Advance Engineering Mathematics
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Lecture 55 - Correlation and Regression - I

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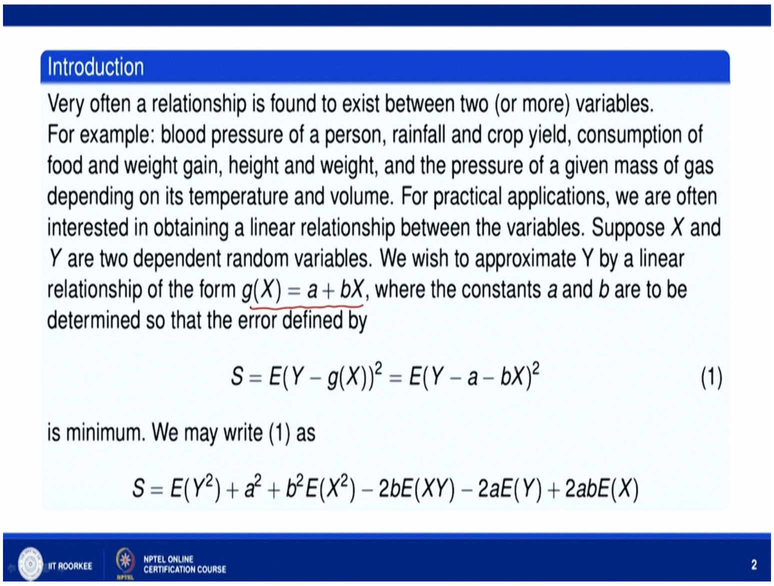

Hello friends! Welcome to my lecture on Correlation and Regression, this is my first lecture on correlation and regression. Very often a relationship is found to exist between two or more variables, for example, blood pressure of a person and his age, rainfall and crop yield, consumption of food and weight gain, height and weight of a person, the pressure of a given mass of gas depending on its temperature and volume. For practical application, for practical applications we often are interested in obtaining a linear relationship between the variables.

Suppose X and Y are two dependent variables. We wish to approximate Y by a linear relationship of the form gX equal to a plus bX, where the constants a and b are to be determined so that the error defined by S equal to expectation of Y minus gX whole square which is equal to expectation of Y minus A minus bX whole square is minimum, that is the error is minimum in the least square sense. So, we may write this equation 1 as S equal to EY square plus a square plus b square into EX square, minus 2bE XY minus 2 a EY plus 2ab EX.

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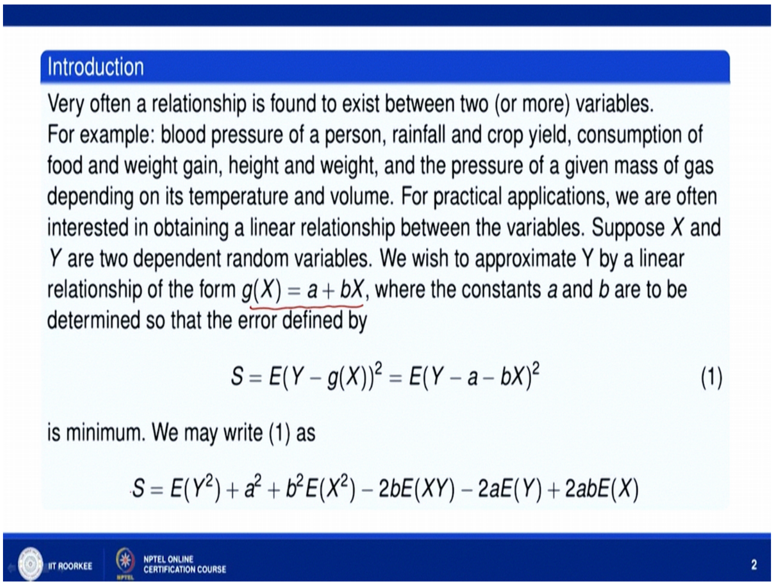

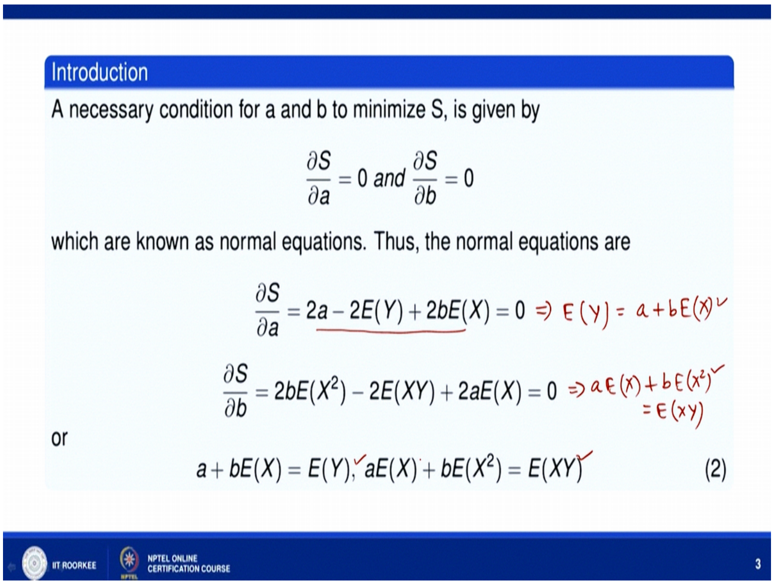

Now a necessary condition for a function of two variables, say here they are a and b okay, you can see we want to minimize S, S is a function of two parameters a and b, so in order for S to be a minimum, we must have the partial derivative of S with respect to a equal to 0 and partial derivative of S with respect to b equal to 0. These 2 equations are known as normal equations. Thus the normal equations are if you differentiate S with respect to a partially okay, then you get partial derivative of S with respect to a equal to 2a minus 2EY plus 2b EX, okay.

So we get partial derivative of S with respect to a as 2a minus 2EY plus 2b EX and we put it equal to 0. Similarly when we differentiate S partially with respect to b, we get 2b EX square minus 2E XY plus 2a EX okay, so we get 2b EX square minus 2E XY plus 2a EX equal to 0, now this equation. Okay, this equation gives us a minus EY plus b EX equal to 0 or I can say EY equal to, this equation gives you, EY equal to a plus b times EX okay, and from this equation what we get? We get a EX plus b EX square equal to E XY okay.

So we have two equations, this one and this one. Okay, which are given by number 2, so there are two equations connecting the two unknown values a and b, we can, they are linear equations a and b, so we can solve them for the values of a and b.

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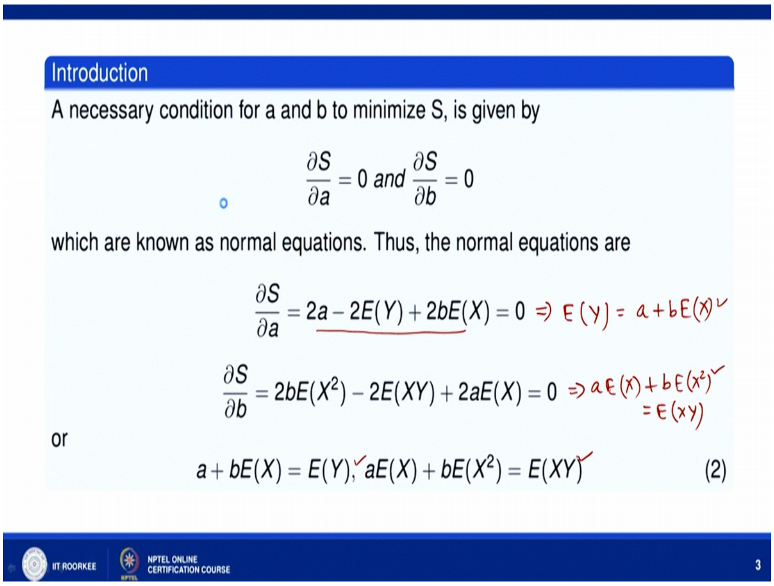

