Advanced Matrix Theory and Linear Algebra for Engineers

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Lecture No. # 18

Linear Transformations part 2

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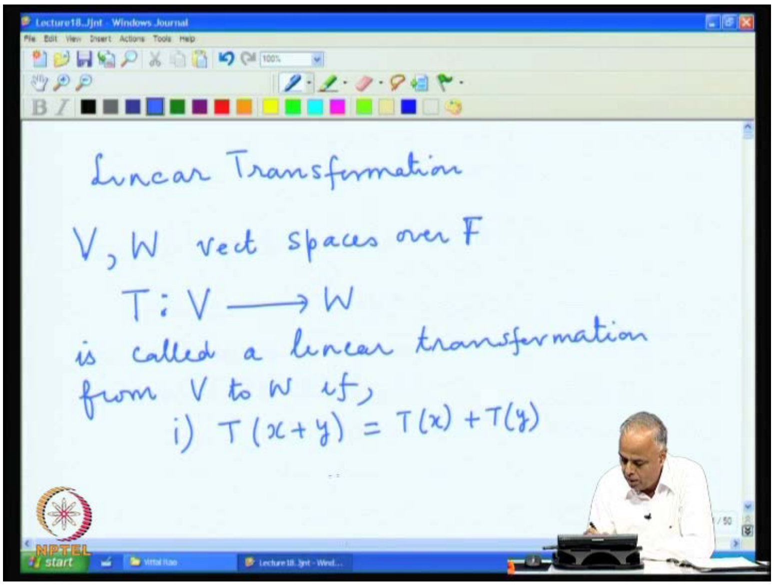

In the last lecture they introduced a very important notation know as linear transformations between two vector spaces. Let us recall the definition let V and W be vector spaces over a field F. Then a transformation which converts V vectors to W vectors is called a linear transformation. It is a transformation it is a linear transformation of course, it transforms V to W. So it is a linear transformation from V to W if what should it do since V and W both have vector space structures both have addition and scalar multiplication and we want this T to preserve the structure of addition and scalar multiplication. So is called a linear transformation if one it preserves addition that means T of x plus y must be equal to T of x plus T of y for every x y in V.

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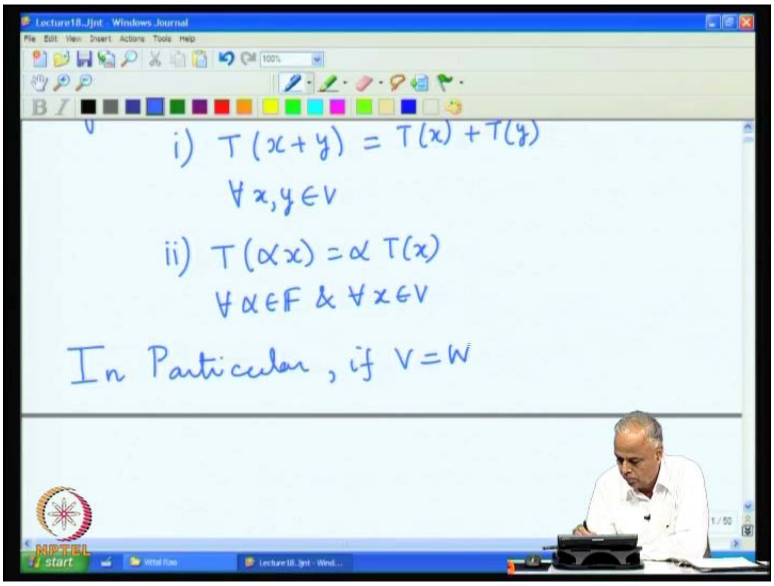

Similarly, it should preserve scalar multiplication that is T of alpha x must be equal to alpha T of x for every alpha in F and for every x in V. So thus something is linear transformation from V to b W. First if the transformation is V to W and if it preserves the algebraic structure of the vector spaces namely the additional scalar multiplication operations. Note that the addition on the left hand side of this definition refers to the addition in V. Because we are adding V vectors and the addition on the right hand side of this definition refers to the addition in W. Because we are adding W vectors similarly, the scalar multiplication on the left hand side. Here refers to the scalar multiplication in V and the scalar multiplication on the right hand side refers to the scalar multiplication in W in particular if V is equal to W.

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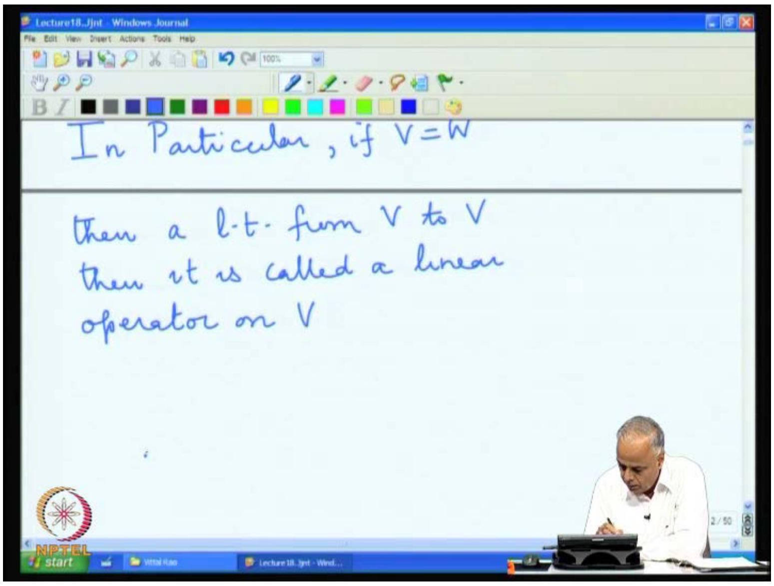

Then a linear transformation from V to V that means it again a transformation from V V into V that is V vectors are again encoded again as V vectors and it preserves the addition and scalar multiplication. Then it is called a linear operator on V it is called a linear operator on V so linear operator on V is nothing but, a linear transformation from V to V.

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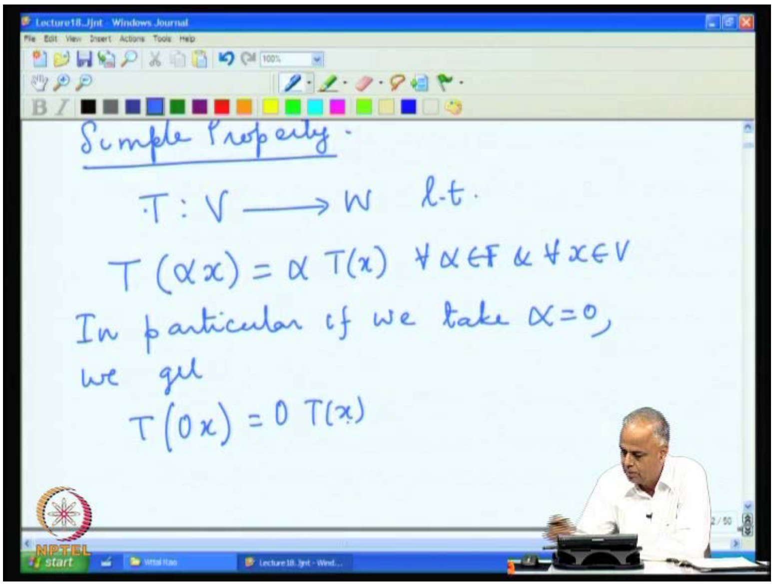

Now we will look at a very simple property of linear transformation. Before we see some examples this is a property which is every linear transformation possess. So we have a linear transformation from V to W write l t for linear transformation. So we have a vector space V and a vector space W over a field F and we have a linear transformation V to W. Now we have because it is a linear transformation T of alpha x is equal to alpha T of x for every alpha in F and for every x in V this is because a linear transformation preserves scalar multiplication and in particular. If we take alpha to be 0 we get T of 0 times x is equal to 0 times T of x.

Now the left hand side what we are doing is we are taking a vector x and V and we are multiplying it by the scalar 0 and so we did the 0 vector in the V space on the right hand side we are multiplying 0 a W vector. So did the 0 vector of the W space on the right hand side.

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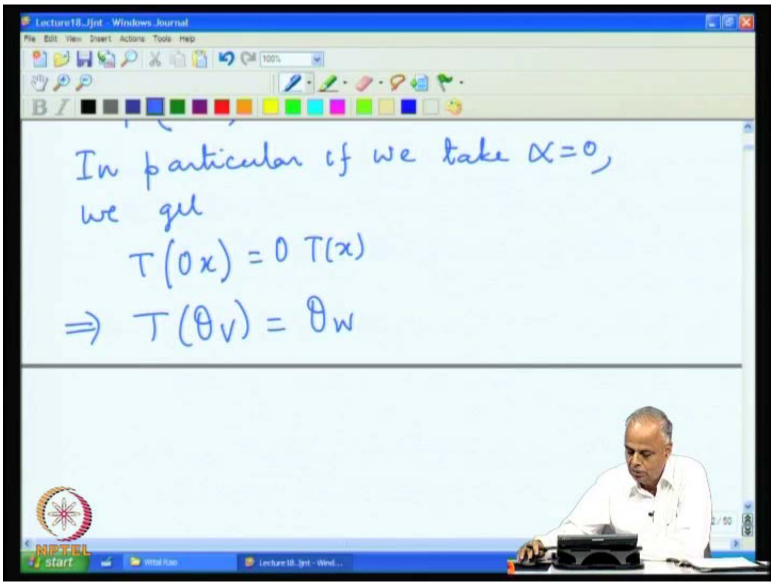

So we get T of theta V is equal to theta of W which means that a linear transformation always maps the 0 vector into the 0 vector.

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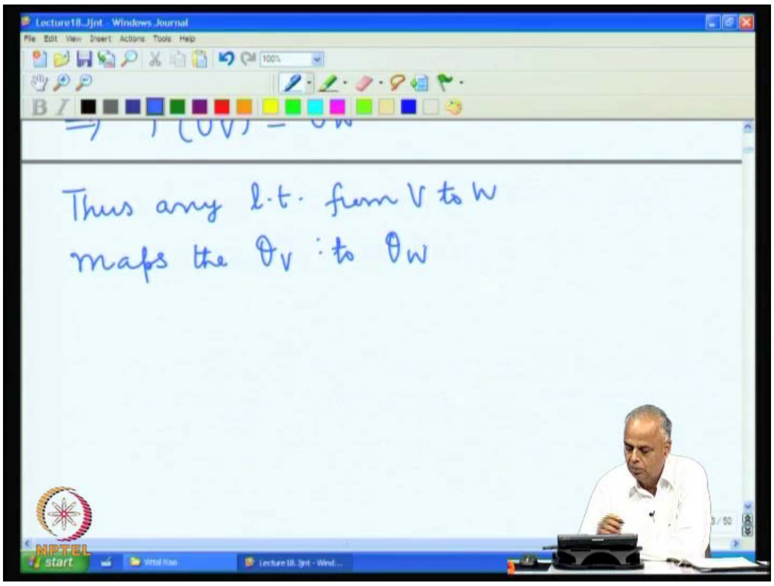

So thus any linear transformation from V to W maps the 0 vector theta V to the 0 vector theta W in W. This is a very important property every linear transformation must do it and therefore, if some transformation from V to W does not take the 0 vector through 0 vector it cannot be a linear transformation.

