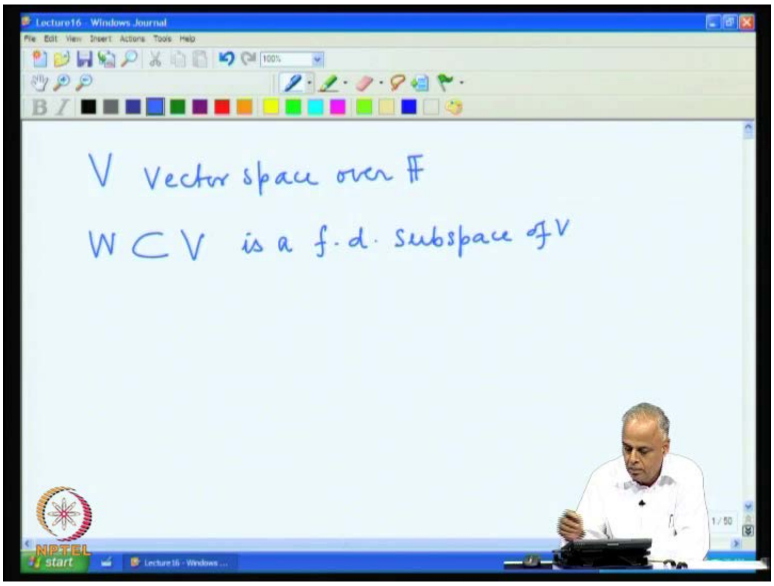
Advanced Matrix Theory and Linear Algebra for Engineers

Prof. R. Vittal Rao
Centre for Electronics Design and Technology
Indian Institute of Science, Bangalore

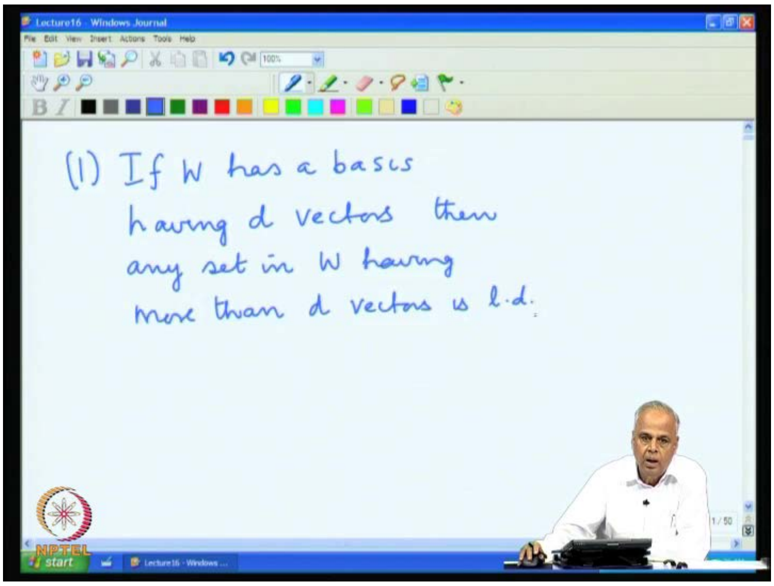
Lecture No. # 16
Basis - Part 3

(Refer Slide Time: 00:24)



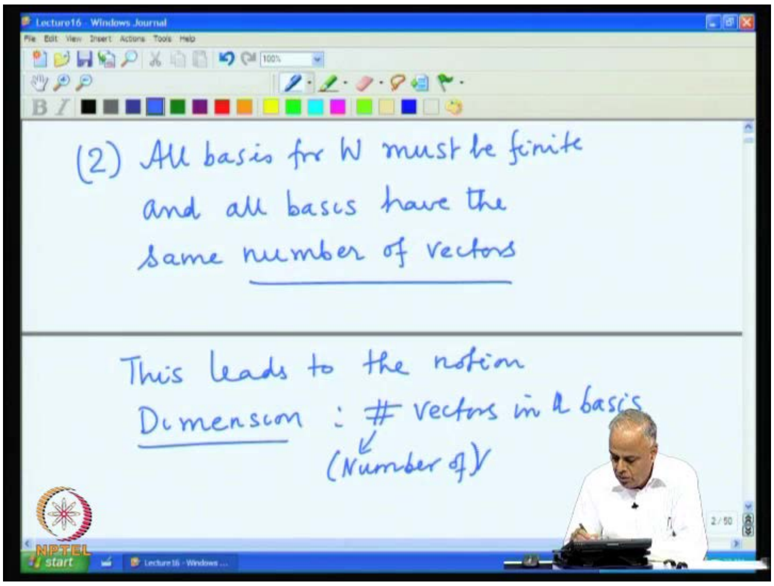
In the last lecture, we studied finite dimensional spaces, and we observed the following simple properties. Suppose, V is a vector space over F, and W is a finite dimensional sub space of V.

(Refer Slide Time: 00:46)



What we observed for the following facts? If W had a basis having d vectors, then anything about the d must be linearly depended, then any set in W having more than d vectors is linearly dependent.

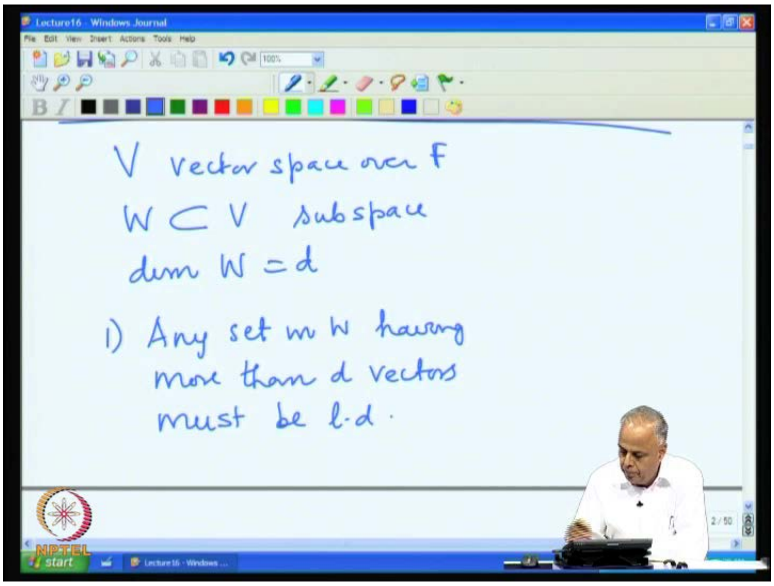
(Refer Slide Time: 01:28)



Now using this fact, we establish that all bases for W must be finite the moment, the W is a finite dimensional, it is one finite basis, and consequently all bases must be finite and all basis have the same number of vectors. So, the vectors may be different in different basis, but the number of vectors must be the same in every basis. This leads to the notion of the dimension of a sub space; the dimension was just the number of vectors in a basis.

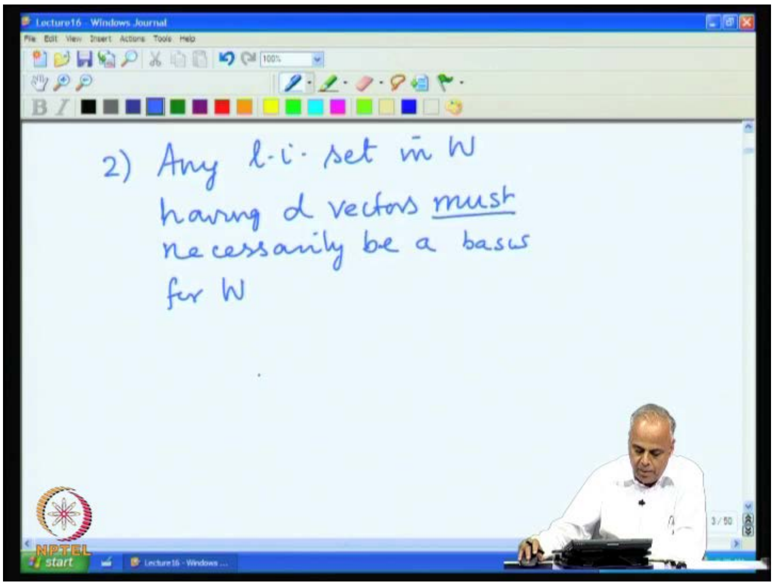
The dimension of any finite dimensional sub space is equal to the number of vectors in a basis. This symbol means number of vectors. So, the number of vectors in a basis is call the dimension of that space. In particular, if V itself is finite dimensional, we call it finite dimensional vector space, and a number of vectors in the basis will be called the dimension of that sub space.

(Refer Slide Time: 03:06)



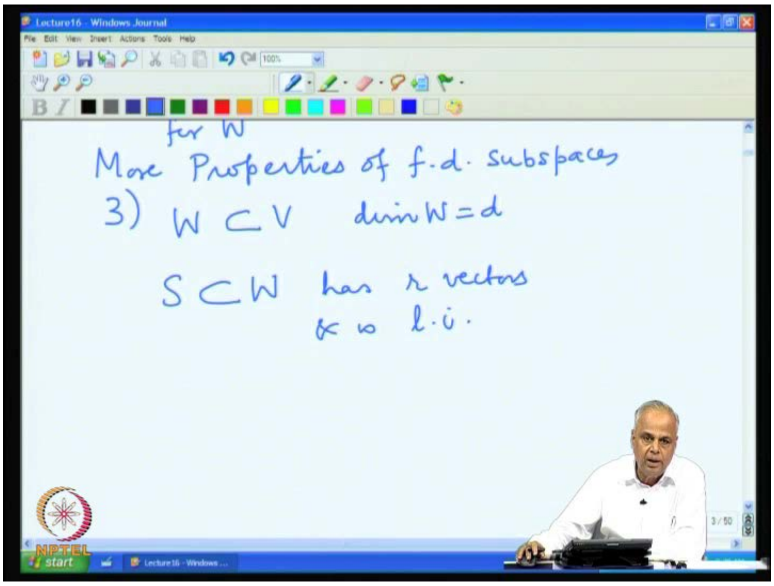
Now, suppose V have a vector space, over a field F and we have a sub space W and dimension of W is d. Then, we observed by the same principle is above. Since, the dimension is d any d plus 1 vectors must be linearly depended. So, any set in W having more than d vectors must be linearly depended.