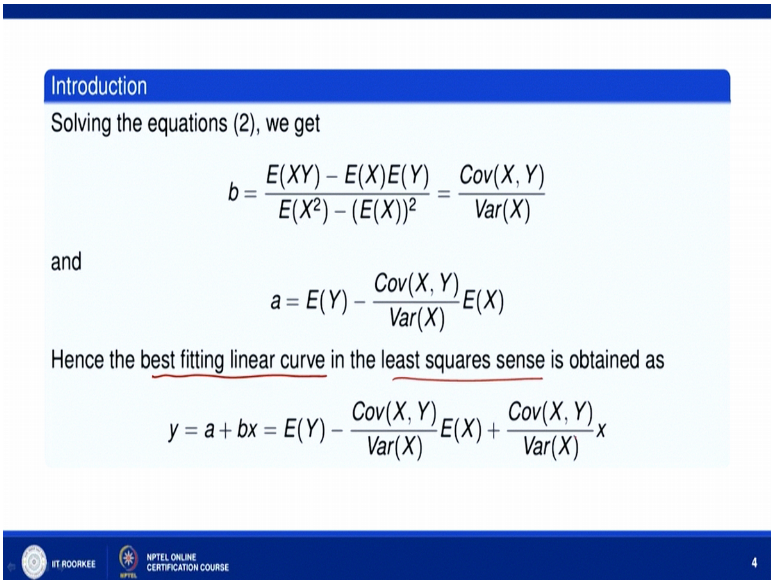

And when the solve them we get b equal to EX, Y minus EX into EY upon EX square minus EX whole square. Now, E XY minus EX EY gives us the covariance of the random variables X and Y, so covariance of XY divided by EX square minus EX whole square is variance of X, so b equal to go variance of XY divided by variance of X and when we put the value of b, in one of the two equations, say for example, EY equal to a plus b E X, we get the value of a okay, the value of a comes out to be EY minus covariance of XY divided by variance of X into EX, hence the best fitting linear curve in the least square sense is given by Y equal to a plus bX, where a is EY minus covariance of XY divided by variance of X into EX plus b, b is covariance of XY divided by variance of X into X.

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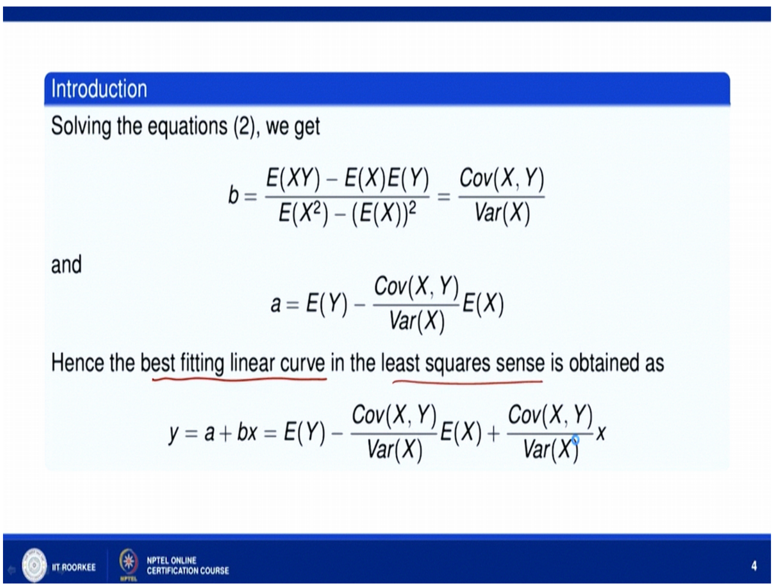

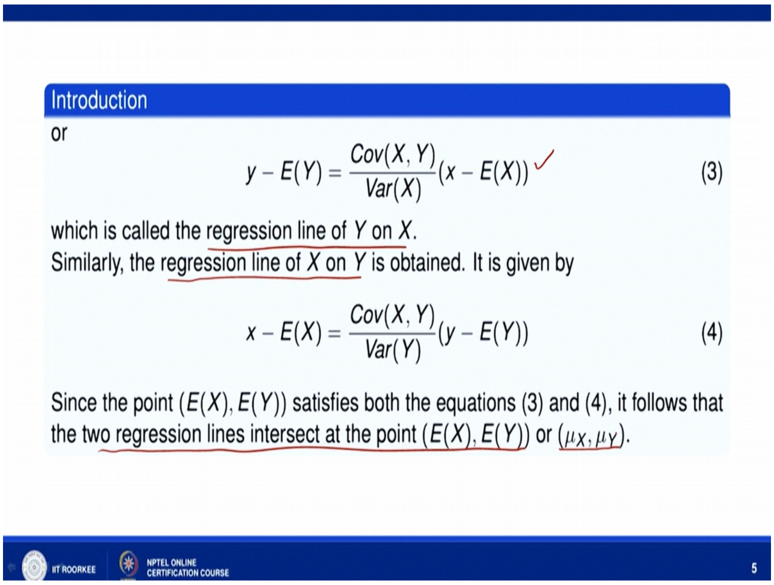

We can rewrite this equation as Y minus EY equal to covariance of XY over variance of X into X minus EX okay, so we have this equation, variance of Y minus EY equal to covariance XY over variance of X into X minus EX, this equation is known as the regression line of Y on X okay. For a given value of X, you can get be approximate value of Y using this question, so it is called as the regression line of Y on X. Similarly, the regression line of X on Y, if you are given the value of Y and you want to estimate, get an estimate of the value of X, then we need the regression line of X on Y.

So in a similar manner we can find the regression line of X on Y, it is given by X minus EX equal to covariance of XY over variance of Y into Y minus EY. Now since the point EX, EY okay, satisfies both the equations 3 and 4 okay, now you can see if you put here the point EX, EY in this equation, then you see EY minus EY equal to 0, EX minus EX equal to 0, so 0 equal to 0, so EX, EY satisfies this equation. Similarly here, EX, EY satisfies this equation and therefore it follows that the two regression lines intersect at the point EX, EY. EX, EY will also denote by mu X, mu Y, so they meet at the point mu X mu Y.

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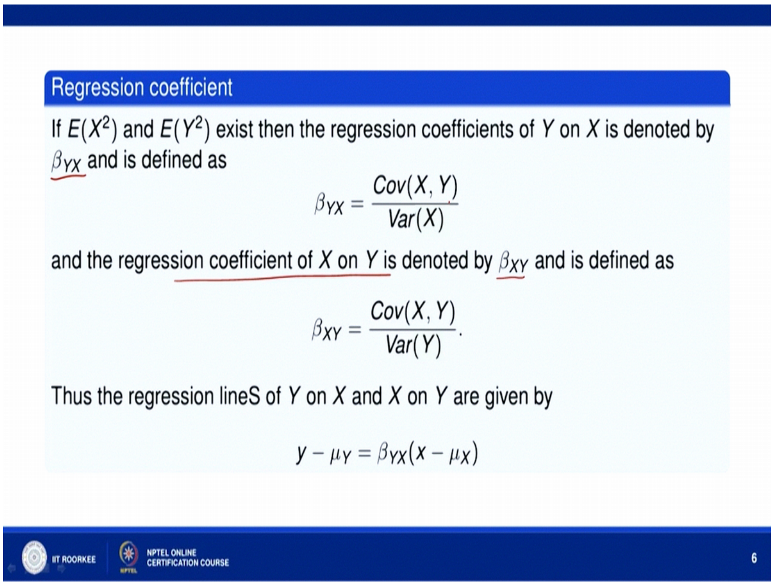

