Advanced Engineering Mathematics
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Lecture – 59
Testing of Hypotheses - III

Hello friends, welcome to my lecture on testing of hypotheses, this is third and final lecture on testing of hypotheses.

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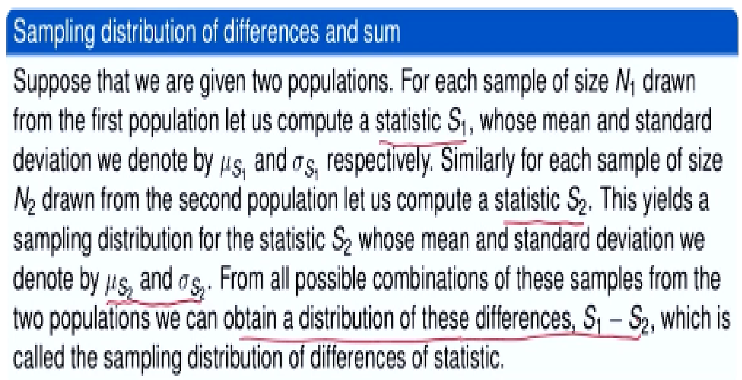

Suppose that we are given 2 populations for each sample of size N1 drawn from the first population, let us compute a statistic S1 whose mean and standard deviation we denote by Mu S1 and sigma S1 respectively. Similarly, for each sample of size N2 drawn from the second population, let us compute a statistic S2, this yield a sampling distribution for the statistic S2 whose mean and standard deviation we denote by Mu S2 and sigma S2.

From all possible combinations of the samples from the 2 populations, we can obtain a distribution of these differences; S1 –S2 which is called the sampling distribution of differences of statistic.

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Sampling distribution of differences and sum cont...

The mean and standard deviation of this sampling distribution, denoted respectively by $\mu_{S_1-S_2}$ and $\sigma_{S_1-S_2}$ are given by

$$\mu_{S_1-S_2} = \mu_{S_1} - \mu_{S_2} \ \text{and}\ \sigma_{S_1-S_2} = \sqrt{\sigma^2_{S_1} + \sigma^2_{S_2}} \qquad (1)$$

provided that the sample chosen do not in any way depend on each other, i.e. the samples are independent.

If S_1 and S_2 are the sample means from the two populations, which we denote by $\bar{X}_1$ and $\bar{X}_2$, then the sampling distribution of the differences of means is given for infinite populations with mean and standard deviation μ_1, σ_1 and μ_2, σ_2 respectively by

The mean and standard deviation of the sampling distribution denoted by Mu S1- S2 and sigma S1 – S2 are given by Mu S1 – S2 = Mu S1 – Mu S2 and sigma S1 – S2 = square root of sigma S1 square + sigma S2 square provided that the sample chosen do not in any way depend on each other that is the samples are independent, if S1 and S2 are the sample means, from the 2 populations which we denote by X1 bar and X2 bar.

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Sampling distribution of differences and sum cont...

$$\mu_{\bar{X}_1-\bar{X}_2} = \mu_{\bar{X}_1} - \mu_{\bar{X}_2} = \mu_1 - \mu_2 \ \text{and}\ \sigma_{\bar{X}_1-\bar{X}_2} = \sqrt{\sigma^2_{\bar{X}_1} + \sigma^2_{\bar{X}_2}} = \sqrt{\frac{\sigma_1^2}{N_1} + \frac{\sigma_2^2}{N_2}} \qquad (2)$$

using

$$\mu_{\bar{X}} = \mu \ \text{and}\ \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}$$

the result also holds for finite populations if sampling is with replacement. Similarly, results can be obtained for finite populations in which sampling is without replacement, by using

$$\sigma_{\bar{X}_1} = \frac{\sigma_1}{\sqrt{N_1}}\sqrt{\frac{N_p-N_1}{N_p-1}}$$

$$\mu_{\bar{X}} = \mu \ \text{and}\ \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{N}}\sqrt{\frac{N_p-N}{N_p-1}}.$$

Then the sampling distribution of the differences of means is given for infinite populations with mean and standard deviation Mu1 sigma 1 and M2 sigma 2 respectively by Mu X1 bar – X2 bar = Mu X1 bar – mu X2 bar which is = Mu1 – Mu2 sigma X1 bar – X2 bar =square root sigma X1 bar square + sigma X2 bar square = square root of sigma 1 square / N1 + sigma 2 square/ N2.