(Refer Slide Time: 03:58)



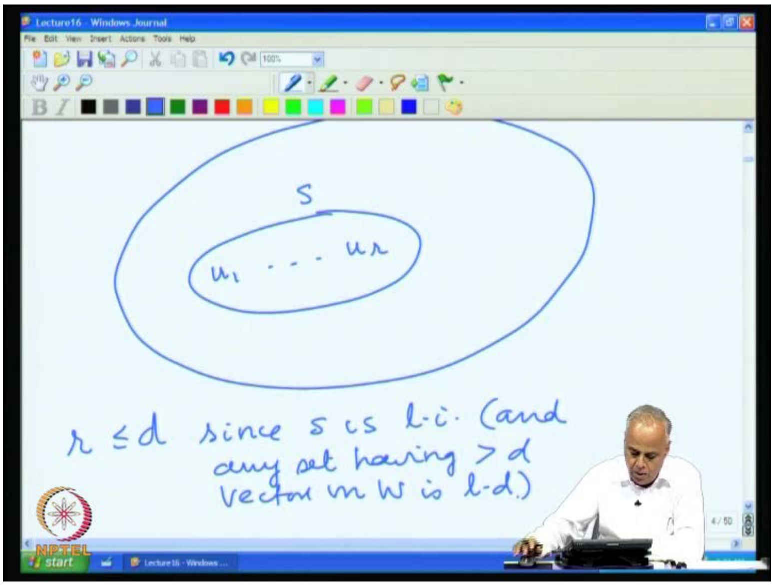
And more importantly, if we have n linearly d linearly independent vectors, they must necessarily form a basis for d. So, any linearly independents set in W having d vectors must be must necessarily be a basis for W. So, in d dimensional space any d linearly independent vectors will automatically for a basis.

(Refer Slide Time: 04:44)



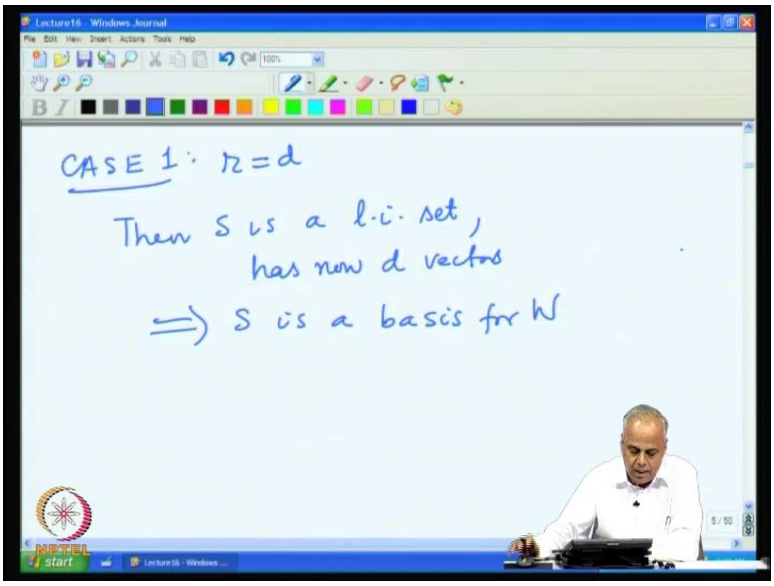
We shall now look at, some more properties of finite dimensional spaces. More properties, these will the properties, which we saw the last lecture. Now, we look at more properties of finite dimensional sub spaces. So again, we have W contained in V, and dimension of W is said d. And suppose, we have a set S in W, which has r vectors and is linearly independent. I am considering a set S in W, which is r vectors and which is linearly independent.

(Refer Slide Time: 05:34)



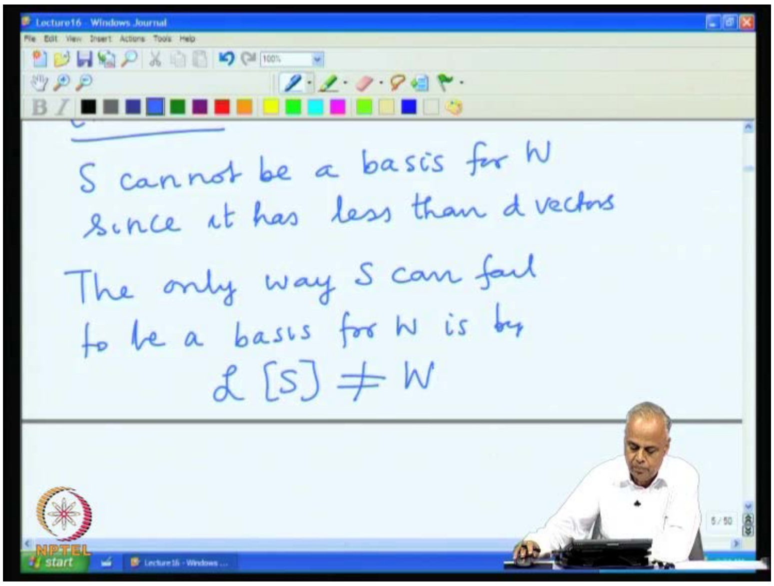
So here is W and here is S and there are r vectors in this. And they are all linearly independent and we take dimension of W, we have take it to be d. Now since, W is d dimensional any d plus 1 vectors must be linearly dependent and therefore, since S is linearly independent, it cannot have more than d vectors. So, r is less than or equal to n. Since, S is linearly independent, and any set having greater than r is less than d, because we have taken d to be the dimension, any set having greater than d vectors, in W is linearly dependent. So, r is less than or equal to d. So therefore, the moment of we have a d dimensional sub space and if we take any linearly independent set it cannot have more than d vectors.

(Refer Slide Time: 06:56)



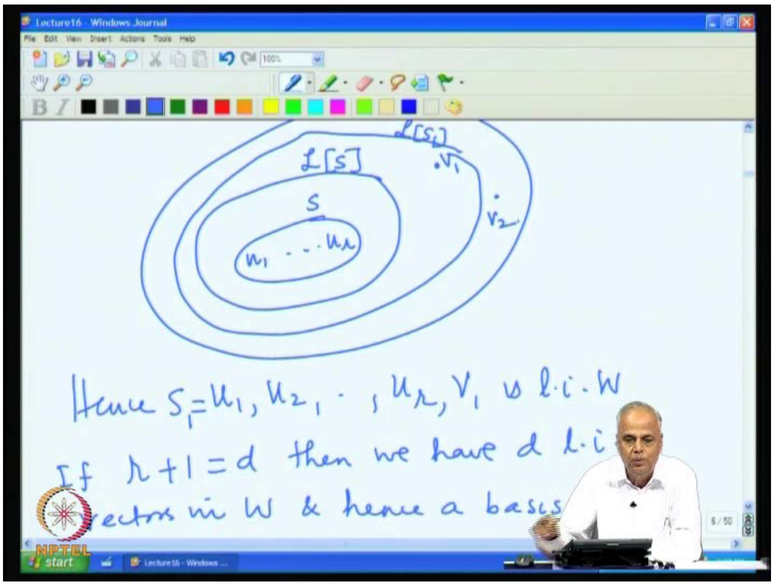
Now, let us look at this situation, r is less than or equal to d, so the two possibilities; 1 is r equal to d, the other 1 is r less than d. Let us look at this case r equal to d. If r equal to d then, S is a linearly independent set, because we have already assumed we are started with the linearly independent set, and it us now d vectors so, if we have set of d vectors which are linearly independent in a d dimensional space, we have said it must be a basis. That is S is a basis for W.

(Refer Slide Time: 07:44)



The next case is when r is less than d. If r is less then d, then obviously S cannot be a basis because any basis for W must have d vectors, but S has only r vectors, which is less than d. So, the first thing is S cannot be a basis for W, since it has less than d vectors. To form a basis, we need exactly d vectors, because the dimension is d and any basis must contain d vectors. So, S cannot be a basis. Why did it fail to a basis then, to be a basis a set has to be linearly independent and span this space. We have already assumed it is linearly independent, so the only way S can fail to be a basis is for not spanning W. So, S can only the way, the only way S can fail to be a basis for W is by L S being not equal to W.

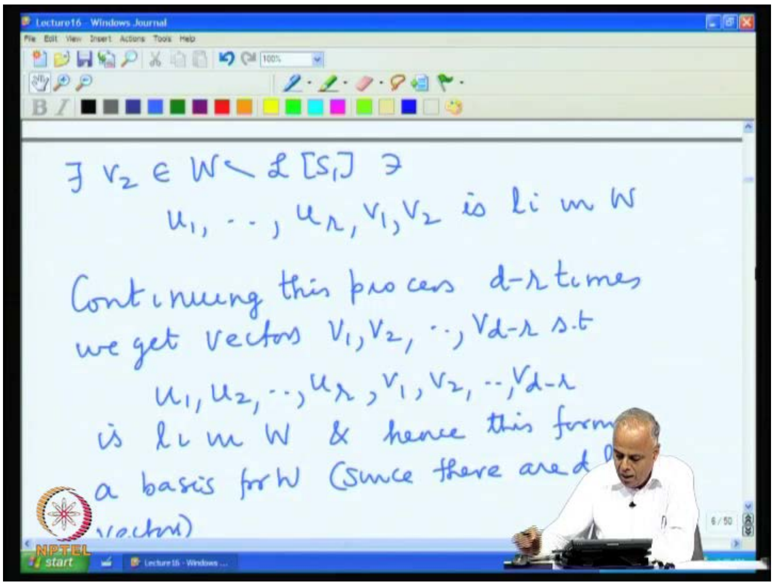
(Refer Slide Time: 09:18)



So, what does it mean? We have W, in that we have S, which consists of u 1, u 2, u r and then, we look at L S, L S cannot be W. It has to be only a small part of W. And therefore, if L S is not W, there is something sitting in W, outside L S. Let us say, 1 such vector is V 1. So, there exist a V 1, which is in W, but which is not in L S.W minus L S. There is exist vector V 1 which is in W, but not in L S. But we have seen that, whenever we have a sub space, inside which we have a linearly independent set and we pick a vector outside the sub space, then these together must form still linearly independent set in the bigger space. So, we take this bigger space to be W, the sub space of W to be L S, something outside that and something linearly inside linearly independent inside that together, they must be linearly independent. Hence, we have u 1, u 2, u r with V 1, now, upended is linearly independent in W.