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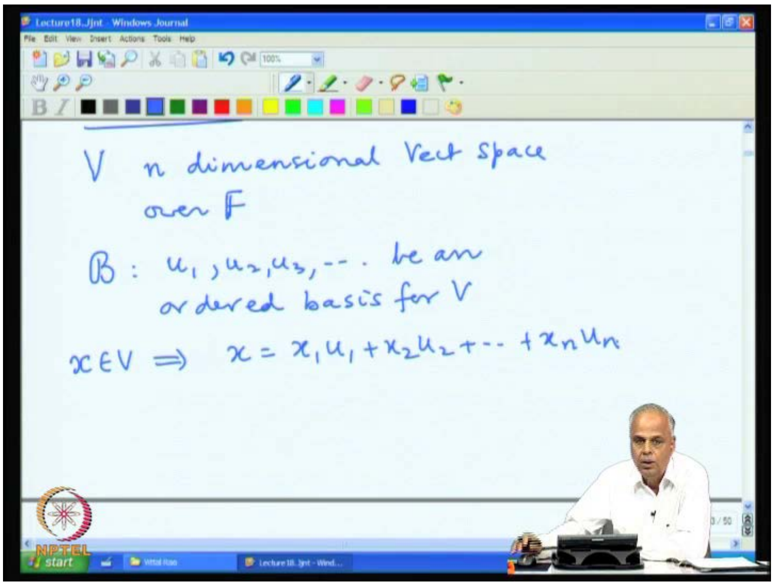

We shall look at some simple examples of linear transformation. Some of which we have already seen we just recall them. The first example we see the following this is the example which motivated us to the definition of linear transformation. There are many ways of motivating linear transformation. We choose this example to motivate the definition of a linear transformation. What is this example let us take V at n dimensional vector space over the field F so we have n dimensional vector space over the field F.

And let us choose a basis for V an ordered basis for V any basis must contain n vector because the dimension on the space is B. So let B u 1 and u 2 u 3 etc be an ordered basis for V. So once we have an ordered basis for V we knew that any vector x can be expressed as a linear combination of the basis vector. So x belongs to V implies x is equal to x 1 u 1 plus x 2 u 2 plus x n u n.

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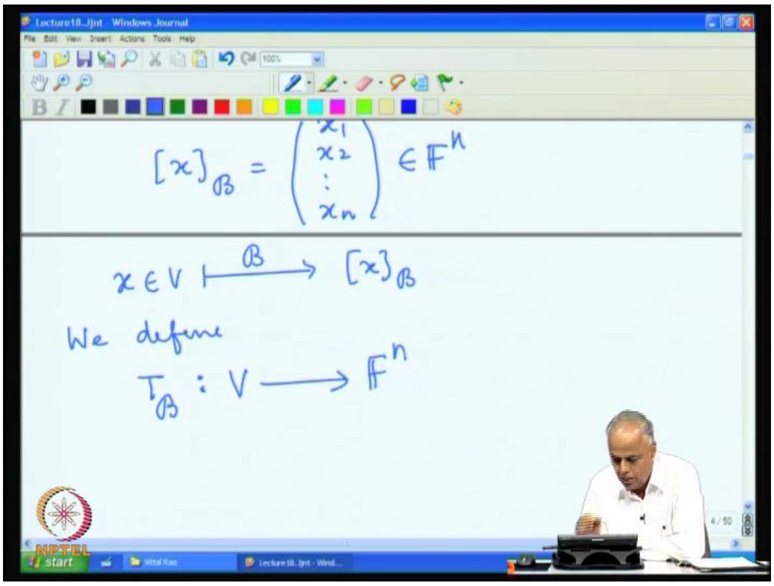

This let us as to the identification of x to a vector x B which we define as x 1 x 2 x n which belongs to F n. Therefore, every vector x in V starting from this we can convert using the ordered basis B of vector x in B. Now we consider this transformation. So we define T it comes out because of the basis B. So we call it as T B , T B a transformation from V to W, W in this case is F n.

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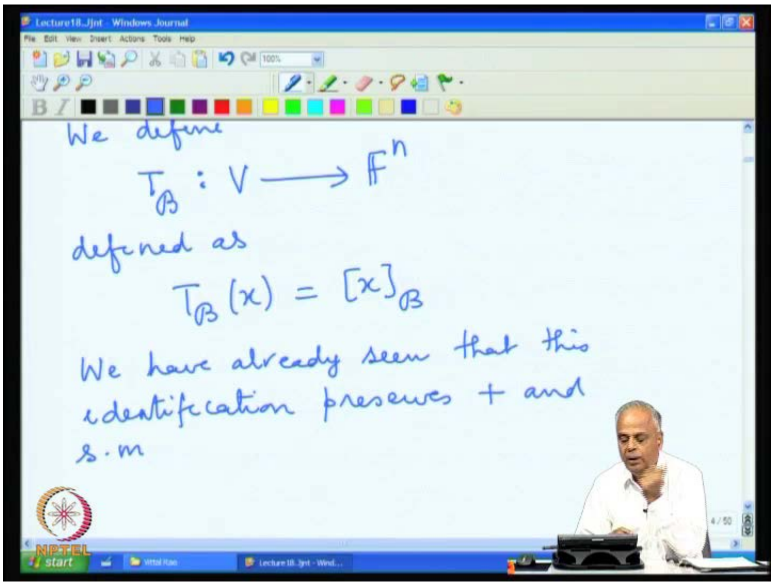

So V to F n is defined as T B of x is x B. So start from a vector x in B comes to the column vector F n and there is a unique column vector. Because every vector x is a unique representation in terms of the basis and we have already verified that this identification preserves addition and scalar multiplication. Already seen that this identification preserves plus and scalar multiplication. I will write s m for scalar multiplication. So this preserves addition and scalar multiplication and therefore, it is a linear transformation the moment it preserves addition and scalar multiplication it qualifies to be called as linear transformation.

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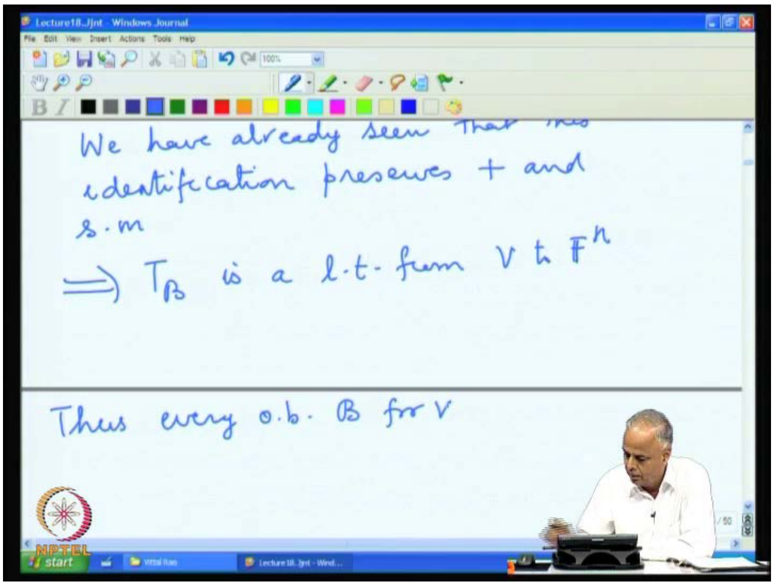

That implies T B is a linear transformation from V to F n. Therefore, the moment you start with order basis B it automatically generates a linear transformation from V to F n.

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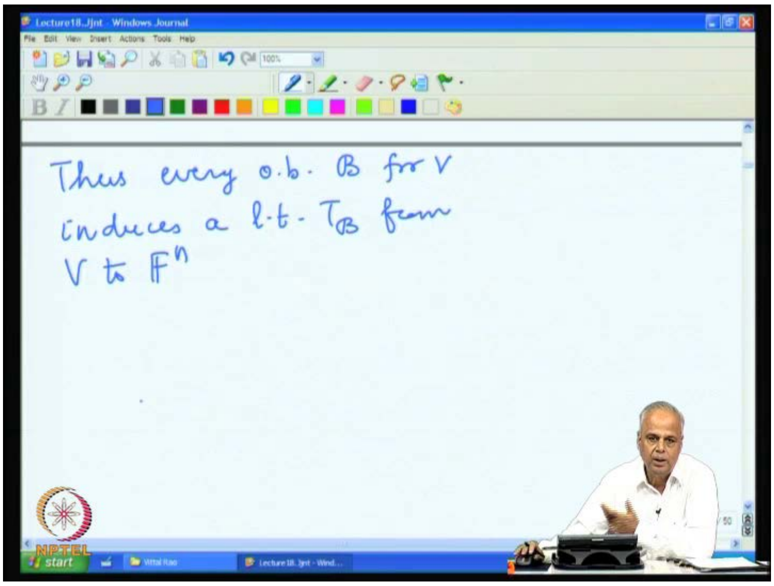

So thus every ordered basis B for V induces a linear transformation T B from V to F n. Therefore, there are infinite number of linear transformation from an n dimensional spaces to F n. Because we could choose any basis and any order of that basis and every time we get a ordered basis. We have a T B corresponding to it so we already have lot of examples of linear transformation.

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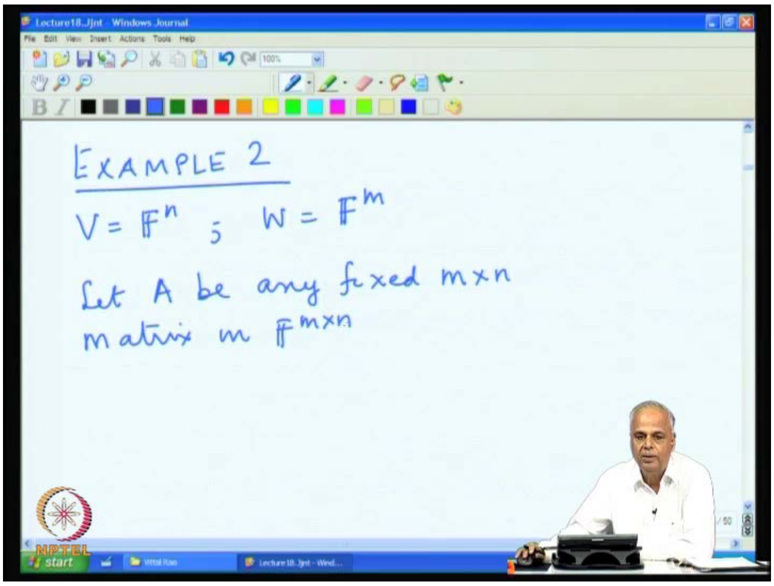

The second example we look at is a very important example from the point of view of various questions that we raise in the beginning of course, about linear systems of equations. So we take V to be the vector space F n where F is a field and W it is a vector space F m. Where n and n are positive integers then let A be any fixed m by n matrix in F m n.

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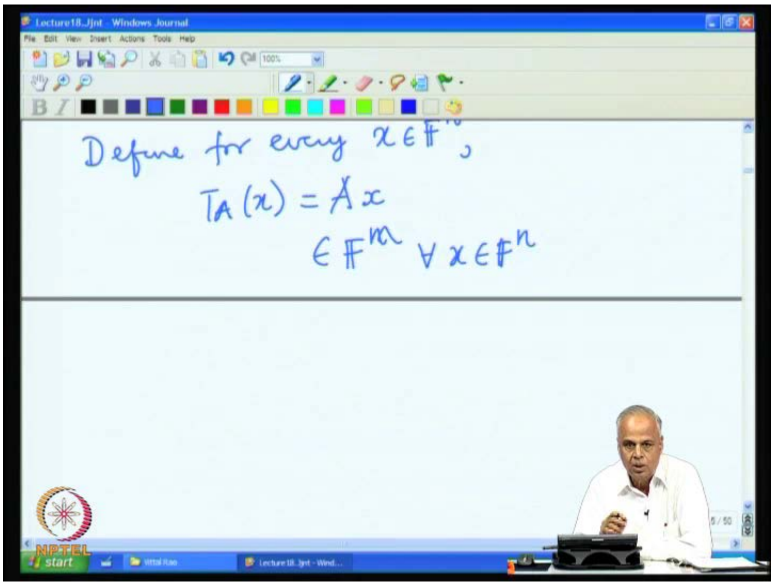

So take any fixed m by n matrix over F consider the vector space F m and F m may defined for every x in F m , T A x as follows. Simply pre multiply the vector x by A x. So for every x in F m we constrict A x and we called it T A of x. This is a function which converts vector x to a vector A x and this function is generated out of the matrix A. So we call it T sub A so T sub A of x is A of x. Now we know that A is a n by m matrix x is a n by 1 matrix. So the product will be a n by 1 matrix so it will belong to F m for every x in F m. Therefore, this is a transformation which converts the F m vector x to F m vector A x.

