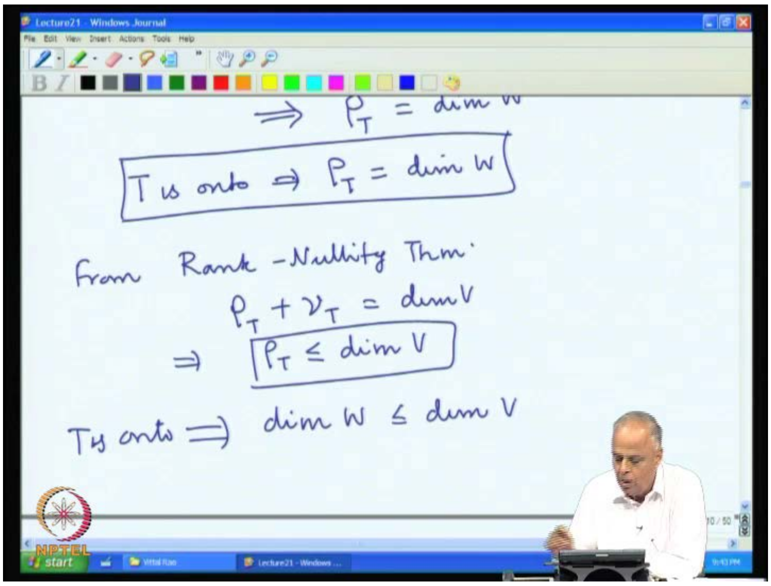

So, therefore, we have T is onto implies the rank of T is dimension of W. We also had from the rank nullity theorem; what does the rank nullity theorem says? The rank of T plus the nullity of T is the dimension of V and therefore, the rank of T if we add something non negative; namely new T if we get dimension of V. Therefore, the rank of T must be less than or equal to the dimension of V because the dimension of V is rank of T plus something which is non negative and hence comparing this statement with this statement; we see that the T is onto implies the dimension of W must be less than or equal to dimension.

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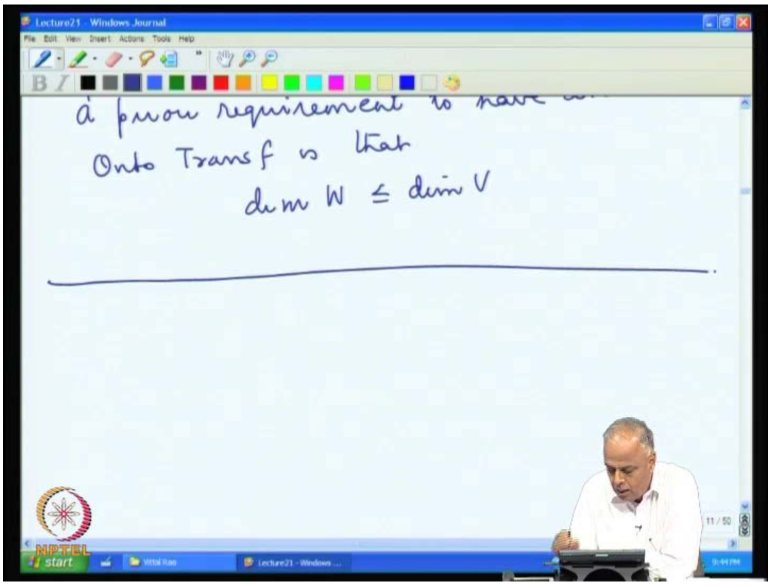

So, an a priori requirement, an a priori requirement to have an onto transformation is that dimension of W equal less than or equal to dimension of V. Recall that we found that an a priori requirement to have a one one transformation was the other way round dimension of V was less than or equal to dimension of w. So, unless dimension of W is less than or equal to dimension of V; we cannot expect to have an onto linear transformation from V to W.

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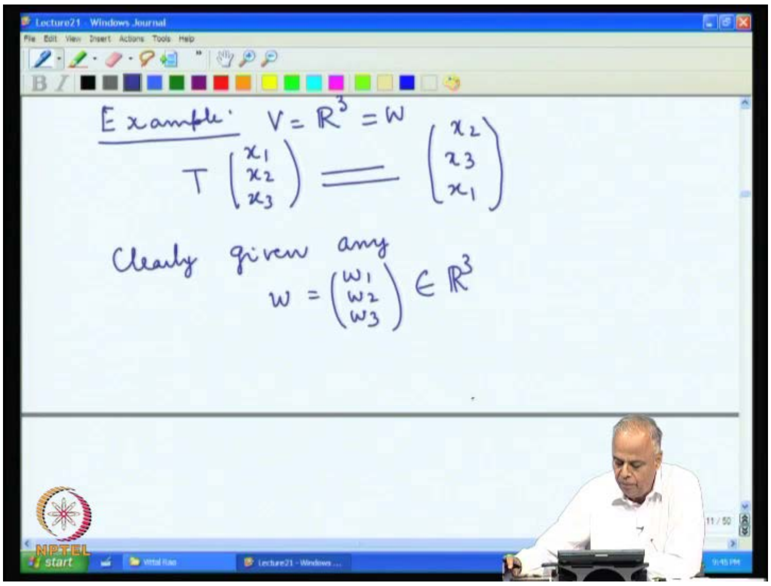

Now, let us look at a simple example. Let us consider the first example. We had for one one transformation again. So, we have V equal to R 3 and we take W also R 3 and we define T as x 1 x 2 x 3 going to the vector x 2 x 3 x 1, a permutation of the coordinates. We had this already as a one one transformation and let us check whether this is an onto transformation.

So, clearly to check an onto transformation, we want to see whether every vector in W can be written in the form T of some x in v.

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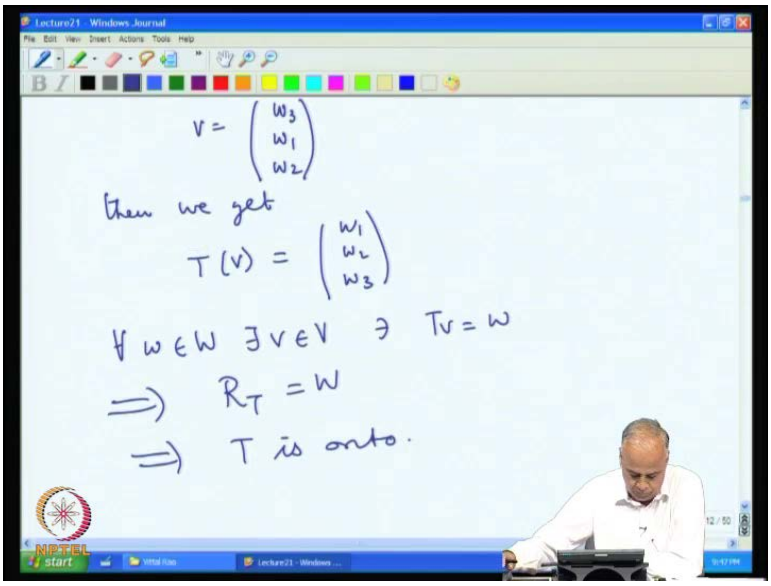

So, clearly given any W equal to W 1 W 2 W 3 in R 3 if we define V equal to W 3 W 1 W 2 then we get T of V. What does T do? It permutes the components of V in an order and when you do that permutation; we get let us see whether we have the W3. Let us check very carefully, we would like to have the third component to be the first component of the pre image vector and the first component to be the second component of the pre image vector and so on so forth. The third the. So, if we look at it we get T V equal to W 1 W 2 W 3. Hence we have for every W in W there exist V and V such that T V equal to W that implies range of T is W that says T is onto.