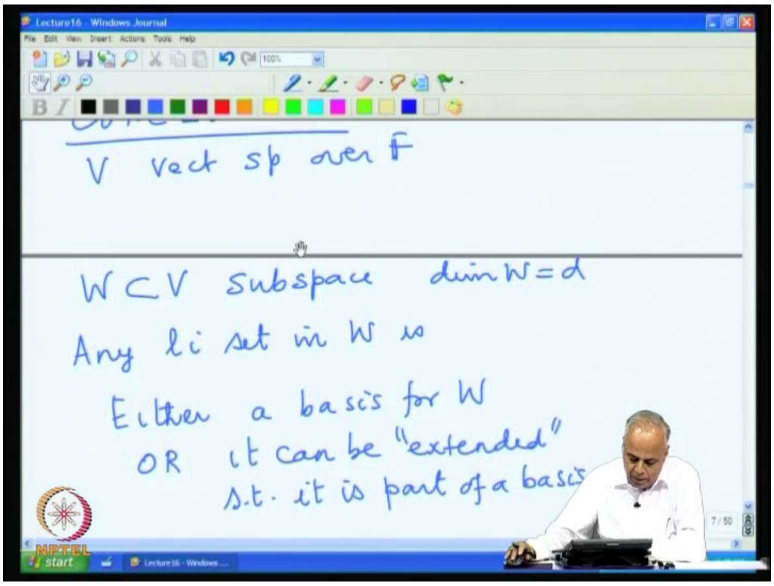
Now, we started with r vectors linearly independent in W, since r was less than d, it was not adequate to span the whole space and therefore, it could not become a basis, it needed a help. So, we needed to supply more vectors. Now, we are supply one more vectors to the set S and made it us likely bigger linearly independent set. Now, if r plus 1 is equal to d, then we have d linearly independent vectors in W and the moment we have d linearly independent vectors in a d dimensional space and hence a basis for W. If r plus 1 is less than d, what we do is, we now look at this space span by, all this fellows together. If r plus 1 is less than d, then if I call this set S 1, L S 1 will still not span W. That means, I can pick another vector V 2 outside this.

(Refer Slide Time: 12:20)



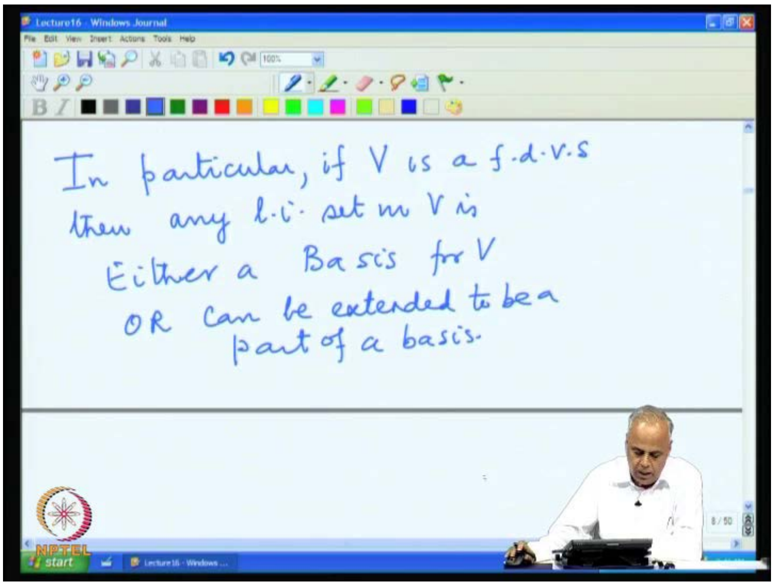
So, there exits V 2 belonging in to W minus L S 1 such that u 1, u 2, u r, V 1, V 2 is linearly independent in W. Thus, we can go on expanding this set, 1 by 1 adding vectors from outside. So, continuing this process, d minus r times, we get vectors V 1, V 2, V d minus r, such that u 1, u 2, u d, u r, V 1, V 2, V d minus r is linearly independent in W. But now, there are d vectors there are r of them in the use in d minus r in the V they add up to totally d vectors and the moment we have d linearly independent vectors you have got a basis. And hence, these forms a basis for W since; there are d linearly independent vectors. So, therefore, either r is equal to d as in case 1, we had r equal to d, in which case the starting set itself was a basis or the case r less than d, we are able to go on supplying vectors V 1, V 2, V d minus r and thereby, making it a basis. So, either it is already begin of linearly independent set form a basis and if it is not begin of we can supply adequate numbers of vectors, that it is form a basis.

(Refer Slide Time: 14:36)



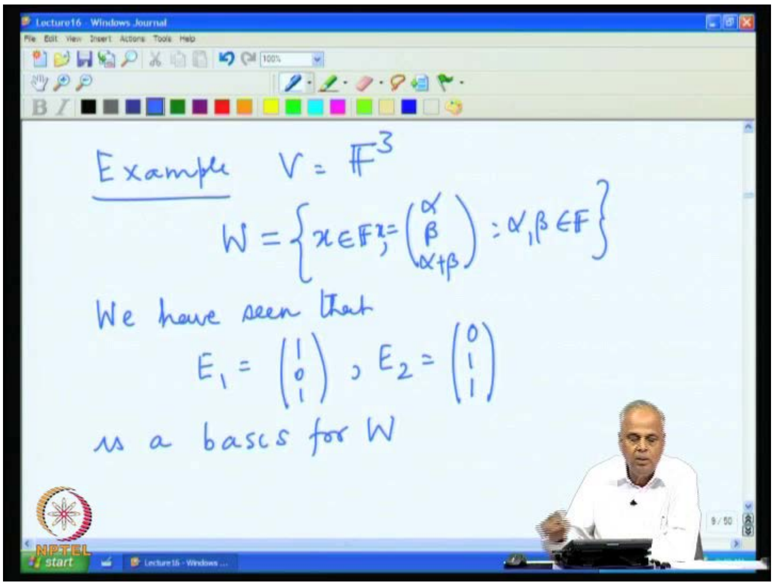
So, how do we conclude? So, the conclusion is that V vector space over F and we have W in V sub space dimension of W equal to d, the finite dimensional sub space then, any linearly independent set in W is, as in case one either already a basis, either basis for W or if it fails to be a basis. We can upend vectors to it and enough number of them, so that it forms of a basis. We get either this or it is can be extended, then we say extended we mean, we could upend more vectors to it, such that it is part of a basis. It is already a full basis or it is made a part of a basis is this is whole thing of basis and the u vectors are part of it. So either, the set is a basis any linearly independent set is the basis or it can be extended to be a part of a basis. That every linearly independent set can be slowly strengthened to become a basis. At least, we have a seen this works in the finite dimensional sub sets.

(Refer Slide Time: 16:25)



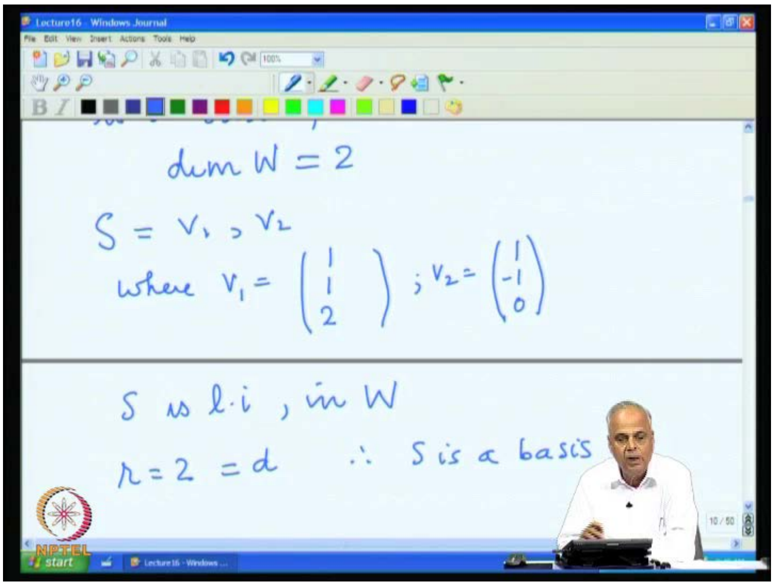
In particular, if V is a finite dimensional vector space, then any linearly independent set in V is either a basis for V or can be extended to be a part of a basis.

(Refer Slide Time: 17:12)



Let us, look at a simple example; let us take the space very simple space V to be F 3 and let us take this sub space W. We have seen this sub space before it consists of all the vectors in F 3, of the form alpha beta alpha plus beta, where alpha and beta belong to F. If we take F to be r, this is what be geometrically interpreted as, the z equal to x plus y plain. The third component is equal to the sum of the first two components. Consider this sub space, we have already seen, that say E 1 equal to 1 0 1 by taking alpha equal to 1 and beta equal to 0, and E 2 by taking beta equal to 0 and beta equal to 1 and alpha equal to 0 is a basis for W is the basis for W. Because it is obviously, linearly independent and every vectors in W it is alpha times E 1 plus beta times E 2 for suitable alpha and beta.

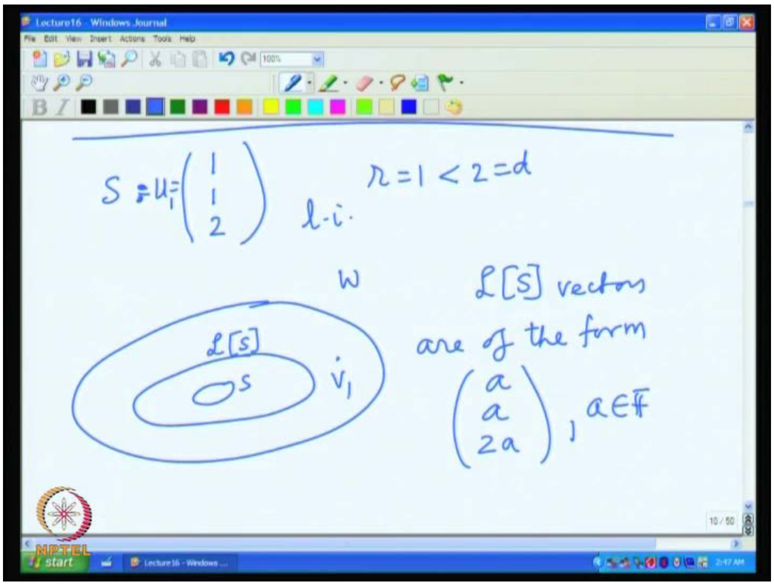
(Refer Slide Time: 17:12)



So, this is a basis for W and how many vectors are there in this basis vectors and exactly two vectors and therefore, dimension of W is equal to 2. Now, suppose I take this set S, which is V 1 and V 2, where V 1 is the vector 1 1 2 and V 2 is the vector 1 minus 1 0, let us observe first of all S is linearly independent. Because you cannot get V 2 as a multiple of V 1, because V 1 has non 0 component as third to get V 2, we can only multiply V 1 by 0, so we cannot get V 2 as a linear combination of V 1 similarly, we cannot get V 1 as linear combination of V 2 and therefore, V 1 and V 2 linearly independent. And these are vectors in W, because V 1 is obtained by taking alpha equal to 1 beta equal to 1 and V 2 obtain by alpha equal to 1 and beta equal to minus 1.