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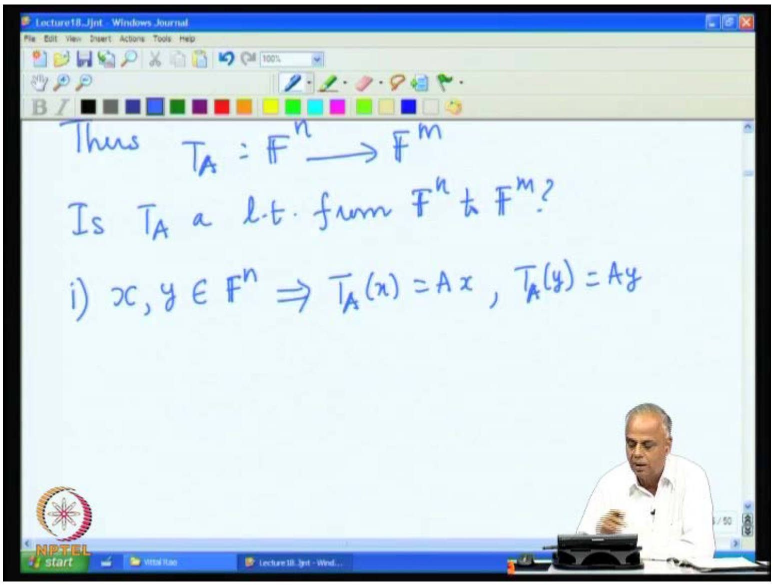

Thus T A maps F n to F m. Since we have a transformation between these two vector spaces it is natural to ask. Whether it is a linear transformation is T A a linear transformation from F n to F m. Let us check this in order to verify whether it is a linear transformation from F n to F m. We have to verify the two basic conditions that whether T a preserves addition whether T A preserves scalar multiplication. So let us check addition suppose we have to vectors x y in F n then by the definition of T A T sub A of x is A x. That is how we define the transformation T A T A of y is A y so what the transformation does it just pre multiplies the vector by matrix a.

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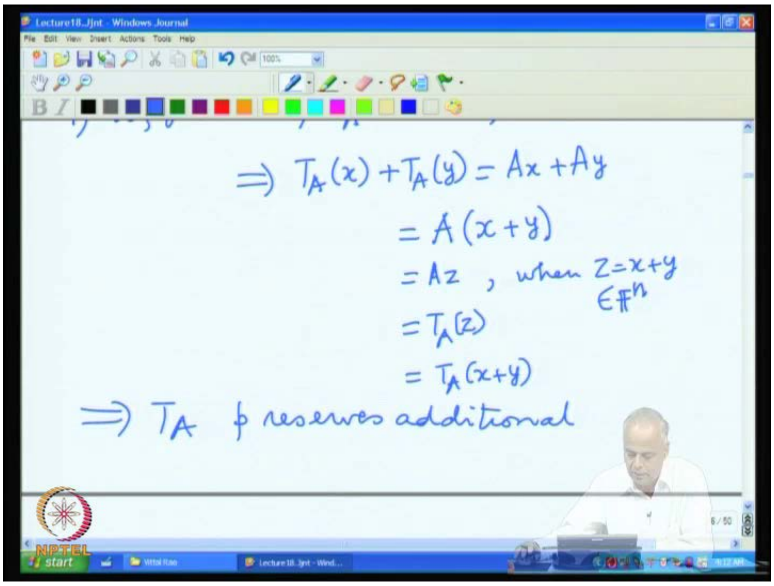

And therefore, we get T A of x plus T A of y is A x plus A y .Now the right hand side we have the matrix A times the vector x plus the matrix A times the vector y .We know that the matrix multiplication is distributive so we can write it as A of x plus y. Now if we call the vector x plus y as z where z is x plus y. Then we get A z now since x is in F n and y is in F n so z is also in F n. So the moment we take the vector z in F n and pre multiply it by a that means we are taking T A of z. Which means we are taking T A of x plus y so thus we see T A of x plus y is the same of T A of x plus T A of y and therefore, T A preserves addition so cross one a due for being qualified being a linear transformation the next addition.

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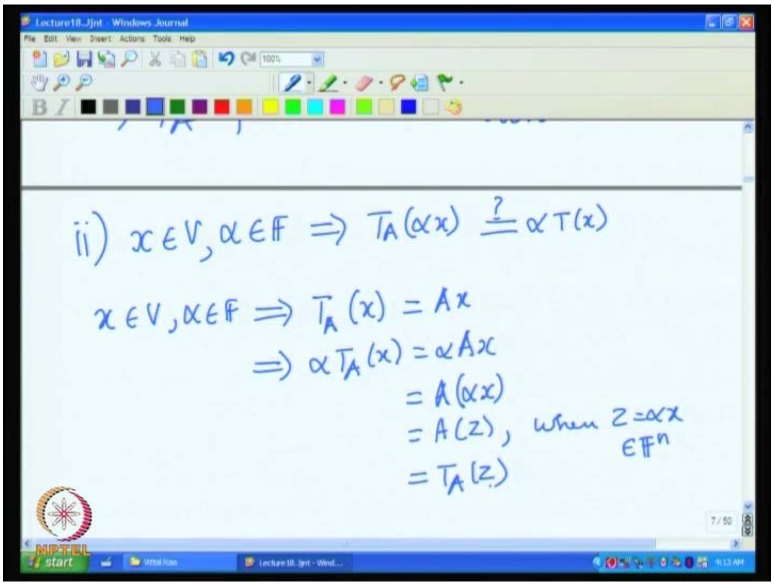

The next thing we have to check is whether T A preserves scalar multiplication .So we take a vector x in V, we take a vector scalar F and then we look at T A of alpha x and ask whether it is equal to alpha T A of x. If this is satisfied then we will have the T a preserves scalar multiplication. Now we have x belonging to V alpha belongs to F now since x belongs to V by our definition of the transformation T A x will be equal to A x which means alpha T A x will be equal to alpha times A x. On the right hand side were the matrix A the vector x and the scalar alpha in matrix multiplication the scalars can be moved in and out of the multiplication .So this will be same as a of alpha x. If we now call as A of z where now z is alpha x now x is in F n therefore, alpha times x is in F n so the z vector is in F n. Now the moment you take a vector in F n A z means we are taking T A of z.

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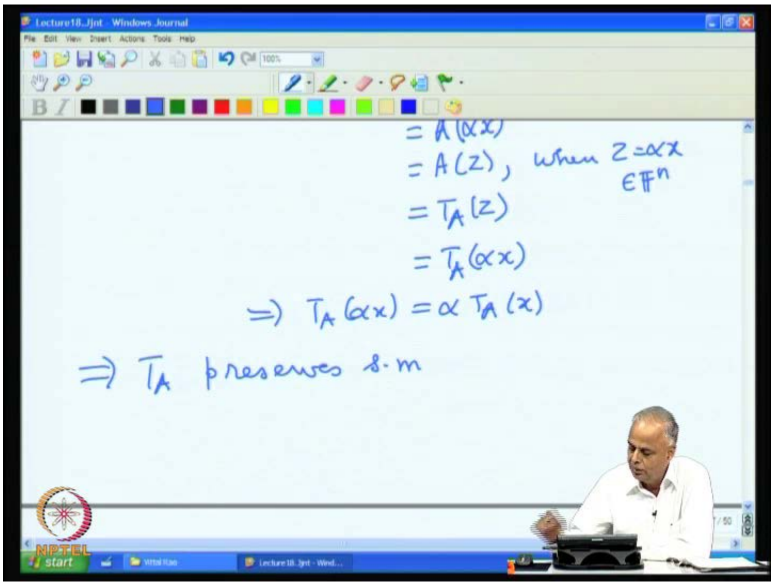

Now z is alpha x. So that is equal to T A of alpha x, so thus we see T A of alpha x is equal to alpha times T A of x and hence T A preserves scalar multiplication. Therefore, T A preserves scalar multiplication. So we have seen that T A preserves addition we have seen that T A preserves scalar multiplication.

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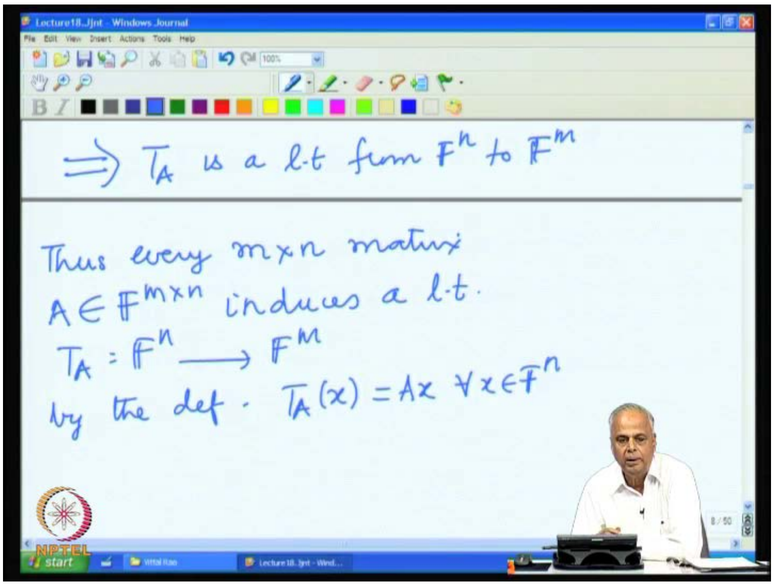

Hence these two together implies T A is a linear transformation from F n to F m. So we start with a matrix A and a m by n matrix A and from that n by m matrix A we generated a linear transformation T A from F n to F m. So thus every m by n matrix a belonging to

F m n induces a linear transformation T A from F n to F m by the definition T A x is equal to A x for every x in F n. So every matrix gives rise to a linear transformation from F n to F m.

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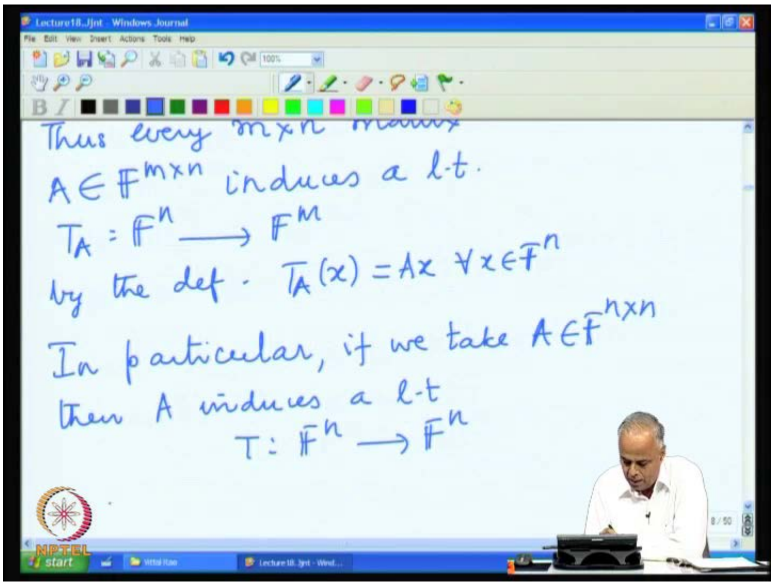

In particular if we take A to be a n by n matrix. now this case n is equal to n then a induces a linear transformation T from F n to F n by the definition T A x is equal to A x for every x in F.

