Advanced Engineering Mathematics
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Lecture – 55
Correlation and Regression – I

Hello friends welcome to my lecture on correlation and regression, this is my first lecture on correlation and regression. Very often our relationship is found to exist between 2 or more variables, for example blood pressure of a person and his age, rainfall and crop yield, conjunction of rood and weight gain.

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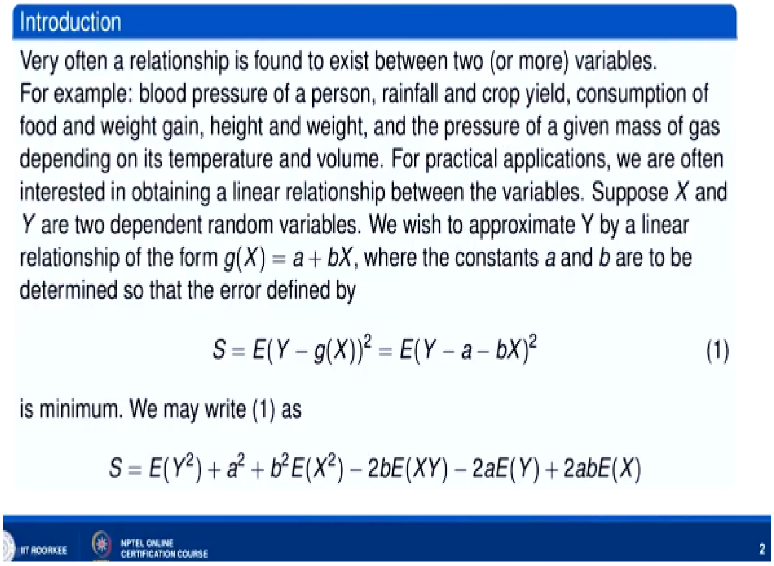

Height and weight of a person, the pressure of a given mass of gas depending on it is temperature and volume. For practical applications we often are interested in obtaining a linear relationship between the variables. Suppose x and y are 2 dependent variables, we wish to approximate Y by a linear relationship of the form gX = a + bX, where the constants a and b are to be determined.

So that the error define by S = expectation of Y – gx whole square which is = expectation of Y – a –bx whole square is minimum, that is the error is minimum in the least square sense. So we may write this equation 1 as S = E Y square + a square + b square * EX square – 2bEXY – 2aEY + 2 abEX.

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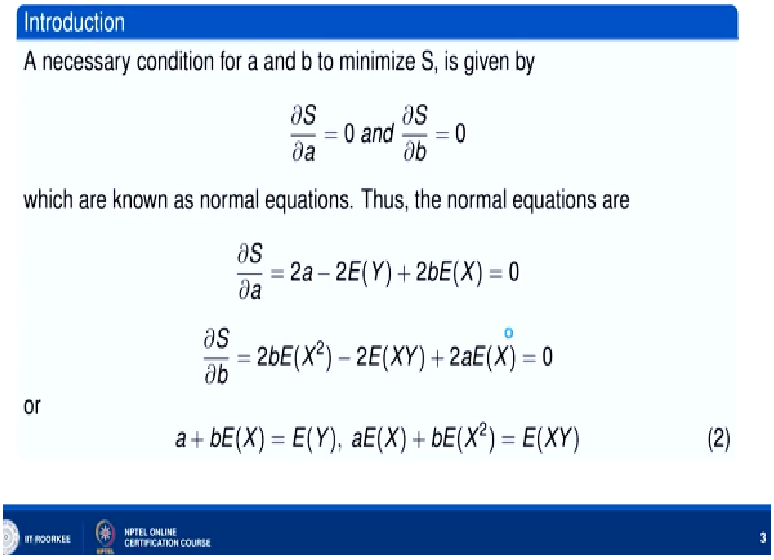

Now a necessary condition for a function of 2 variables say here they are a and b okay, you can see we want to minimize S, S is the function of 2 parameters a and b, so in order for S to be a minimum we must have the partial derivative of S with respect to a = 0 and partial derivate of S with respect to v = 0. These 2 equations are known as normal equations. Thus the normal equations rae if you differentiate S with respect to a partially okay.

Then you get partial derivate of S with respect to a = 2a – 2 EY + 2b EX okay. So we get partial derivate of S with respect to aS, 2a – 2EY + 2VEX and we put it = 0. Similarly, when we differentiate S partially with respect to V we get 2VEX square – 2EXY + 2a EX okay. So we get 2VEX square – 2E XY + 2 EX = 0. Now this equation okay, this equation gives us a – ey + bEX = 0.

