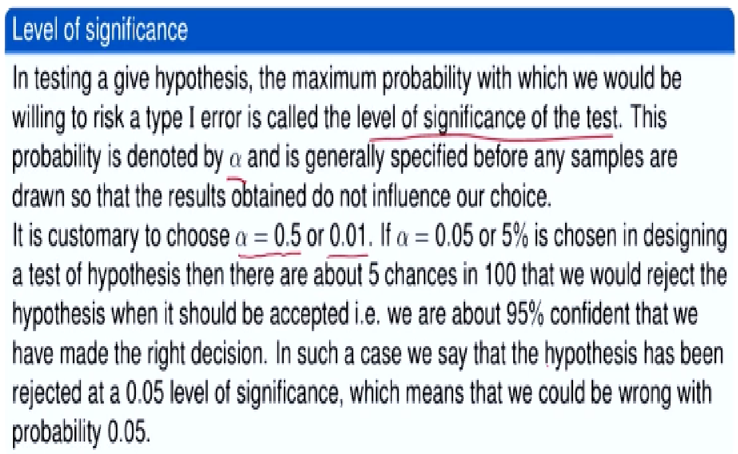
Advanced Engineering Mathematics

Prof. P. N. Agrawal
Department of Mathematics
Indian Institute of Technology - Roorkee

Lecture – 58
Testing of Hypotheses - II

Hello friends, welcome to my lecture on testing of hypotheses, this is second lecture on testing of hypotheses, let us discuss first level of significance.

(Refer Slide Time: 00:43)



In testing, a given hypotheses, the maximum probability with which we would be willing to risk a type 1 error is called the level of significance of the test, this probability is dented by alpha and is generally specified before any samples are drawn, so that the results obtained do not influence our choice. Now, it is customary to choose alpha = 0.5 or 0.01, if alpha = 0.05 or 5% is chosen in designing a test of hypothesis, then there are about 5 chances in 100 that we would reject the hypotheses, when it should be accepted.

That is, we are about 95% confident that we have made the right decision in such a case, we say that the hypothesis has been rejected at a 0.05 level of significance which means that we could be wrong with probability 0.05.

(Refer Slide Time: 01:32)

Simple hypothesis

It is a statistical hypothesis which completely specifies a parameter. Null hypothesis is always a simple hypothesis stated as an equality specifying an exact value of the parameter (includes any value not stated by alternative hypothesis).

Example

(i) $N.H = H_0; \mu = \mu_0$ i.e. the population mean equals to a specified constant μ_0.
(ii) $N.H = H_0; \mu_1 - \mu_2 = \delta$ i.e. the difference between the samples means equals to a constant δ.

Now, let us define simple hypothesis, it is a statistical hypothesis which completely specifies a parameters, null hypothesis is always a simple hypothesis is stated as an equality specifying an exact value of the parameter, it includes any value not stated by alternative hypothesis. Null hypothesis is denoted by H0 as we have seen earlier, so Mu = Mu0 means, the population means equal to a specified constant Mu0.

Null hypothesis = H0, Mu1 – Mu 2 = delta, it means that the difference between the sample means equals to a constant delta.

(Refer Slide Time: 02:16)

Composite hypothesis

It is stated in terms of several possible values i.e. by an inequality.

Alternative hypothesis

It is a composite hypothesis involving statements expressed as inequalities such as $<, >, \neq$.

Example

(i) $A.H = H_1; \mu > \mu_0$,
(ii) $A.H = H_1; \mu < \mu_0$,
(iii) $A.H = H_1; \mu \neq \mu_0$.

Now, let us discuss some composite hypothesis; composite hypothesis is stated in terms of several possible values that is by an inequality, alternative hypothesis is the complete; is a composite hypothesis involving statements expressed as inequalities such as <, >, !=, because <. >, != will involve several values, okay, so alternative hypothesis is a composite hypothesis. Now, alternative hypotheses, AH is denotes alternative hypotheses, alternative hypotheses we denote by H1, okay.

So, Mu > Mu0, it involves infinitely many values, so Mu > Mu0 or Mu < Mu0 or Mu ! = Mu0, they are all alternative hypothesis.