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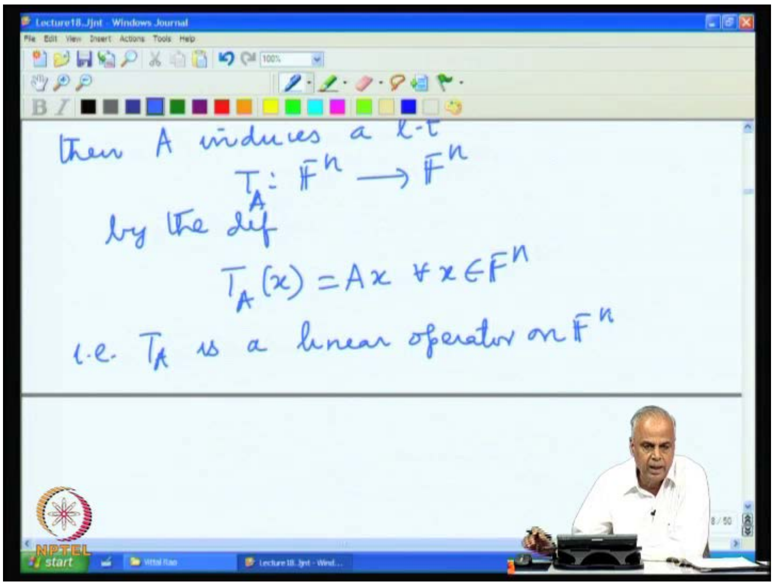

Now since this is a linear transformation from the vector space F n to itself it becomes a linear operator that is T A is a linear operator on F n.

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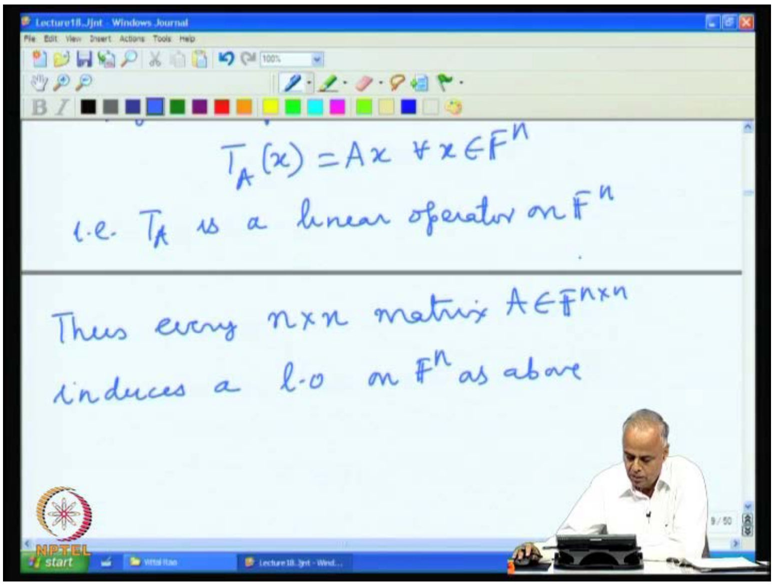

So thus every n by n matrix induces a linear operator on F n. Thus every n by n matrix A in F n by n all the entries are from the field F induces a linear operator. We will write lo for linear operator on F n how does it induce as above that is T A x is equal to A x so every n by n matrix induces a linear transformation from F n to F n every n by n matrix induces a linear transformation from F n to F n and hence a linear operator on F n.

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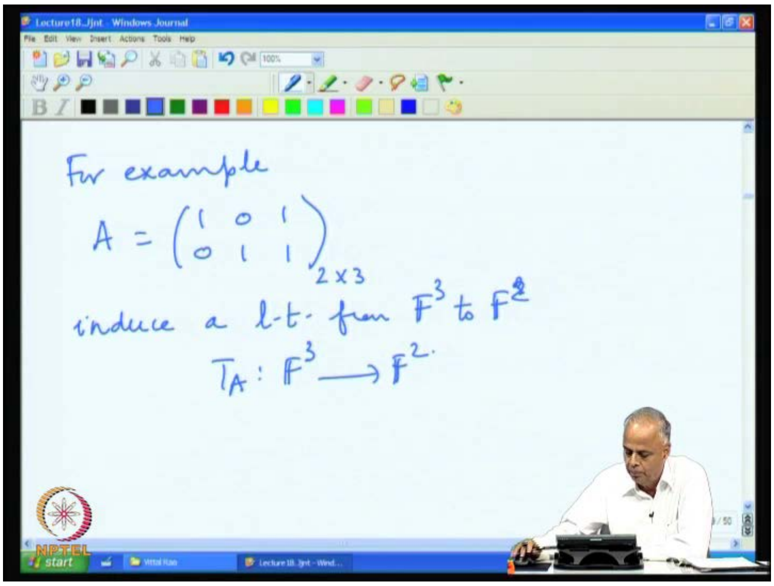

For example, look at a is equal to 1 0 1 0 1 1. Now this is a 2 by 3 matrix so this is m is equal to 2 and n is equal to 3. Therefore, this will induce a linear transformation from F 3 to F 2 let us call this as T A mapping F 3 to F 2.

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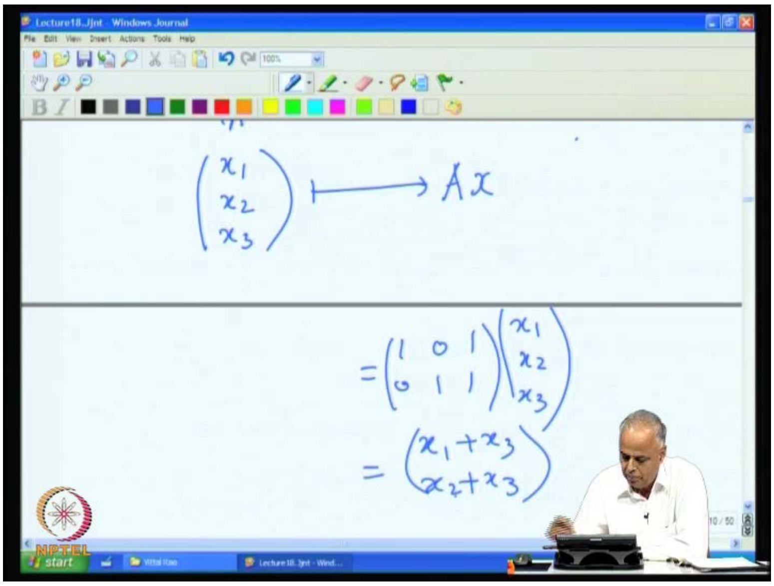

How is it defined it has to take a three component vector and it should map it to a two component vector. How do I get the two component vector I have to take the matrix A and multiply it by the vector x. which is the same as A is 1 0 1 0 1 1 and a vector is x 1 x 2 x 3 and therefore, we get x ` plus x 3 x 2 plus x 3.

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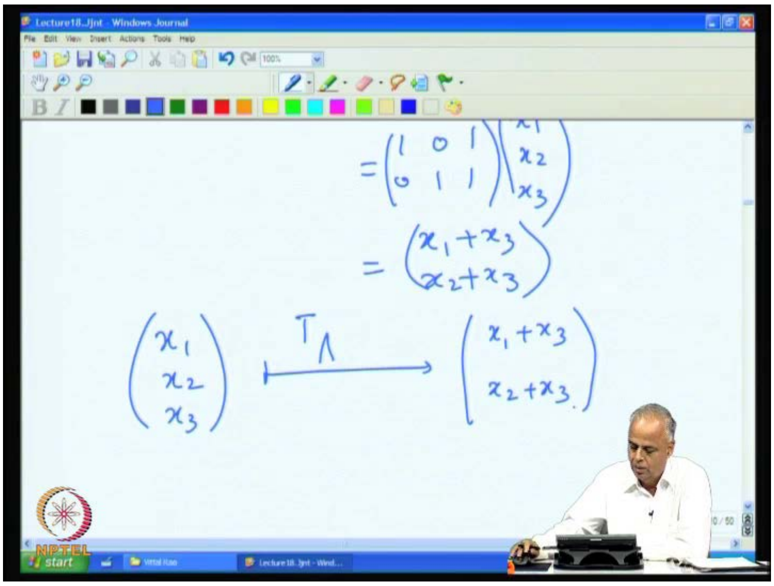

So this matrix induces this transformation which takes the vector x 1 x 2 x 3 this is the matrix transformation T A which takes it to x 1 plus x 3 x 2 plus x 3.

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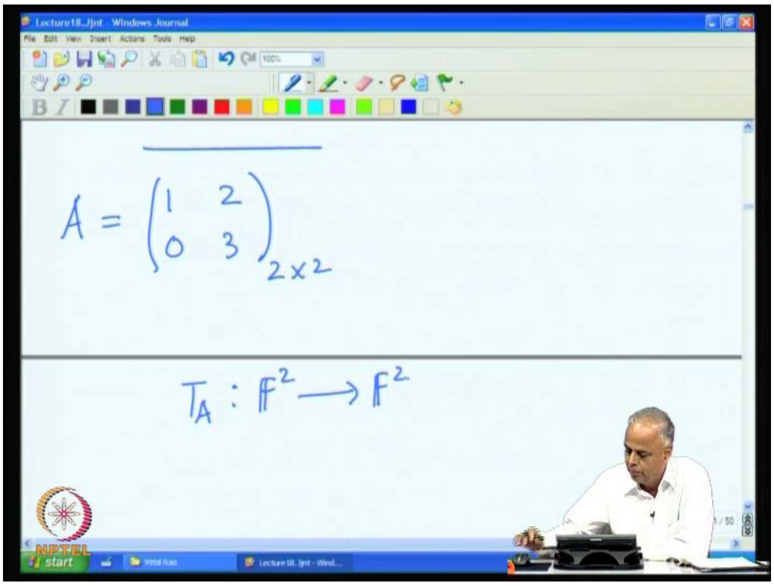

Similarly, consider A now to be 1 2 0 3 then this is a 2 by 2 matrix. So it will have to generate a transformation which will be now a linear operator because m is equal to n so it should be a linear operator from F 2 to F 2.

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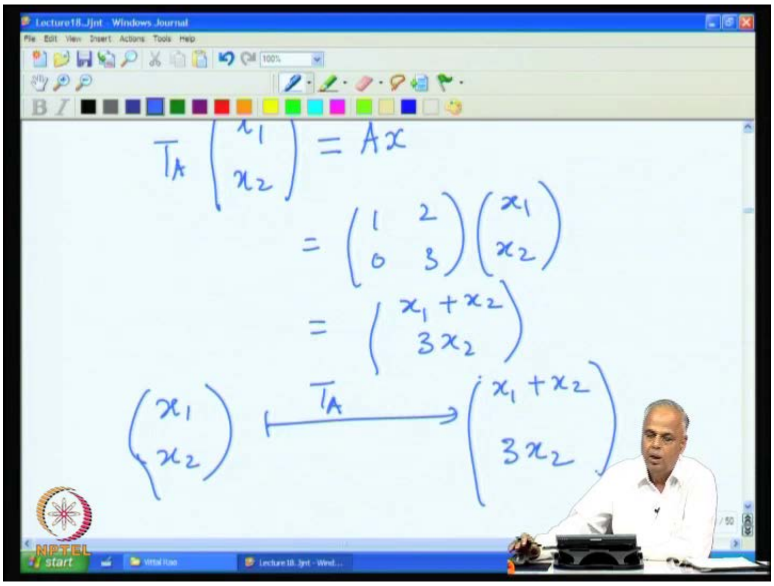

And what should it do T A of a vector x 1 x 2 must be again a vector in x 1 F 2. So it has two component. How do I get it I have to take the matrix a and multiply it by the vector x. which means 1 2 0 3 into x 1 x 2 which is x 1 plus x 2 plus and 3 x 2. So this transformation defined by this matrix takes the vector x 1 x 2 x 3. this is a 2 by 2 matrix so x 1 x 2 maps it to the vector x 1 plus x 2 and 3 x 2. Thus every n by n matrix gives us a linear transformation from F m to F n F n to F m every n by n matrix gives us a linear transformation from F n to F n and therefore, a linear operator on F n.