So, S is linearly independent. It is in W and there are exactly 2 vectors r in this case is 2, which is equal to d and therefore, S is a basis. Then never we have to linearly independent vectors it is automatically a basis. This is a case 1 that we discuss, whenever you take are 2 be 2, it is automatically a basis.

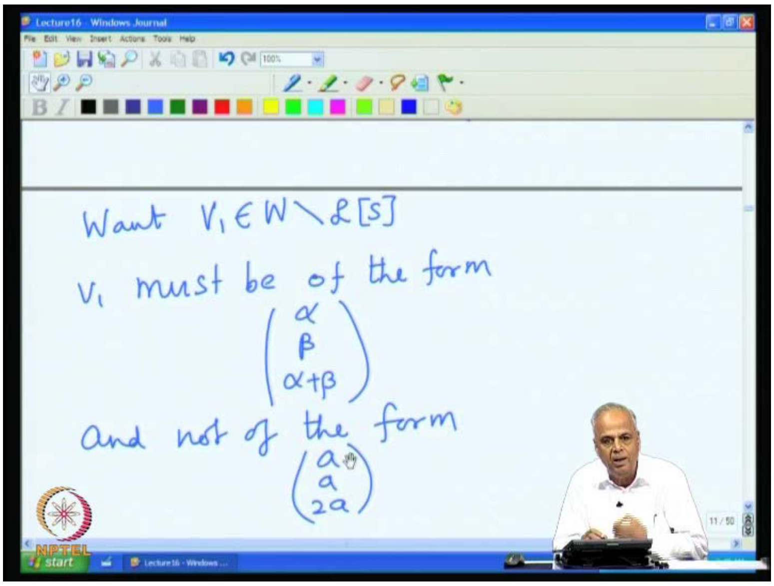
(Refer Slide Time: 20:38)



If we take now, S to be only this vector 1 1 2, let we call it as u 1, then this s cannot be a basis. Here, r is 1, which is less than 2 which was d but this is linearly independent. We have now started with linearly independent set, which is smaller in size then the given dimension. So therefore, how many vectors do we have to upend in order to expand it or extend it to a basis. The total dimension is 2, we already have 1 vector. So, r is 1, d is 2. We d d minus r vectors which is 2 minus 1, so we need to upend exactly 1 vector to the set to the get a basis. How do we upend this? We have this W; we had this S, now we are looking at L S.

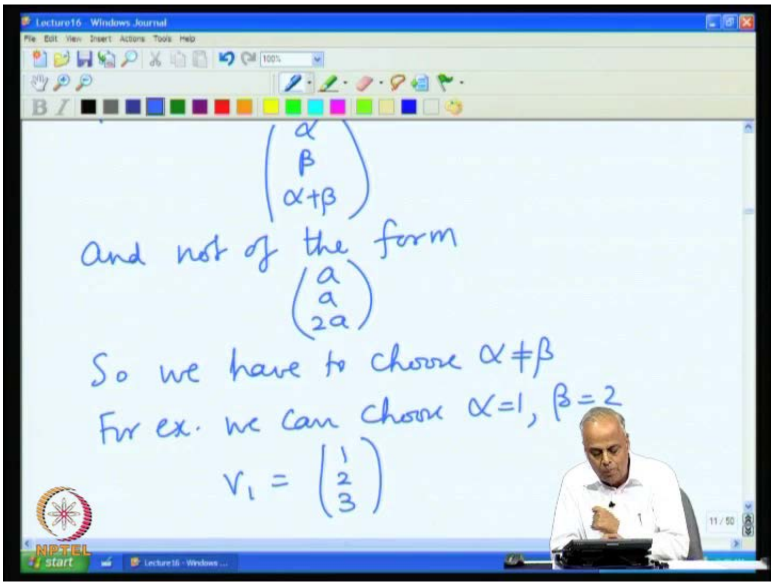
Our upending vector must come from outside L S. How does L S look like? This space span the S consists of all multiples of u 1, so L S vectors are of the form a a 2 a. Where a is in F. Because we have to multiples of u 1. That is only vectors that available in span L S, All must be just multiples of these vectors, u 1. L S vector of this form and we looking for V 1 outside L S.

(Refer Slide Time: 22:23)



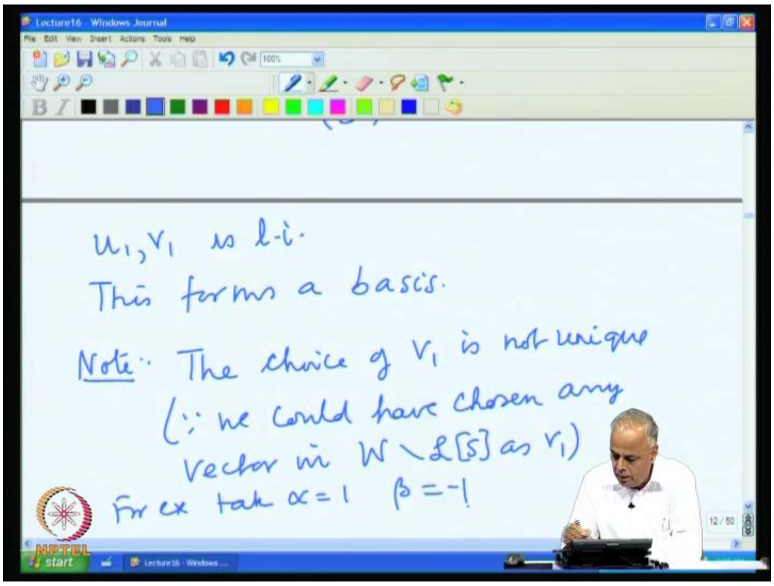
So, we want V 1 belong in to W, but not in L S, W minus L S. So, it must be a vector in W, and therefore it must be a vector of this form alpha beta alpha plus beta, but it should not but be a vector in L S, therefore it should not be of the form a 2 a a a 2 a. So therefore, V 1 must be of the form alpha beta alpha plus beta for some alpha and beta, and not of the form a a 2 a. If it as to be of this form, and not of this form this means alpha and beta must be different, because the moment you take alpha and beta same, we get a vector of this form. So therefore, we must choose alpha and beta such that there are different.

(Refer Slide Time: 23:38)



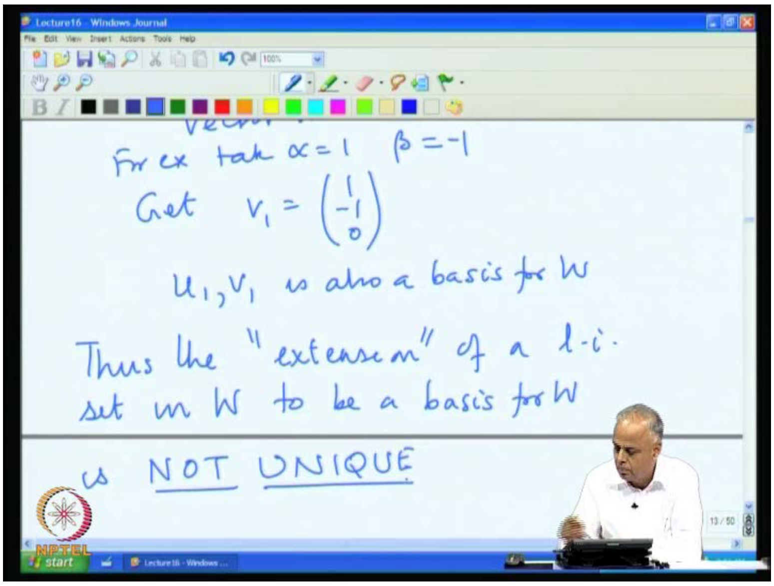
So we have to choose, we can choose any alpha and beta as longer their alpha and beta different, we will get a vector outside L S. For example, we can choose alpha equal to 1, beta equal 2 and get V 1 as 1 2 3. This is certainly not in L S, because for something to be in L S, the first and the second components in must be equal, the third component is must be double in the first component.

(Refer Slide Time: 24:18)



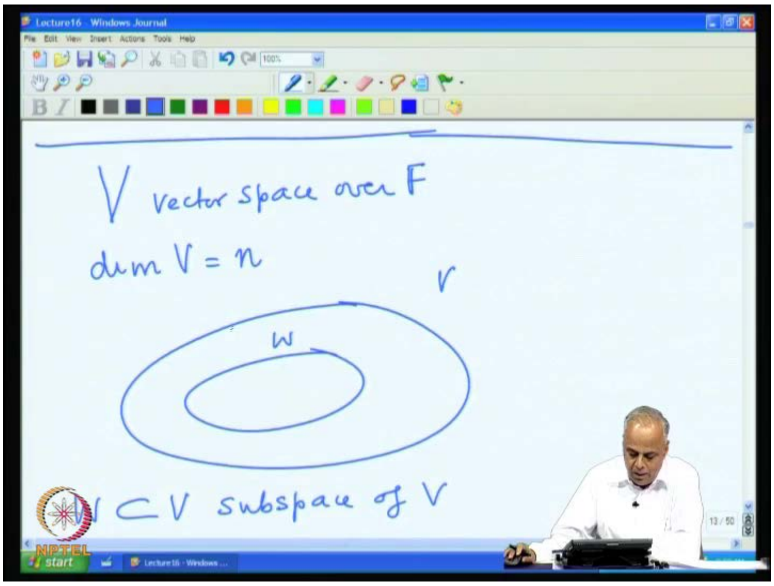
This and therefore, u 1 v 1 now, is linearly independent. There are two vectors to form a basis, this form a basis. Now, we see that u 1 is part of this basis whatever linearly independent set you start with, you can make it a part of this basis. Note: our choice of v 1 was not unique. Because always we to do was we can chosen alpha and beta that the alpha not equal to beta. We could chosen alpha is 1, beta is 0 or alpha is 36, beta is 27, whatever values we choose as long in alpha and beta are not equal, we will get a valid V 1. Therefore, the choice of v 1 is not unique. Because we could have chosen any vector in W minus L S as V 1.