Mu X bar = Mu and sigma X bar = sigma/root N as we know already for the infinite population Sigma X bar is given by Sigma/ root N.

The result also holds for finite populations if sampling is with replacement because when there is sampling with replacement, we can theoretically consider it as an infinite population because any number of samples can be drawn from the population without exhausting it. Similarly, some results can be obtain for finite populations in which sampling is without replacement, in that case Mu X bar is again Mu and sigma X bar is Sigma over root N * square root Np - N over Np – 1.

Now, these formulas Mu X1 bar – X2 bar = Mu X1 bar – Mu X2 bar, this follows from Mu S1 – S2 = Mu S1 – Mu S2, okay, S1 bar is; S1 is X1 bar, S2 is X2 bar, so we have Mu X1 bar – X2 bar = Mu X1 bar – Mu X2 bar and sigma X1 bar – X2 bar = square root X sigma X1 bar square + sigma X2 bar square, okay and we know that sigma X1 bar square is sigma 1 square/ N1 in the case of infinite population.

In the case of finite population sigma X1 bar square will be = sigma 1 square/ N * Np - N divided by Np – 1, sample size is N1, so we will have Np – N1 divided by Np – 1, so in the case of finite population of size; if you are trying samples of size N1 then sigma X1 bar will be = sigma 1/ root N1 * square root Np – N1 divided by Np – 1, okay.

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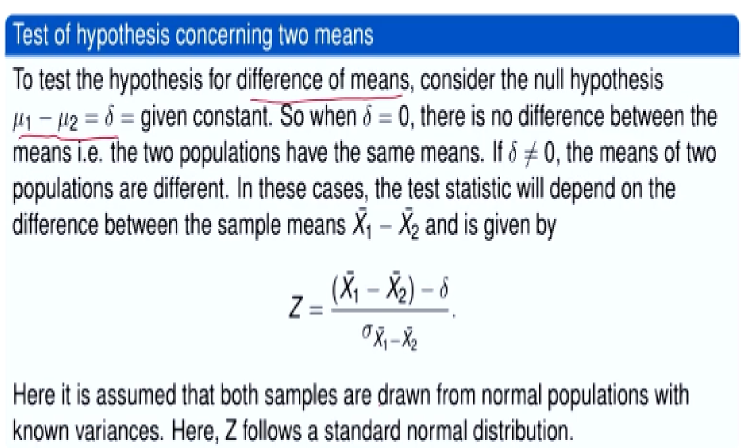

So, now, let us consider testing of hypotheses concerning 2 means, we have earlier done the case of testing of hypotheses for a single population, now we will consider the testing of hypotheses when there are 2 populations. So, to test the hypotheses for differences of means, okay, so we are considering the test static to be mean of the population. So, to test the hypotheses for difference of means consider the null hypothesis, Mu1 – Mu 2 = delta, okay where delta is some given constant.

So, when delta =0, there is no difference between the means, the 2 populations have the same means, if delta is != 0, the means of the two populations are different, in these cases the test static that is the mean will depend on the difference between the sample means X1 bar – X2 bar and it is given by Z = X1 bar – X2 bar - delta divided by sigma X1 bar – X2 bar. Here we are assuming that both the samples are drawn from normal populations with known variances.

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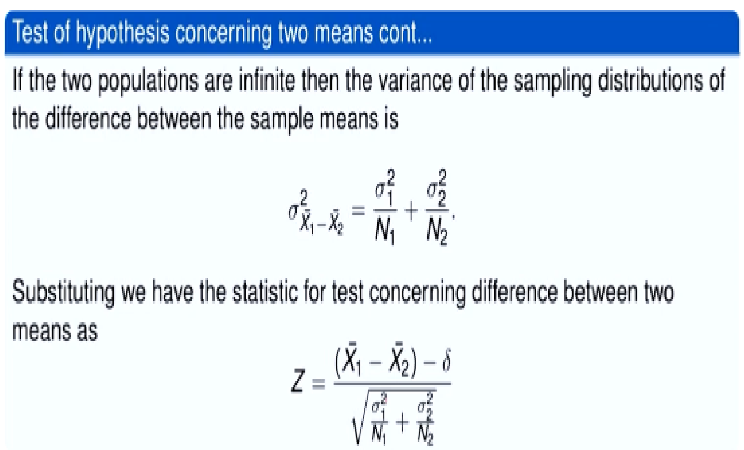