(Refer Slide Time: 03:09)

Critical region

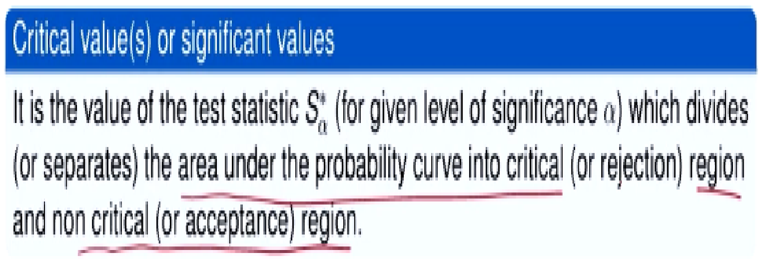
In any test of hypothesis, a test statistic S^*, calculated from the sample data,is used to accept or reject the null hypothesis of the test. Then we consider the area under the probability curve of the sampling distribution of the test statistic S^* which follows some known (given) distribution. This area under the probability curve is divided into two regions, namely the region of rejection (significant region or critical region) where N.H. is rejected, and the region of acceptance (non-significant region or non critical region) where N.H. is accepted. Thus the critical region is the region of rejection of N.H. The area of the critical region equals to the level of significance α. Note that the critical region always lies on the tail(s) of the distribution. Depending on the nature of A.H.,C.R. may lie on one side or both sides of the tail(s).

Now, in any test of hypothesis, a test statistic S star calculated from the sample data is used to accept or reject the null hypothesis of the test then, we consider the area under the probability curve of the the sampling distribution of the statistic S star which follow some given distribution, these area under the probability curve is divided into 2 regions, namely the region of rejection. The region of rejection is called as significant region or critical region, okay, where NH is rejected.

And the region of acceptance that is non-significant region, the region of rejection is called as the critical region and then we have region of acceptance that is called as non-significant region where NH is not accepted. So, thus the critical region is the region of rejection of NH, these, the area of the critical region equals to the level of significance alpha, okay. The area of the critical region equals to the level of significance alpha.

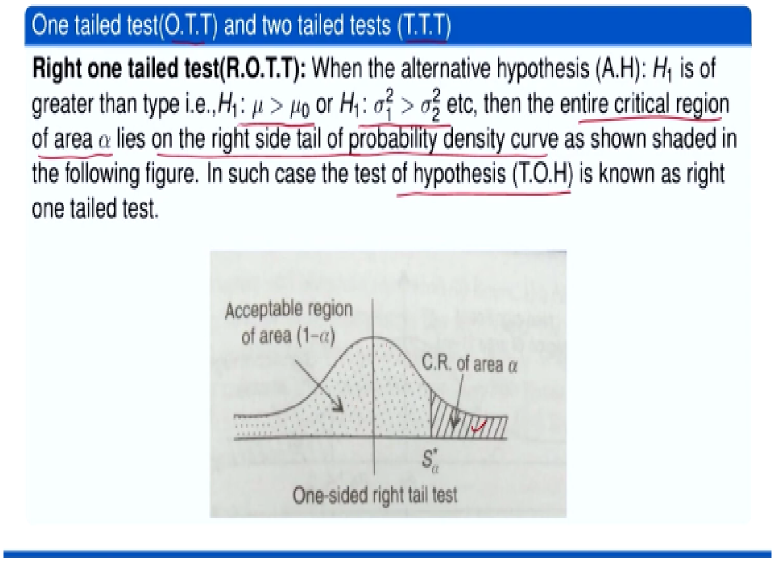
Note that the critical region always lies on the tails of the distribution, depending on the nature of alternative hypothesis; critical region may lie on one side or both sides of the tails.

(Refer Slide Time: 04:31)



Now, let us discuss the critical value or significant values, it is the value of the test statistic S alpha star for given level of significance alpha, when you are given the level of significance alpha from that we can decide the value of S alpha star, this S alpha star divides the area under the probability curve into critical or critical region and non-critical region, critical region is the region of rejection, non-critical region is the region of acceptance.