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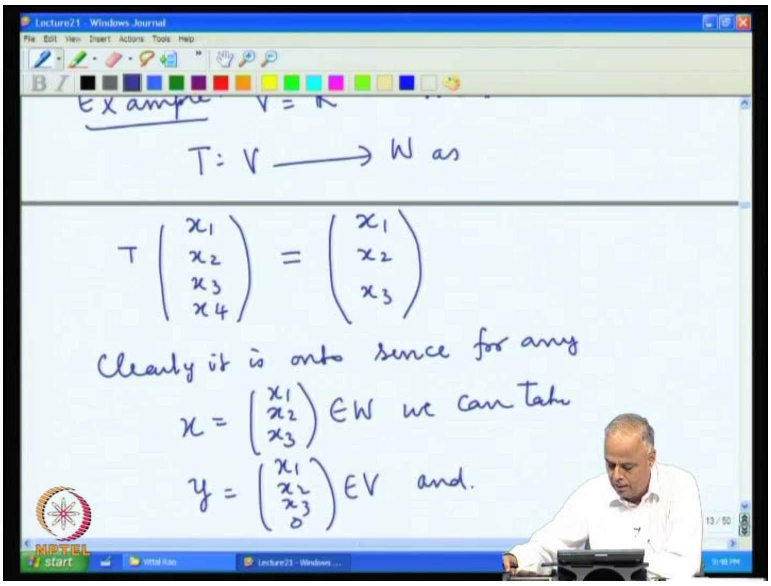

Let us take another example. Let us take V to be R 4 and W to be R 3. Now, notice that the dimension of V is strictly greater than the dimension of W. Now, we define T mapping V to W as what should T do? It should take a vector in R4 say x 1 x 2 x 3 x 4. It should map it to a vector in R3. So, let us say x 1 x 2 x 3. Then clearly it is onto since for any x is equal to x 1 x 2 x 3 belonging to W, we can take y to be x 1 x 2 x 3 0 in V.

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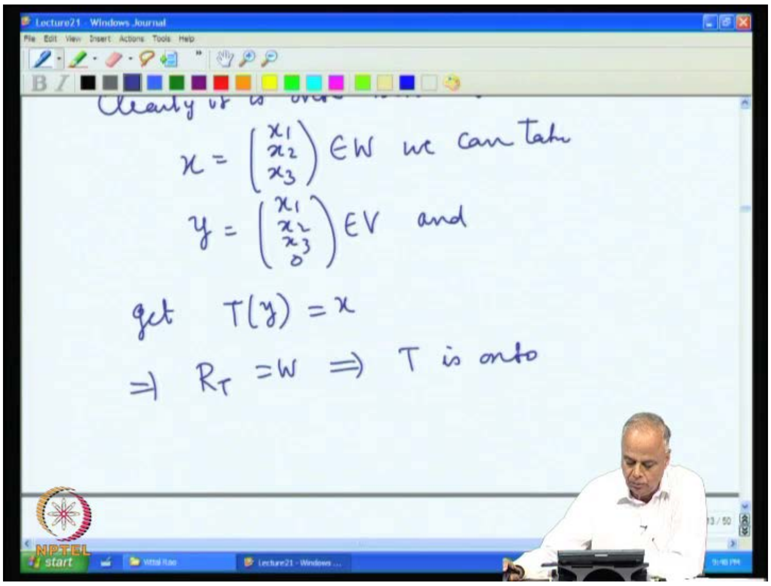

And get T y equal to x and therefore, range of T is W which simply says T is onto and as observed earlier in this case we have dimension of V is greater than or equal to dimension w.

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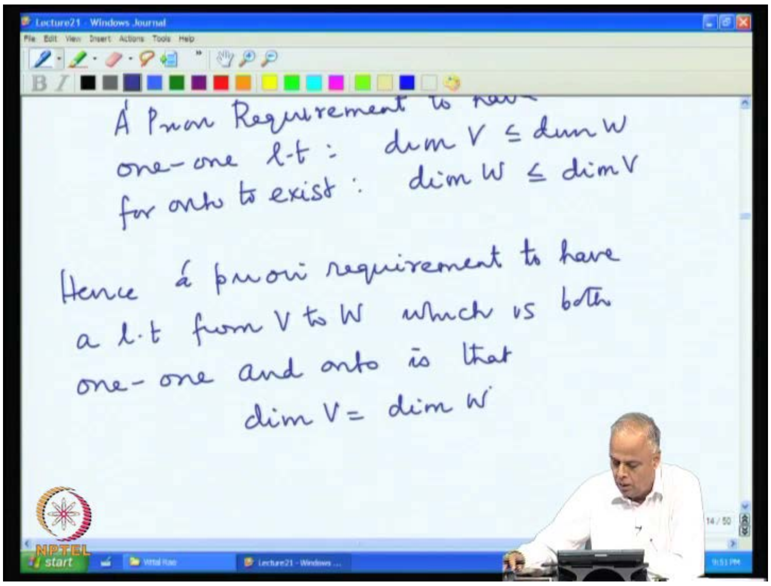

So, now, therefore, recall a priori requirement to have one one linear transformation was that the dimension of V, dimension of W was less than or equal to the dimension of v. So, this is onto. So, let us summarize both of them carefully.

Let me lets go back and look at what exactly we had. Remember we had shown that an a priori requirement for a one one transformation is that dimension of V is less than or equal to dimension of W. This is what we had in order to have one one transformation we must have dimension of V is equal to dimension of w.

So, that is a very important requirement for a linear transformation to exist that. So, we should have dimension of V should be less than or equal to dimension of W for the one one transformation to exist. And then for onto to exist; the requirement was dimension of W is less than or equal to dimension of V consequently if you want to expect both one one and onto then it is, you better start with V and W which both have equal dimension.

So, hence a priori requirement to have a linear transformation from V to W which is both one one and onto is that dimension of V is equal to dimension of w.

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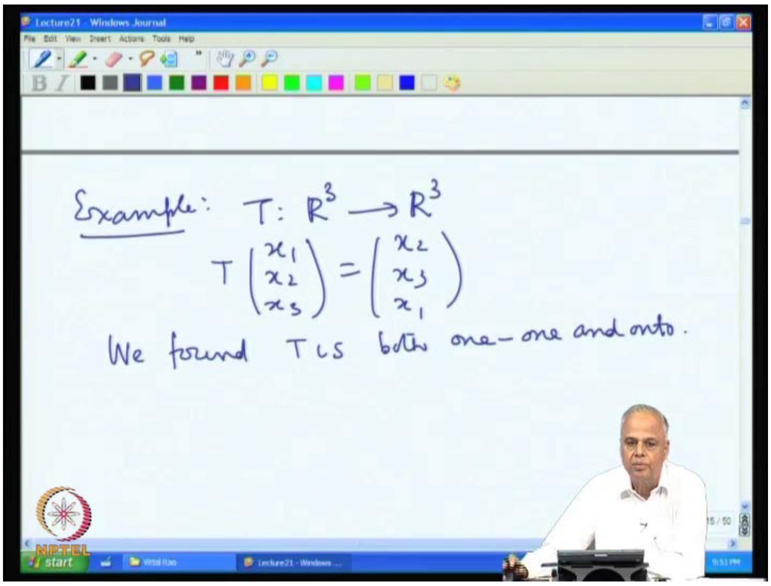

Recall the example we had, T mapping R 3 to R 3 where we had the permutation of the coefficients or the components. We found T is both one one and onto. Notice that in this case we have dimension of V is equal to dimension of w.