(Refer Slide Time: 25:36)



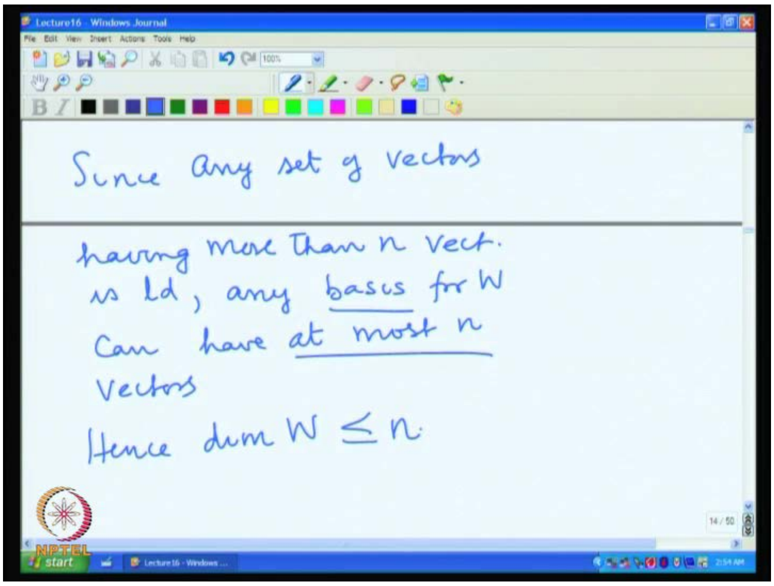
For example, take alpha is equal to 1, beta equal to minus 1, you get V 1 equal to 1 minus 1 0. Then, this u 1, u 1 come back with this V 1 is also a basis. And therefore, when we say, that any linearly independent set can be extended to be a basis, the extended part is not unique. The starting part the given set as is a unique that is we want it to be a part of the basis and what we upend to make it basis that upended part can be chosen in any arbitrary manner. There are many ways are choosing it. Thus, the extension of a linearly independent set in W to be a basis for W is not unique. However, the fact of the matter is, that we can always extend. There should be at least one extension that is what is important for us.

(Refer Slide Time: 27:08)



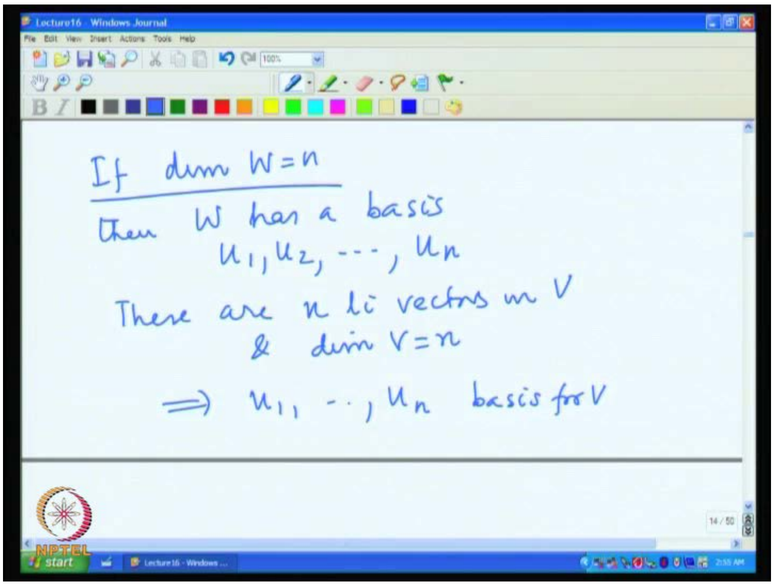
Now, let us look at another property of finite dimensional spaces. Suppose, V is the vector space, over F and V itself it is finite dimensional, say dimension of V is equal to n. So, I start with the original mother space. The basic vector space, to be a finite dimensional vector space and we take the dimension to be equal to n. Now, suppose I taken sub space of W. So, W subspace, then any r plus 1 any n plus 1 vectors in V must be linearly dependent, because of dimension of V is n. So, any basis for W cannot have more than n vectors, because if you have more than n vectors it will automatically become linearly dependent.

(Refer Slide Time: 28:17)



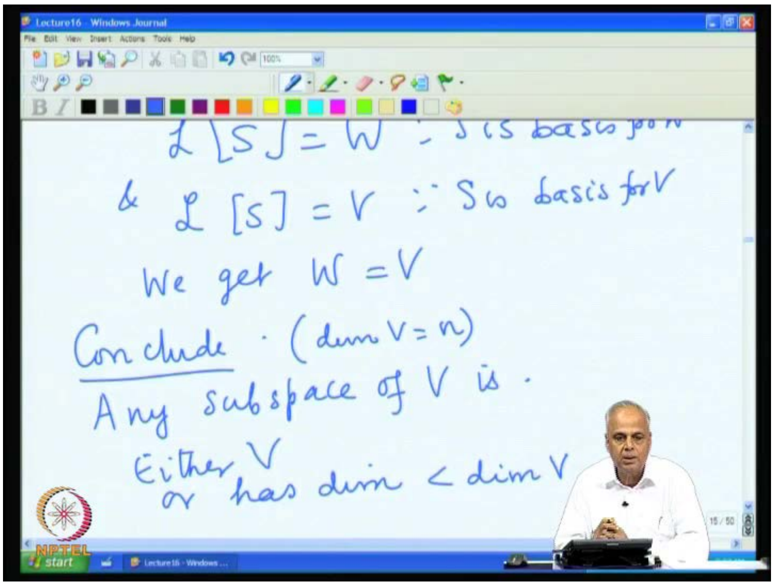
So, since any set of vectors having more than n vectors is linearly dependent, any basis for W can have at most n vectors. Because the basis has to be the linearly dependent, if you more than n vectors become the linearly basis as to be linearly independent. If you go more than n vectors, it will become linearly dependent. And therefore, we cannot more than n vectors in still be linearly independent. So, to form a basis, we must have at most n vectors. That means, a basis for W can have at most n vectors that means the dimensional W can be at most n. Because dimension is the number of vectors in a basis and we have observe the number of vectors in a basis can be at most n. And hence, dimension of W has to be less than or equal to n.

(Refer Slide Time: 29:41)



If dimension of W is equal to n, then W has a basis consisting of how many vectors, because the dimension is n it has to have n vectors. Say u 1, u 2 extra u n. But then, the whole spaces of dimension n and we have seen that n dimensional space any and linearly independent vectors will form a basis, so these are n linearly independent vectors in V also, because there in W and W is part of V. Therefore, these are n linearly independent vectors in V and dimension of V is n and therefore, any linearly independent vectors n of them will form a basis for V, so that says u 1, u 2, u n basis for V. Now, on the 1 hand is basis for W and therefore, it is spans W and the other hand is a basis for V and therefore, it is spans V. It spans W, it span V and since it spans same W must be equal to V.

(Refer Slide Time: 31:05)



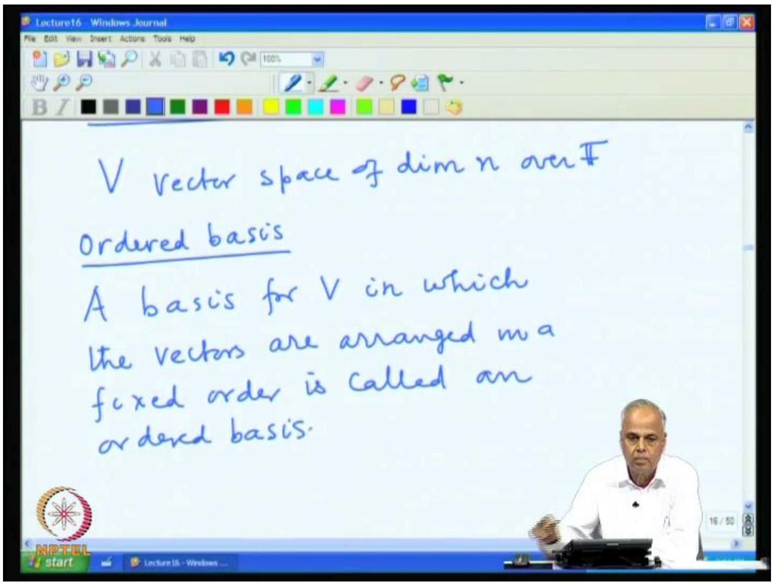
Hence, L W is equal to this note L W; let us call this set as S. Hence, S is equal to u 1, u 2, u n is such that L S is W, because S is basis for W and L S is V, because S is basis for V. Comparing the 2, we get W equal to V. So therefore, what is that we can conclude, if we take any sub space of an n dimensional space, it is dimension is less than or equal to n, but if it is dimension is n it is must be the whole space, otherwise is dimension is should be strictly less than n.

So, we conclude that an any sub space of V, where V is dimension n, we are assuming that V is the vector space of dimension n is either V or has dimension less than dimension V. So, we let us assume the dimension of V equal to n. Suppose, we have finite dimensional space then any sub space of that vector space, must be either all of V are must have dimension much smaller at least 1 dimension smaller than V.

Whenever, we want to conclude, that sub space is exactly equal to V, one way of showing is the both of them have the same dimension. Because we know now that if a sub space a as the same dimension of the whole space, it must be exactly equal to the whole space. These are the some of the simple basic properties of finite dimensional spaces and we shall be using them very regularly without ever mentioning them again and again, because these are so genetic and fundamental properties of a finite dimensional space.

Now, we are going to look at this basis, remember we started with the notion of basis, from the idea that we are going to look at a sampling set and then we need set that we want a sampling set, we want to do proper sampling. We do not want to do over sampling; we do not want to do under sampling; we do not want to do over sampling means, we did not want redundant information, which means we do not want to do linearly dependent set. We do not want to do under sampling means, we want to do span to the whole space, we do not want to do miss any information, which simply means want to linearly independent spanning set and that let us to the notion of basis. Now, what is this sampling going to u and how does it help us. This is what we are going to study.