So the linear operators in F n will have odd number of examples of the square matrix is with n rows and n columns. So this is a huge class of examples which will be interesting to us because we are trying to solve a system of m equations in n unknown is through a matrix A which boils down to looking at the linear transformation generated by the matrix A that is a transformation T A.

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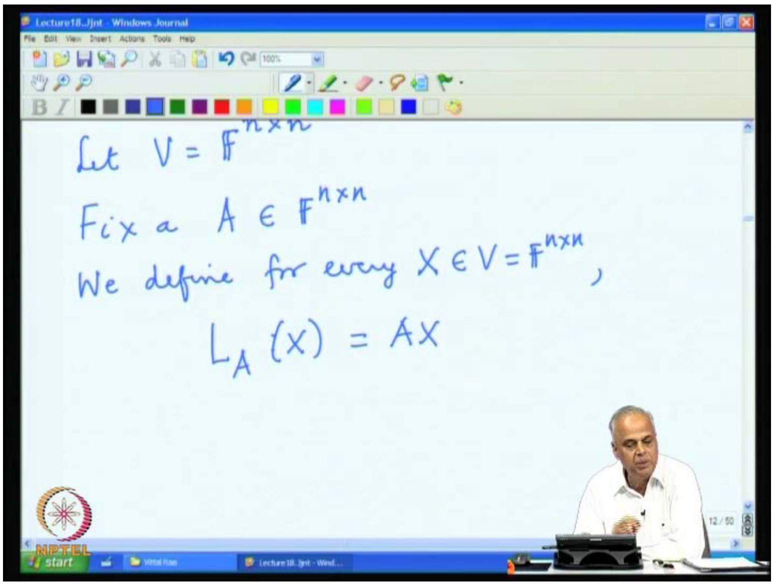

Now let us look at another example another class of examples. We will be looking at a lot of examples in the world of matrices. Because that is our interest all our questions that we have raised in the beginning of the course are essentially concerning matrices. now let us look at vector space V to be the set of all n by n matrix over F. So the vector space of all square matrix is of size n by n over the field F. We have already seen that this is a vector space with usual matrix addition and scalar multiplication rules for matrices. Now look at this vector space fix a vector V here what are vectors here they are all matrices.

So fix a matrix A in F n n so once you fix a matrix a in F n n through this again we are going to generate a linear transformation on F n cross n. Which means we are going to generate a linear operator on V how do we do this so we define for every x in V what are the elements of V they are all matrices what type of matrices n by n matrices and the entries are all over the field F. So V is a F n cross n here for any matrix x we convert it to another matrix n by n matrix. How do we do this I will use a notation which will be useful to generalize L A of X to be A X so we are left multiplying any vector any matrix x by the matrix A. That is why we use the symbol L that is means left multiplication by what the matrix A. So the transformation is denoted by L A left multiplication by x. Now we see that A is an n by n matrix because we have chosen a in F n cross n and x is an n by n matrix because I chosen x is in V so both are n by n matrices.

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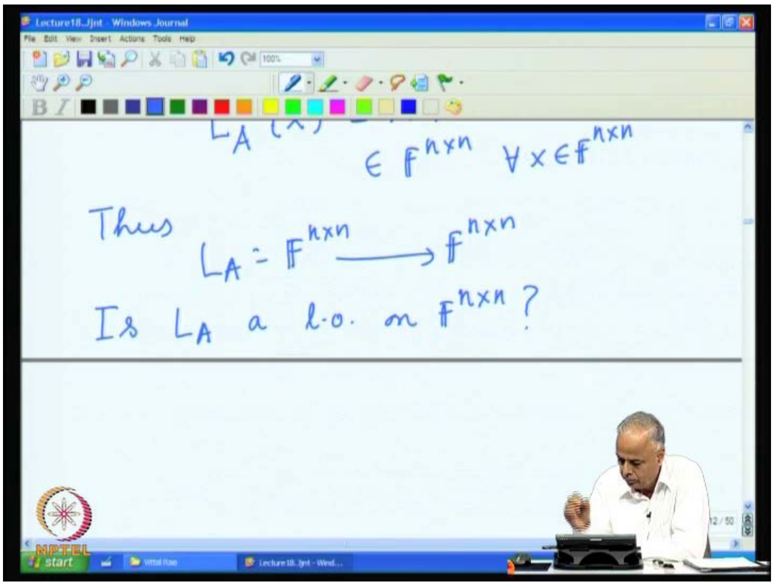

So that product will be an n by n matrix and therefore, this belongs to F n cross n for every x in F n cross n. And therefore, L A converts n by n matrix into another n by n matrix and therefore, L A is definitely a transformation from F n cross n to F n cross n in other words it encodes an n by n matrix by another n by n matrix. This a is the hazing operation or the encoding operation by pre multiplying the matrix x by a we are hiding the original matrix A and disguising it as a new matrix L A of x. Thus we have a linear at least we have a transformation from F n to F n. So the moment we have a transformation between two vector spaces we always ask whether is a linear transformation in particular. If we have a transformation from a vector space into itself we ask whether it is a linear operator is L A a linear operator on Fn cross n now L A is qualified to be called a linear operator on F n cross n. If it preserves that to be always linearity comes from the two fundamental things namely it preserves addition and it preserves scalar multiplication.

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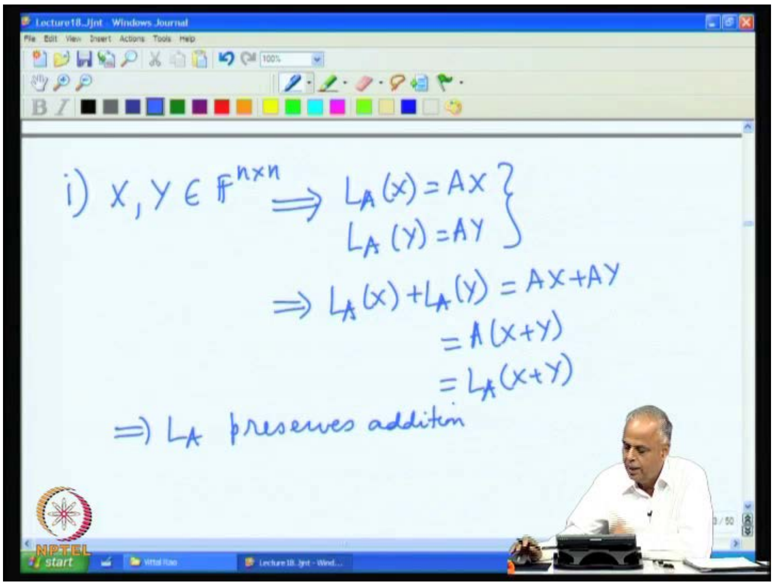

So let us verify whether L A does this two so if you have X and Y in F n cross n we saw the two vectors in the vector space. Because we have chosen the vector space to be the vector space of all n by n matrices and by our definition L A of X is left multiplication of X by A L A of Y is left multiplication of Y by A and that says L A of X plus L A of Y is A X plus A Y again matrix multiplication is distributive. It is A X plus Y X is n by n Y is n by n therefore, X plus Y is n by n. Whenever we take a n by n matrix and pre multiply it by a it boils down to taking l a of that first multiplier. Therefore, L A of X plus Y is L A X plus L A Y so that says L A preserves addition the next thing that is required for L A to be qualified to be called as a linear operator is that it preserves scalar multiplication.

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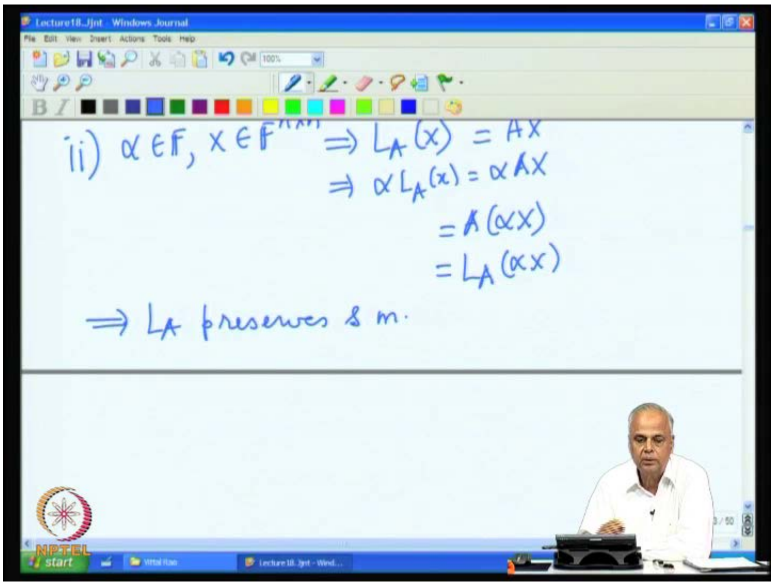

So let us look at a scalar and a vector and the vector space is n cross n so it is a matrix vector. Now it is a matrix again so we have L A of X is equal to A X by definition and therefore, alpha times the L A X is equal to the alpha A X again matrix multiplication constants can be moved in and out. So it is a alpha X so again when we take X is a n by n matrix and multiply it by a scalar we get a alpha X is an n by n matrix and whenever an n by n. Matrix is pre multiplied by a that means we are taking L A of alpha X and thus we see that L A of alpha X is alpha L A X. That means L A preserves scalar multiplication thus L A preserves both addition and scalar multiplication one and two to get a L A is linear and since it is a transformation from F n to itself it becomes a linear operator.

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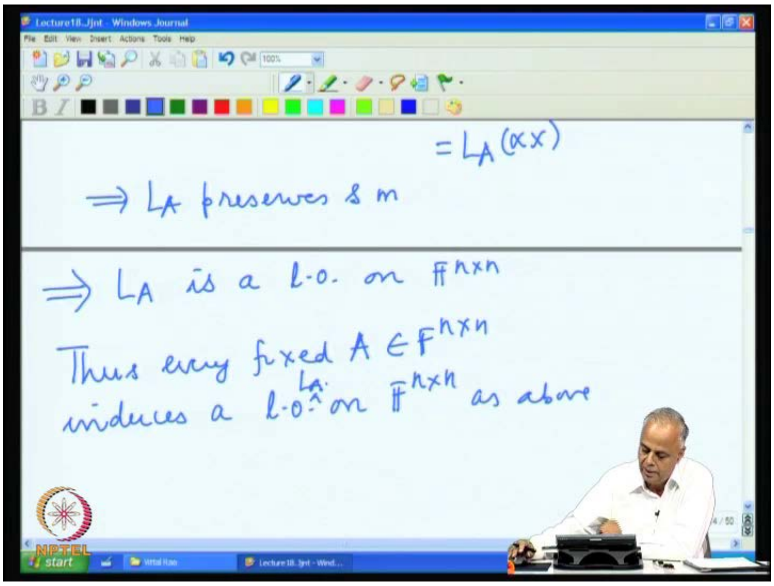

So L A is a linear operator on F n cross n. Thus every we started with a fixed matrix A and generated a linear transformation. So thus every fixed A in F n cross n if we fix one A you will get one resolution induces a linear operator on F n cross n. As above the linear operator is what we denote by L A. Every a generates an L A as above we can start with an n by n matrix and go on left multiplying by a fixed matrix and then we go on getting a q r n by n matrix and that is the transformation and that transferred to be a linear operator. Now you can obviously guess instead of left multiplication we could have also done right multiplication.