Or I can say EY = this equation gives you EY = a + b times EX okay, and from this equation what we get, we get aEX + bEX square = EXY okay. So we have 2 equations, this one and this one okay, which are given by number 2 okay. So there are 2 equations, connecting the 2 unknown values a and b, they are linear equations in a and b, so we can solve them for the values of a and b and when we solve them we get b = EXY – EX * EY upon EX square – EX whole square.

Now EXY- EX EY gives us the covariance of the random variables X and Y, so covariance of XY/EX square – EX whole square is variance of X. So b = covariance of XY/variance of X and when we put the value of V, in one of the 2 equations, say for example EY = a + b EX, we get the value of a okay.

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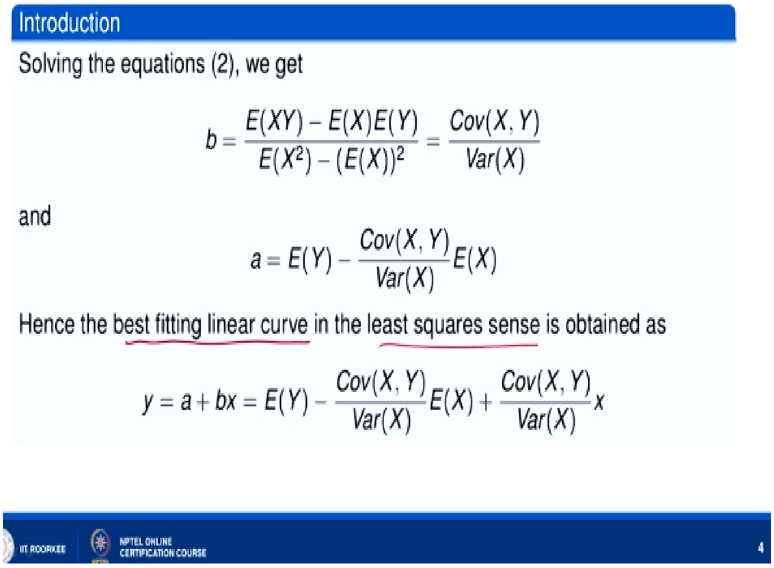

The value of a comes out to be EY – covariance of XY/variance of X * EX hence the best fitting linear curve in the least square sense is given by y= a+ bx where a is EY – covariance of XY/variance of X * EX + b, b is covariance of XY/variance of X * X.

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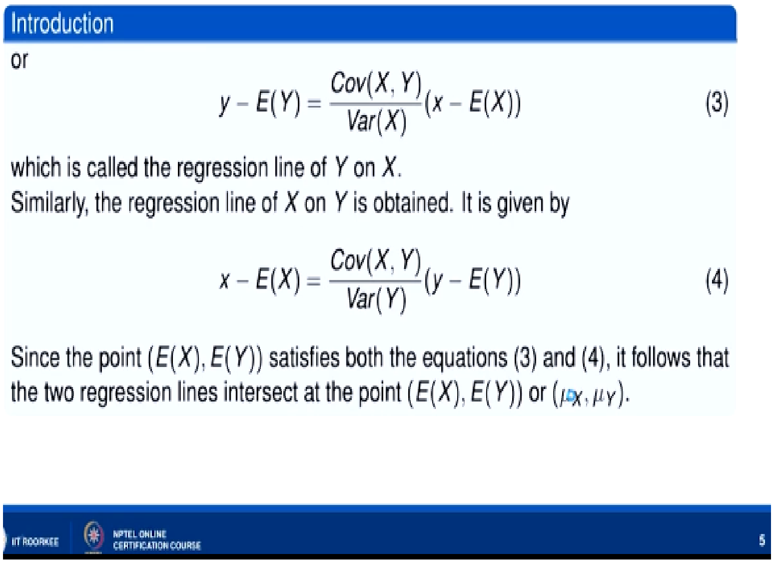

We can rewrite this equation as Y – EY = covariance of XY/variance of X * X-EX okay. So we have this equation, variance of Y - EY = covariance XY/variance of X * X-EX, this equation is known as the regression line of Y on X okay, for a given value of X you can get the approximate value of Y, using this equation, so it is called as the regression line of Y on X. Similarly, the regression line of X on Y.

If you are given the value of Y and you want to get an estimate of the value of X, then we need the regression line of X on Y. So in a similar manner we can find the regression line of X on Y, it is given by X – EX = covariance of XY over variance of Y * Y-EY. Now since the point EX EY okay satisfies both the equation 3 and 4 okay. Now you can see if you put here, the point EX EY in this equation then you see EY – EY = 0, EX – EX = 0, so 0 = 0.