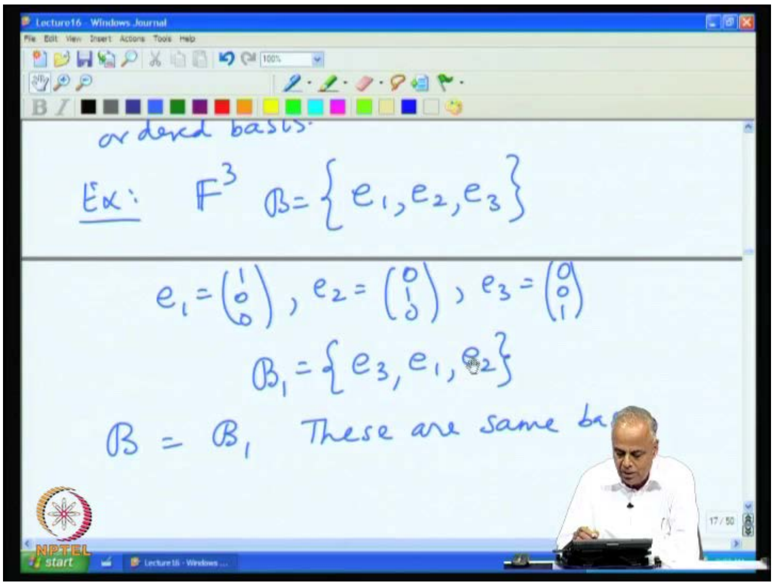
(Refer Slide Time: 34:32)



Now the role of a basis, will see what sort of help it does in our analysis. So, first of all let us say, V is a vector space of dimension n over F. We have a finite dimensional vector space; it is dimension is in n. Now, it can have any basis, but any basis must have exactly n vectors, because of the dimension is n. We now introduce the notion of an ordered basis. Well as the words just see is it is a basis the only difference is when we say basis we say a linearly independent set.

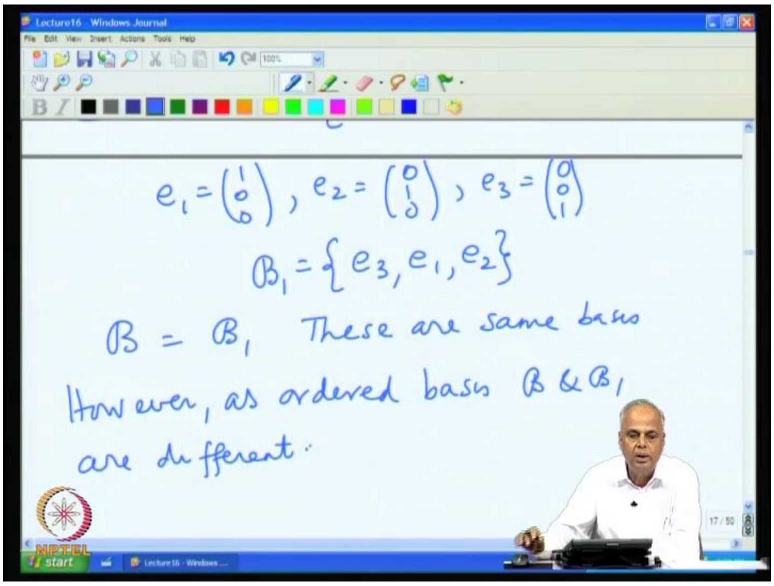
It a set of vectors, when we say a basis we mean as set of vectors. In a set of vectors it does not matter, how we list the vectors in what order we list the vectors. For example, if we list the vectors as u 1, u 2, u 3 or we put u 2 first, u 1 next and u 3 later, it does not matter. In a set the order will which we list the vectors does not matter. However in a order basis, we not only have it as a set, we in is on a particular order in which will list the set. A basis for V in which the vectors are arranged in a fixed order is called an ordered basis.

(Refer Slide Time: 36:40)



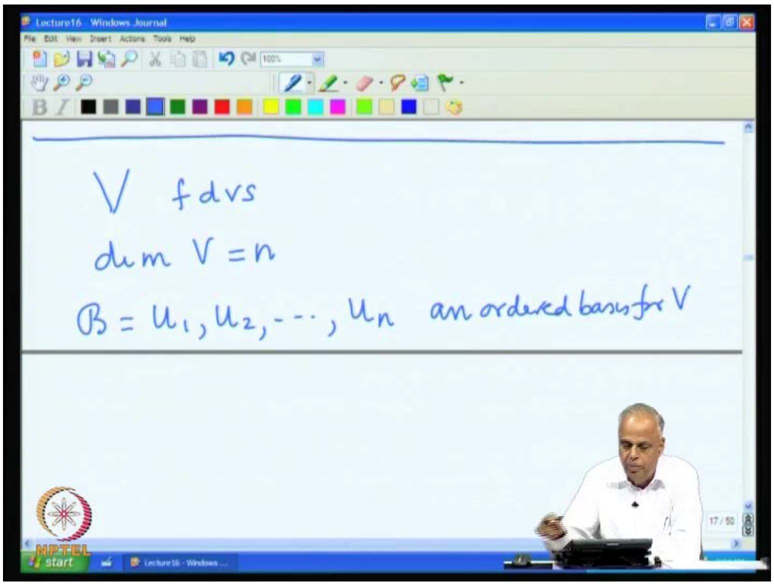
For example, let us look at F 3, the vector space F 3. Now, we know that the set of vectors e 1, e 2, e 3 is a basis, where e 1 is 1 0 0, e 2 is 0 1 0, e 3 is 0 0 1. We know that this is a basis for this the space F 3. Now, if you look at this set, this B 1 as a set is the same as B, when not 2 sets equal if every element of B is also in B 1 and every element of B 1 is also in B. We look at that way these two sets are equal. So, B is equal to B 1. These are same basis. However, if we look at order in this there relate listed, the order is totally different. Here, e 1 comes first, in this basis e 3 comes first, in this basis e 2 come second, in this basis e 1 come second, in the basis e 3 comes third and here e 2 comes third.

(Refer Slide Time: 38:13)



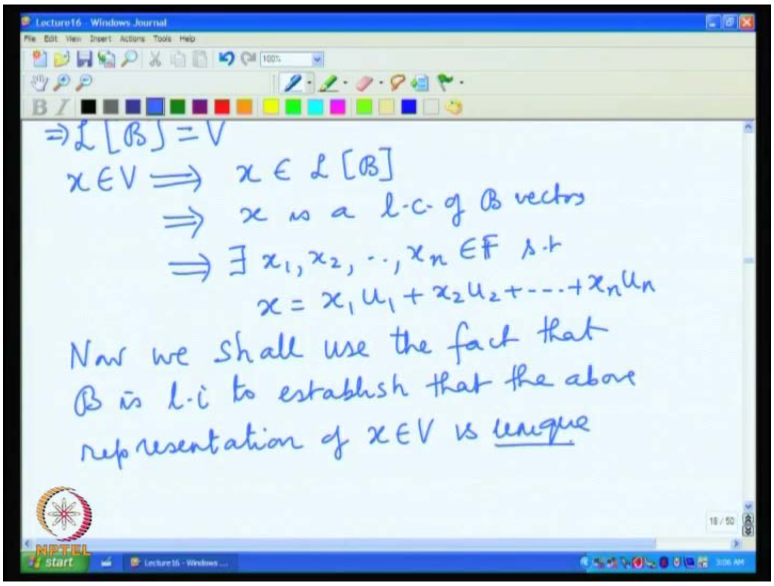
However, as ordered basis B and B 1 are different. Because the vectors in the basis are ordered in a different order. So, thus we will be dealing with ordered basis going to want to be ordered basis.

(Refer Slide Time: 38:41)



Now, let us look at where this is leads to us. V finite dimensional vector space, dimension of V equal to n, now let us look at ordered basis how many vectors will be there in a ordered basis well, any basis will have n vectors, and in addition to that v level order. So let us put B as u 1, u 2 extra u n in ordered basis for B from now on will write ob for a ordered basis. So, the short form will be ob. So, let B - u 1, u 2, u n be an ordered basis for V.

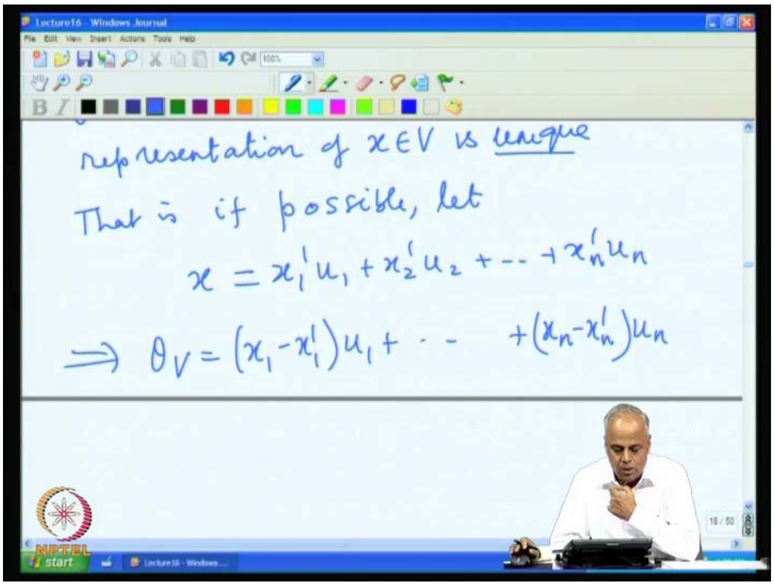
(Refer Slide Time: 39:29)



Let us, take any vector x in V. Now, what you mean by being an ordered basis in V. First of all it is a basis. If it ever a basis, it must be linearly independent and it must be span the whole space. So in particular therefore, it is span must be equal to V. B is the ordered basis means, L B must be equal to B. Now, consider any vector x in V, that says since V is equal to L B, x is in L B. If x is in L B, that means x is a linear combination of B vectors. And therefore, there must exist suitable coefficient. So, there exist x 1, x 2, x n in F, such that x can be written as x 1 u 1 plus x 2 u 2 plus extra x n u n.

Every vector in V, can be expressing the linear combination of this basis vectors. Now, we shall see that we have use that fact B is a basis, only up to this part namely that L B is V we have not yet use the fact that B is linearly independent. We will use the fact that B is linearly independent to establish that for every vector, this representation is unique. Now, we shell use the fact that B is linearly independent; because it is a basis it must be linearly independent. We shell use this pack the base linearly independent, to establish that the above representation, as a linear combination of the B vectors. The above representation of any x of x in V is unique.