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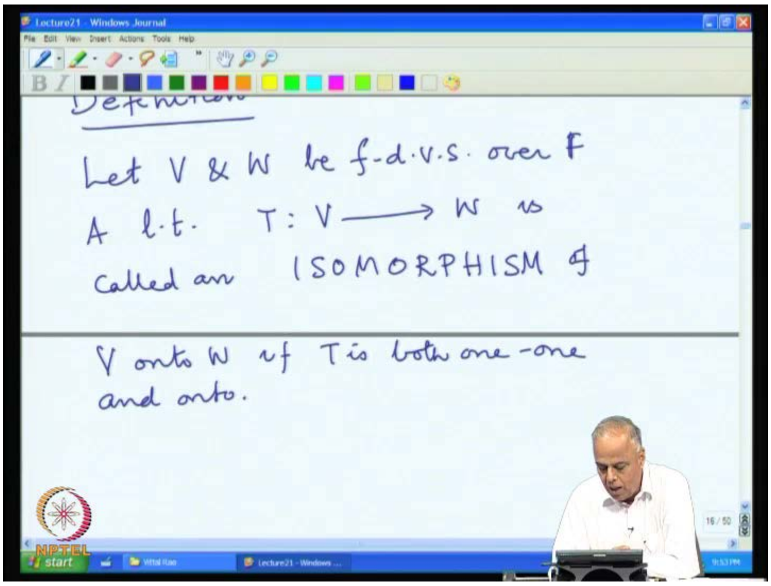

Now, this leads us to a following definition; let V and W be finite dimensional vector spaces over F then a linear transformation T from V to W is called an isomorphism of V onto W if T is both one one and onto.

So, you have a linear transformation from a finite dimensional vector space V to a finite dimensional vector space W which is both one one and onto then, we call such a linear transformation as an isomorphism from V onto w.

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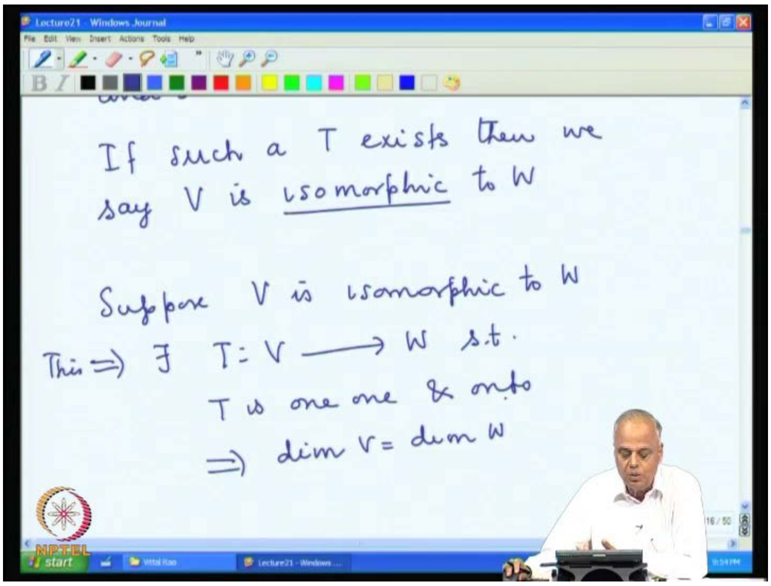

If such a T exists, then we say V is isomorphic to w. Now, let us examine this notion of isomorphism a little bit carefully. Suppose V is isomorphic to W then, this says there exists a linear transformation from V to W such that T is one one and onto. Now, we have seen that an a priori requirement for a linear transformation which is both on one one and onto to exist is that the dimension of V is equal to dimension of W. And therefore, if V is isomorphic to W, dimension of V must be equal to dimension of w.

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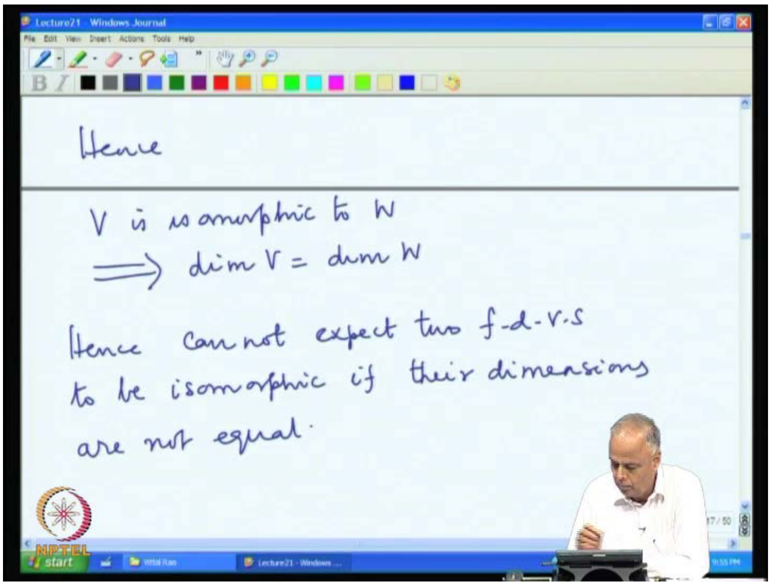

Hence, V is isomorphic to W implies dimension of V is equal to dimension of W. Therefore, we cannot expect two vector spaces to be isomorphic if their dimensions are not equal. So, hence cannot expect two finite dimensional vector spaces to be isomorphic if their dimensions are not equal. Now, therefore, the question arises if the dimensions are equal can we expect them to be isomorphic?

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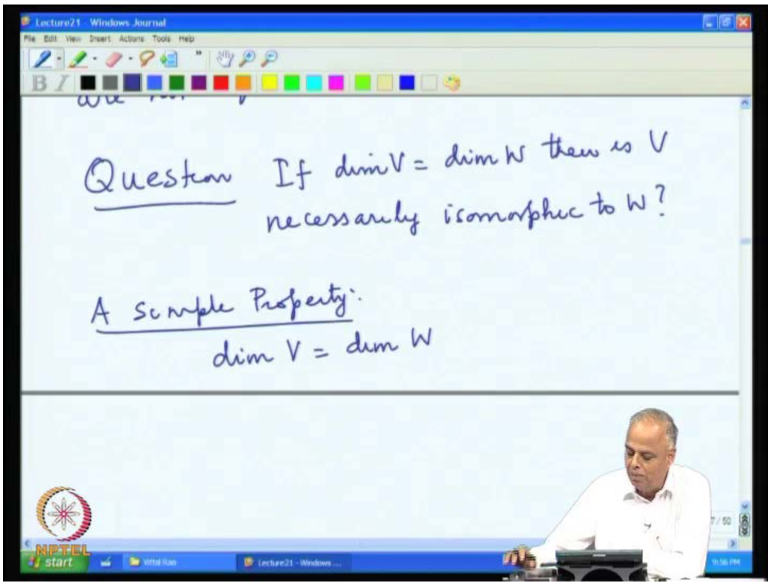

So, the question: If dimension of V is equal to dimension of W then is V necessarily is V necessarily isomorphic to W? This question arises. Now, before we answer this question let us look at a very simple property of such isomorphism's from two spaces of equal dimensions. So, a simple property. So, again we start with dimension of V equal to dimension of W because there is no hope of an isomorphism if dimension of V is not equal to dimension of W.