So EX EY satisfies this equation, similarly here EX EY satisfies this equation and therefore it follows that the 2 regression lines intersect at the point EX EY, EX EY be also denoted by mu x, mu y. So they meet at the point mu x mu y.

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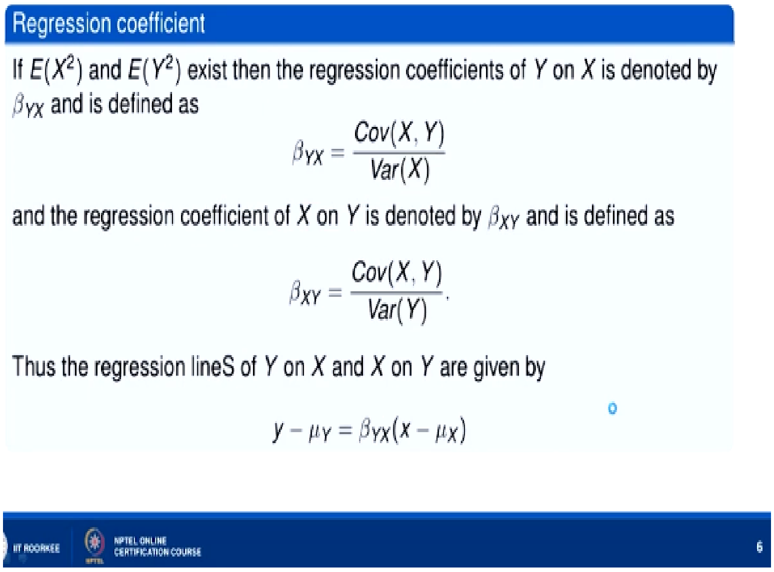

Okay now if EX square and EY square exist okay, then the regression coefficient of Y on X is denoted by beta yx and is defiend as beta yx = covariance of XY/variance of X and the regression coefficient of X on Y is defined as beta xy, denoted by beta xy and is defined as beta xy = covariance of XY/Variance of Y. Now using these regression coefficient, the regression line of Y on X okay.

The regression line of Y on X which is Y – mu y = covariance of xy/variance of X * X- EX, I can write it as Y – mu y = covariance of xy/variance of x * x-mu x. I can write it as beta covariance of X/covariance of X is the regression coefficient of Y on X. So beta yx x- mu x and this equation which is the regression line of X on Y can be written as X – mu x = beta xy * y- mu y okay.

So using the notations for regression coefficient of Y on X and X on Y we can write the regression lines of Y on X and X on Y in this manner okay. So this is your regression line of Y on X okay and the other one is the regression line of X on Y this one.

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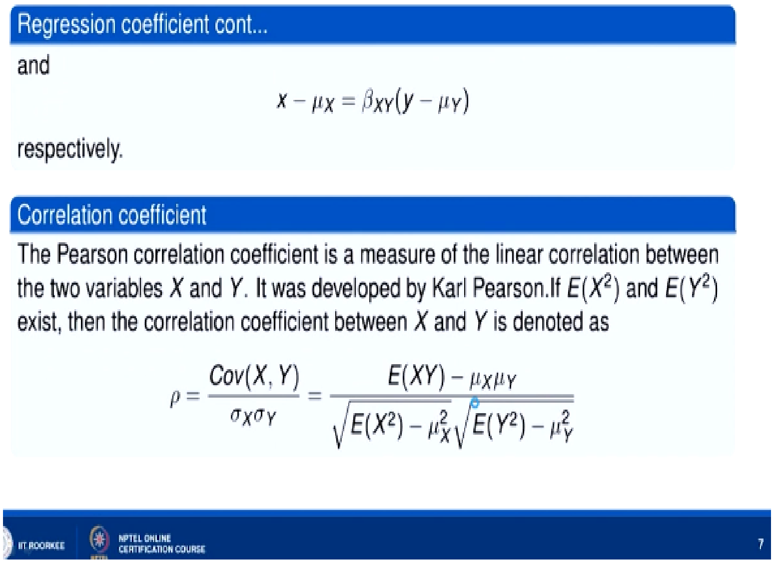

The regression line of X on Y, now correlation coefficient, the Pearson correlation coefficient is a measure of the linear correlation between the 2 variables X and Y. It was developed by Karl Pearson. If EX square and EY square exist then the correlation coefficient between X and Y is denoted as rho or we also write it as rho XY and it is = covariance of XY/sigma x, sigma y.