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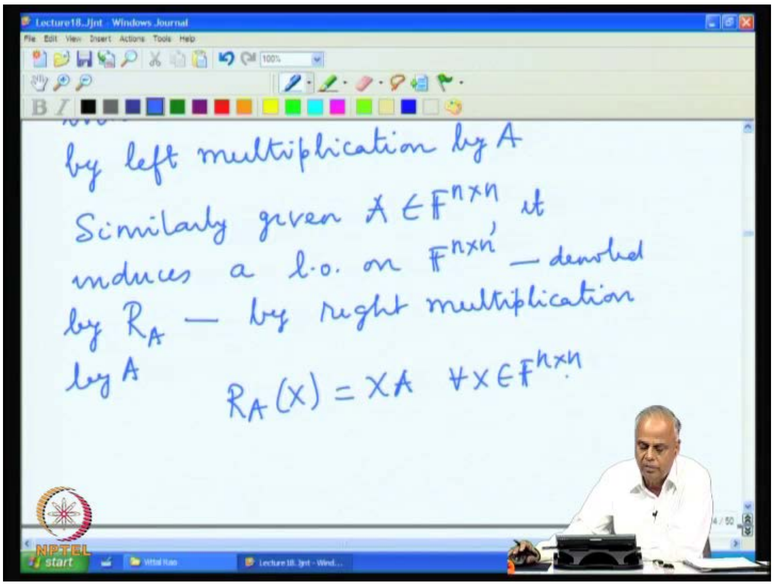

We will write here as above by left multiplication left multiplication by a similarly, given a in F n cross n it generates or induces a linear operator on F n cross n. Which we denote by R A by R stands for right multiplication by right multiplication by A to be a R A X equal to X A for every X in F n cross n by the same arguments in the above you can verify this is also a linear operator. So thus we have starting from every fixed n by n matrix we can generate a linear operator on the vector space of n by n matrices.

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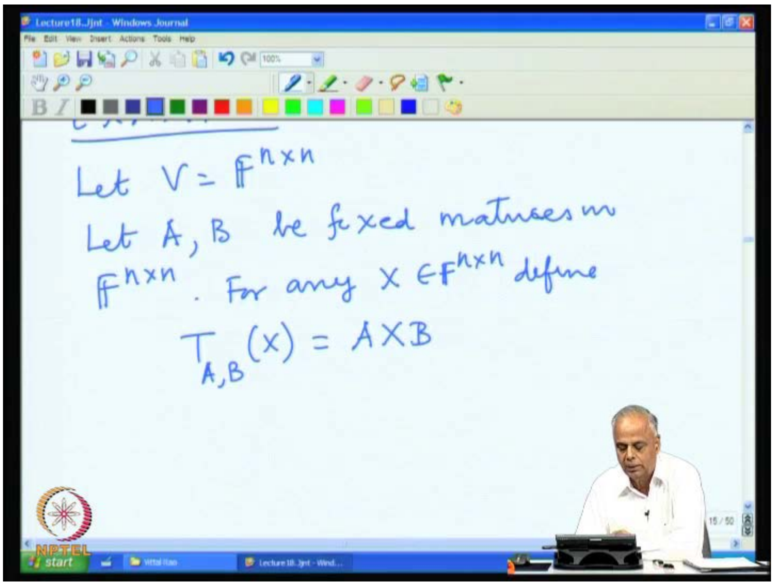

Let us now do a little more of this example in the first case we took a and left multiplied it by A and in the second case we took a and right multiplied it a to get R A. We could have done both we could have left multiplied as well as right multiplied and we could have chosen one A to left multiply and one B to right multiply. So that what we do now so let V is equal to F n cross n for F left multiplying fix one matrix A and for right multiplying you fix another matrix B. So let A B be fixed matrices Fn cross n take any two fixed matrices in F n cross n now we are going to generate a transformation starting from these two matrices.

We will call it as T A B so for any X in F n cross n define T A coma B of X to be use a for pre multiplying and B for post multiplication. Now we leave it as an X first of all we observe that everything is n by n matrix. So the product is going to be in n by n matrix and therefore, A B of X is also a n by n matrix.

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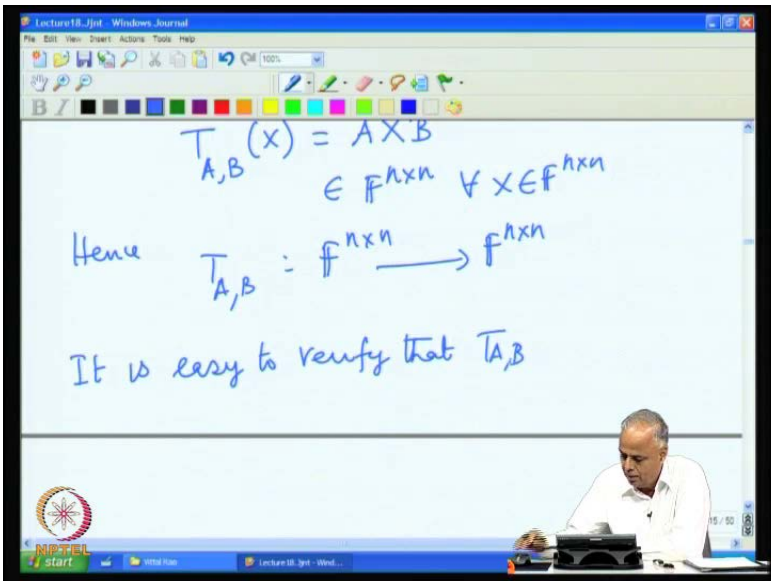

This belongs to F n cross n matrix for every X in F n cross n and hence¸ T A B is a transformation from F n cross n to F n cross n. Now we leave it as an exercise to verify simply write it is easy to verify the same arguments we have to carry on to verify that T A B is a linear operator on F n cross n.

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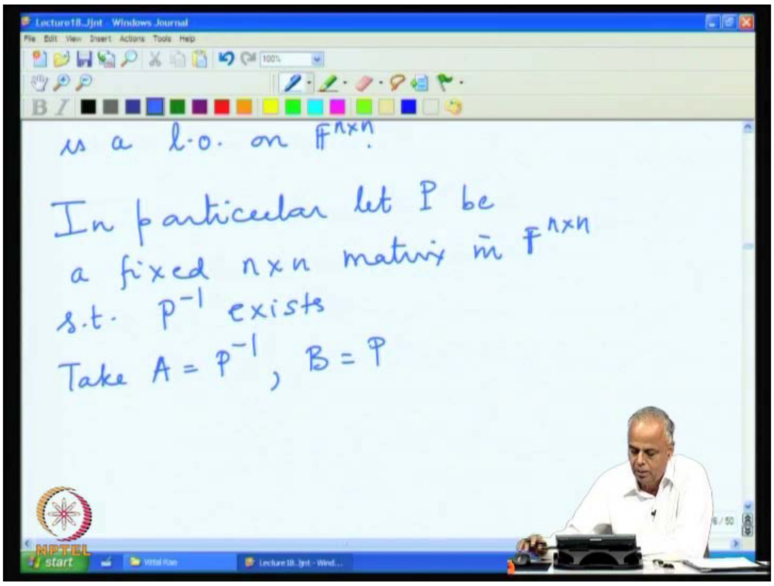

We have to again verify that T A B of X plus Y is equal to T A B of X plus T A B of Y that is T A B preserves addition and T A B of alpha X is alpha times T A B of X. That is T A B preserves scalar multiplication. So we have now a handle on the left and a handle on the right or a coding from the left or a coding from the right to change a matrix X to the newer matrix an encoded. Matrix A X B in particular let p be a fixed n by n matrix in F m cross n such that inverse exist p inverse exists. Suppose I start with an n by n matrix which is invertible then we take a to be p inverse and b to be p in the above. So if we can take A to be p inverse and B to be p and what we get is linear transformation.

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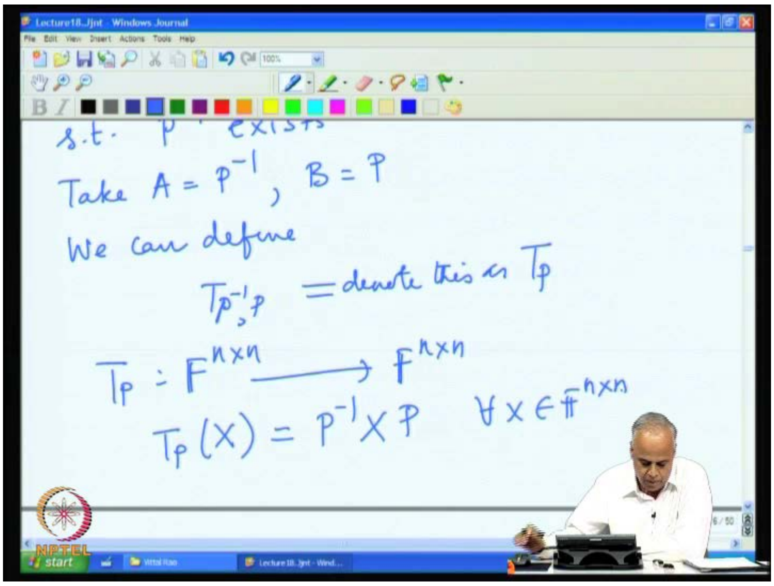

Then we can define T A B what is T A B now T p inverse p by sort we denote this by denote this a just as T p. Because there is only one matrix involved and is inversed. So we call it as T p how is T p defined T p is a transformation from F n cross n to F n cross m and it is defined as T p of X is A. In this case it is p inverse x B in this case it is p for every X in F n cross n. Therefore, it converts the vector or matrix X to newer matrix p inverse X p and since for any A and B this will generate a linear operator in particular for this A and B it will generate a linear operator.

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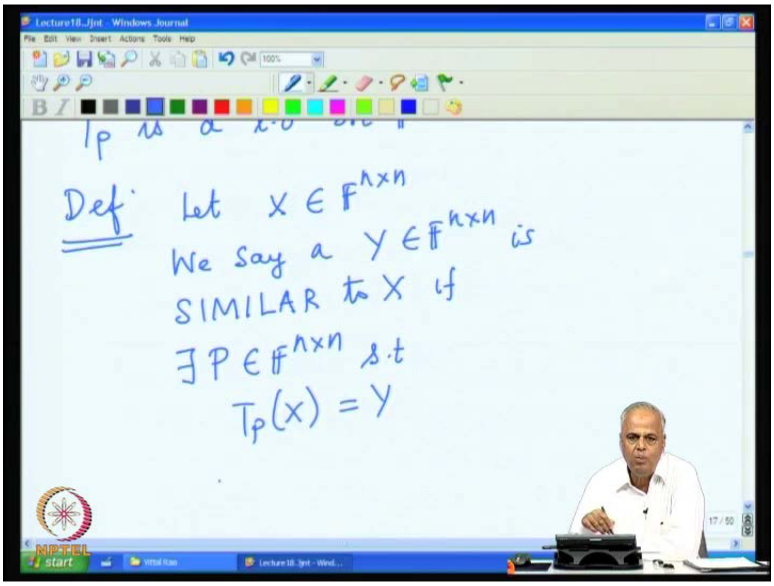

T p is a linear operator on F n cross n. We will introduce a definition which will be useful later. Let X be any n cross n matrix we say a y belonging to F n cross n matrix is similar to X if there exist a p in F n cross n that is a n by n matrix. Such that T p of X is Y in other words that Y is a coded version of X is some code of p. The code is generated by p because it covers the vector the matrix X as p inverse X p. Therefore, for the coding of matrix T p X we have used the matrix p. Therefore, we say Y is similar to X if Y is a coded version of X in some code p in some code generated by some p. So then we say X is similar to Y or Y is similar to we say we will look at things later but we simply follow that Y is similar to X if it happens. This is very important notion which will come in handy which will come in our later analysis and therefore, any set T p is called a singularity transformation.