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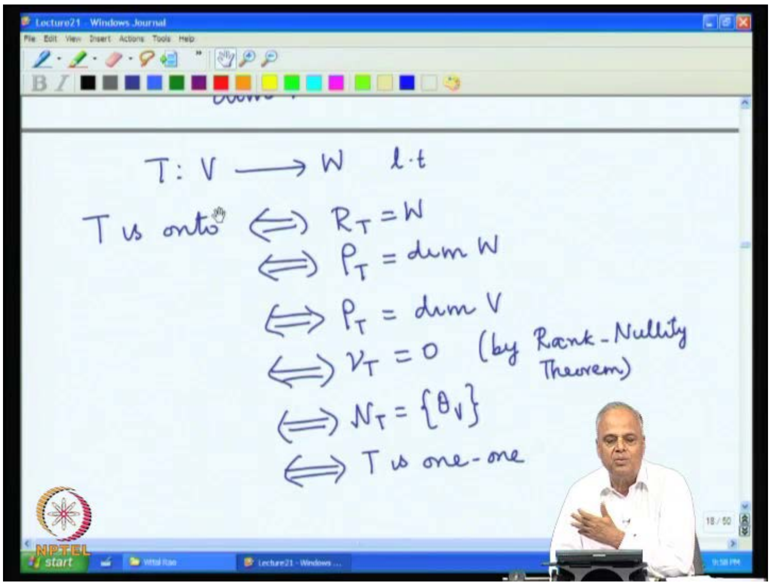

And suppose T is a linear transformation from V to W, a linear transformation. Now, suppose T is onto, when is T is onto? This means if and only if the range of T is W if and only if the rank of T is equal to the dimension of w. But, the dimension of W is the same as the dimension of v. So, if and only if the rank of T is equal to the dimension of v. But, by the rank nullity theorem the rank plus nullity is equal to the dimension of v. So, therefore, the if that the rank of T has to be dimension of V the nullity of T is forced to be 0. So, the nullity of T must be equal to 0 by rank nullity theorem.

Now, the nullity being 0 is what exactly the meaning of the fact that null space of T consists of only by 0 vector and this is exactly what is the meaning of T is one. So, what we have observed is that if we have two vector spaces of equal dimension then, the moment a linear transformation is onto; it is also one one and the moment the linear transformation T is one one and it is also onto.

So, an onto transformation is automatically one one and onto. A one one transformation is automatically one one and onto. Therefore, everyone one transformation will become an isomorphism, every onto transformation will become isomorphism.

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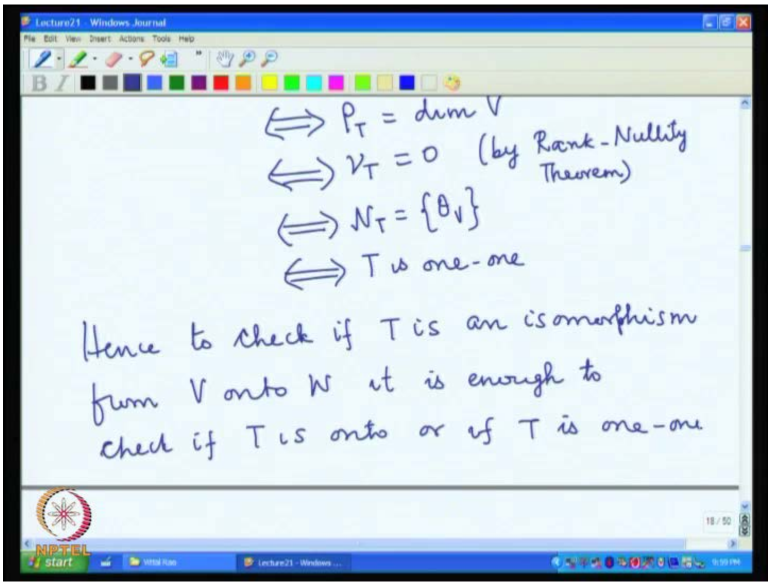

Hence, to check if T is an isomorphism from V onto W it is enough to check if T is onto or if T is one one. Of course, we are assuming that the dimension of V is equal to dimension of w.

And let us now go back to the question that we raised. The question that we raised was if dimension of V is equal to dimension of W; are they necessarily isomorphic to W? What does this question mean? This means can we generate a linear transformation from V to W which is both one one and onto? But, now I do not have to worry whether one one and onto either one one or onto is enough because one of them implies the other. So, to check isomorphism it is enough if you check one of them.

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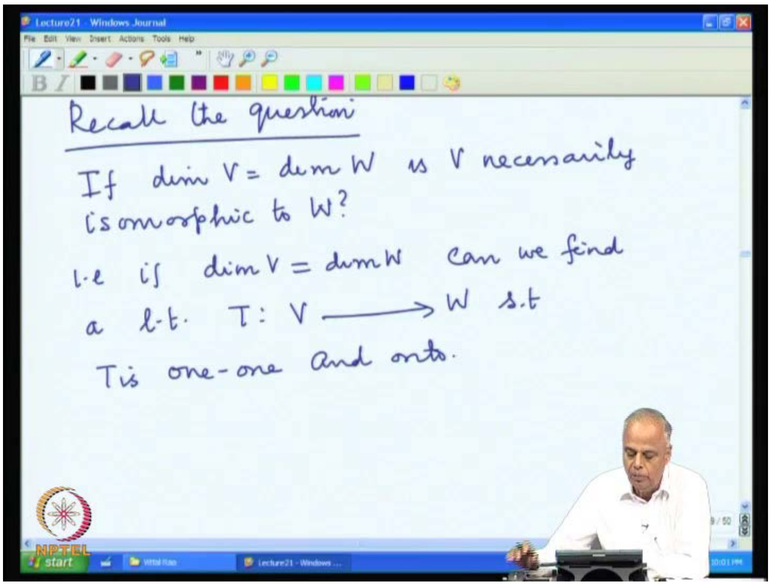

So, let us go back that again lets recall that question recall the question. If dimension of V is equal to dimension of W and both are finite dimensional vector spaces over F; is V necessarily isomorphic to w?

This question means that is if dimension of V is equal to dimension of W; can we find a linear transformation T from V to W such that T is one one and both one one and onto? Let us see whether we can generate such a linear transformation.

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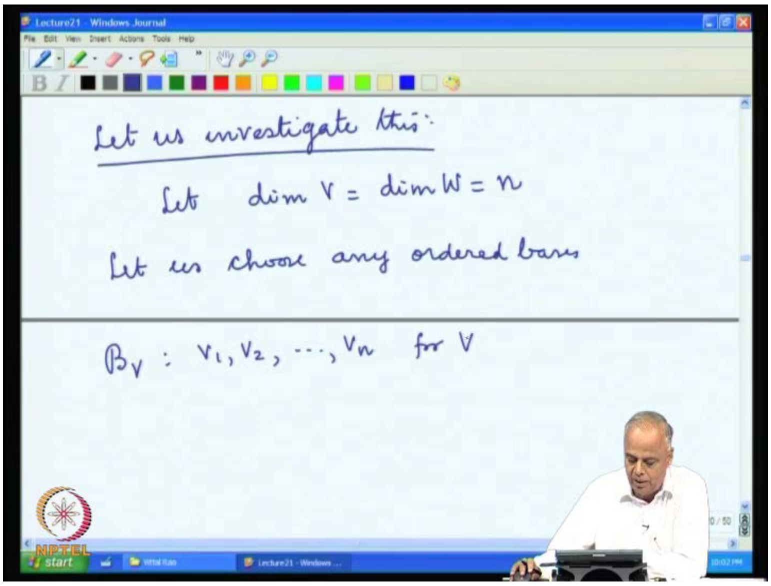

So, we are given that the dimension. So, so lets investigate this whether we can construct such a T. The information given to us is that V and W are finite dimensional and that their dimensions are equal. So, let dimension of V is equal to dimension of W. Let us call that dimension as n.

Now, since V the finite dimensional space and has a dimension n; any order basis will have n linearly independent vectors. So, let let us choose any ordered basis. Let us call it as B v. V 1 V 2 V n for V this we can do because V is a finite dimensional space. Its dimension is n. Therefore, any basis will have n linearly independent vectors.