Now covariance of XY by definition is EXY – mu x, mu y and sigma X square root variance of X that is EX square – mu x square and sigma/square root of variance of Y which is square root of EY square – mu y square.

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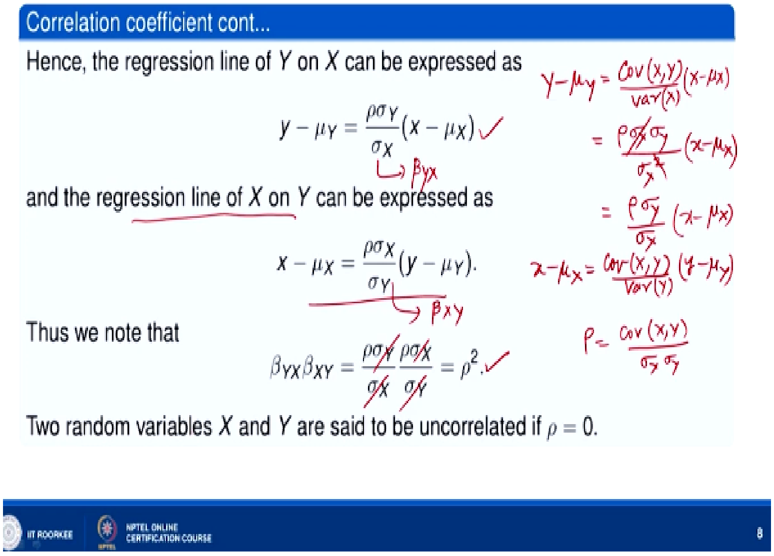

Now the regression line of Y on X okay, using the definition of rho, we can write the regression line of Y on X in this form, you see we had y- mu y = covariance of xy/variance of x * x- mu x okay and rho = rho is covariance of xy, be rho = covariance of xy/sigma x sigma y. So covariance of xy is rho times sigma x sigma y. So I can write it as rho times sigma x, sigma y/variance of x is sigma x square.

So we have sigma x square and then x – mu x okay, this cancels with this and we get rho times sigma y/sigma x, x – mu x. So y – mu y = rho sigma y/sigma x * x – mu x. In a similar manner we can express the regression line of X on Y in terms of rho okay. We have x – mu x = covariance of xy/variance of y * y – mu y okay. So when you put for covariance of xy you put rho times sigma x sigma y and then divide by rho y square what you get is rho * sigma x/sigma y.

So we get x – mu x = rho * sigma x/sigma y * y – mu y okay. Thus we note that this is beta/x okay. This is beta/x and by our definition, this is beta xy. So when you multiply beta yx and beta xy what we get rho sigma y/sigma x okay * rho sigma x/sigma y and this is = this cancels with this, this cancels with this, you get rho square. So beta/x * beta xy = rho square. the 2 random variables are x and y are called uncorrelated if the coefficient of correlation coefficient rho = 0.

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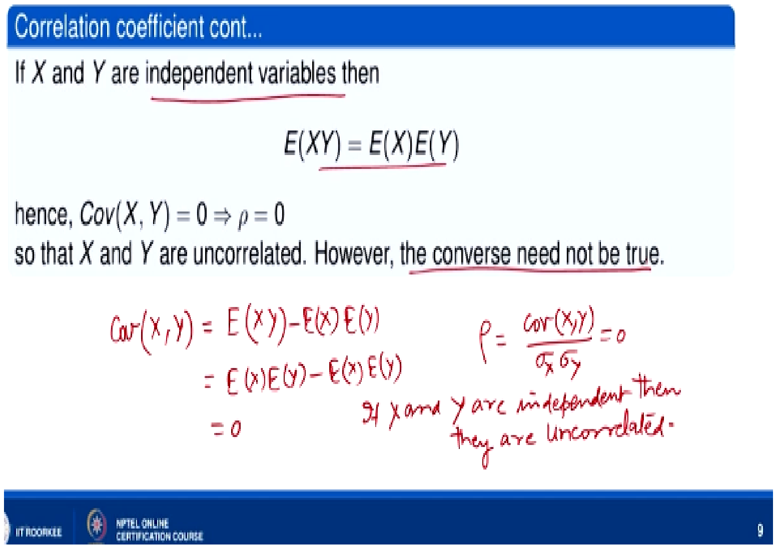

If X and Y are independent variables okay, if X and Y are independent variables then we know that expected value of X * Y is = expected value of X * expected value of Y okay, hence covariance of X Y, covariance of X Y is expected value of XY – expected value of X * expected value of Y okay. This we know, so when X and Y are independent random variables then EXY = EX * EY gives us EX * EY – EX * EY gives us covariance of xy = 0.