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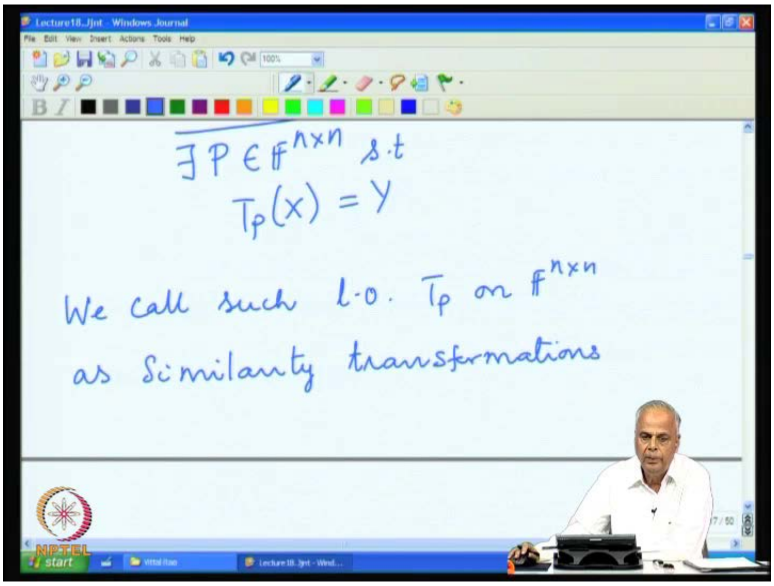

We call such transformation , such linear operator T p on F n cross n as similarity transformation we can generalize likely this notion for rectangular matrices.

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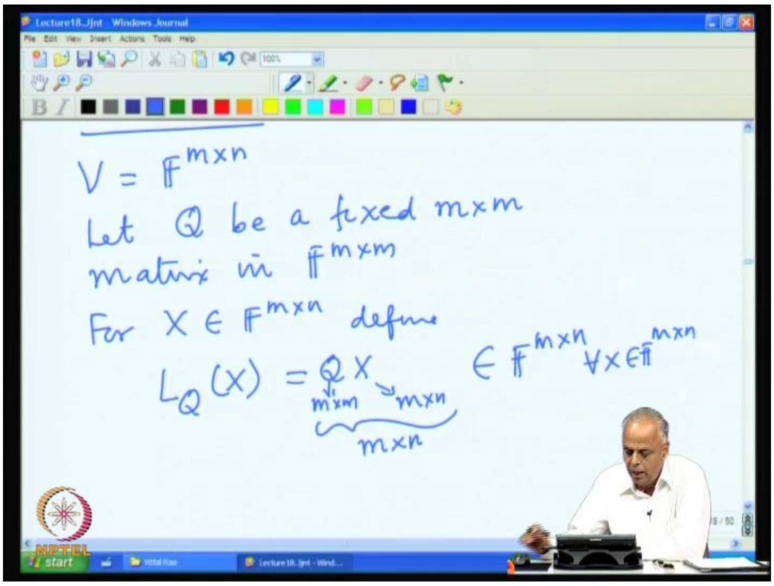

Let us look at a next class of example these are the class of example which will come into play in getting the answer to the various questions. That we raised about matrices and linear systems of equation. So let us now take the vector space V to be rectangular matrices m by n matrices for m. Now I want to pre multiply or left multiply X in F m so I have to take an m by n matrix. So let as take let Q be a fixed m by m matrix in F m cross m then for X in F m n define L Q of X again L is left multiplication is what Q X this is perfectly. Because this is m by m Q is m by m and X is m by n so the product is going to be m by n. This belongs to F m by n for every X is in and therefore, this L Q codes a m by n vector into another m by n vector.

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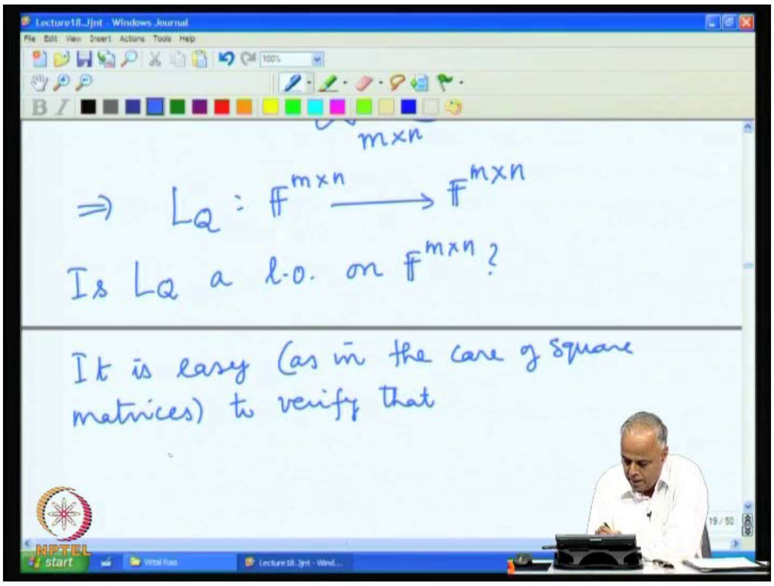

And that means L Q is the transformation from F m cross n to F m cross n. Again whenever we have a linear transformation from a vector space to itself we want to know whether it is linear or not is L Q a linear operator on F m cross n. Along the same lines as we did for the square matrices you can verify L Q is a additional scalar multiplication.

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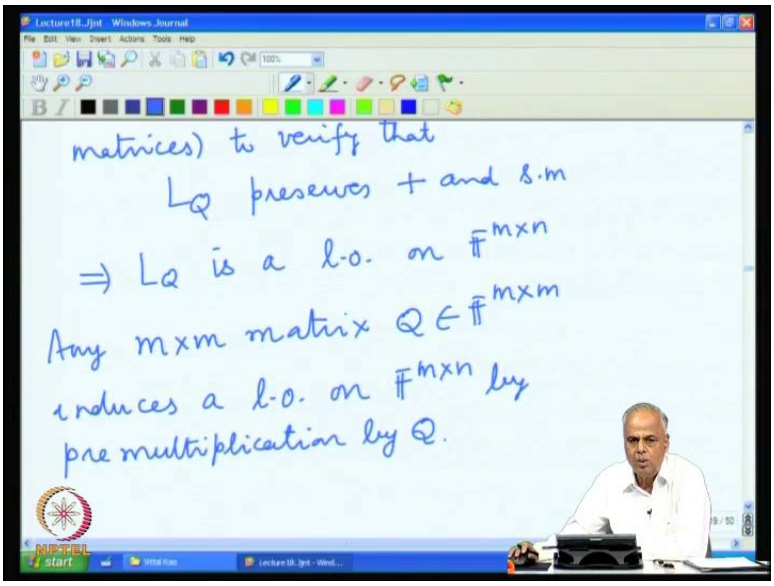

It is easy as in the case of square matrices to verify that L A L Q preserves addition plus and scalar multiplication and therefore, L Q is a linear operator F m cross n. So any m by n matrix so any m by n matrix in Q in F m cross m induces a linear operator on F m cross n by pre multiplication by Q. Now we want to post multiply since we are dealing with m by n matrices we have to take n by n matrix.

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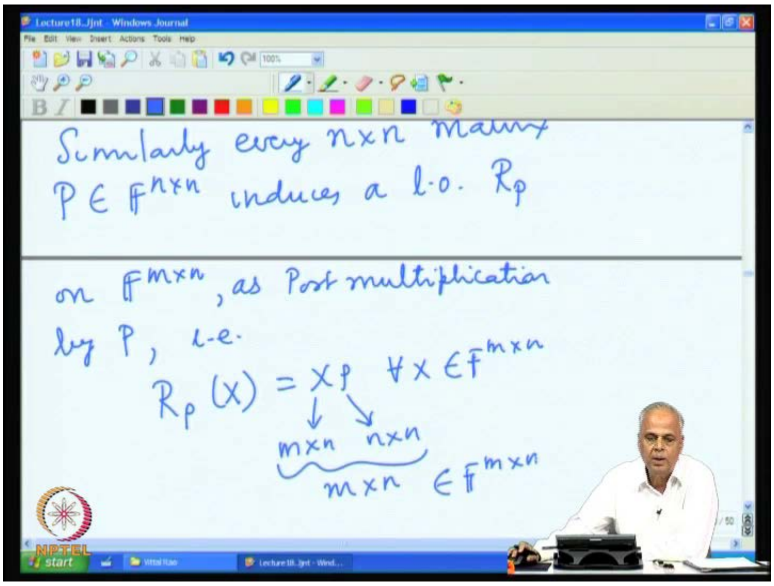

Similarly, every n by n matrix let us call it as p in F n cross n induces a linear operator R p R stands for right multiplication on F m cross n. As post multiplication by p that is R p of any X is X p for every X in F m cross n again note that X is m by n p is n by n. So the product is m by n and therefore, it is going to be in F m cross n. Therefore, it maps F m cross n matrix to n cross m. So we have again left multiplication as well as right multiplication but when we are dealing with rectangular matrices. We must be very careful as to the size we choose for left multiplication and the size we choose for the right multiplication. Now just as we did for square matrices we can combine left and right multiplication to as follows.

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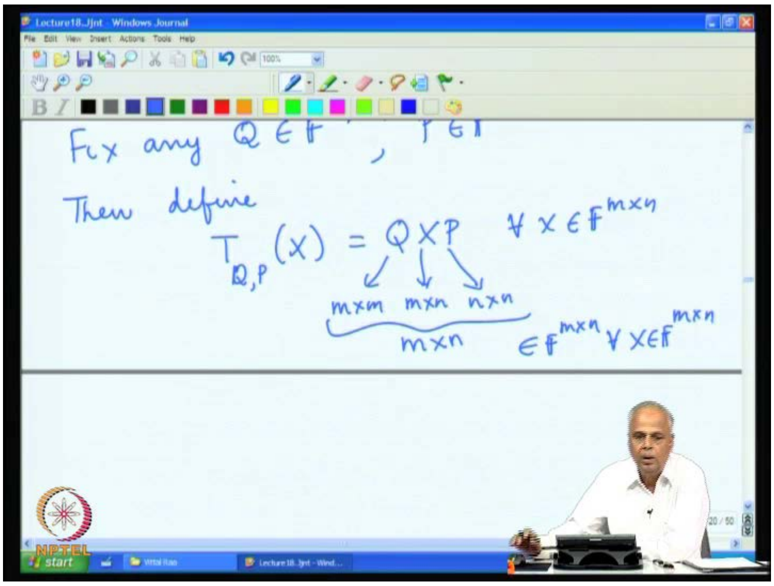

So given or fix any Q in F m cross m and p in F n cross n p and q are both square matrices Q is size m. Row size of the space we are going to choose and the p is of the column size in F m cross n. Then define T Q p of x to be we left multiply by Q and right multiply by p for every X. Now we this is m by m this is m by n and this is n by n and therefore, product is going to be m by n and therefore, it belongs to F m cross n for every X in F m cross n and therefore, it again codes m by n matrices to n m by n matrices.

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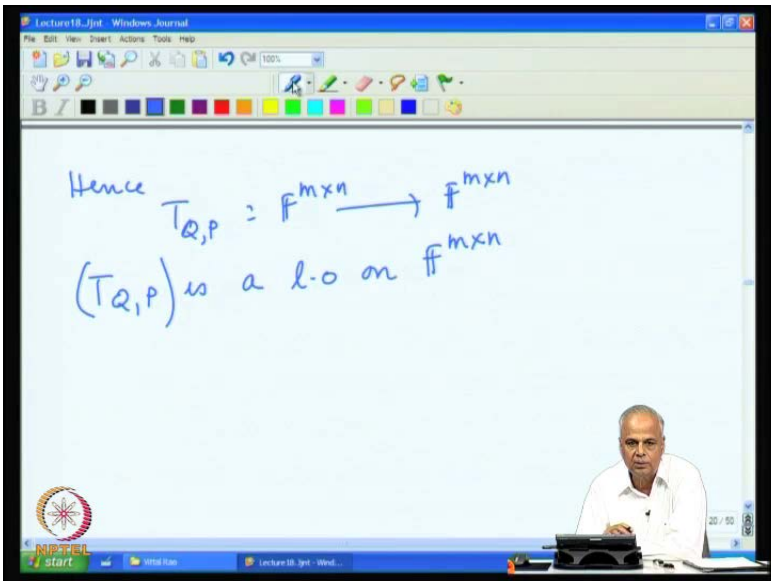

And hence T A T Q p is a linear transformation from m cross n to m cross n matrices and since it is a mapping and linear on the same vector space T Q p is a linear operator. So this T Q p is a linear operator on F m cross n. Now we will see later but, the notion we introduced here for square matrices. Where we took p inverse x p this sort of transformation which we called as similarity transformation are useful in question about diagonalization. That we raised for square matrices in the first two lectures and then we later for the m by n matrices. The type of pre post multiplication transformation that we have here will come in handy in the question also called singular value decomposition. Which we shall be studying and which is the generalization of the question of diagonalization in the case of from the square to the rectangular matrices. So we have several classes of transformation that we can talk about for square matrices and for rectangular matrices.