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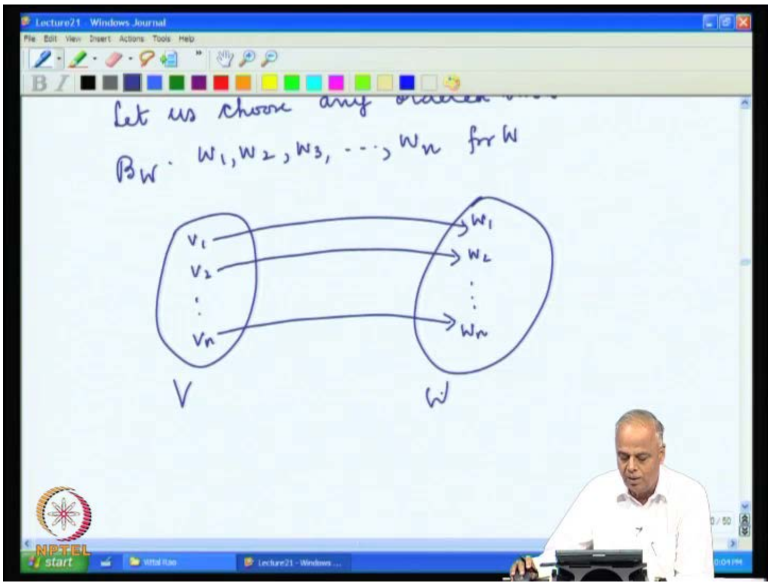

Similarly, let us choose any ordered basis for W now. That W basis should also have n vectors, n linearly independent vectors n linearly independent vectors from W. Let us call them as W 1 W 2 W 3 W n for w.

So, now we have these 2 basis. So, here is V here is W and we have this basis vectors V 1 V 2 V n, W 1 W 2 W n. These V's are the basis for V and the W vectors are the basis for W. Now, we are going to slowly generate a transformation from V to W. The way to generate this transformation is a linear transformation is completely determined by its action on the basis vectors. So, we will first say what the linear transformation touched to these basis vectors V 1 V 2 V 3 V n and then decide what it should do for the other vectors in v.

The easiest way to do this is, to take the first basis vector of V to the first basis vector of W, take the first basis vector second basis vector of V 2 of V to the second basis vector W 2 and the last basis vector V n of V to the last basis vector W n of w.

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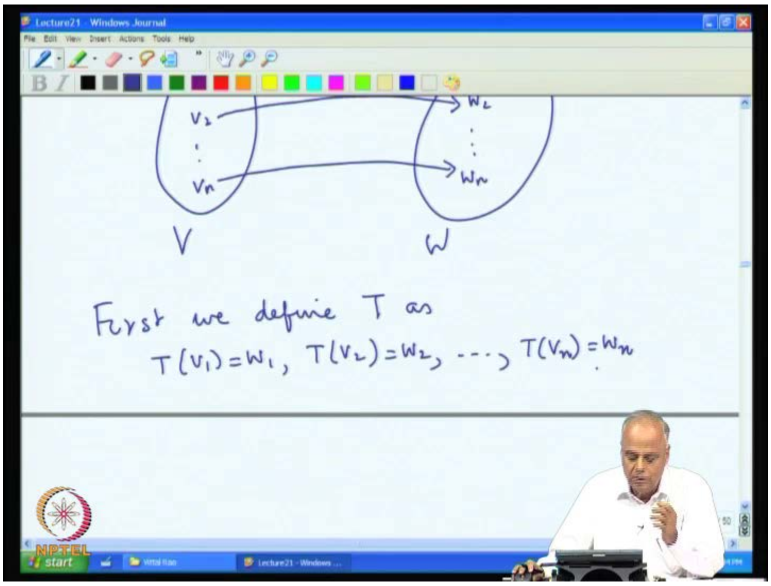

So, what we do is first we define T as T V 1 is equal to W 1, T V 2 equal to W 2 and so on, T V n equal to W n. Now, once we have said what it does to the basis vectors. We shall now see what it does to the other vectors.

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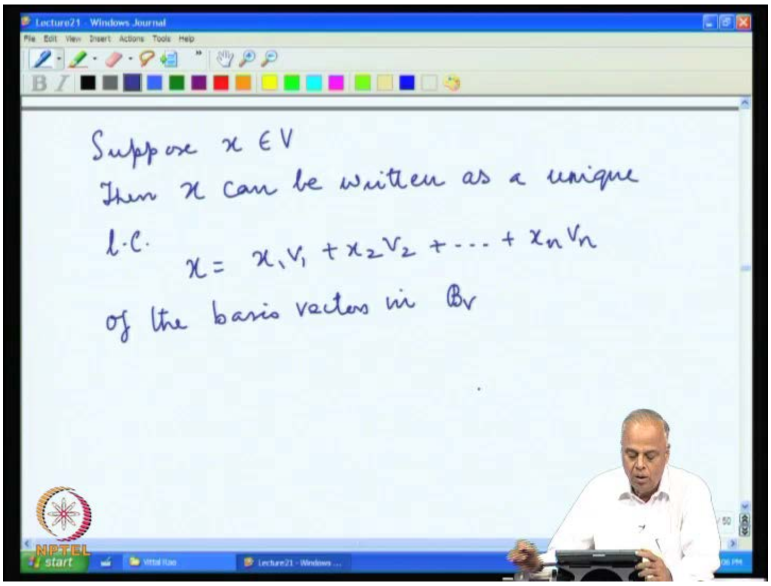

Suppose, x belongs to V. If x belongs to V then any vector in V can be written in terms of the basis vectors as a unique linear combination. So, we should able to write as x is a linear combination of V 1 V 2 and V n. Let us do that. So, then x can be written as a unique linear combination x is equal to x 1 V 1 plus x 2 V 2 plus x n V n of the basis B v, of the basis vectors in B v.

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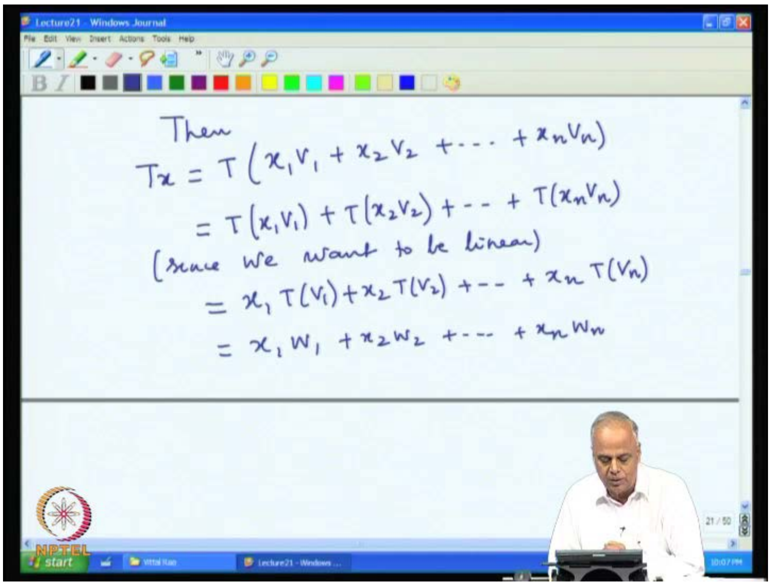

Now, once we do that we want to know what is T x. Then T x should be T of x 1 V 1 plus x 2 V 2 plus x n V n. Now, we would like to have T to be linear. We were trying to define a linear transformation. So, for a linear transformation T of a super position is the super position of the T's or T of a sum it is sum of the T's. So, we can write this as T x 1 V 1 plus T x 2 V 2 plus so on. T x n V n since we want T to be linear.