That is now covariance of xy = 0 means rho, rho is given by covariance of xy/sigma x, sigma y okay. So when covariance of xy is 0, rho = 0, so if x and y are 2 independent random variables then x and y are uncorrelated okay, so x and y, if x and y are independent then they are uncorrelated, but we shall see that the converse is not true okay. The converse is not true okay. So let us show it by means of an example.

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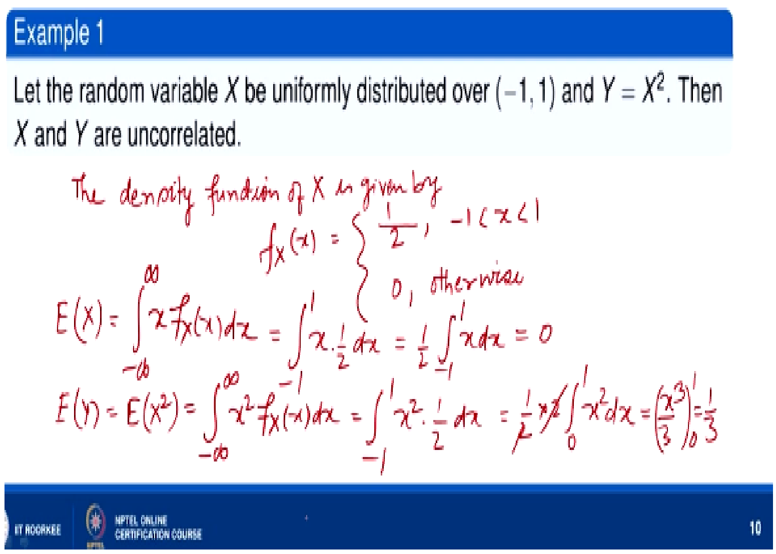

Let us show that rho = 0 okay but x and y are dependent random variables okay. So let us take this example, let us say the random variable x is uniformly distributed over the interval – 1 to 1 okay and Y = X square, then X and Y are uncorrelated. Now we know that if X is uniformly distributed over -1 to 1 then the density function of X is given by the density function of fxx = 1/b-a okay.

If it is uniformly distributed over the interval ab then fxx is 1/b-a, so this is 1 + 1, that is 1/2 when x lies in the interval – 1 1 and 0 otherwise. Okay, now we need to find the value of rho and show that rho = 0 okay. So we have found fxx, now we need to find expected value of x. So expected value of x is integral over x * expected value of x is integral over – infinity to infinity x * fxx dx. Now it is half over the interval – 1 to 1.

So this is integral over -1 to 1 x * 1/2 dx okay. So this is 1/2 * x is in odd function of x. So integral over -1 to 1 x dx will be = 0 is expected value of x = 0. Now expected value of y = expected value of x square okay. So expected value of x square means integral over – infinity to infinity x square fxx dx, which will be = integral over -1 to 1 x square * 1/2 dx which is = 1/2 * x square is an even function of x.

So 2 times 0 to 1 x square dx, so what we get is x cube/3, integral of x square is x cube/3 over the interval 0 to 1 and this gives me value 1/3. So we have got the value expectation of x, expectation of y. Now let us find the value of, because we want the value of rho. So we need to find expected value of xy.

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$$E(XY) = E(X^3) = \int_{-\infty}^{\infty} x^3 f_X(x)\,dx = \int_{-1}^{1} x^3 \cdot \frac{1}{2}\,dx = \frac{1}{2} \times 0 = 0$$

Thus,

$$Cov(X,Y) = E(XY) - E(X)E(Y) = 0 - 0 = 0$$

Hence $\rho = \frac{Cov(X,Y)}{\sigma_X \sigma_Y} = 0$

So let me find expected value of xy, this is expected value of xy is y = x square. So we get expected value of x cube. So integral over – infinity to infinity x cube fxx dx. This is = integral over -1 to 1 x cube * 1/2 dx and we get 1/2, x cube is an odd function of x so the value of the integral is 0 and we get E xy = 0, thus covariance of xy = E xy – Ex * Ey = Exy is 0, okay, we have found Ex = 0 okay.

So this is 0 – 0 okay, so 0 and hence rho = covariance of x y/sigma x sigma y okay = 0. So coefficient of correlation is = 0 but we are given that y = x square. So coefficient of correlation is 0 but he random variables x and y are dependent, so this si a problem which shows that the converse is not true.