And here Z follows a standard normal distribution now, if the two populations are infinite then the variance of these sampling distribution of the difference between the sample means, Sigma X1 bar – X2 bar square as we have seen already, sigma X1 bar – X2 bar square, okay, sigma x square X1 bar – X2 bar = sigma 1 square/N1 sigma 2 square/N2. Substituting we have these statistic for tests concerning difference between 2 Mu's as Z = X1 bar – x2 bar - delta divided by the square root Sigma 1 square/N1+ Sigma 2 square/N2.

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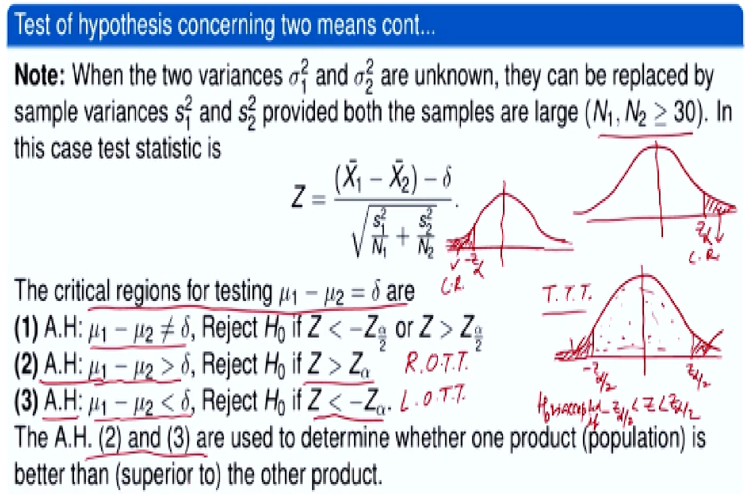

Now, when the 2 variances; sigma 1 square and sigma 2 square are unknown, they can be replaced by sample variances, so if we do not know the variances sigma 1 square and sigma 2 square for the 2 populations then they can be replaced by the sample variances, so sigma S1 square and S2 square provided both the samples are large that is N1, N2 must be > = 30, in this case test static will be Z = X1 bar – X2 bar - delta divided by the square root S1 square/N1 + S2 square/N2.

Now, let us see the critical regions for testing Mu1 – Mu2 = delta are; if alternative hypothesis is Mu1- Mu2 is != delta. then we have 2 tailed test, okay, TTT we have to apply that is we will reject the H0, if Z is < - Z alpha/2,; Z alpha/ 2 and –Z alpha/2 that gives us the critical values of Z, okay, so let us say this is our standard normal curve, so this is say – Z alpha/2 and this is +Z alpha/2, so this is our critical region, okay.

So, in this case when Mu1 – Mu2 is! = delta, okay, we know that we have to apply 2 tailed test, so this is the region which is known as the critical region, okay, so if Z is < - Z alpha /2 or Z is > Z alpha/2 then we have to reject H0, okay, so that means this region is the significant region, okay, the results are significant here, okay and this is the region of acceptance of H0, okay, this region where when Z lies between –Z alpha/2 and Z alpha/2, okay, this is in this region H0 is accepted.

If –Z alpha/2 is < Z < Z alpha/2, now Mu1 – Mu2 is > delta, if Mu1 –Mu2 > delta then we know that we have to apply right one tailed test, ROTT okay, so in this case what happens; if you look at the standard normal curve, the area under when Z is >; this is your critical .point Z alpha, so when Z is > Z alpha, okay, then we reject H0, so this is region which is the critical region, okay, so reject H0 if Z is > Z alpha.