Again, using the same linearity; T of a scalar multiple of a vector in the scalar multiple of the T of the vector. So, we can write it as x 1 T V 1 plus x 2 T V 2 plus x n T V n and we have already said what T V 1 T V 2 T V n are we said T V 1 we are going to map it to W 1 T V 2 we map it to W 2 T W T V n we are going to map it to W n.

So, what we are got here is that the moment you know the action of the linear transformation basis vectors; they action on other vector is automatically taken care of by the super position principles.

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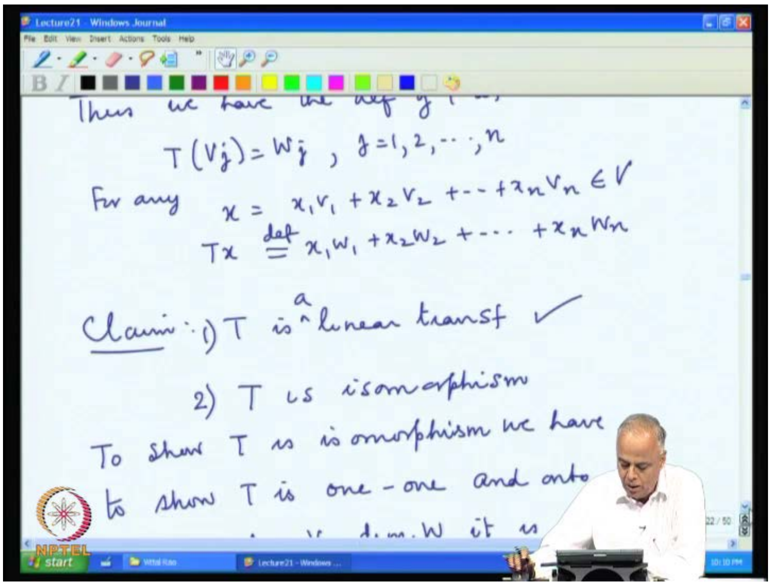

So, thus we have the definition of T as T of V j is W j for j equal to one 2 n and for any x equal to x 1 V 1 plus x 2 V 2 plus x n V n belonging to V T x is by definition is x 1 W 1 plus x 2 W 2 plus x n W n.

Now, all we will do now is, we claim T is linear and this see the linear transformation. Put it now, this we have already taken care of in the way we define T we brought in the linearity to get the definition of x. So, this has already been taken care of. So, we have to going to claim that T is isomorphism.

To show T is isomorphism, we have to show that T is one one and onto we have to show T is one one and onto and we have observed earlier that when you have dimension of V is equal to dimension of W; you check onto it automatically forces one one. If it is one one it automatically forces onto and therefore, it is enough to check one of them and since dimension of V is equal to dimension of W; it is enough to check if T is onto because the moment it is onto, we know it is going to be one one because dimension of V is w.

So, now let us check whether T is onto. Let us look at the definition. If you look at the mapping now, we have W 1 belongs to the range of T because W 1 is a image of V 1. Similarly, W 2 is the image of V 2. So, W 2 belongs to the range of T and finally, W n belongs to the range of t. So, we have W 1 W 2 W n.

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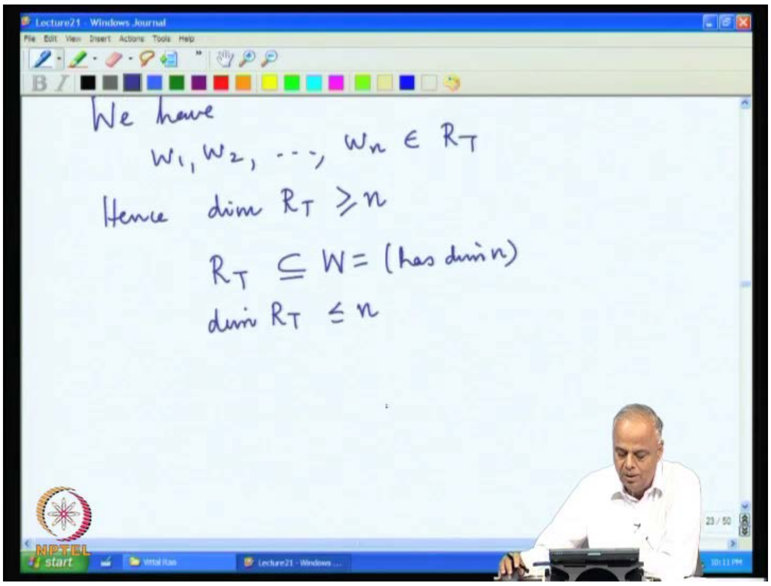

So, we have W 1 W 2 W n all belong to range of t. But these are all linearly independent vectors and hence dimension of range of T is at least n because we have n linearly independent vectors in the range of t. But, then range of T is contained in W which is dimension n and therefore, dimension of range of T must be less than or equal to n.

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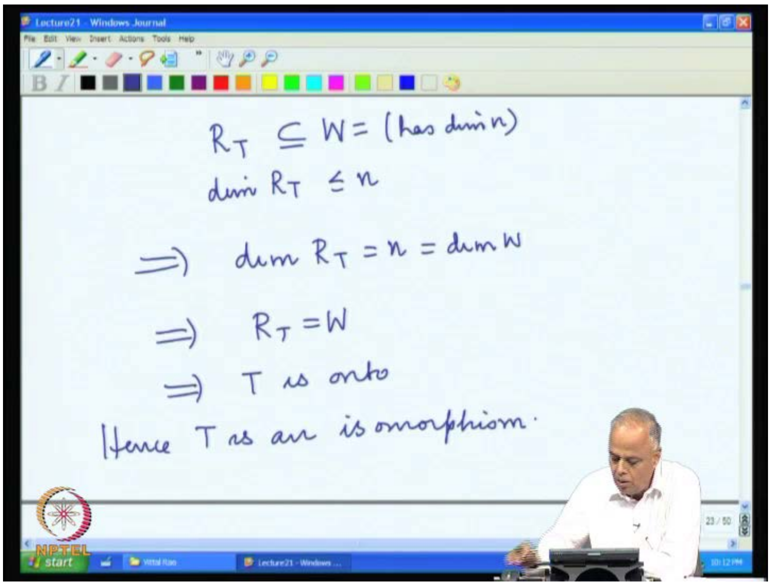

So, comparing these two, we get dimension of range of T must be exactly equal to n which is equal to the dimension of w. But, if R T is a sub space of dimension n in an n dimensional space it implies range of T must be exactly equal to W which says T is onto. Hence, T is an isomorphism.