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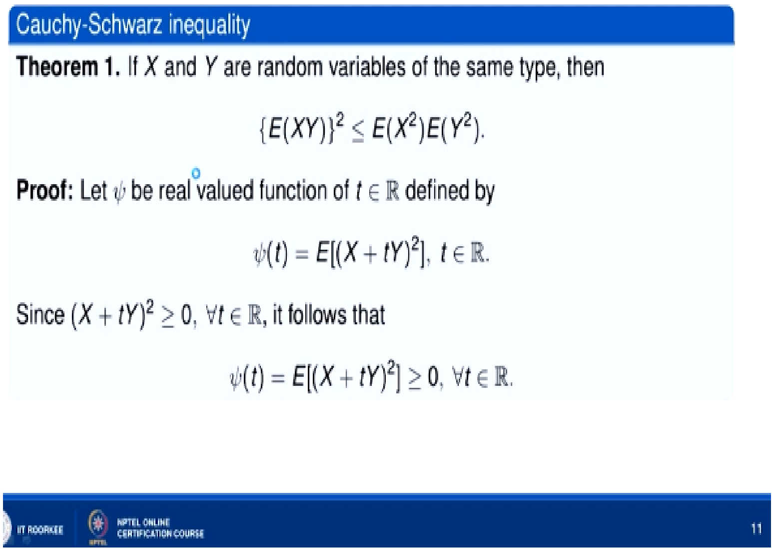

Now let us prove the Cauchy-Schwarz inequality, we shall need this Cauchy-Schwarz inequality to show that the value of the coefficient of correlation that is rho lies between -1 and +1. So if x and y are random variables of the same type that means either both of them are discrete random variables or they are both continuous random variables. So then expected value of x/whole square is <= expected value of x square * expected value of y square.

Let us take psi to be a real valued function okay, of the real number t okay, so let psi be a real valued function of a real variable t defined by psi t = expectation of x + ty whole square, where t belongs to R. Now x + ty whole square is >= 0 for every value of t belonging to R, therefore it follows that psi t is also a nonnegative function of t okay, psi t = expectation of x + t/whole square is also >= 0 for every value of t belonging to R.

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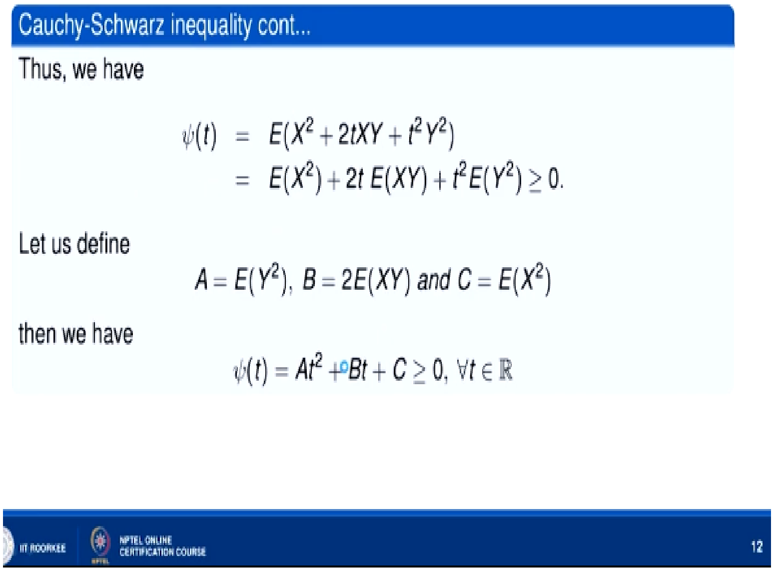

And therefore we can write psi t as, we can write psi t = expectation of x square + 2 t XY + t square Y square, okay and this is = expectation of X square 2t is scalar so + 2t times expectation of XY + t square times expectation of Y2 which is >= 0. Now let us denote expectation of Y square/a, expectation of XY * 2/B and expectation of X square/C. Then psi t = At square + Bt + C which is >= 0 for every value of t belonging to R.