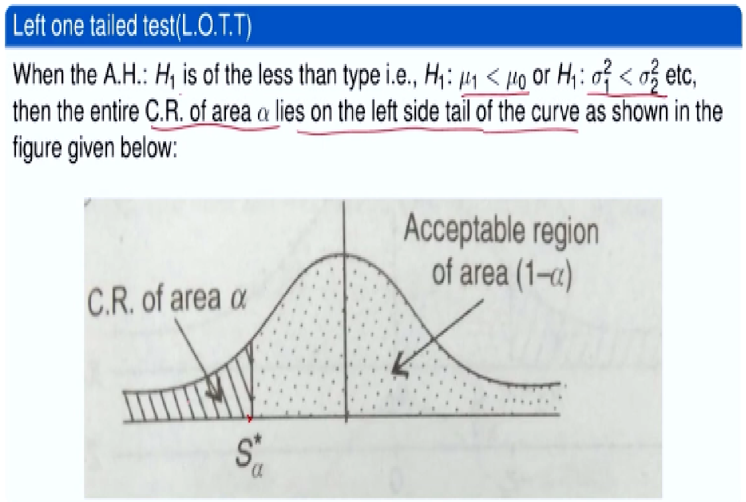
(Refer Slide Time: 05:06)



In the case on one tailed test; one tailed test we denote by OTT, over two tailed test we denote by TTT, so write; let us say there are 2 one tailed test, right one tailed test and left one tailed test. Right one tailed test is the following; when the alternative hypothesis H1 is of > tail, Mu > Mu0 or sigma 1 square > sigma 2 square, then the entire critical region of area alpha lies on the right hand side, right side tail of the probability density curve as shown in the following figure.

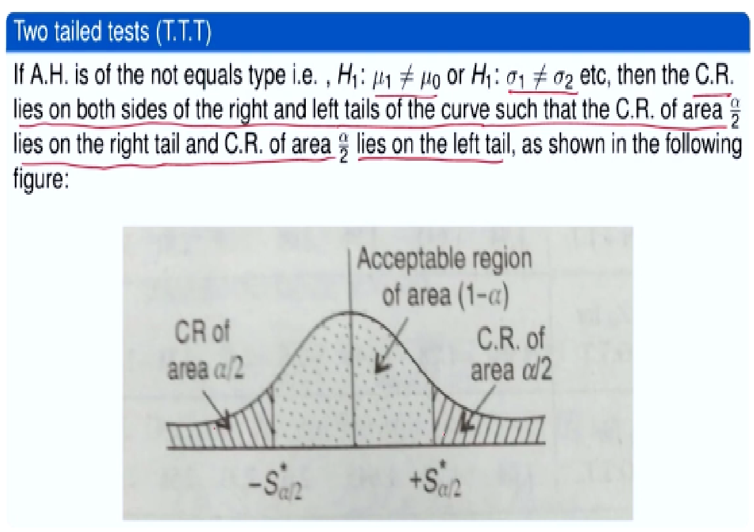
You can see in the following figure this area, okay, this is the area which is = alpha, the level of significance alpha and it lies to the right of the probability density curve. In such case the test of hypothesis TOH is known as right one tailed test, so this area is alpha and the remaining area is 1- alpha, this is normal distribution.

(Refer Slide Time: 06:09)



Now, in the case of left one tailed test, when the alternative hypothesis H1 is of the < type that is Mu1 < Mu0 or sigma 1 square < sigma 2 square then the entire critical region of area alpha lies on the left side tail of the curve as shown in this figure, you see, this is normal; curve of the normal distribution, this area which is = alpha, the level of significance lies to the left of the probability density curve and this is the test statistic S alpha star, which is decided from the level of significance alpha.

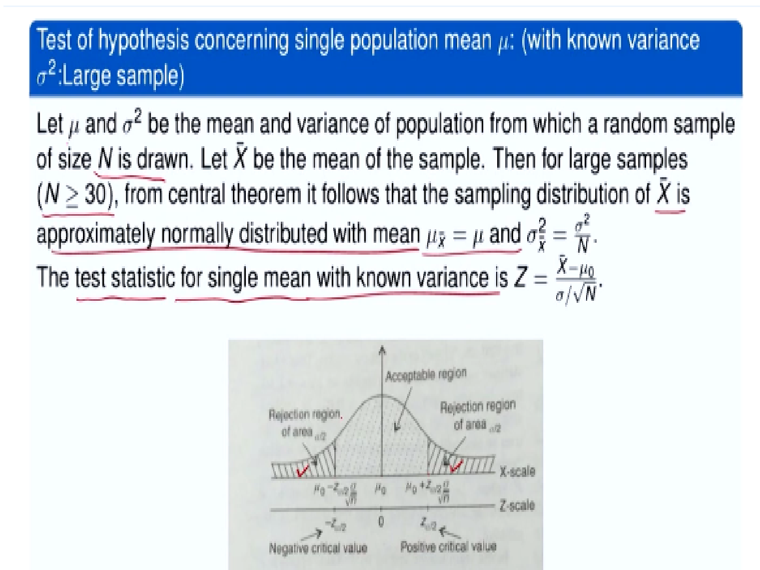
(Refer Slide Time: 06:49)



Now, 2 tailed test, if AH is not of the; not equals type that is Mu1 is != Mu 0 or sigma 1 is != sigma 2, when Mu1 is != Mu 0, we will have to consider both the cases, Mu1 < Mu0, Mu1> Mu0, so there are; there we have 2 tailed test, okay, so then the CR region; CR critical region lies on both sides of the right and left tails of the curve such as the CR of area alpha/2 lies on the right tail and CR of area alpha/2 lies on the left tail.