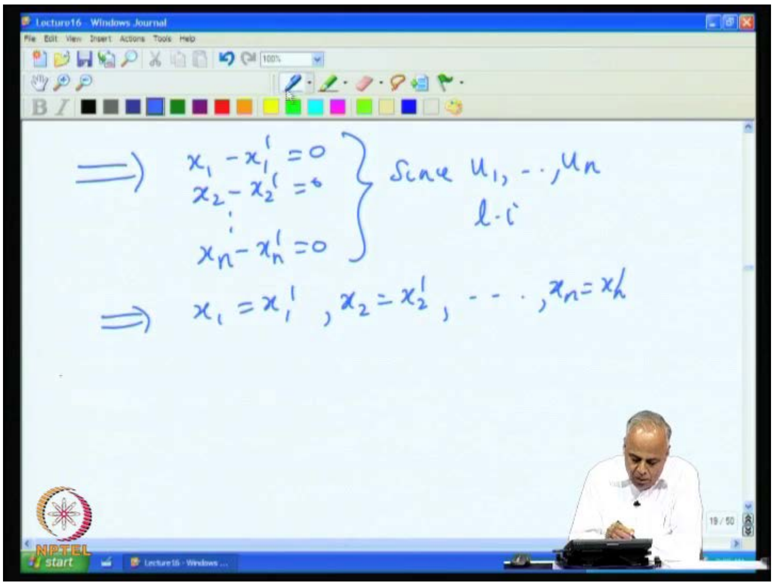
(Refer Slide Time: 42:01)



What you mean by that? That is if that is if possible, let x have another representation. What do you mean by another representation? In the representation, we have the u 1, u 2, u n but that basis is fix we cannot change that the only thing that can change are this coefficients x 1, x 2, x n. We have another representation means some other co efficient times u 1 plus, some other co efficient times u 2 plus, some other co efficient times u n. Suppose, there is another such representation, on the 1 hand x is x 1 u 1 plus x 2 u 2 plus x n u n and the other hand x is equal to x 1 prime u 1 plus x 2 prime u 2 plus x n prime u n. Now, let us subtract these two representation let us subtract this from this, what you get the left hand side is x minus x which gives theta v.

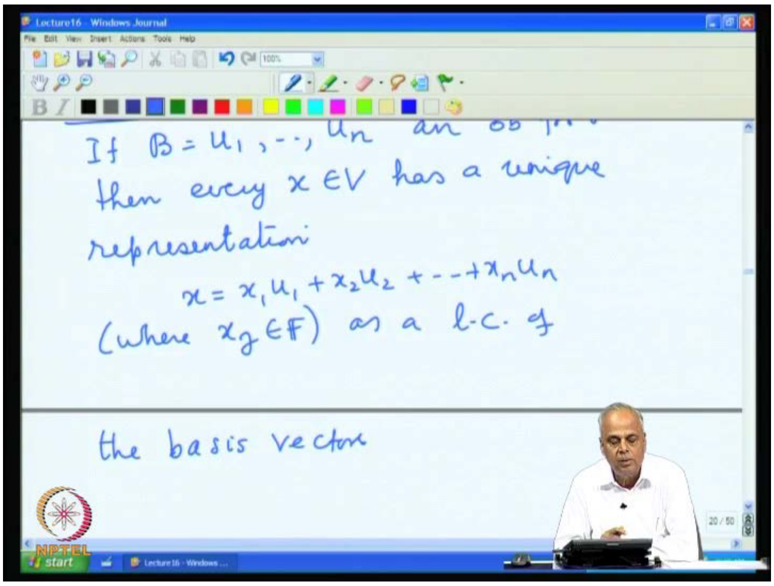
We get theta v, the right hand side we get x 1 minus x 1 prime u 1, we have x 1 minus x 1 prime u 1 plus etcetera x n minus x n prime u n. However, we now know that B is a basis, any basis must be a linearly independent and therefore, u 1, u 2, u n are linearly independent vectors, but then once you have linearly independent vectors, the only linear combination that will give the 0 vector, if when you take all the co efficient as 0.

(Refer Slide Time: 43:49)



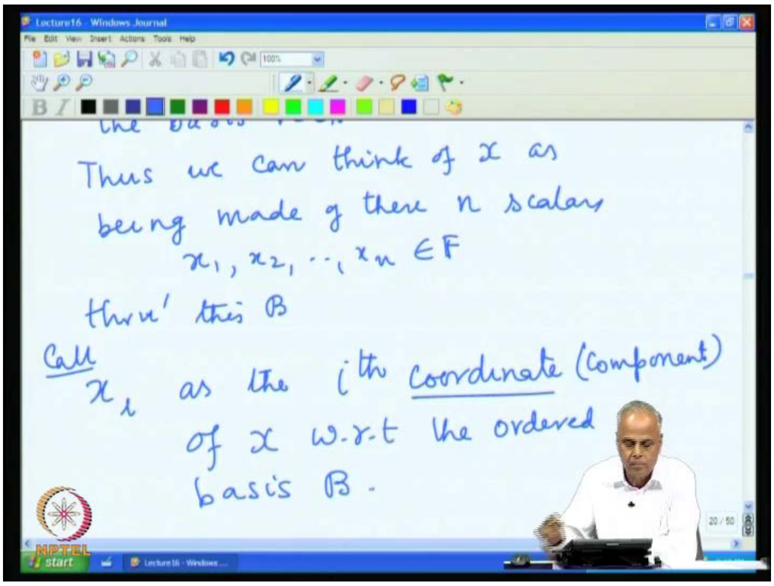
So, that implies x 1 minus x 1 prime equal to 0, x 2 minus x 2 prime is 0 and x n minus x n prime is 0, since u 1, u 2, u n are linearly independent. So, that says x 1 equal to x 1 prime, x 2 equal to x 2 prime and so on x n equal to x n prime. So, these two representations are the same. x 1 u 1 again x 1 prime is x 1 only. So, there cannot be any other different representation, both must be the same representation.

(Refer Slide Time: 44:32)



So therefore, what is the conclusion that we have? The conclusion is if B equal to u 1, u 2, u n and ordered basis for V. Then, every x in V has a unique representation x equal to x 1 u 1 plus x 2 u 2 plus x n u n, where the x j are all in F as a linear combination of the basis vectors. So, every vector as a linear combination of the basis vectors and there is one and only one way, what does that mean? It is something like if you know the x at the sampling points, these are this sampling points of the space V a basis is consider as the sampling set, if you know how the vector x behaves in this at the sampling point essentially, I can reconstructlly vectors. So, just that we do in signals. Here, we get a digitization of an abstract vector.

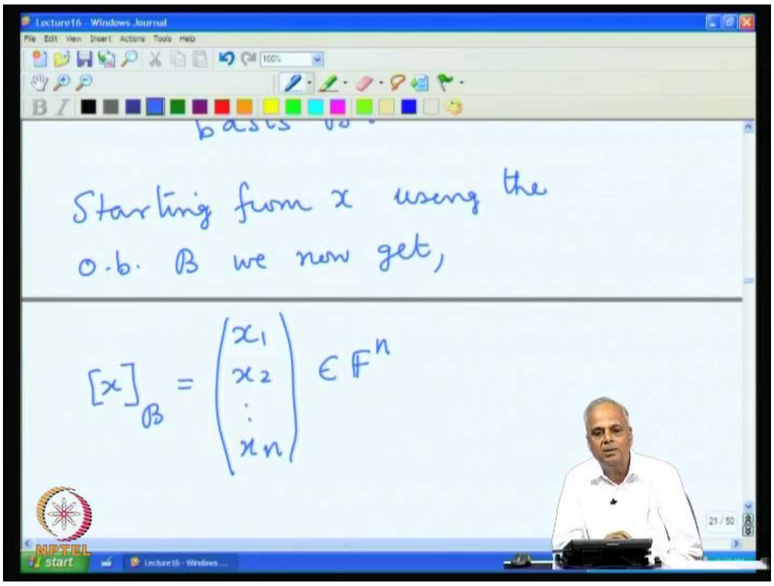
(Refer Slide Time: 46:06)



Thus, we can think of x has been made of these n scalars x 1, x 2, x n in F, through this basis B. Through this in other words, we had a vector which is an abstract quantity could be any type of vector space, i could be abstract vector space over a field F, but we could it always digitized and bring in back to the scalar level, there are n scalar, which has stored the information about this vectors. The moment is n scalar, the basis the sampling set B is known, when we can reconstruct the vector x as x 1 u 1 plus x 2 u 2 plus x n u n.

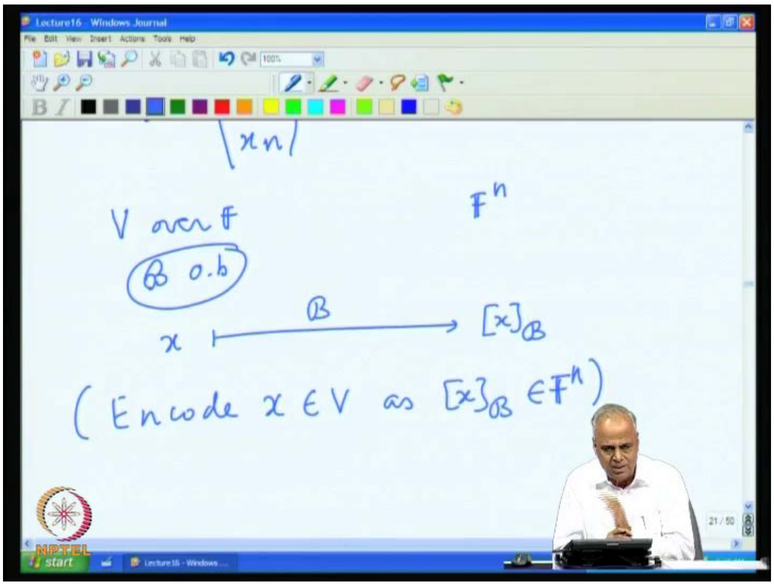
So, we call x i as, so we call x i as the i th co ordinate as the i th co ordinate of the vector x, with respect to the ordered basis, because the order is important the moment which change the second to the third, then the second component will become a third component. The co ordinate or the component, it is also call the component co ordinate of x with respect to the ordered basis B. So once, we have an ordered basis for B, then every vector in V can be converted in to a set of n numbers or n scalar or n elements in the field F elements in the field are called scalar.