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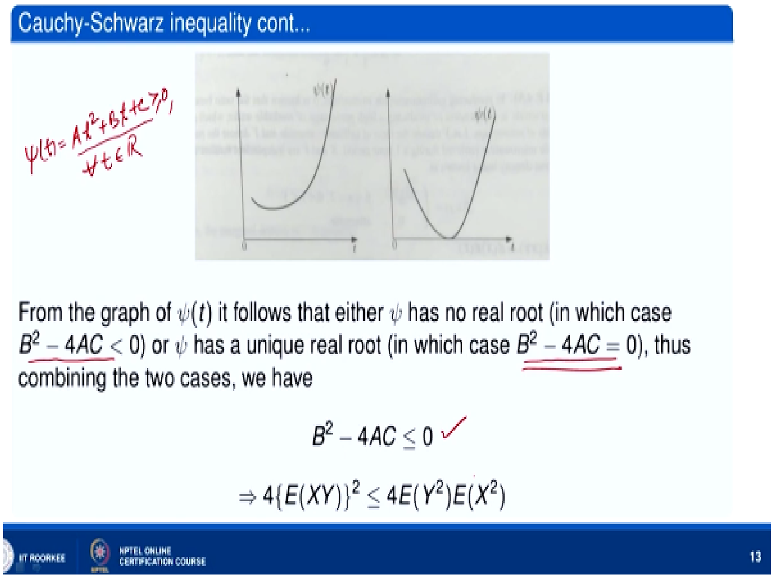

Okay so psi t = At square + Bt + C which is >= 0 for every value of t belonging to R. Now we have to see 2 graphs okay. This graph and this graph, they are both see At square + Bt + C okay is a parabola okay it is a parabola, so this is parabolic curve okay and from the graph of psi t it follow that either, now since the psi t is always >= 0 either xi has no real root okay in which case B square – 4 AC will be < 0 because it is quadratic equation in t.

Or psi has a unique real root, in which case B square – 4 AC = 0 in this graph, you can see it has a unique real root. So in that case B square – 4 AC will be = 0 and therefore combining this case and this case okay, we have B square – 4 AC <= 0 okay. So B = 2 times E XY, so B square is 4 times EXY whole square and <= 4 times A that is EY square * C which is EX square.

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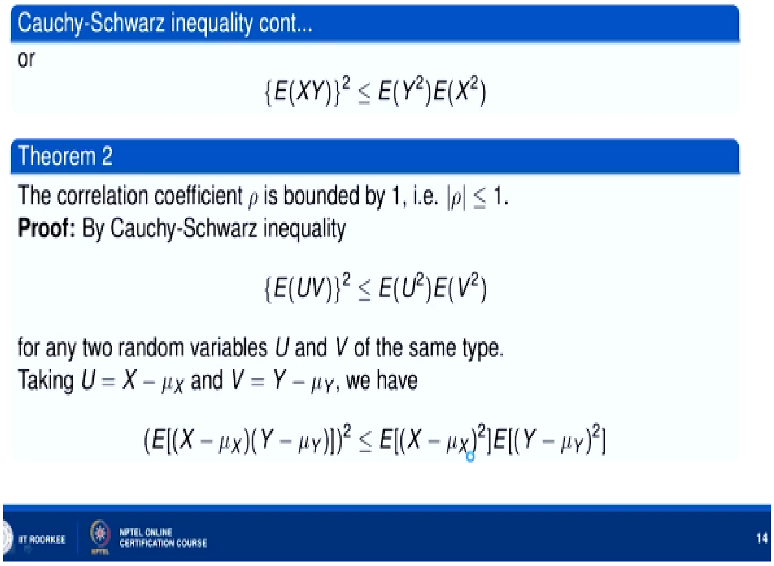

So we get EX/whole square <= EY square * EX square which proves the Cauchy-Schwarz inequality for the random variables X and Y. Now let us show that the correlation coefficient rho is bounded by 1, that is mod of rho is, or = 1. So by Cauchy-Schwarz inequality, if you take any 2 random variables U and V, then expectation of UV whole square is <or = expectation of U square * expectation of V square.

Now let us define U to be = X – mu x and V = Y – mu y. Then from this equation okay, we have expectation of X – mu x * Y – mu y whole square <= expectation of X – mu x whole square * expectation of Y – mu y whole square. Now this is what you can see. This is nothing but covariance of XY okay. So covariance of XY whole square, this is sigma X square, this is sigma Y square, that is their variances of X and Y.

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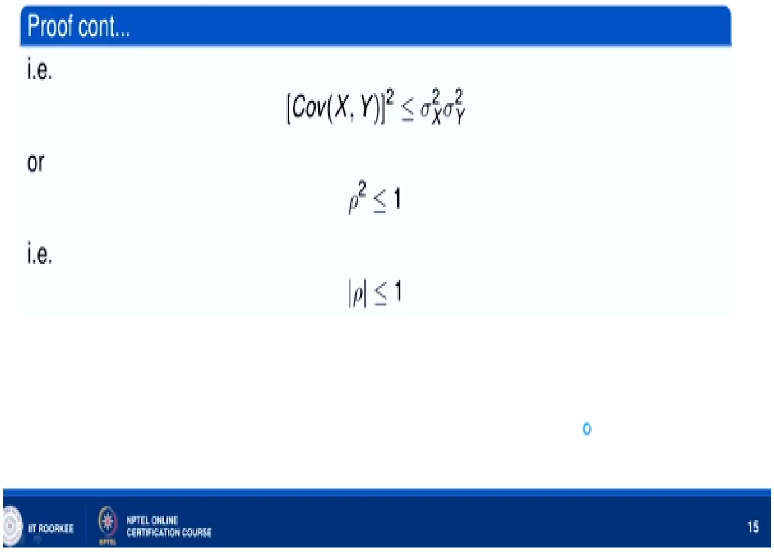