Because this is the curve symmetric with respect to y axis, so half of the area of the curve lies on one side, half lies on the other side, so the total area under the curve for the level of significance is alpha, so alpha/2 lies to the left of the curve, to the left side and alpha/2 lies to the right of the; right side, so this is your alpha/2 area and this area is alpha/2 and this is – S star alpha/2, + S star alpha/2.

(Refer Slide Time: 08:00)



Now, let us discuss test of hypothesis concerning single population mean, let us consider single population, okay and we discuss test of hypothesis concerning the single population with mean Mu and known variance sigma a square. So, let Mu and sigma a square be the mean and variance of the population from which a random sample of size N is drawn, now let X bar be the mean of the sample then for large samples that is N > = 30, from central limit theorem, it follow that the sampling distribution of X bar is approximately normally distributed with mean Mu X bar = Mu.

And sigma X bar = sigma a square/N, okay the test statistic for single mean with known variance is Z = X bar – Mu0 divided by sigma y root N, okay, this is the curve, so here it is 2 tailed test, so that means alpha/2 area is here, this is the value of Z for this alpha/2 area, minus is denoted by –

Z alpha/2 and this is the value of Z here + Z alpha/2 for this alpha/2 which lies on the right side, okay, this Mu0, okay.

This is negative critical value, -Z alpha/2, this is positive critical value Z alpha/2 on the Z scale okay, this is acceptable region, the value; I mean this region is the region of acceptance and this is the region where your H0 is rejected, okay the null hypotheses rejected, here in this portion, null hypotheses is accepted so region where the null hypotheses is rejected is called the critical region.

So, this region and this region, their critical region, okay, they are called significant regions and this is called non-significant region, where the null hypotheses H0 is accepted.

(Refer Slide Time: 10:05)

Compute the test statistic Z, by

$$Z = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{N}}$$

Here $\bar{X}$, the mean of the sample of size N, is calculated from the sample data.

Conclusion: Reject H_0 if Z_{cal} or Z falls in the critical region i.e. observed sample statistic is probably significant or highly significant at α level. Otherwise accept H_0 if $(-Z_{\frac{\alpha}{2}} < Z < Z_{\frac{\alpha}{2}})$.

And let us compute the; we then compute the test statistic Z by Z = X bar – Mu0 over sigma/ root N, here X bar is the mean of the sample size; sample of size N, it is calculated from the sample data. Now, how we will conclude; we will reject the null hypotheses H0, if Z fall in the critical region, okay, if Z falls in the critical region that is observed sample statistic is probably significant, okay.

If Z falls in the critical region, then the null hypotheses will be rejected, okay, so that region is called as significant region, so the observed sample statistic is probably significant or highly significant at alpha level of acceptance otherwise, we accept H0 and we will accept H0, if –Z alpha/2 is < Z < Z alpha/2 in the case of 2 tailed test.

(Refer Slide Time: 11:05)

The following table gives critical values of z for both one tailed and two tailed tests at various levels of significance:

Level of significance α	0.10	0.05	0.01	0.005	0.002
Critical values of z for one tailed tests	-1.28 or 1.28	-1.645 or 1.645	-2.33 or 2.33	-2.58 or 2.58	-2.88 or 2.88
Critical values of z for two tailed tests	-1.645 and 1.645	-1.96 and 1.96	-2.58 and 2.58	-2.81 and 2.81	-3.08 and 3.08

The following table gives critical values of z for both one tailed and 2 tailed test at various levels of significance. Now, these table, this is the critical value of z for one tailed test, it is 0.10, if alpha is 0.10, it is -1.28 or + 1.28, if it is alpha is 0.05, it is -1.645 or +1.645 likewise, we have for alpha = 0.01, alpha = 0.005, alpha = 0.002 and when we have 2 tailed test we have to apply, then when alpha = 0.10, critical values are -1.645 +1.645 and for 0.05, they are – 1.96 +1.96 and so on for alpha = 0.002, they are -3.08 and + 3.08.