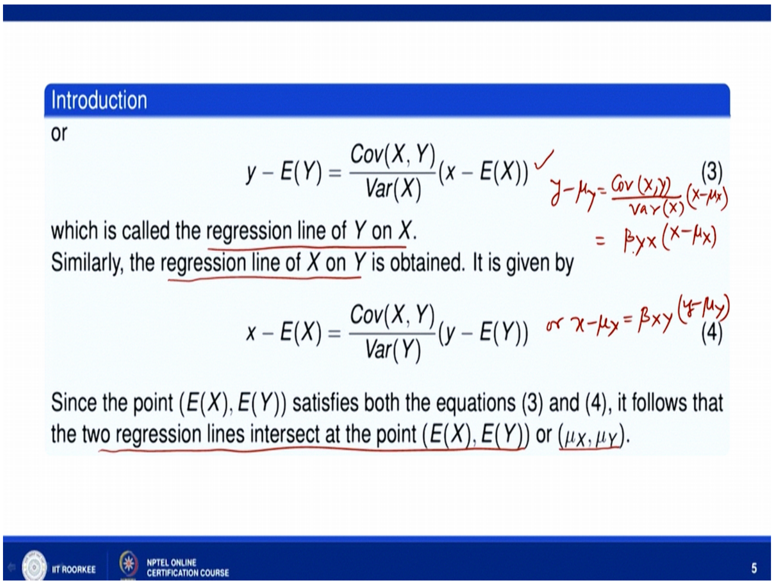

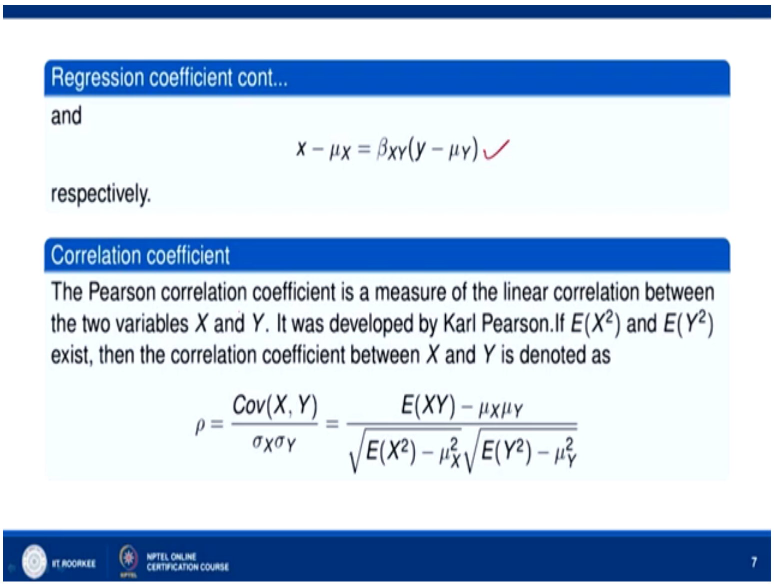

Okay, now if EX square and EY square exist, then the regression coefficients, the regression coefficient of Y on X is denoted by beta YX and is defined as beta YX equal to covariance of XY divided by variance of X and the regression coefficient of X on Y is defined as beta XY and denoted by beta XY and it is defined as beta XY equal to covariance of XY divided by variance of Y. Now using these regression coefficients, the regression line of Y on X okay, this regression line of Y on X, which is Y minus mu Y equal to covariance of XY over variance of X into X minus EX, I can write it as Y minus mu Y equal to covariance of XY divided by variance of X into X minus mu X. I can write is as beta, covariance of XY over variance of X is the regression coefficient of Y on X.

So beta YX, X minus mu X and this equation which is the regression line of X and Y can be written as X minus mu X equal to beta XY into Y minus mu Y okay. So using the notation for regression coefficient of Y on X and X on Y, we can write the regression lines of Y on X and X on Y in this manner okay. So, this is your regression line of Y on X okay and the other one is the regression line of X on Y this one, the regression line of X on Y.

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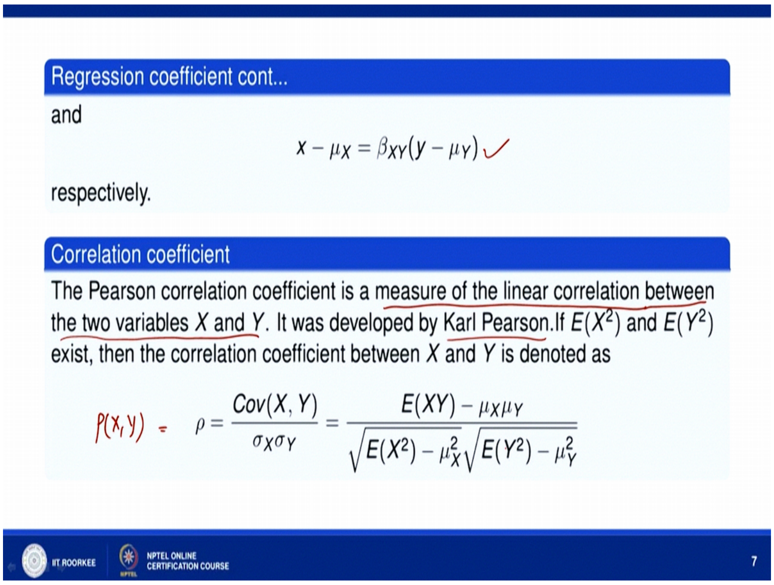

Now correlation coefficient, the Pearson correlation coefficient is a measure of the linear correlation between the two variables X and Y, it was developed by Karl Pearson, if EX square and EY square X exist, then the correlation coefficient between X and Y is denoted as rho or we also write it as rho XY and it is equal to covariance of XY divided by sigma X sigma Y. Now covariance XY by definition is E XY minus mu X mu Y and sigma X is a square root variance of X, that is EX square minus mu X square and sigma Y is square root of variance of Y which is square root of EY square minus mu Y square.

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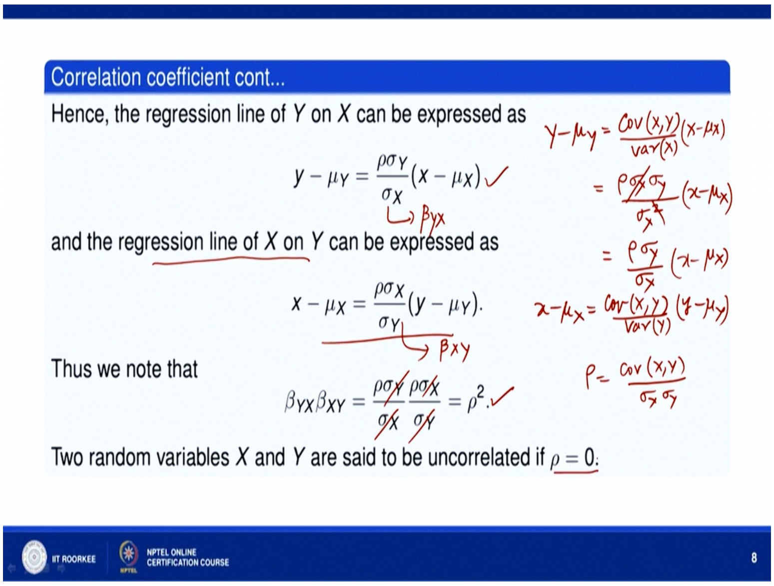

Now the regression line of Y on X okay, using the definition of rho we can write the regression line of Y on X in this form, you see we have, we had Y minus mu Y equal to covariance of XY divided by variance of X into X minus mu X okay and rho is equal to, rho is covariance of XY, the rho is equal to covariance of XY divided by sigma X, sigma Y. So, covariance of XY is rho times sigma X, sigma Y, so I can write it as rho times sigma X, sigma Y divided by variance of X is sigma X square. So we have sigma X square and then X minus mu X okay, this cancels with this and we get rho times sigma Y over sigma X, X minus mu X.

So Y minus mu Y equal to rho sigma Y over sigma X into X minus mu X. In a similar manner we can express the regression line of X or Y in terms of rho okay, we have X minus mu X equal to covariance of XY divided by variance of Y into Y minus mu Y okay. So when you put for covariance of XY, you put rho times sigma X sigma Y and then divide by rho Y square, what you get is rho into sigma X divided by sigma Y, so we get X minus mu X equal to rho into sigma X divided by sigma Y into Y minus mu Y okay.

Thus we note that, this is beta YX okay, this is beta YX and by our definition this is beta XY, so when you multiply beta YX and beta XY what we get, row sigma Y over sigma X okay into rho sigma X over sigma Y and this is equal to, this cancels with this, this cancels which this, you get rho square. So beta YX into beta XY equal to rho square. The two random variables are X and Y are called uncorrelated if the coefficient of, correlation coefficient rho is equal to 0.