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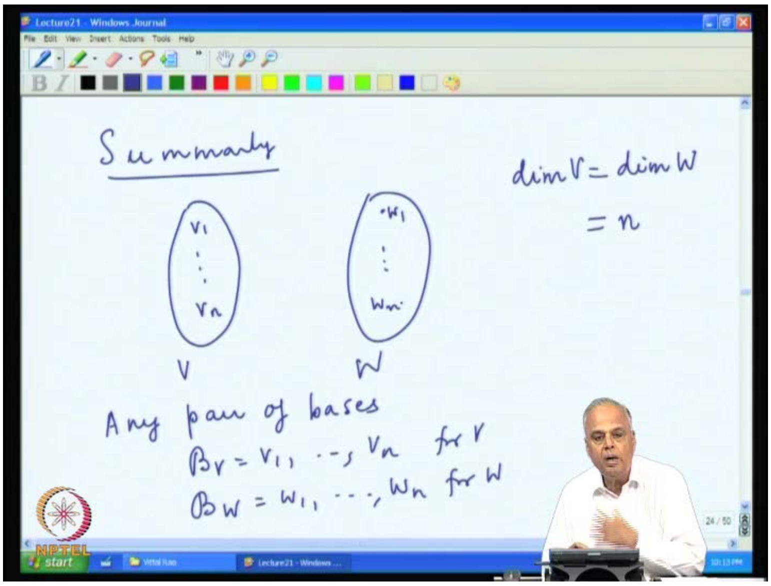

So, that what have we achieved? Therefore, what we have seen we can summarize as follows: You start with a vector space V, another vector space W. Both are vector spaces of same dimension finite dimensions over the same T. So, if we have two vector spaces of the same dimension say equal to n, then any pair of basis B v V 1 V 2 V n for V B w W 1 W 2 W n for w. So, you start with two vector spaces of the same dimension. Take any ordered basis B v for V, any ordered basis B w for W.

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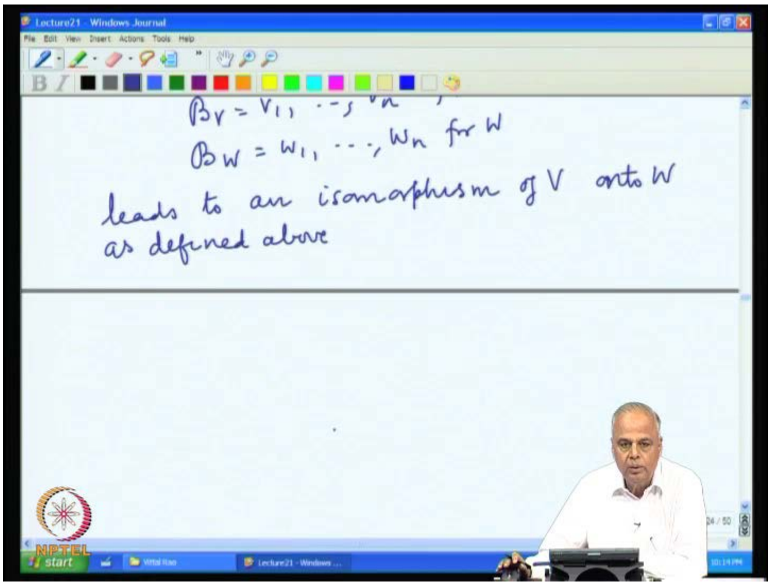

So, you take a pair of basis one for V 1, one for W leads to an isomorphism of T sorry of V onto W as defined above. The isomorphism is obtained by mapping the jth base vector of V to the jth base vector of w.

So, therefore, not only that (( )) finite dimensional vector spaces of the same dimension or isomorphic, there are many isomorphism that go from V to W. Every pair of basis will give raise to an isomorphism from T to W.

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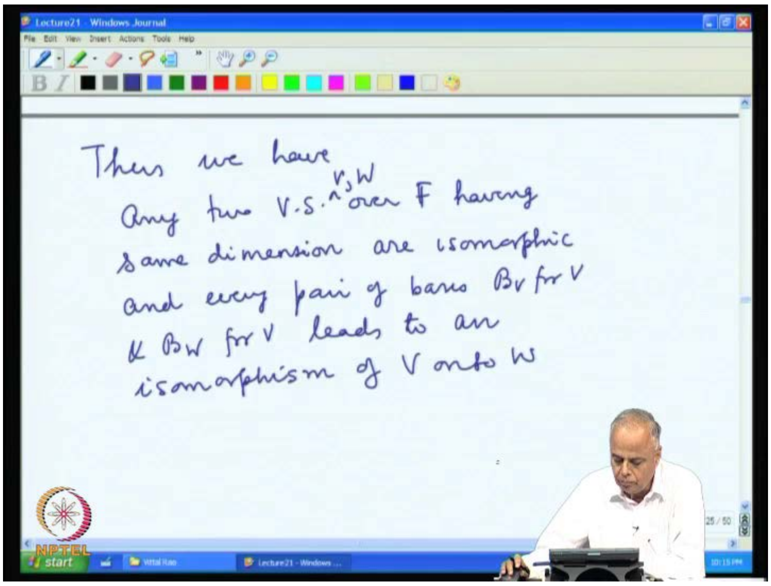

So, thus we have every any two vector spaces over F having same dimension assume we are assuming they are all finite dimension having same dimension or isomorphic and every pair of basis B v for V and b W for w. So, lest say any two vector spaces V W leads to an isomorphism of V onto W.

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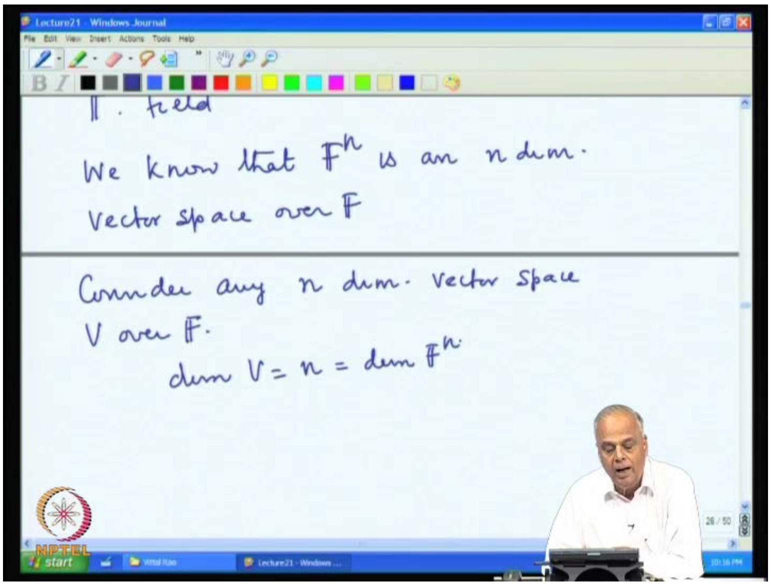

So, what does this all imply? These all implies the following; suppose we have a field F, do we know at least one n dimensional vector space over F? We already know namely the F n where you write the n components with the n coordinates coming from F.

So, we know that F n is an n dimensional vector space over F. Now, consider any n dimensional vector space V over F. Now, since dimension of V is equal to n is equal to dimension of F n; we have seen that any two vector space of the same dimension are isomorphic.

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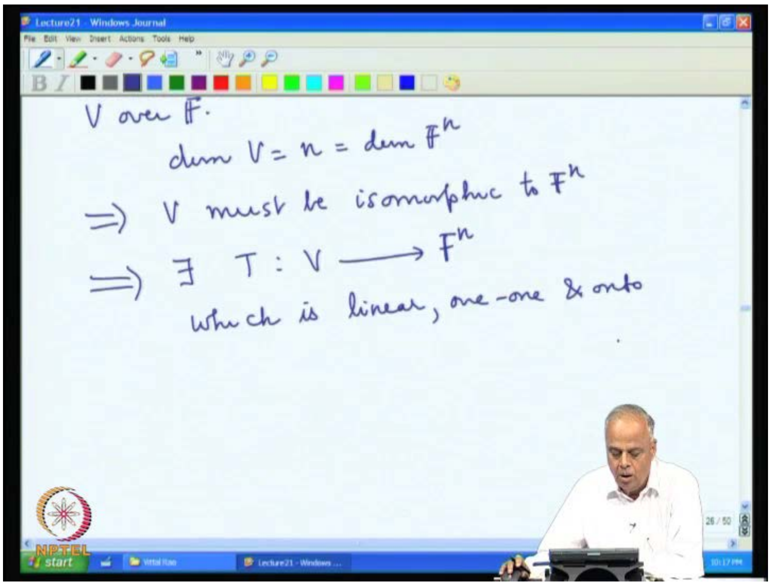

This implies V must be isomorphic to Fn. What does that mean? This implies there exist a transformation from V to F n which is linear one one and onto.