(Refer Slide Time: 12:03)

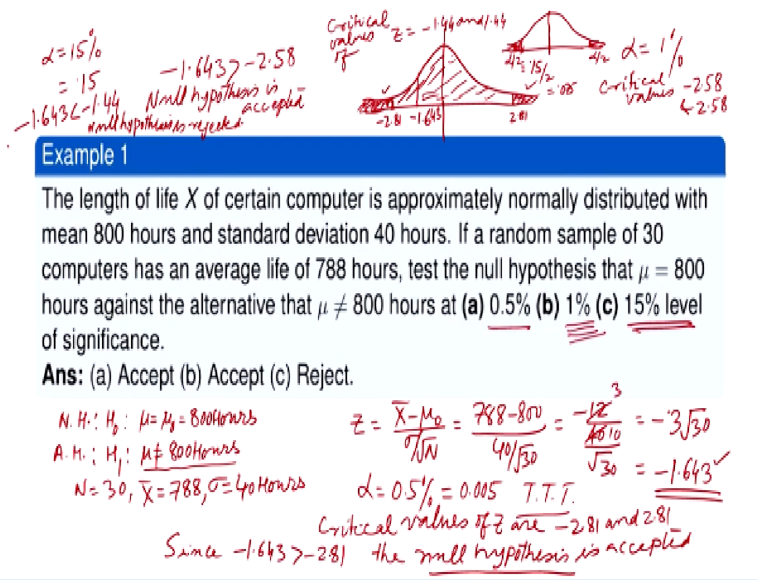
Thus the test of hypothesis or test of significance or rule of decision consists of the following six steps.

1. Formulate N.H.: H_0
2. Formulate N.H.: H_1
3. Choose L.O.S.: α.
4. C.R.: is determined by critical value S^*_α and the kind of A.H. (based on which the test is R.O.T.T. or L.O.T.T. or T.T.T.).
5. Compute the test statistic S^* using the sample data.
6. Decision: Accept or reject N.H. depending on the relation between S^* and S^*_α.

Now, let us see how will make the test of hypotheses, the test of hypothesis or test of significance or rule of decision consists of this 6 steps, so we first formulate the null hypotheses H0, then reformulate the alternative hypothesis H1 okay, then we choose the level of significance alpha, then the critical regions, CR is determined by the critical review S alpha star, okay and the kind of AH based on which the test is or write one tailed test or left one tailed test or 2 tailed test, okay.

So, this has to be decided, they compute the test statistic S star using the sample data then accept or reject NH, null hypothesis NH depending on the relation between S star and S alpha star.

(Refer Slide Time: 12:58)



Now, let us consider the length of life X of a certain computer is approximately normally distributed with mean 800 hours and standard deviation 40 hours. If a random sample of 30 computers has an average life of 788 hours, test the null hypotheses that Mu = 800 hours. So, null hypotheses NH that is H0 is Mu = Mu0 = 800 hours, again the alternative hypotheses that Mu0 is != 800 hours.

So, alternative hypothesis is h1 is Mu0 = 800 hours, okay, now we are given N = sample of size is 30, so N = 30, we are given okay, X bar random sample of size 30 computers has an average of 788 hours, so X bar = 788, so we know X bar, we know Mu0, we know sigma, sigma = 40 hours, so let us calculate the value of Z; Z = X bar – Mu; X bar - Mu0 divided by sigma/ root N, so this is 788 - 800 divided by sigma is 40 divided by root 30.

So, this is = -12 divided by 40 root 30, okay so this is -0.3, this is divided by root 30, okay, -0.3 * root 30, okay, so this comes out to -1.643 okay, now what we have; we have alpha = 0.5%, alpha is given to be 0.5%, so this is = 0.0005, okay now and this is Mu is != 800 hours, so it is a 2 tailed test we have to apply, okay, 2 tailed test we have to apply, so let us see for 2 tailed test, when alpha = 0.005, when alpha =0.005, critical values for Z are -2.81, okay.

So, -2.81 we have okay, so critical values of Z are -2.81 and +2.81, okay, now this is approximately normally distributed so we can see here, okay this is -2.81 and is +2.81, this is the critical region, here we are getting the value -1.643, okay, 1.643 means, this value of Z lies in the region of acceptance, okay, since -1.643 is > -2.81 okay, the null hypothesis is accepted or this is the region of rejection that is critical reason, this one, this alpha/2, this alpha/2, this critical region and this is the region of acceptance, non-significant region.