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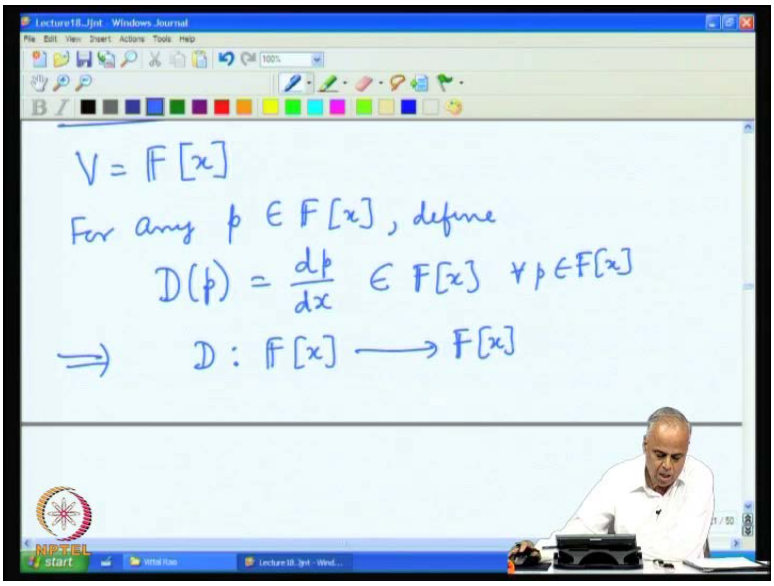

We shall continue to look at more examples. Because linear algebra one of the most things in the linear transformation. So we look at more simple example let as consider the example of all polynomial over F in the variable x. So F x transude for vector space of all polynomials and now for any what are the vectors in V they are polynomial denoted by p. For any p in F x that is any polynomial define D of p to be dp by dx that is the derivative that is why we use the symbol capital D. So the transformation we are thinking of, now is differentiation so take a polynomial p and just differentiate it and we know that if we differentiate the polynomial. We will again get a polynomial and therefore, it belongs to F x for every p in F x and hence D is a transformation from F x to F x the moment.

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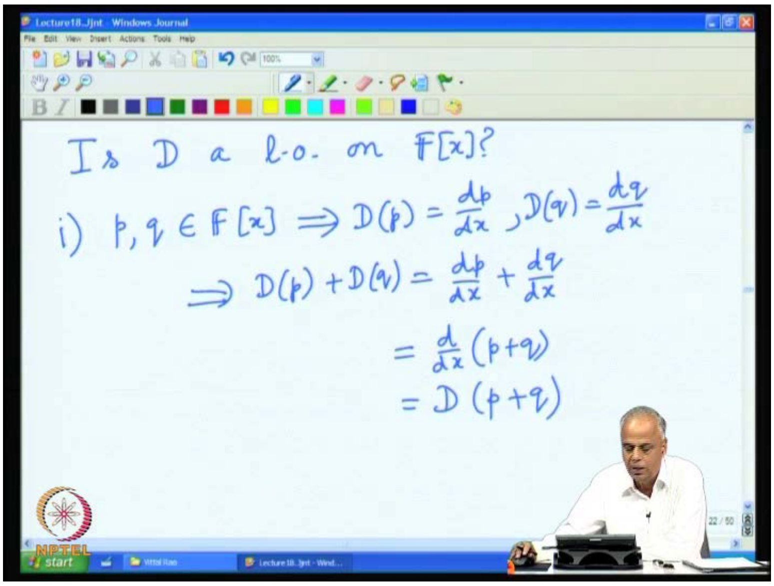

We have transformation from vector space itself natural question is the D linear operator on and again when does it qualified to be a linear operator it has to preserve the two basis operation. Let us check this p and q are in F x means by definition d of p is dp dx and d of q is dq dx that means d of p plus d of q is dp dx plus dq dx but, the derivative of a sum is the sum of the derivates and hence this is equal to d dx of p plus q now p is the polynomial and q is the polynomial and therefore, p plus q is the polynomial and therefore, we are taking the derivate of polynomial the moment. We take a polynomial we mean d of the polynomial so this is equal to p plus q so thus we see dp plus dq is dp plus q.

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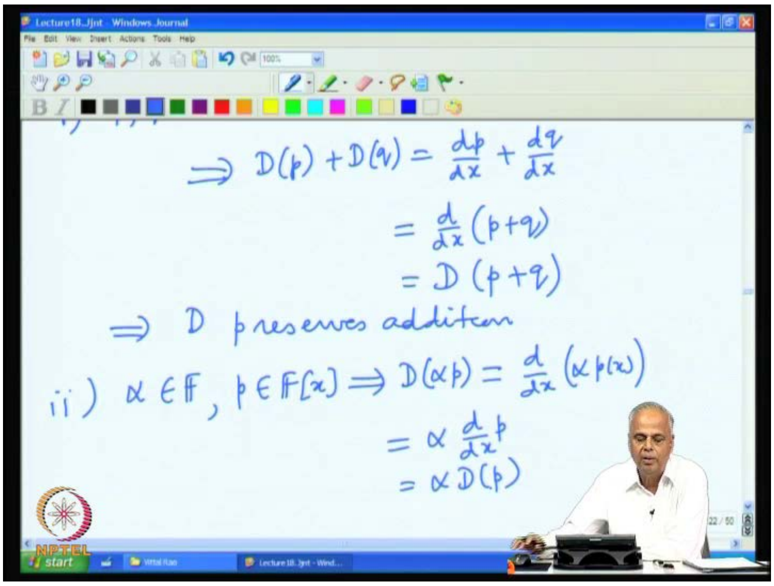

And hence D preserves addition and the next thing we have to verify is that whether D preserves scalar multiplication. So if alpha is any scalar and p is any vector and vector is are now polynomial we have D of alpha p by definition. It should be D dx of alpha p x but, you know derivate of constant time a function. The constant can be pulled out so this will be alpha d dx of p but, d dx of p is just d of p therefore, we have d of alpha p is alpha dp.

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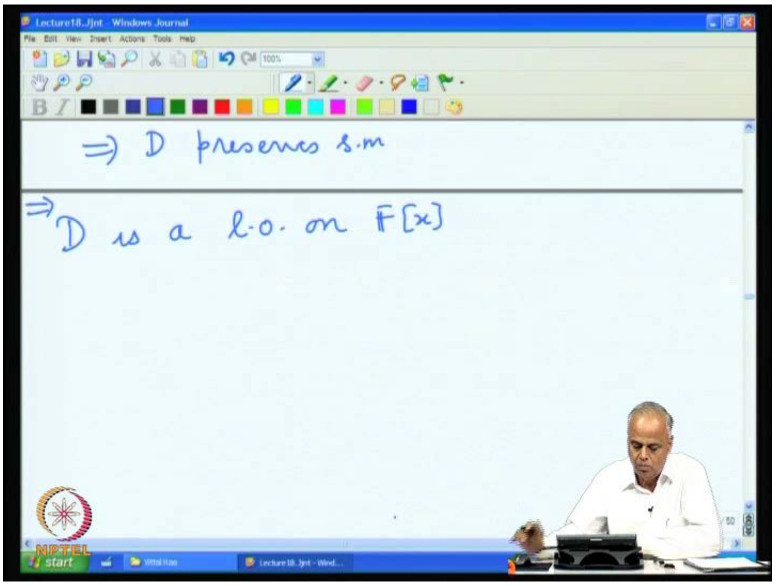

And hence D preserves scalar multiplication and thus we have D preserves addition and scalar multiplication. It is a transformation from F x to F x all these put together gives that D is a linear operator on F of x and thus the differentiation operator is linear operator on the space of all polynomial.

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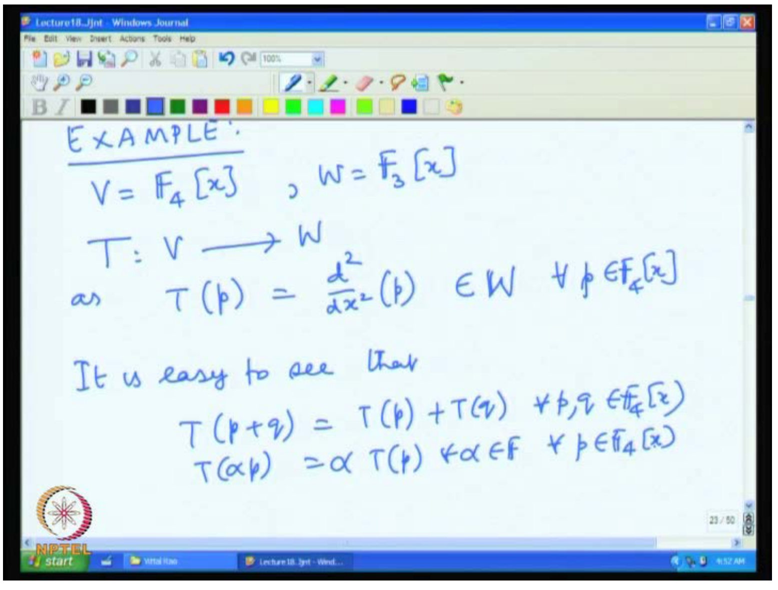

Now we have one more example, we extend this differentiation operator now we take F to be F 4 x and W to be F 3 x with space of all polynomials of degree less than or equal to 4 is V less than or equal to three is W. Then we define T mapping V to W as T of p is second derivative of p once again derivate splits addition into separately and scalar multiplication separated out and therefore, it is easy. Now first of all you see that if you take polynomial degree less than or equal to 4 and we differentiate it twice to be a polynomial of degree to two. Since W contain all polynomial of degree less than or equal to 3 all this will belong to W for every p in F 4 x. Therefore, it will be a mapping from V to W and now it is easy to see that T p plus q is T p plus T q for every p q in F 4 x and T of alpha p is alpha T p for every alpha in F and every p in F 4 x.

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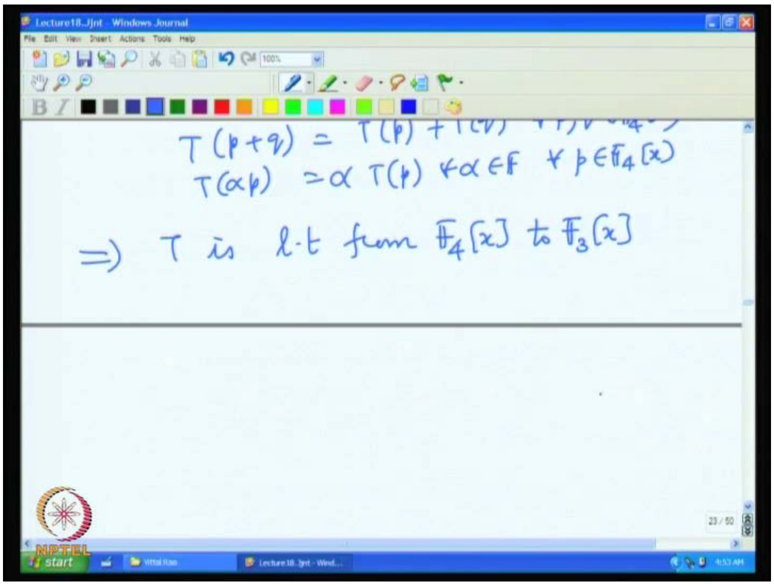

And therefore, T is the linear transformation from F 4 x to F 3 x. We shall study more examples of linear transformation in the next lecture and continue study in the structure of linear transformation. It is a study of structure of linear transformation which gives us the answer to all question that we have raised.