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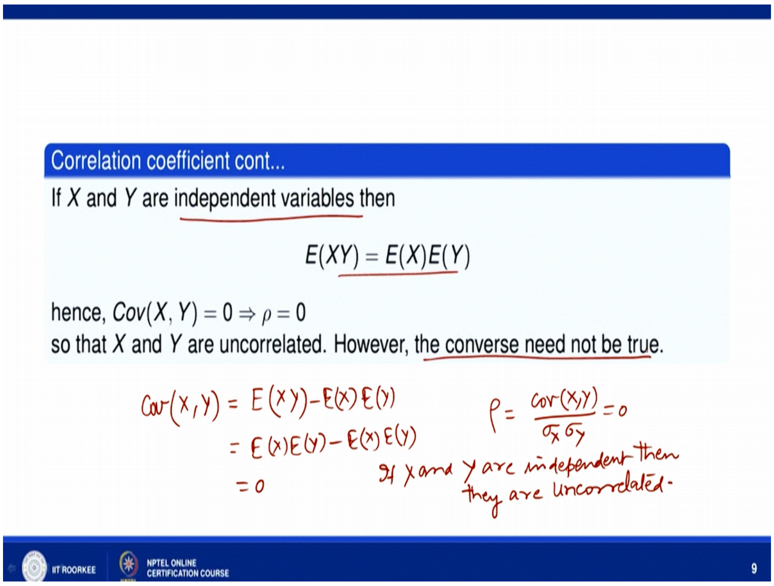

If X and Y are independent variables okay, if X and Y are independent variables, then we know that expected value of X into Y is equal to expected value of X into expected value of Y okay, hence covariance of XY is expected value of XY minus expected value of X into expected value of Y okay, this we know.

So when X and Y are independent random variables, then E XY equal to EX into EY gives us, EX into EY minus EX into EY gives us covariance of XY equal to 0, that is now covariance of XY is equal to 0 means, rho, rho is given by covariance of XY divided by sigma X sigma Y okay. So when covariance of XY is 0, rho equal to 0, so if X and Y are two independent random variables, then X and Y are uncorrelated okay, so X and Y, so if X and Y are independent then they are uncorrelated, but we shall see that the converse is not true okay, so let us show it by means of an example.

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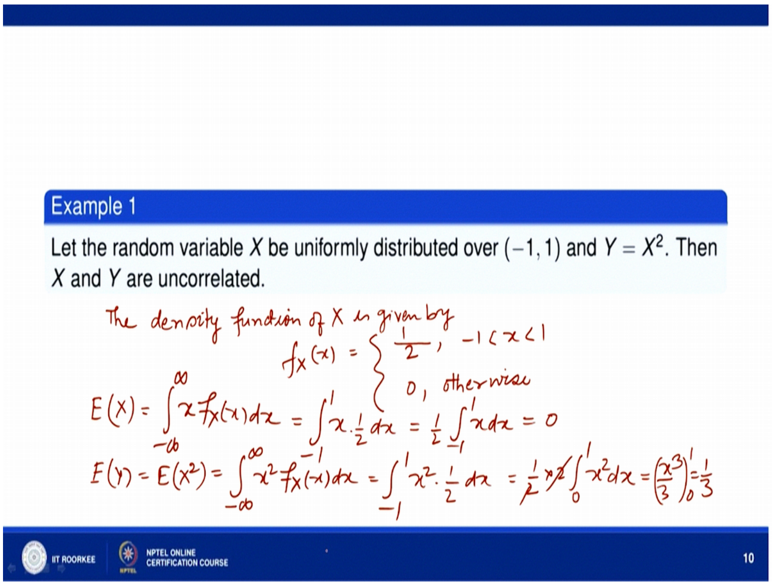

Let us show that rho is equal to 0, okay, but X and Y are dependent random variables okay, so let us take this example, let us say that the random variable X is uniformly distributed over the interval minus 1 to 1 okay and Y is equal to X square, then X and Y are uncorrelated. Now we know that if the X and Y, if X is uniformly distributed over minus 1 to 1, then the density function of X is given by, the density function of FX X equal to 1 over B minus A okay, if it is uniformly distributed over the interval AB, then FX X is 1 over B minus A. So this is 1 over, plus 1 that is 1 over 2, when X lies in the interval minus 1, 1 and 0 otherwise.

Now we need to find the value of rho and show that rho is equal to 0. Okay, so we have found FX X, now we need to find expected value of X, so expected value of X is integral over X into, expected value of X is integral over minus infinity to infinity, X into FX X DX, now it is half over the interval minus 1 to 1, so this is integral over minus 1 to 1, X into 1 by 2 DX okay, so this is half into X is an odd function of X, so integral over minus 1 to 1, X DX will be equal to 0, so expected value of X is equal to 0.

Now expected value of Y is equal to expected value of X square. Okay, so expected value of X square means integral over minus infinity to infinity, X square FX X DX, which will be equal to integral over minus 1 to 1, X square into 1 by 2 DX, which is equal to 1 by 2 into, X square is an even function of X, so two times 0 to 1, X square DX, so what we get is X cube by 3, integral of X square is X cube by 3 over the integral 0 to 1. And this gives me value 1 by 3, so we have got the value or expectation of X, expectation of Y. Now let us find the value of, because we want the value of rho, so we need to find expected value of XY okay.

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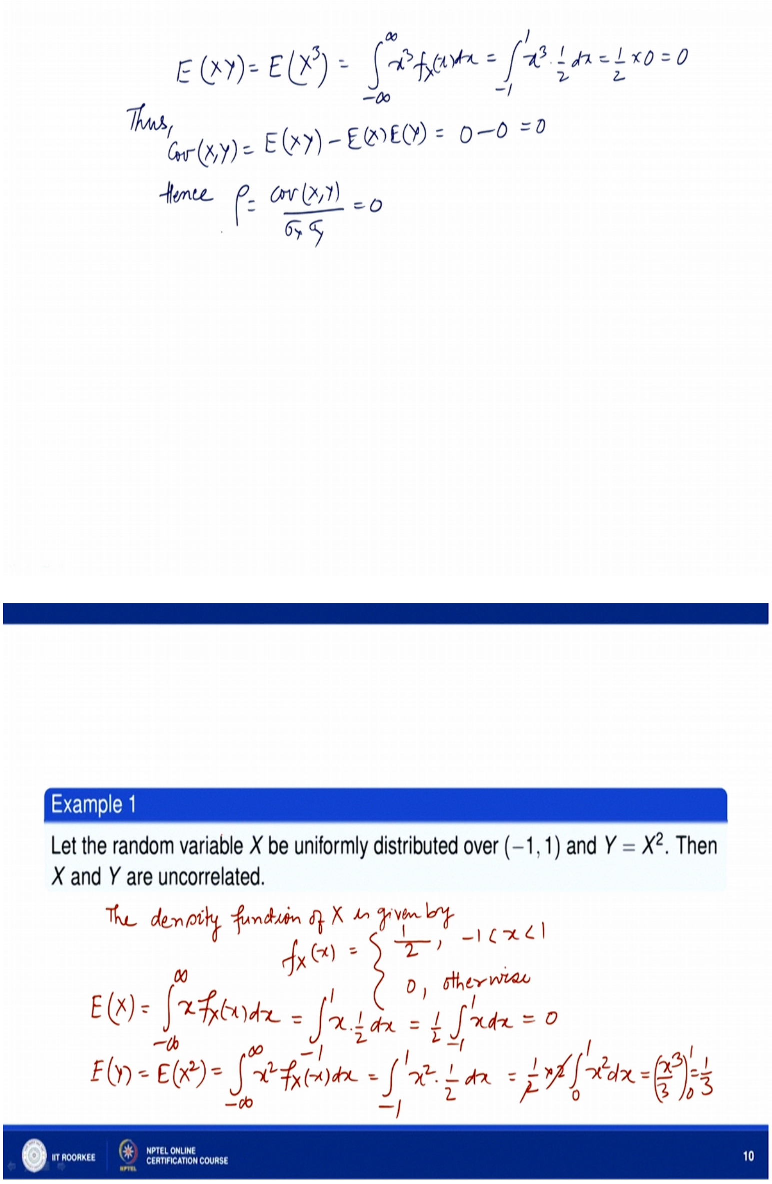

So let me find expected value of XY, this is expected value of XY is, Y is equal to X square, so we get expected value of X cube, so integral over minus infinity to infinity, X cube FX X DX, this is equal to integral over minus 1 to 1, X cube into half DX and we get 1 by 2, X cube is an odd function of X, so the value of the integral is 0 and we get E XY equal to 0. Thus covariance of XY equal to E XY minus EX into EY equal to, E XY is 0. Okay, we have found EX equal to 0. Okay, so this is 0 minus 0, so 0 and hence rho equal to covariance of XY divided by sigma X sigma Y okay is equal to 0.