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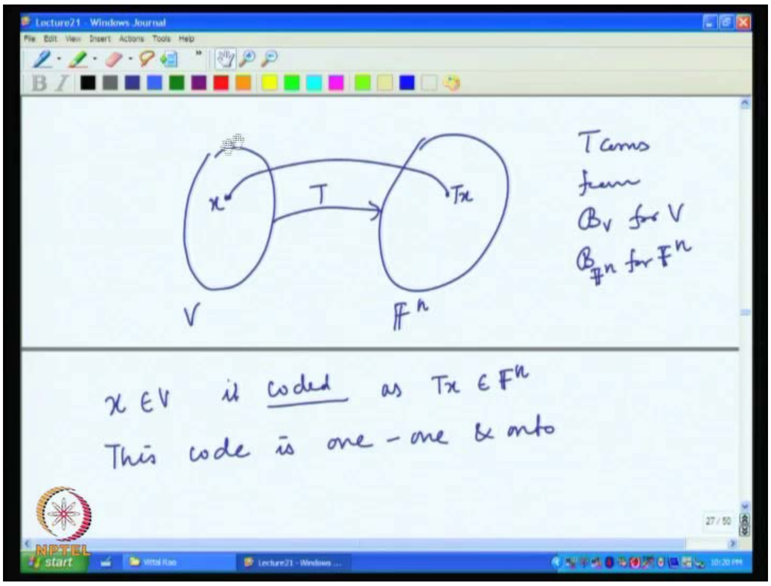

What does this mean? This means the following: You have V, you have F n, you can get a transformation T from here which takes any vector x in V to the vector T x in F n and this transformation is obtained from T comes from two basis, V for V for V and V F n for F n. Choose any two basis one for V and one for F n as we have seen earlier we can construct a isomorphism T from V to F n. Suppose x is mapped to T x under this transformation.

What does that mean? The vector x in V has is coded, we can interpret this is coded or encrypted as the vector T x in F n and since T is one-one; this coding is without any confusion. Distinct vectors get coded as distinct codes and it is onto means all the codes are exhausted that is every vector in F n is an encryption of a vector in x. So, this code is one-one and onto.

And therefore, there is a translation of the language of V to the language of F n. All the statements in V can be now translated to the statements of F n by translating every vector x to its corresponding code in F n. Now, this translation is actually transliteration because every word gets translated to a unique meaning. Every vector gets translated to a unique meaning. In F n this code is onto.

But now, in V we are doing some algebra additional scalar multiple, but, since T is one one this code also maintains that algebra because the coded version of a sum is the sum of the coded version. That is what is meant by saying T of x plus y is T of x plus T of y the coded version of a scalar multiple is a scalar multiple of the coded version. This is means T of alpha x is alpha T n.

So, this code does not create any confusion. It translates every word uniquely and every word in the new language as a meaning in the old language and the translation preserves all the algebra this means instead of working in V, we could as well work in F n.

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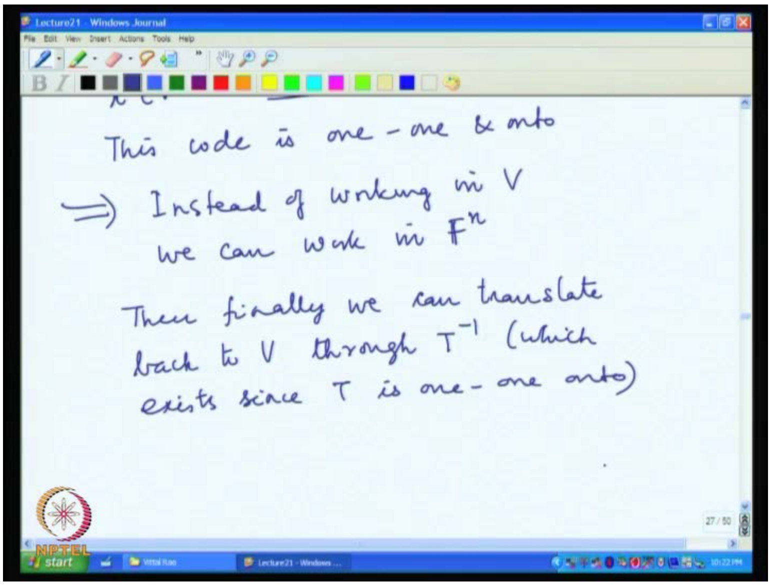

So, that which boils down to saying all this means that, instead of working in V we can work in F n and once they have worked in F n, we have got our answers in F n. Whatever problem is in V it can be translated to a problem in F n and once you have solved the problem in F n, the answer can be translated back to the language of V. Why can we translate back to that language? This is because T is one one and onto. So, it will automatically have an inverse map and that inverse map will carry the decryption part of it. It will carry the coded T x to the original x and therefore, then finally, we can translate back to V through T inverse which exists since T is one-one onto.

Therefore, what this means is the following; it just means that if you have a field F any vector space of dimension n over F is essential F n itself spoken in some other language. And for the translation, we need this isomorphism and the dictionary that provides this translation is the ordered basis B v and b W through which we generate this isomorphism dictionary. And the reverse dictionary is T inverse. T is the translation from V to F n and T inverse is the translation from F n to v.

So, this all amounts to saying that essentially we have to study only F n. Whenever you want to study n dimensional vector spaces over F, the only thing that we have to study are F n and once we study F n any other n dimensional test space can be converted to this values.

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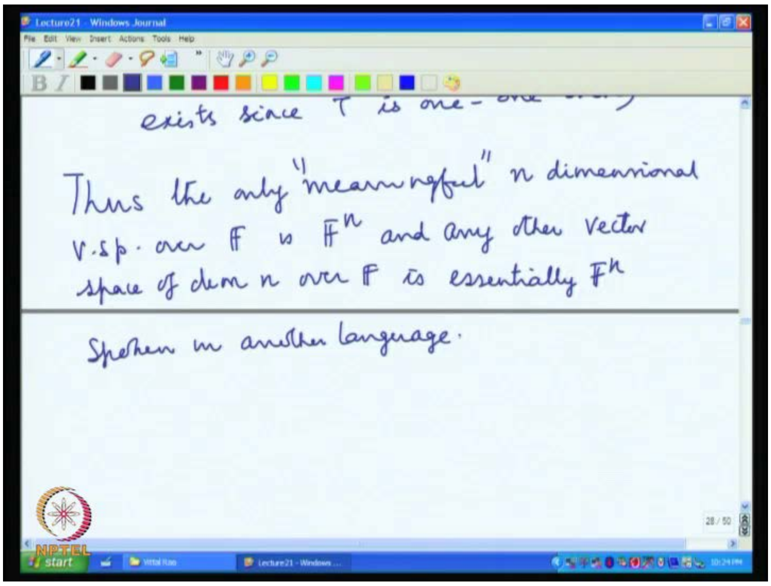

So, we conclude by saying that thus the only meaningful n dimensional; when we say meaningful it is easiest one to analyze, n dimensional vector space over F is F n and any other vector space of dimension n over F is essentially F n spoken in another language.

The isomorphism is the translation dictionary and therefore, it boils down to saying we study F n carefully. If you know Fn, you know the whole universe of n dimensional vector space is over F. Therefore, we shall continue looking at F n more carefully. We shall look at this in the next lecture.