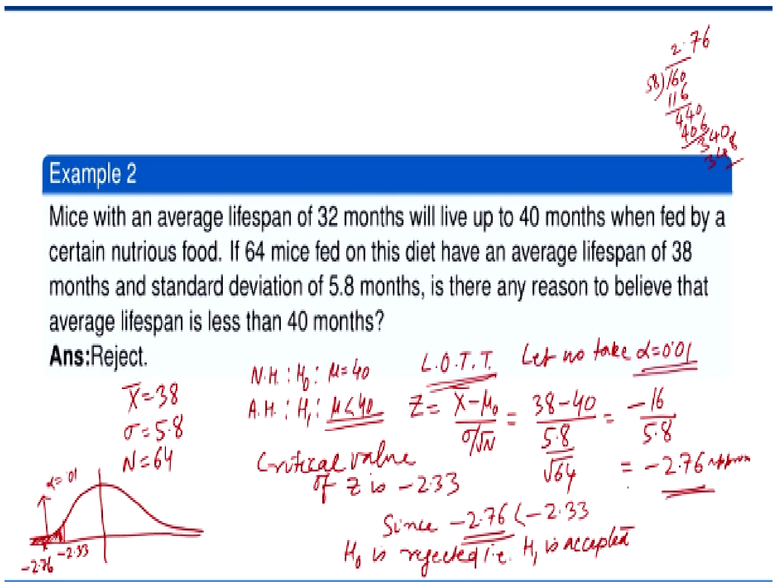
So, Z = -1.643, okay, this is > the value of -2.81 so we have this result that null hypothesis is accepted at 0.5% level of significance. Now, when alpha = 1%, okay, alpha = 1% then what happens; let us see, so alpha = 1% means, alpha = 0.01, then the critical values of Z are -2.58 and +2.58, so here the critical values are -2.58 and + 2.58 and now, -2.; alpha and now this Z, okay, this computed value of Z is -1.643; -1.643 is also > -2.58, okay.

So, again the null hypotheses is accepted, now let us consider the third case when alpha = 15%, so alpha = 15% means, alpha = 0.15 okay so, in this case what will happen; 0.15 area, okay will lie under the probability density curve, so half of this 0.15 area will lie to the left, half will lie to the right, okay, so this will be 0.075 okay and here also it will be 0.075 okay, so alpha/2 area lies on this side, alpha/2 lies on this side that is 0.15/2 means 0.075 lies on each side.

For this 0.075 the value of Z is; critical value of Z is -1.44, okay, so critical values of Z are -1.44 and 1.44, okay, so in these c part, okay, what is happening is the critical values of Z are -1.44 and +1.44 and be notice that the -1.643 is < -1.44 okay, so -1.643 is < -1.444, -1.643 is < -1.44 and therefore, the value of Z -1.643 lies in the critical region, so the null hypothesis is rejected that is we accept the alternative hypothesis.

So, here null hypotheses is rejected, so null hypothesis is rejected in the c case, okay, so in the case a, where alpha is 0.005, null hypotheses is accepted, in the case alpha = 0.01 null hypotheses is again accepted, when alpha = 0.15 since the critical value of Z is more than the value of the Z which is calculated from the data that is -1.643, okay, the null hypotheses is rejected, okay. So, in the a and b part, we are accepting the null hypotheses, in the c part, we are rejecting the hypotheses.

(Refer Slide Time: 22:04)



Now, let us consider example 2, mice with an average life span of 32 months will live up to 40 months when fed by a certain nutritious food, if 64 mice fed on this diet have an average life span of 38 months, so X bar here is 38, sigma is 5.8, N = 64 now, we have to test is there any region to believe average life span of mice is < 40 months, so Mu, null hypothesis is that Mu = 40, okay while alternative hypothesis is that.