So coefficient of correlation is equal to 0, but we are given that, Y is equal to X square, so coefficient of correlation is 0 but the random variables X and Y are dependent. So this is a problem which shows that the converse is not true.

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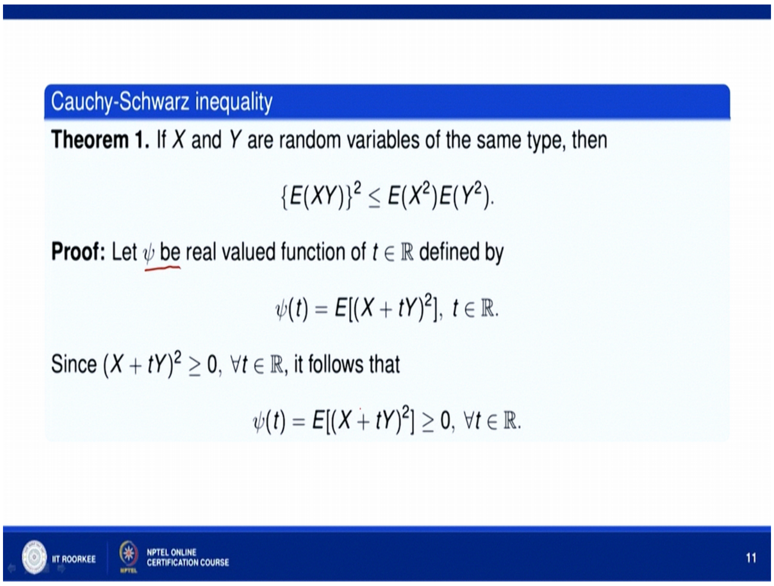

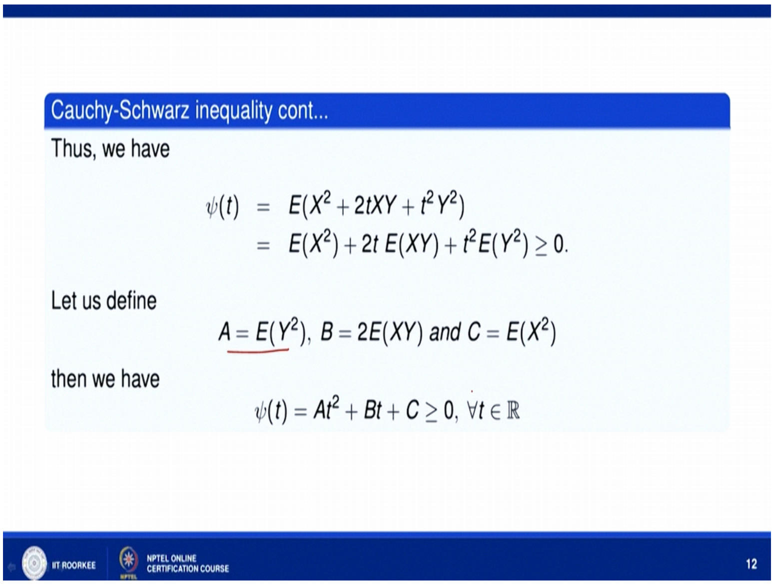

Now let us prove the Cauchy-Schwarz inequality, we shall need this Cauchy-Schwarz inequality to show that the value of the coefficient of correlation that is rho lies between minus 1 and plus 1. So if X and Y are random variables of the same type, that means either both of them are discrete random variables or they are both continuous random variables, so then expected value of XY whole square is less than or equal to expected value of X square into expected value of Y square. Let us takes psi to be a real valued function of the real number t. Okay, so let psi be a real valued function of a real variable t defined by psi t equal to expectation of X plus t Y whole square, where t belongs to R.

Now X plus t Y whole square is greater than or equal to 0 for every value of t belonging to R, therefore it follows that psi t is also a negative function of t. Okay, psi t equal to expectation of X plus t Y square is also greater than or equal to 0, for every value of t belonging to R and therefore we can write psi t as we can write psi t equal to expectation of X square plus 2t XY plus t square Y square. Okay and this is equal to expectation of X square 2t is a scalar, so plus 2t times expectation of XY plus t square times expectation of Y square which is greater than or equal to 0.

Now let us denote expectation of Y square by A, expectation of XY into 2 by B and expectation of X square by C, then psi t is equal to At square plus Bt plus C, which is greater than or equal to 0 for every value of t belonging to R.

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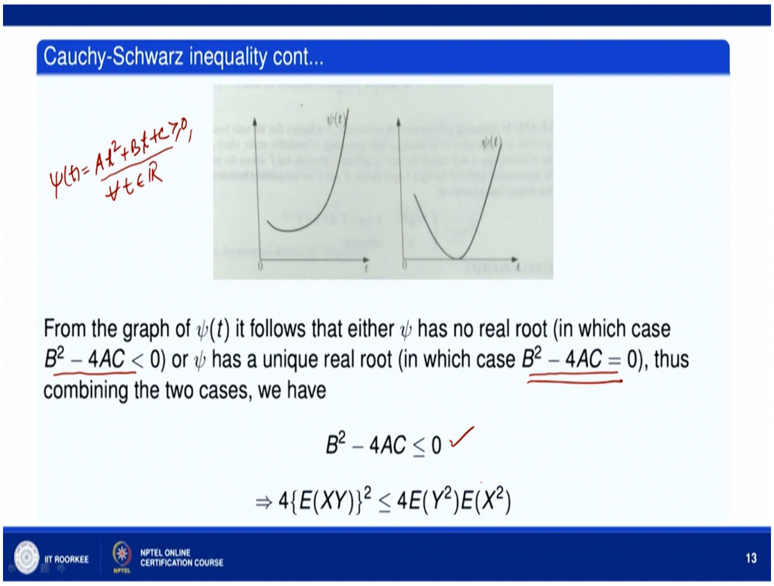

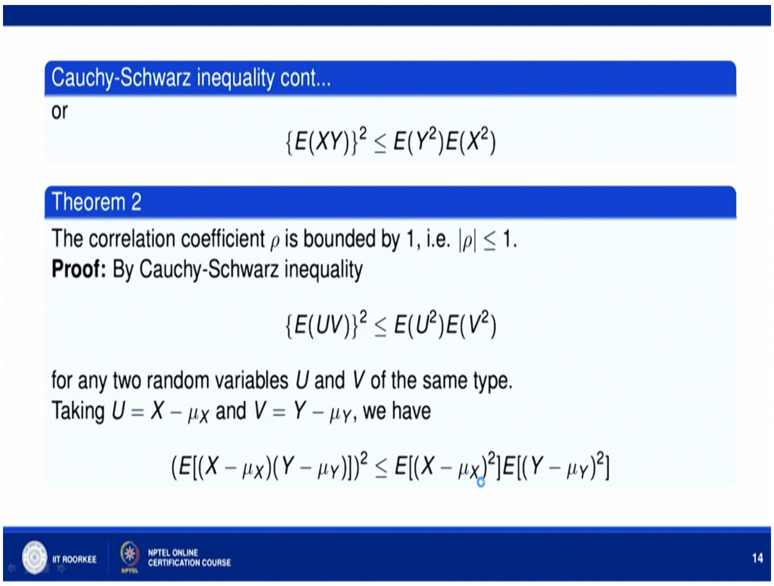

Okay, so psi t is equal to At square plus Bt plus C, which is greater than or equal to 0 for every value of t belonging to R. Now we have to see two graphs okay, this graph and this graph they are both, see At square plus Bt plus C is a parabola, it is a parabola, so this is a parabolic curve. Okay and from the graph of psi t, it follows that either, now since this psi t is always greater than or equal to 0, either psi has no real root in which case B square minus 4 AC okay will be less than 0 because it is a quadratic equation in t or psi has a unique real root, in which case B square minus 4 AC equal to 0. In this graph, you can see it has a unique real root.