And when Mu1 – Mu2 is < delta, we apply LOTT, okay, in the case of LOTT, we have let us say this is –Z alpha okay, the critical value here, so in this case, so this is the region of critical region here okay, CR, so when Z is < -Z alpha we will reject the null hypothesis H0, if Z is > -Z alpha, then we will accept the null hypothesis and we will say that the results are not significant. If Z is < -Z alpha, then we say that the result is significant, okay.

And we have to accept the alternative hypothesis, so the cases 2 and 3, okay, this case and this case okay are used to determine whether one product or population is better than or superior to the other product.

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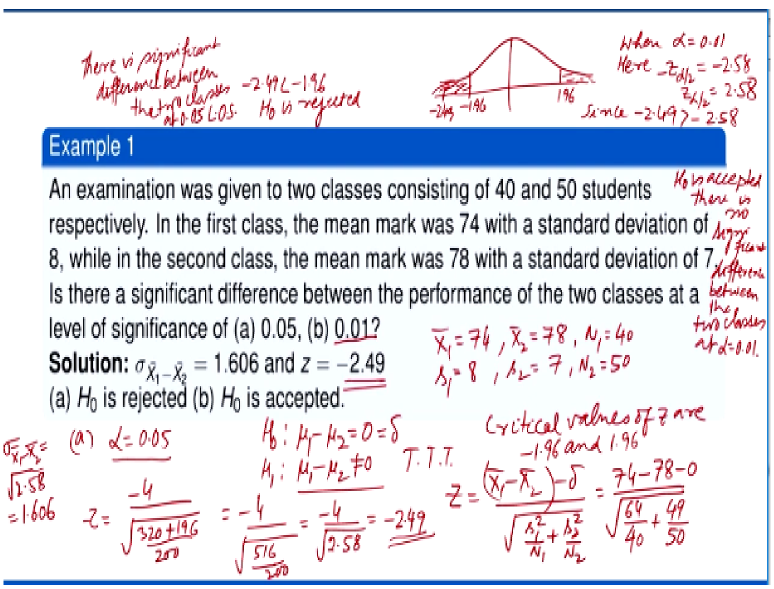

Now, let us consider this problem; an examination was given to 2 classes consisting of 40 and 50 students respectively, in the first class, the mean mark was 74, so X1 bar here; X1 bar = 74, okay, a standard deviation S1, we are given the sample standard deviation, so S1 = 8 and then X2 bar, okay X2 bar= mean mark is 78 for the second class and S2 = 7, we have N1 =; first class has 40 students, so N1 = 40 and N2 = 50, okay.

Now, we have to decide is there a significant difference between the performance of the 2 classes at a level of significance 0.05, 0.01, so let us first consider the case a, here alpha = 0.05 okay now, let us so our null hypothesis is that there is no significant difference between the performance of the two classes that is the difference whatever it is, it is just due to a chance okay, so H0 is Mu1 – Mu2 = 0, okay, delta = 0.

We are assuming that the null hypothesis is that there is no significant difference between the two classes, okay, while alternative hypothesis is that Mu1 – Mu2 is != 0 that is there is significant difference between the two classes, okay. When alpha =; now, so here Mu1 – Mu2 is ! = 0 it means that we have to apply 2 tailed test, okay, we have to apply to 2 tailed test and let us see for alpha = 0.05, what are the critical values of Z?

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The following table gives critical values of z for both one tailed and two tailed tests at various levels of significance:

Level of significance α	0.10	0.05	0.01	0.005	0.002
Critical values of z for one tailed tests	-1.28 or 1.28	-1.645 or 1.645	-2.33 or 2.33	-2.58 or 2.58	-2.88 or 2.88
Critical values of z for two tailed tests	-1.645 and 1.645	-1.96 and 1.96	-2.58 and 2.58	-2.81 and 2.81	-3.08 and 3.08

They are given enough in a table at the end of this lecture, so let us see, okay, so 0.05, okay 0.05 for the 2 tailed test, the critical values are .-1.96 and 1.96, okay, so let us go there, so critical values of Z are -1.96 and 1.96, okay, so this is Z alpha/2 – Z alpha/2, this is +Z alpha/2, okay, so if Z lies between -1.96 to 1.96 okay, then H0 is accepted otherwise it is rejected, okay. So, let us find the value of Z?