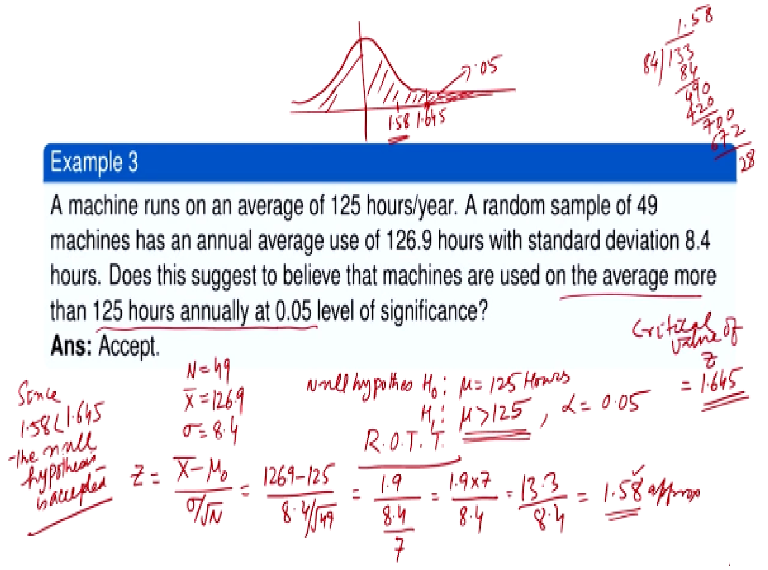
Mu is < 40, so since Mu is < Mu0, it is left tailed test, left one tailed test, so LOTT; so LOTT we have to apply and it is customary to take alpha = 0.05 or 0.01, so let us take alpha = 0.01, so level of significance we are taken to be 0.01, so let us take alpha = 0.01, okay, now let us calculate the value of Z from X bar – Mu0 divided by sigma over root N, so X bar = 38, 38 – Mu0 = 40 divided by sigma = 5.8 and then divided by root 64.

So, we have -2 here multiplied by root 64 is 8, so -16 divide by 5.8, okay and this comes out to be, okay, so roughly it is -2.76, okay, approximately, so now for alpha = 0.01 in the case of LOTT, let us find the value of Z, okay so this can be seen from this table, 0.01, okay for one tailed test, it is -2.33, okay, left one tailed test, -2.33, let us see the critical value of Z is -2.33 and this is -2.76, so since -2.76 is < -2.33, okay, so this is our critical region, not here on the left side only.

So, this is the critical region lies to the left side only, so this is a critical region is alpha = 0.01 and the value of Z is -2.33 critical value and Z = calculated value is - 2.76 so this is <; -2.76 is< -2.33, so we are in the critical region, okay, so it follows that H0 is rejected, meaning that H1 is accepted, alternative hypothesis is accepted and alternative hypothesis is that Mu is < 40 that means there is reason to believe that the average lifespan is < 40 months, okay.

So, that is the conclusion that is the reason to believe that the average lifespan of mice when fed by certain nutritious food is < 40 months.

(Refer Slide Time: 26:28)



Now, let us discuss a machine runs on an average of 125 hours for year, a random sample of 49 machine, so N = 49 has an annual average use of 126.9 hours, so X bar = 126.9 with standard deviation sigma = 8.4. Does this suggest to believe that machines are used on the average more than 125 hours, so MU null hypothesis H0 none is Mu = 125 hours, Mu0 is 125 hours, H1 alternative hypothesis is that Mu is more than 125 hours, okay, they are used on the average more than 125 hours, so Mu is > 125.

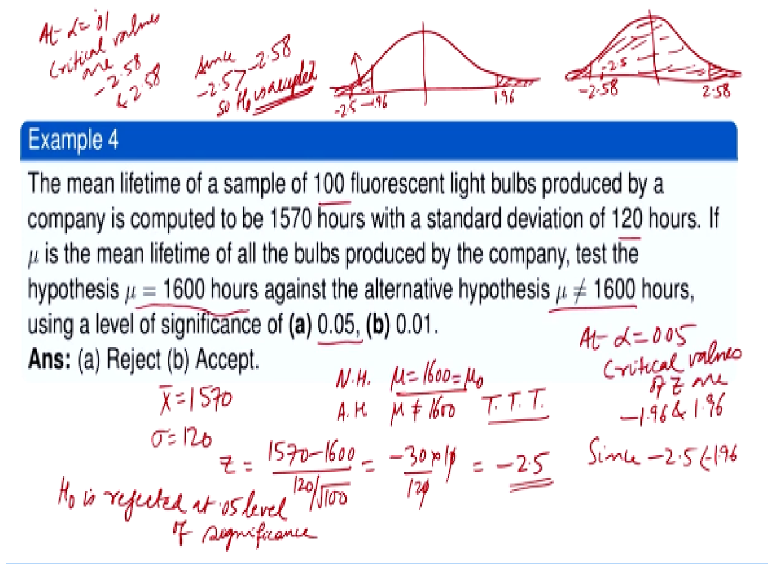
So, since we have Mu > 125, this is right one tailed test, we have to apply okay, right one tailed test we have to apply, now let us calculate alpha is given to be here 0.05, at 0.05 level of significance we have to see, we have to test our hypothesis now, let us day find the value of Z, Z = X bar – Mu0 divided by sigma over root N. So, X bar = 126.9 – Mu0 is 125 divided by sigma = 8.4 divided by root N, N = 49, so we have this.