So in that case B square minus 4 AC will be equal to 0 and therefore combining this case and this case, we have B square minus 4 AC less than or equal to 0. Okay, so B is equal to 2 times E XY, so B square is 4 times E XY whole square and less than or equal to 4 times A that is EY square into C, which is E X square, so we get E XY whole square less than or equal to EY square into EX square, which proves the Cauchy-Schwarz inequality for the random variables X and Y.

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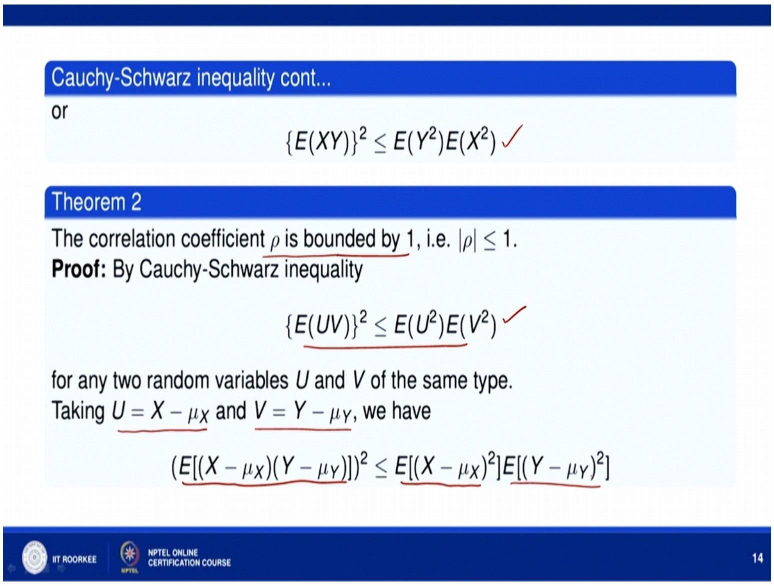

Now let us show that the correlation coefficient rho is bounded by 1 that is mod of rho is less than or equal to 1, so by Cauchy-Schwarz inequality if you take any two random variables U and V, then expectation of UV whole square is less than or equal to expectation of U square into expectation of V square. Now let us define U to be equal to X minus mu X and V equal to Y minus mu Y, then from this equation. Okay, we have expectation of X minus mu X into Y minus mu Y whole square less than or equal to expectation of X minus mu X whole square into expectation of Y minus mu Y whole square. Now this is what you can see, this is nothing but covariance of XY okay, so covariance of XY whole square, this is sigma X square, this is sigma Y square, that is the they are variances X and Y.

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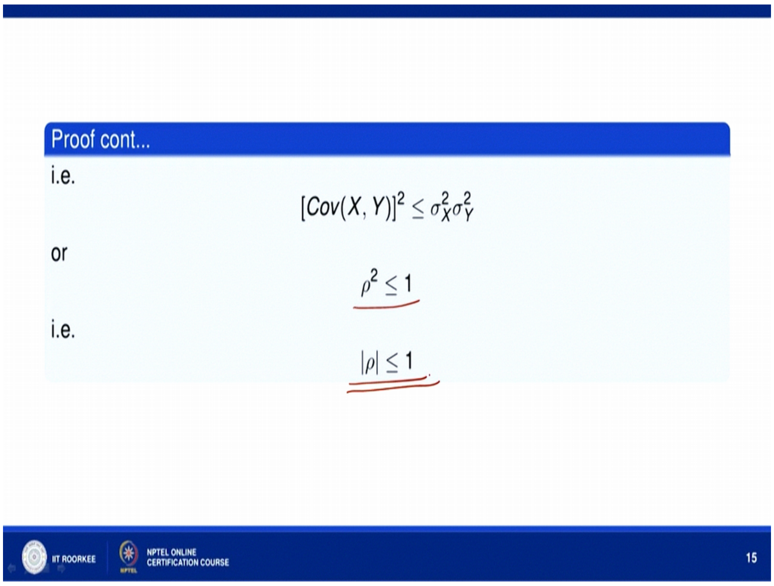

So we get covariance of XY whole square less than or equal to sigma X square into sigma Y square. Dividing by sigma X square sigma Y square, we get covariance of XY whole square divided by sigma X square sigma Y square less than or equal to 1, or rho square is less than or equal to 1 which implies that mod of rho is less than or equal to 1. So, this proves the result that the coefficient of correlation is bounded by 1.

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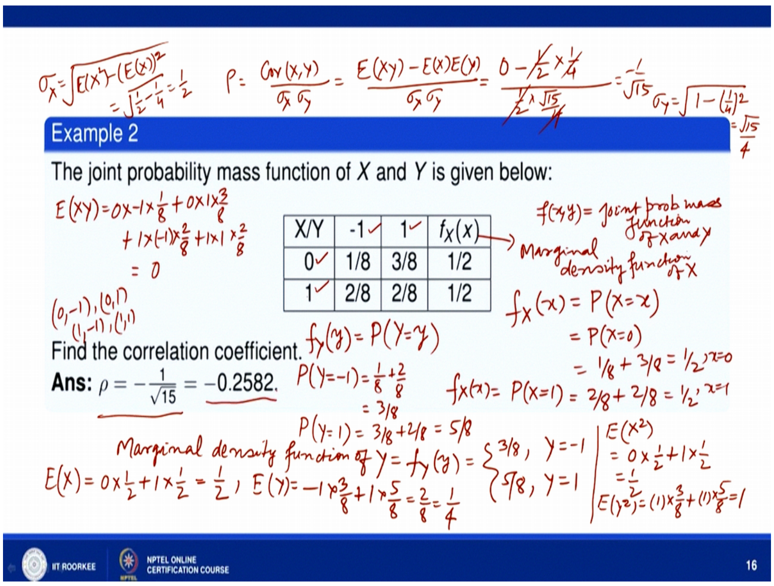

Now let us take an example, the joint probability mass function of X and Y is given in this table. Okay, these are the values of X and these are the values of Y okay, so when X takes the value 0, Y takes the value of minus 1 okay, F XY the joint probability mass function of X and

Y that is F XY, this is the joint probability mass function, I am denoting the join probability mass function of X and Y by F XY, this is joint probability mass function of X and Y okay.

So this is the probability that X takes the value 0, Y takes the value minus 1, this is the probability that X takes the value 0, Y is the value 1 and this is the probability that X takes the value 1, Y takes the value minus 1, this is the probability that X takes the value 1, Y takes the value 1. So the marginal density function, FX X, this is marginal density function of X, so FX X is equal to probability that X takes the value X, so X takes the value 0.

Okay, probability that X takes the value 0 will be equal to 1 by 8 plus 3 by 8 okay, which is equal to 1 by 2 and then probably that X takes the value 1 okay is equal to 2 by 8 plus 2 by 8 which is equal to 1 by 2. So FX X for X equal to 0. Okay, this is the case when X is equal to 0, this is the case when X is equal to 1, so FX X is equal to 1 by 2 when x takes the value 0 and 1 by 2, again, when X equal to 1.

Now let us find FY Y okay, FY Y is probability that Y takes the value Y okay, so let us first find the probability that Y takes the value minus 1. So Y takes the value minus 1, so this will be equal to 1 by 8 plus 2 by 8 which is equal to 3 by 8 and probably that Y takes the value 1, which is equal to 3 by 8 plus 2 by 8, this is equal to 5 by 8 okay, so the marginal density function of Y equal to FY Y is equal to 3 by 8 for Y equal to minus 1 and 5 by 8, 3 by 8 for Y equal to minus 1 and for Y equal to 1 it is 5 by 8. We have to find the correlation coefficient, so we need to find the expected value of X and expectation value of Y okay, so expectation of X.

Let us first find the expectation of X, it is the values of X multiplied by the corresponding probabilities okay, so X takes the two values. Okay, X takes the value 0, 0, multiplied by its probability FX okay, so X is equal to 0, multiplied by half okay plus 1 multiplied by 1 by 2, so we get expectation of X equal to half. Expectation of Y we can get similarly, values of Y multiplied by their corresponding probabilities, so value of Y is minus 1 okay, multiplied by 3 by 8 and then value of Y is 1 multiplied by 5 by 8, so 5 by 8 minus 3 by 8 is 2 by 8 which is equal to 1 by 4, so expectation of X is 1 by 2, expectation of Y is 1 by 4.