Z = X1 bar – X2 bar – delta divided by the square root of S1 square/ N1 + S2 square/ N2, X1 bar – X2 bar; X1 bar = 74, okay 74 – X2 bar is 78, delta = 0 divided by the square root of now, S1 is square, S1 square is 8 square, means 64, 64 divided by N1 = 40 and then +N2, sorry S2 = 7, okay, so 7 square, S2 square is 7 square means 49 divided by N2 that is 50, okay. Now, let us calculate this value, so Z = 74-78 that is -4 divided by the square root.

Here the LCM is 200, okay, so 200 and then we have here 320 + 196, okay so this is -4 divided by the square root, this one is 516, okay, 51 6 divided by 200, okay 516 divide by 200 and this is then = -4 upon the square root of; when you divide by 200, this is 258 okay, so 2.58, okay -4 upon square root of 2.5, it comes out to be -2.49, okay, so this is -2.49 and actually, this square root of 2.58, okay which is; which gives us the value of the Sigma X1 bar – X2 bar.

This is = 1.606, so square root of 2.58, actually this is the value of Sigma X1 bar – X2 bar, Sigma X1 bar – X2 bar, this is = square root S1 square/N1 + S1 square/N2, this is = 2.58 which is 1.606, so this value 1.606 is the value of this denominator, okay, so this is -4 upon 1.606 which comes out to be -2.49 okay. Now, this value of Z, let us see whether it lies between -1.96 and +1.96 know.

Because if we draw this curve, okay this is -1.96 to 1.96, okay and Z is -2.49, so -2.49 is here, okay, so -2.49 < -1.9, okay, therefore, this is the region; this is the critical region, okay, this critical region, so H0 is to be rejected, okay, H0 is at rejected which means that alternative hypothesis is accepted that is there is significant difference between the two classes, okay, so we have the result; we can conclude that there is significant difference between the two classes at 0.05 level of significance, okay.

Now, let us look at 0.01, what happens if you take alpha = 0.01, okay, so when alpha = 0.01, let us see the values of Z, okay, the critical values of Z, so when 0.01, critical values of Z are -2.58 and +2.58, so here –Z alpha/2 is -2.58 and Z alpha/2 is 2.58, okay, so here the value of Z, which we have calculate is -2.49, so since -2.49 is > -2.58, okay, H0 is accepted, okay, so H0 is accepted, which means that at 0.01 level of significance, okay, we can say that there is no different between the two classes, okay.

So, there is no significant difference between the two classes at alpha = 0.01, okay that means both the classes performances the same, okay, wherever is the difference, it is due to just due to a chance.

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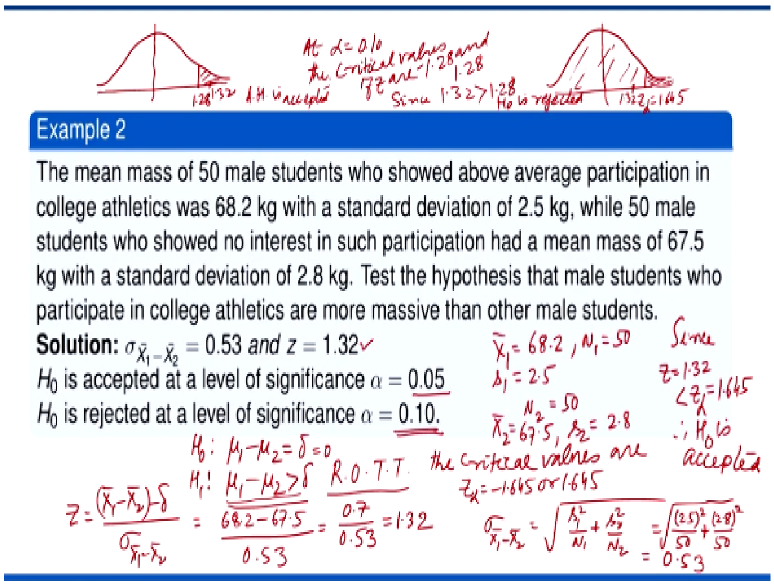

Now, let us go to example 2, okay, the mean mass of 50 male students who showed above average participation in college athletics was 68.2, okay, so here X1 = 68.2 and N1 = 50 with as standard deviation 2.5 kilogram, so S1 = 2.5, while 50 male students who showed no interest so N2 = 50, okay, while 50 male students who showed no interest in such participation as a mean mass of 67.5, okay.