So, this is -1., no +1.9 divided by 8.4 over 7, okay, so this is how much; 7; 1.9 * 7 divide by 8.4 so this is 13.3 divided by 8.47, so 133 divided by 84, so roughly 1.58, okay, this 1.58, now let us see at 0.05 level of significance, what is the value of Z, okay from the table, critical value of Z, so we have alpha = 0.05, okay and critical value of Z is 1.645, okay, 1.645, so here critical value of Z 1.645, now 1.645 is > 1.58, okay.

So, since 1.58 is < 1.645, okay the so this value of Z lies in the region of acceptance, okay, so the null hypothesis is accepted, okay because you can see this is your curve, this we are applying right tailed test, okay, this area is 0.05, and we have found the value of critical value of Z as 1.645, so 1.645, critical value of Z is here, 1.645 okay, oh sorry, critical value of Z we have found to be 1.645, so okay this is 1.645, okay, this value is 1.645, okay, 1.58 is here.

So, it lies in the region of acceptance, so null hypothesis is accepted, this mean that alternative hypothesis is rejected, okay, so this Mu is > 125, okay, so alternative hypothesis is rejected, okay that is to say that machines does this suggest to believe that machines are used on the average more than 125 hours annually, okay this is not accepted, okay, so machines are not used on an average more than 125 hours annually at 0.5 five level of significance.

(Refer Slide Time: 31:43)



Now, the mean lifetime of a sample of 100 fluorescent light bulbs produced by a company is computed to be 1570 hours, so X bar = 1570 hours with a standard deviation sigma = 120 hours, we have Mu =; Mu is the mean lifetime of all the bulbs produced by the company, test the hypothesis Mu = 16 hours, so we have null hypotheses Mu = 1600 hours, so this is my Mu0, okay and then again the alternative hypothesis that Mu is != 1600.

So, we are considering 2 tailed test that is TTT because we are taking Mu0 = 1600 hours and now, let us find the value of Z, so Z = 1570 -1600 divided by sigma; sigma is given to be 120; 120 divided by root 100 because they are 100 bulbs, okay, so how much is this; this is -30 divide by 120 and then we have 10 here, so this is 2.5 right, -2.5, okay, so sigma comes; Z comes out to be -2.5.

Now, let us see for the 2 tailed test, what are the critical values of Z, on the level of significance, 0.05 and 0.01, okay at 0.05 yeah, 0.05, for the 2 tailed test, the critical values are -1.96 and +1.96, so critical values at alpha = 0.05 critical values are -1.96 and +1.96 okay, now we can see since -2.5 is < -1.96 okay, this is critical value -1.96 and this is 1.96 okay, this area alpha/2, this area alpha/2, okay.

Now, -2.5 lies here somewhere, so -2.5 is < -1.96 and therefore, this is region of rejection of H0, okay, so H0 is rejected okay, null hypothesis that is Mu is != 1600 hours is rejected, this is called critical region, okay. So, since this value computed value - 2.5 lies here okay, it follows that H0 is rejected at 0.05 level of significance, okay. Now, if take 0.01, what happens, 0.01; the values of Z are -2.58 and 2.58.

So, at alpha = 0.01, okay, critical values are -2.58 and +2.58, okay, now -2.58 is < -2.5 so since -2.5 is > -2.58 okay, we can draw the figure this is -2.58, this is +2.58, alpha/2 area is here and -2.5 is here, so since -2.5 is more than -2.5 okay, 58 it follow that we are in the region of acceptance of the null hypothesis, so H0 is accepted and so the alternative hypotheses is rejected, okay.

So, the mean Mu = 1600 hours is the mean life of the; Mu = 1600 hours is accepted at 0.01 level of significance while in the case of 0.05 level of significance, Mu = 1600 hours is rejected, so this is what I had to say in this lecture, thank you very much for your attention.