Now let us find expectation of X square, so expectation of X square, X is taking value 0 and 1 okay, so 0 square means 0, multiplied by 1 by 2 plus expected value of sorry, we are getting values of X as 0 and 1, so 1 square that is 1 multiplied by the corresponding probabilities that is half okay. So expected value of X square is half, expected value of Y square we can find.

Now Y is taking minus 1 and plus 1, so minus 1 square is 1, 1 into the probability is 3 by 8, plus 1 square means 1 multiplied by 5 by 8, so we get it as 8 by 8 that is equal to 1.

So we have got the values of expectation of X square, expectation of Y square, now let us find the expectation of XY. So expectation of XY okay, so values of X are, X and Y take values X is equal to 0, Y equal to minus 1, so 0 minus 1 okay, then 0 and 1, then X is equal to 1, Y is equal to minus 1, so 1 minus 1. X is equal to 1, Y equal to 1, so 1, 1 okay. So expected value of XY is multiply the values of X and Y with the joint probability, that is joint probability mass function of XY, so X is 0, so 0 into minus 1 okay, X into Y multiplied by 1 by 8 okay.

Then 0 into 1 multiplied by the joint probability 3 by 8, then 1 into minus 1 multiplied by 2 by 8 and then 1 into 1 multiplied by 2 by 8 okay, so how much is this? This is 0, this is 0, and here what we get? Minus 2 by 8, here we get 2 by 8 okay, so expectation of XY equal to 0. Okay and thus what we get? Thus we have rho equal to covariance of XY divided by sigma X sigma Y, covariance of XY is expected value of XY minus EX EY divided by sigma X sigma Y. Now this is expected value of XY equal to 0, so 0 minus expectation of X, expectation of X is half, expectation of Y is 1 by 4 okay, divided by sigma X, sigma X is equal to square root EX square minus EX whole square.

Okay, expectation of X square, we found that is equal to half, minus expectation of X is equal to 1 by 2, so 1 by 2 square means 1 by 4, so this is 1 by 4, square root of 1 by 4 is 1 by 2, so we get 1 by 2 here. Okay, now let us find sigma Y okay, so sigma Y equal to square root expectation of Y square, it is equal to 1 okay, expectation of Y is equal to 1 by 4, so 1 by 4 whole square, so this is equal to square root 15 by 4 okay. So we get here square root 15 by 4 okay, this cancels with this, this cancels with this and we get it as minus 1 by root 15 okay. So rho is equal to minus 1 by root 15 which is equal to minus 0.2582.

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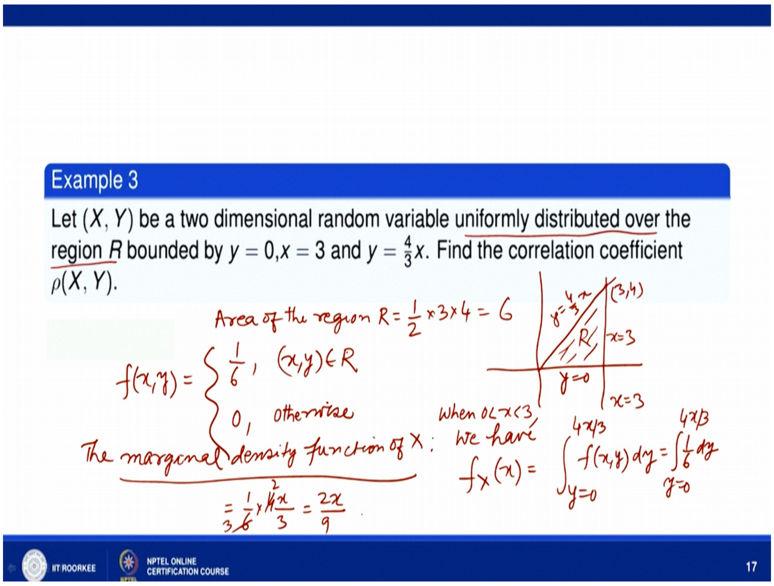

Now let us consider another problem, let XY be a two-dimensional random variable uniformly distributed over the region R bounded by Y equal to 0, X equal to 3, Y equal to 4 by 3 X okay. So this point of intersection is 3, 4 okay, now this is the region R. Okay, region R is bounded by Y equal to 0, X equal to 3 and Y equal to 4 by 3 into X, now area of the region R is equal to 1 by 2 into base. Because it is a triangle, so base is equal to 3 into height, height is 4, so we get 6. Okay, now since the random variable is, since XY is two-dimensional random variable which is uniformly distributed over the region R okay, so we have F XY equal to 1 by R, means 1 by 6 when XY belong to R okay and 0 otherwise.

Okay, we need to first find the marginal density functions okay, the marginal density function of X let us find first okay, so we have FX X equal to Y varies from 0 to 4X by 3, F XY into DY okay, this is the probability that X takes the value X, so this is equal to Y varies from 0 to 4X by 3, 1 by 6 DY, so this is equal to and here X, when 0 is less than X, less than 3 okay, we have FX X equal to this. So this is equal to 1 by 6, 4X by 3 and this is equal to 2X by 9, so FX X equal to 2X by 9, when 0 is less than X, less than 3 and otherwise it is 0.

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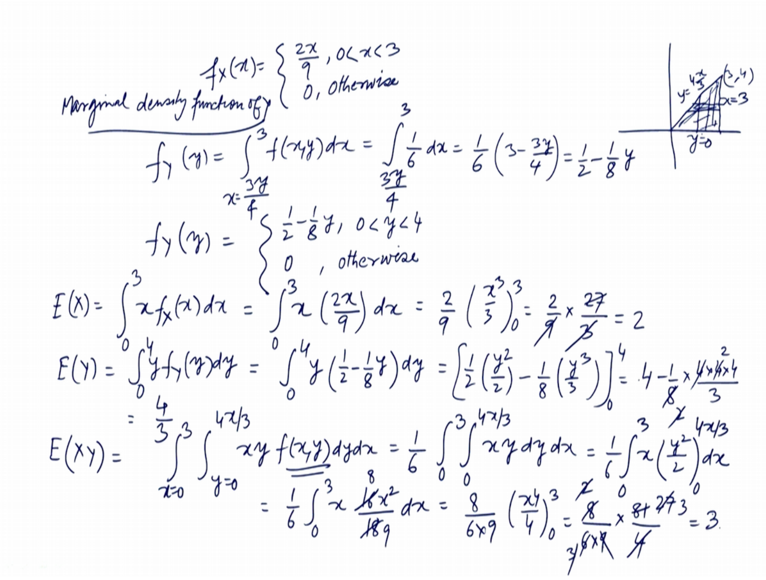

So we write it like this FX X equal to 2X by 9 when 0 is less than X, less than 3 and 0 otherwise. Let us now find the marginal density function of Y, FY Y, so FY Y equal to, now this is the probability that Y takes the value Y, so X varies, we have this region okay, this is Y equal to 4X by 3, this is X equal to 3 and this is Y equal to 0, so X varies from 3Y by 4 to 3, so 3Y by 4 to 3 okay. And F XY DX, so this is 3Y by 4 to 3, 1 by 6 DX okay, so 1 by 6 times 3 minus 3Y by 4 okay, so this is 1 by 2 minus 1 by 8 Y okay. So FY Y is given by 1 by 2 minus 1 by 8 Y when 0 is less than Y less than 4 okay, Y lies between 0 and 4, this is 3, 4 point and 0 otherwise okay.

Now let us find expected value of X, so expected value of X is X multiplied by its probability density function and X varies from 0 to 3 okay, so X varies from 0 to 3, so 0 to 3 X times FX X. FX X is 2X by 9 DX, so this is 2X square by 9 okay, so 2 by 9, integral of X square is X cube by 3, so we put the limits and we get 2 by 9 into 3 cube, 3 cube means 27 divided by 3, so we cancel this and get expected value of X as 2.

Now expected value of Y, so integral over Y, FY Y DY, Y varies from 0 to 4 and what we get is integral over 0 to 4 Y times, FY Y is 1 by 2 minus 1 by 8 Y DY. So this equal to 1 by 2 Y square by 2 minus 1 by 8 Y cube by 3. Let us put the limits and we get this is 4, 4 square by 4, so we get 1 by, so we get 4 okay, minus 1 by 8, Y cube is 4 cube, so 4 into 4 into 4 divided by 3 okay. So this will be 4 minus 8 by 3, so this is 3, 4 is 12, 12 minus 8, so 4 by 3, so this is expected value of Y.