So, X2 bar = 67.5 with the standard deviation, Sigma S2 = 2.8 kilogram, test the hypothesis that male students who participate in college athletics are more massive than other male students, so let us say our H0 is the null hypothesis is the Mu1 – Mu2 = delta = 0, okay that is the male students who participate in college athletics are the same as the other male students, okay on average.

So, alternative hypothesis is Mu1 – Mu2 > delta okay that is Mu1 - Mu2 is > 0, okay, delta is 0, okay, so that is our other hypothesis that is the male students who participate in college athletics, okay are more massive than other male students. Now, we are to consider alpha = 0, alpha is not given to us here, okay at what level of significance we have to test the hypotheses, so on our own, let us consider alpha = 0.05.

So, let us see when alpha = 0.05 since Mu1 – Mu2 is > delta, we have to apply ROTT right one tailed test, okay, so in the case of ROTT, when alpha is 0.05, let us see what is the value of Z, so at 0.05, the value of Z is 1.645, -1.645 or 1.645, okay, so the critical values of Z are Z alpha = -1.645 or 1.645, so let us find the value of Z; Z = X1 bar - X2 bar - delta divided by Sigma X1 bar – X2 bar, okay.

And we know that sigma X1 bar – X2 bar = square root S1 square/ N1 + S2 square/N2, okay, this is = square root S1 square; S1 square is 2.5 square divided by 50 + 2.8 square divided by 50, okay, so if you calculate this value, it comes to be 0.53, okay, now this is = X1 bar is 68.2 - 67.5 divided by 0.53, okay, delta = 0, so this is = 0.7 0. divided by 0.53 okay, it comes out to be 1.32, okay. Now, let us see if Z lies between, if okay, we have to apply right ROTT, okay. If Z is > 1.645, okay, if Z is > 1.645, then H0 is rejected otherwise H0 is accepted.

So, here Z = 1.32 since, Z = 1.32 is < Z alpha = 1.645, okay, here you see this is your critical region, see this is your Z alpha = 1.645, this is critical region, okay, so this is your region of acceptance of H0, okay, so Z = 1.32, so 1.32 lies here, okay, 1.32, so 1.32 is, Z alpha, therefore H0 is accepted, H0 is accepted means there is no significant difference between the 2 classes of students.

Those who participate in college athletics and those who have no interest in college athletics, so there is no significant difference between the two, okay, now, let us take alpha = 0.10, so when alpha = 0.10, the values of Z are; let us see we have to find the values of Z from here, okay with values of Z are -1.28 and 1.28, okay. So, at alpha = 0.10 okay, 0.10 the critical values of Z are -1.28 and 1.28, okay.

Since we are applying ROTT, okay, we are applying ROTT here, okay and 1.32 is > 1.28, this is your 1.28, okay so and 1.32 is here, so we are in the critical region, so since 1.32 is > 1.28, H0 is rejected, okay, which means that at the level of significance Z = 0.10 okay, there is significant difference that it is the male students who participate in college athletics are more massive than other male students.

So, we accept the alternative hypothesis Mu1- Mu2 >0 that is the male students who participate in college athletics are more massive than other male students, so H0 is rejected means AH is accepted, alternative hypothesis is accepted at alpha = 0.10.

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Example 3

In a random sample of 100 tube lights produced by company A, the mean life time of tube light is 1190 hours with standard deviation of 90 hours. Also in a random sample of 75 tube lights from company B the mean life time is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean life times of the two brands of tube lights at a significance level of (a) 0.05 (b) 0.02 ?

Solution: $\sigma_{\bar{X}_1-\bar{X}_2} = 16.5227$ and $Z = -2.421$

For $\alpha = 0.05$ reject H_0

For $\alpha = 0.01$ accept H_0.

Now, let us do the problem number 3, in a random sample of 100 tube lights produced by company A, the mean lifetime of tube light is 1190 hours with the standard deviation of 90 hours.