(Refer Slide Time: 48:11)



Now therefore, starting from x using the ordered basis B, we now get the n scalars, which I am going to write x 1 x 2 x n at the column matrix and this is obviously in F n. And this we call, we started from x, so x has some to do this. We use the basis B. So, B has something to do this. We denoted by x B. So, starting from an dimensional vector space F any vector x can be through an ordered basis converted to a vector in F n.

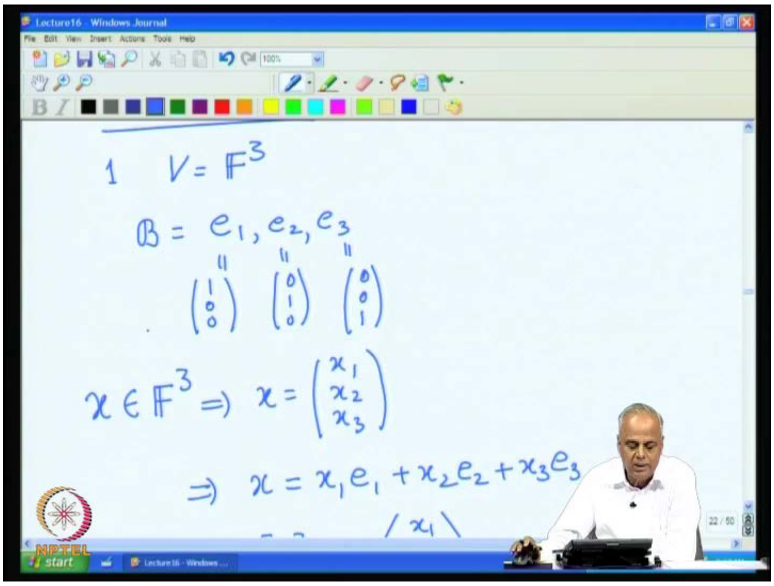
(Refer Slide Time: 49:08)



So, we have on the 1 hand we have vector space V over F, then we have this F n, we start with the basis B for this ordered basis, then once we have this any vector here, can be converted to a vector x B, in through this basis B in this basis F n. In other words, this is some kind of encoding of any vector in x as a vector in F n, are you can call it digitizing a vector in V, to a vector as a vector in F n. So, we encode x in V, as x B in F n.

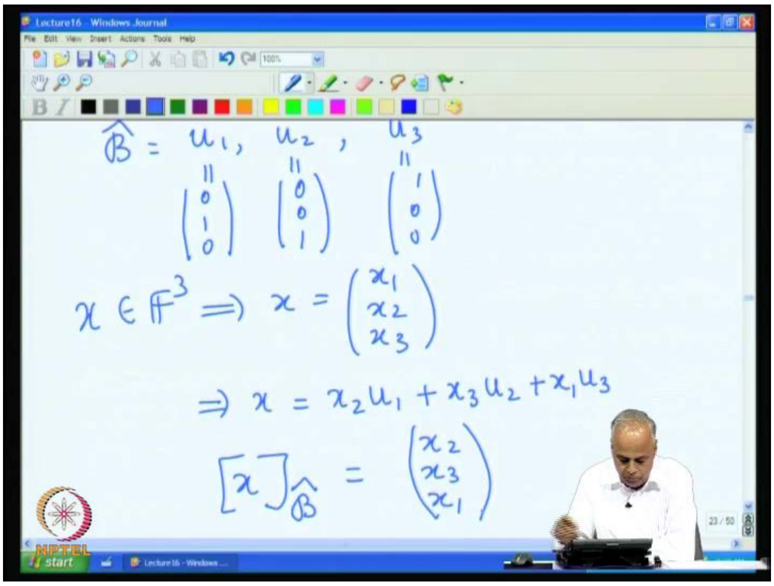
However, abstract the vector space V may be, as long as it is dimension is n. It can always we can converted to a concrete level of F n. F n is simply the standard n component matrix space. So, any abstract vector space, if it is dimension n by choosing suet any arbitrary ordered by basis of V. The every vector x can be translated; can be en coded; can be digitize. As a vector, x B in F n. Let us, look at some simple examples. Will begin with few simple examples.

(Refer Slide Time: 50:49)



Let us, look at the space V to be F 3. Always the starting example is F 3; because that is the if you take F to be a r. We get r 3 our standard 3 dimensional (( )) where, we are let us first take a basis B, which is ordered basis e 1, e 2, e 3 where e 1 is 1 0 0, e 2 is 0 1 0 and e 3 is 0 0 1. This is our standard ordered basis. If, you now take any vector x in F 3, it must be of the form x is equal to x 1 x 2 x 3. Now, how does this encoding take place, through a basis? We must express, this vector x as the linear combination of the B vectors. We have here, B x is equal to x 1 e 1 plus x 2 e 2 plus x 3 e 3. The first component of x with respect to this basis is x 1, second component is x 2 and third component is x 3 and therefore, the encoding in just it this is standard encoding.

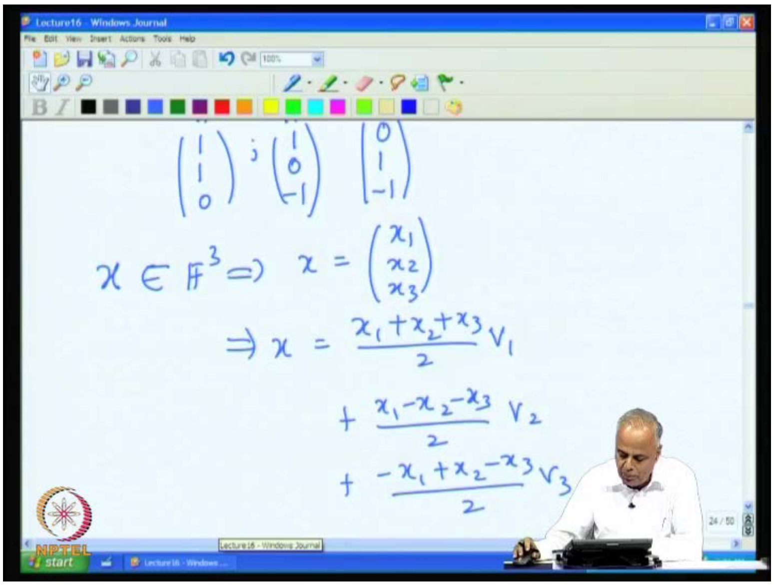
(Refer Slide Time: 52:23)



Suppose, I take the basis same vectors, but now I change the order. So, this is the same basis. I am going to choose, but now I am going to change the order vectors will be the same. So, I am going to take u 1, u 2, u 3, where u 1 is 0 1 0, u 2 is 0 0 1 and u 3 is 1 0 0. So, this is the e 2 of the previous case, the second vector of the previous basis is the first vector become of this basis. The third vector of the previous basis, become the second vector of this basis and the first vector of the previous basis, become the third vector here. We have the same basis set, but we have now a different order.

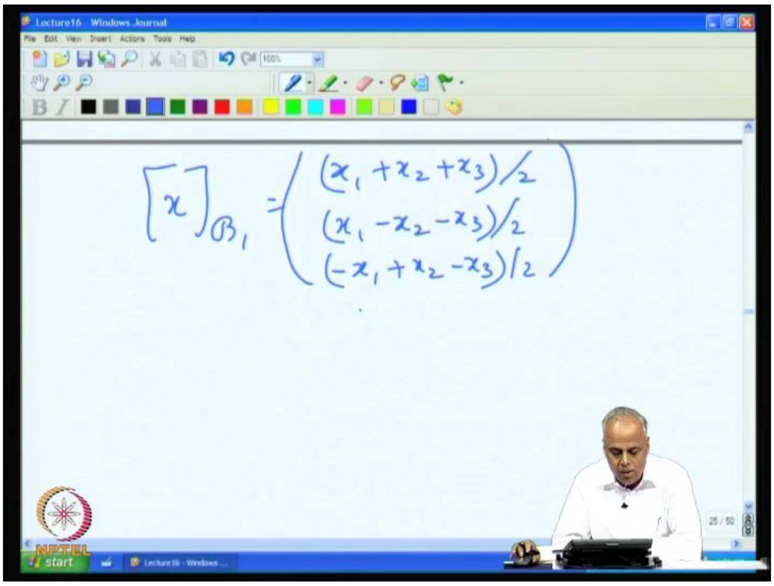
Now if we take any vector x in F 3, x is the form of x 1 x 2 x 3 then, x is now x 2 times u 1 plus x 3 times u 2 plus x 1 times u 3. So, what is the first component of x with respect to this ordered basis it is x 2. The second component of this vector, now the same vector, but now we are asking for the second component with respect to the new basis. It is now, x 3 and third component is x 1 and therefore, the digitization of this vector, with this ordered basis is x 2 x 3 x 1.

(Refer Slide Time: 54:22)



Let us choose another basis. Let us take V 1 to be V 1, V 2, V 3. Where V 1 is 1 1 0 and V 2 is 1 0 minus 1 and V 3 is 0 1 minus 1. Now, if we take any vector x, which is in F 3, x is of the form x is equal to x 1 x 2 x 3 and we can verify, that such a vector x will be equal to x 1 plus x 2 plus x 3 by 2 in to V 1 plus x 1 minus x 2 minus x 3 by 2 in to V 2 plus minus x 1 plus x 2 minus x 3 by 2 in to V 3. And therefore, the new components in terms of this, new basis are x 1 plus x 2 plus x 3 by 2 x 1 minus x 2 minus x 3 by 2 and minus x 1 plus x 2 plus x 3 by 2.

(Refer Slide Time: 55:44)



And therefore, we get x of B 1 is x 1 plus x 2 plus x 3 by 2, x 1 minus x 2 minus x 3 by 2, minus x 1 plus x 2 minus x 3 by 2 this vector. So, the same vector will have different digitization, different encodings, in terms of different ordered basis. Of course, the natural thing therefore is look for a basis, which give as simplest digitization where the representation becomes simple.

Nevertheless there is a fundamental question that remains. After all if you look at the 3 representation that we have, it is the same vector x 1 x 2 x 3, but we have used 3 different basis, we have use first the u 1 u 2 u 3, then we have used this basis, and then we use this V 1 basis, and we get different representations. After all they all represent the same vector. x B - x B had x be 1, all of them represent the same vector x. And therefore, they must be some have related, what is this relation? This is being the topic for the next lecture.