Now expected value of XY okay, so we take the joint probability mass function here, so for now let us see we have to integrate over this area. Okay, for the joint probability mass function, so Y varies from 0 to 4 X by 3, X varies from 0 to 3 and we have X into Y, F XY DX DY, DY DX. F XY is equal to, we are given F XY equal to 1 by 6, so this is 1 by 6 times integral over 0 to 3, integral over 0 to 4X by 3 and we have X into Y DY DX. This F XY is equal to 1 by 6 okay over region R, so we have 1 by 6 integral over 0 to 3X and then we get Y square by 2 and we have the limits 0 4X by 3 DX. So what we get is 1 by 6, 0 to 3 X times, Y square means 16 X square by 9.

So 16 X square by 9 into 2 that is 18 okay. Y square by 2 means 16 X square by 9 into, 16 X square by 18, so we get here 2 into 8 is 16 and here get 9 okay. This is DX okay, so we have 8 by 6 into 9, 8 divided by 6 into 9. X cube, integral of X cube is X4 by 4, 0 to 3, so we get 8 by 6 into 9 and then we have here 3 to the power 4 that is 81 divided by 4 okay. So 4 into 2 is 8 okay and 2 into 3 is 6 and then we can cancel 3 into 9 is 27, 27 will cancel okay, 3 will cancel it as okay, 3, 27 here and 9 cancels 27 with 3. So we get expected value of XY as 3.

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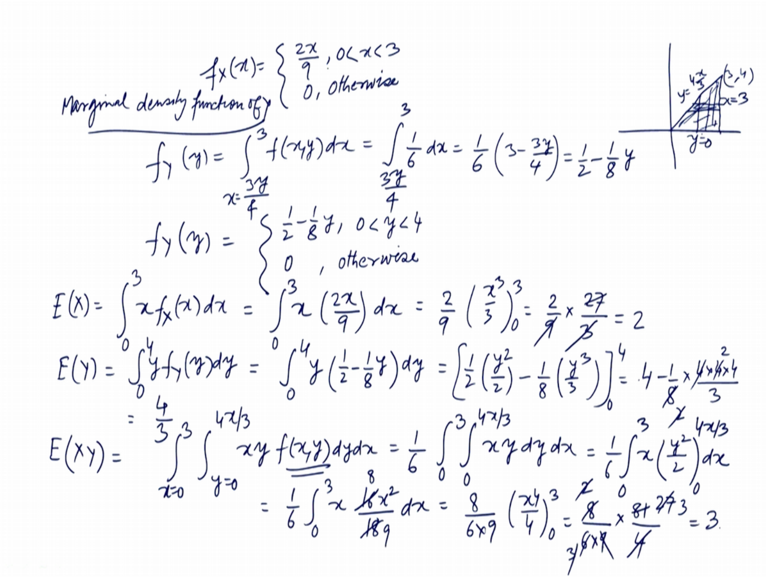

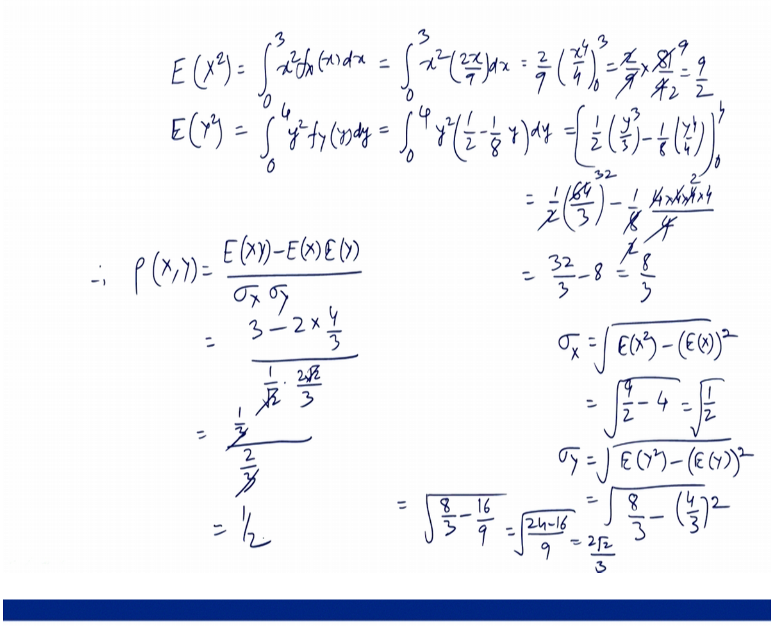

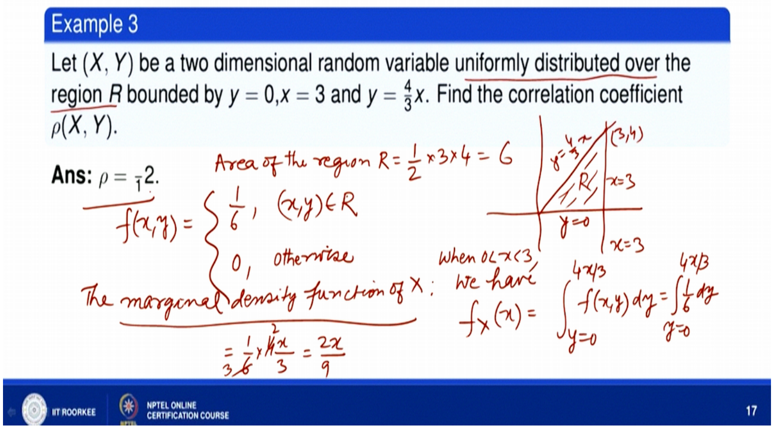

Now we need to find expected value of X square, expected value of Y square okay, so expected value of X square is integral 0 to 3, X square into FX X DX okay and FX X we have found to be equal to 2X by 9 over the interval 0 to 3, so this is 0 to 3 X square into 2X by 9 DX. It comes out to be 2 by 9 integral over of X cube is X4 by 4 0 to 3, we get 2 by 9 into 81 by 4 okay. So 9 into 9 is 81 and we get it 9 by 2, similarly we can find expected value of Y square integral 0 to 4, Y square FY Y DY and it comes out to be integral 0 to 4, FY Y is 1 by 2 minus 1 by 8Y, so 1 by 2 minus 1 by 8Y DY and this is 1 by 2 Y cube by 3 minus 1 by 8 Y4 by 4 and we get the value as 1 by 2. 4 to the power 3, so 64 by 3 minus 1 by 8, 4 to the power 4, so 4 into 4 into 4 into 4 divided by 4 okay, so this cancels and we get this cancels with this, we get 2. This cancels with this, we get 2 okay, so this is 8 and here we get 32.

So 32 by 3 minus 8 okay, so we get 8 by 3. Okay and so rho XY equal to E XY which is E XY minus EX into EY. This is covariance of XY okay, sigma X sigma Y okay, so we found E XY equal to 3 and EX equal to 2, EY equal to 4 by 3 okay. So 3 minus 2 into 4 by 3 divided by sigma X, sigma X equal to square root EX square minus EX whole square. EX square we found is equal to 9 by 2 and EX we found to be equal to 2, so 2 square is 4, so we get this is 1 by 2, so 1 by 2 square root and sigma Y equal to square root EY square minus EY whole square. EY square we found to be 8 by 3, so 8 by 3 minus EY, EY we found to be equal to 4 by 3.

So 4 by 3 whole square, so this is how much? 8 by 3 minus 16 by 9 okay and this is LCM is 9, 24 minus 16, so we get 8 by 9 that is 2 root 2 divided by 3 okay. So we get here 1 by root 2 into 2 root 2 divided by 3 okay, so how much is that? 3 into 3 equal is 9, 9 minus 8, 1 by 3, so 1 by 3 divided by, this cancels with this, 2 by 3. And we get the value as half, so rho XY equal to half. So rho is equal to, correlation coefficient is equal to half. So this is how we solve this problem. With that I would like to end my lecture. Thank you very much for your attention.