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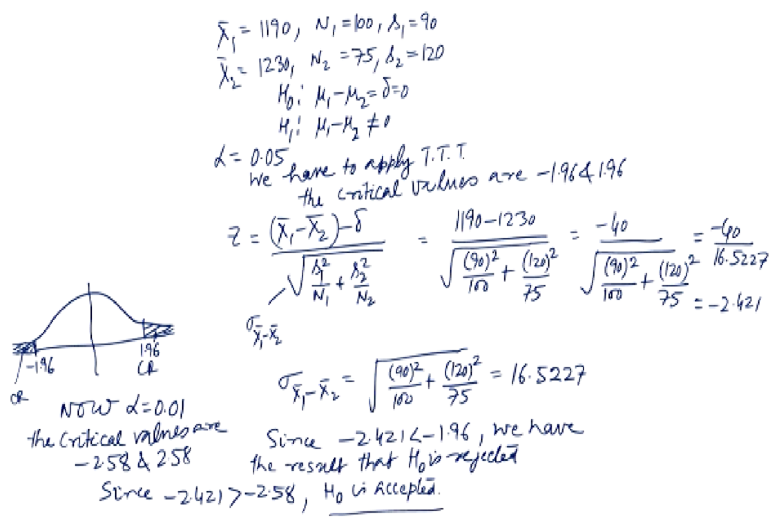

So, we have here X1 bar = 1190, N1 = 100, a standard deviation S1 = 90 hours then we have another random sample of 75 tube lights from Company B where the mean lifetime is 1230 hours, so X2 bar = 1230, N2 = 75, S2 = 120 hours, a standard deviation is 120 hours, so let us now we have to decide is there a significant; is there a difference between the mean lifetimes of the 2 brands of tube lights at significance level of 0.05 and 0.01, so let us assume H0, the null hypothesis to be Mu1 – Mu2 = delta = 0 that is there is no significant difference.

Alternative hypothesis H1 is Mu1 –Mu2 ! = 0 that is there is significant difference between the two samples okay, so let us now find the value of alpha = 0.05, so let us find the critical values; critical values of Z are; now we are to apply 2 tailed test, okay, so in the case of 2 tailed test, the values of Z are -1.96 and 1.96, so we have to apply 2 tailed test, the critical values are -1.96 and 1.96.

If Z lies between -1.96 and +1.96 then we accept H0 otherwise, we reject H0, so let us find the value of Z; Z = X1 bar – X2 bar – delta divided by the square root S1 square over n1 + S2 square over N2, this is nothing but Sigma X1 bar – X2 bar, okay, so X1 bar – X2 bar is 1190 – 1230, delta = 0 divided by square root S1 square, S1 square is 90 square divided by N1 = 100 + S2 is 120, so 120 square divided by N2 that is 75, okay.

Now, let us find these values here, so 1190-1230 is -40 divided by the square root 90 square divided by 100 + 120 square divided by 75, okay. Now, let us see sigma X1 bar – X2 bar, okay which is the denominator in the value of Z, it is 16.5227 and Z comes out to be -2.421, okay, so this sigma X1 bar – X2 bar, okay, sigma X1 bar – X2 bar, okay that is under root 90 square divided by 100 + 120 square divided by 75.

If you calculate this, it comes out to be 16.5227, okay and when we put the value here, we get -40 divide by 16.5227 and this comes out to be -2.421, so -2.421, okay. Now, let us see we have Z lies between -1.96, critical value are -1.96 and +1.96 okay, so this is critical region in the case of the 2 tailed test, okay, we are getting Z = -2.421, so since -2.421 is < -1.96 okay, therefore we are in the critical region, this critical region, this and this, this critical region, okay.

So, we are in the critical region, so this is; so we have the result that H0 is rejected, H0 is rejected here which means that there is significant difference between the two populations, okay, between the tube lights manufactured by company A and tube lights manufactured by company B. Now, when you take alpha = 0.01, let us see what happens, okay, so alpha = 0.0, let us now take, now alpha be = 0.01.

Then for RTT let us see, what are the critical values of Z, so the critical values of Z are -2.58 and +2.58, so the critical values or -2.58 and +2.58, okay now the value of Z is -2.421, so since -2.421 is > -2.58, okay, H0 is accepted which means that there is no significant difference between the tube lights manufactured by company A and the tube lights manufactured by company B at alpha = 0.01 okay, so we are accepting H0.

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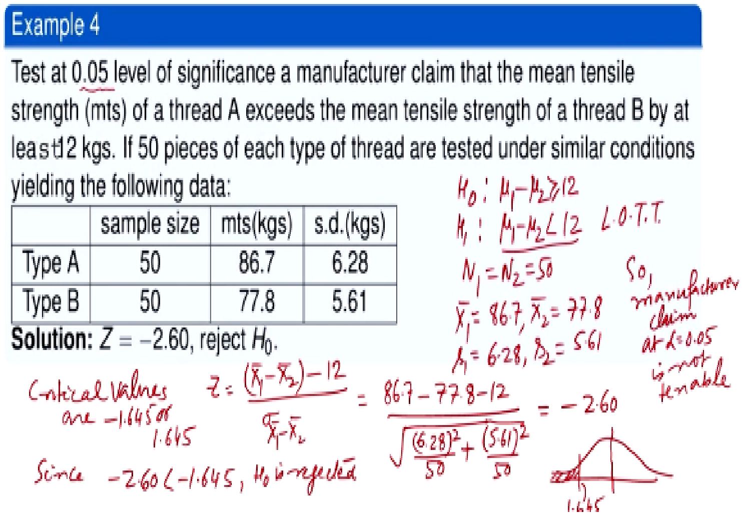

Example 4

Test at 0.05 level of significance a manufacturer claim that the mean tensile strength (mts) of a thread A exceeds the mean tensile strength of a thread B by at least 12 kgs. If 50 pieces of each type of thread are tested under similar conditions yielding the following data:

	sample size	mts(kgs)	s.d.(kgs)
Type A	50	86.7	6.28
Type B	50	77.8	5.61

Solution: $Z = -2.60$, reject H_0.

Now, let us go to last question, test at 0.05 level of significance and manufacturer claim that the mean tensile strength of a thread A exceed the mean tensile strength of thread B by at least 12kgs, okay, so our null hypothesis here is that Mu1 – Mu2 is > = 12, okay, the mean tensile strength of thread A that is Mu 1 and mean tensile strength of thread B that is Mu2, okay, so Mu1 – Mu2 is > = 12.

And then alternative hypothesis H1 is Mu 1 – Mu2 is < 12 okay, now if 50 pieces of each type of thread are tested, so N1 = N2 = 50, okay, are tested under similar conditions yielding the following data. So, for type A, sample size is 50, for type B also sample size is 50, mean strength for type A, X1 bar =86. 7, okay and mean; this X2 bar which is = 77.8 okay and S1 = 6.28, S2 = 5.61, okay.

So, Z = X1 bar – x2 bar – delta; delta is 12, okay divided by Sigma X1 bar – X2 bar, okay, this is = X1 bar – X2 bar, so 86.7 - 77.8 -12 divided by under root S1 square, S1 square means 6.28 whole square divided by 50 + 5.61 square divided by 50, this comes out to be = -2.60 okay, now this is -2.60, we have to test it at 0.05 level of significance okay, now here we are having Mu1 – Mu2 > =12, we are testing Mu1 – Mu2 < 12.

So, this means that we have to apply LOTT, okay, we have to apply LOTT, so left one tailed test we have to apply, so when we have at 0.05 level of significant let us see, we have critical value of Z for one tailed test -1.645 okay and 1.645 so, let us see whether the value of Z that we have got is < -1.645, okay, so critical values are -1.645 or +1.645, okay. Now -2.60 is < -1.645 okay, so this means that we have to to reject H0, okay, this is your critical region, okay, this is -1.645.

So, we are in the critical region okay, so H0 is rejected which means that Mu1 – Mu2 is < 12 that means the manufacturer claim is not tenable, okay, so manufacturers claim is not right, manufacturers claim at 0; alpha = 0.05 is not tenable, okay because we have to reject H0 and we are accepting that Mu1 – Mu2 is < 12, okay so with that I would like to end this lecture, thank you very much for your attention.