So we get covariance of X Y whole square <= sigma x square * sigma y square dividing by sigma x square, sigma y square we get covariance of xy whole square/sigma x square, sigma y square <= 1 or rho square is <= 1, which implies that mod of rho is <= 1. So this proves the result that the coefficient of correlation is bounded by 1. Now let us take an example.

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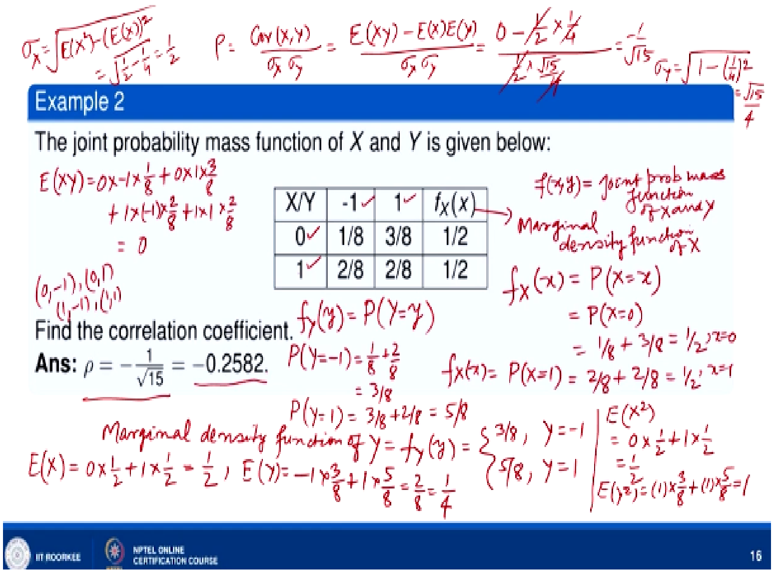

The joint probability mass function of x and y is given in this table okay, these are the values of x and these are the values of y okay. So when x takes the value 0, y takes the value -1 okay, fxy the joint probability mass function of x and y that is fxy, this is the joint probability mass function. I am denoting the joint probability mass function of x and y by fxy, this is joint probability mass function of x and y.

Okay so this is the probability that x takes the value 0, y takes the value -1, this is the probability that x takes the value 0, y takes the value 1 and this is the probability that x takes the value 1, y takes the value -1, this is the probability that x takes the value 1, y takes the value 1. So if the marginal density function okay, fxx, this is marginal density function of x. So fxx = probability that x takes the value x.

So x takes the value 0 okay, probability that x takes the value 0 will be = 1/8 + 3/8 okay, which is = 1/2 and then the probability that x takes the value 1, okay = 2/8 + 2/8 which is = 1/2. So fxx for x = 0 okay, this is the case when x = 0, this is the case when x = 1, so fxx = 1/2 when x takes the value 0 and 1/2 again when x = 1. Now let us find fyy okay, fyy is probability that y takes the value y okay.

So let us first find the probability that y takes the value -1, so y takes the value -1. So this will be = 1/8 + 2/8 which is = 3/8 and probability that y takes the value 1, which is = 3/8 + 2/8 this is = 5/8 okay. So the marginal density function of y = fyy = 3/8 for y = -1 and 5/8, 3/8 for y = -1 and for y = 1 it is 5/8, we have to find the correlation coefficient. So we need to find the expected value of x and expectation value of y okay. So expectation of x.

So let us first find the expectation of x, it is the values of x multiplied by the corresponding probabilities okay. So x takes the 2 values, okay, x takes the value 0, 0 multiplied by the probability fx okay. So x = 0 multiplied by 1/2 okay +1 multiplied by 1/2. So we get expectation of x = 1/2. Expectation of y we can get similarly okay values of y multiplied by their corresponding probabilities.

So value of y is -1, okay multiplied by 3/8 and then value of y is 1 multiplied by 5/8. So 5/8 – 3/8 is 2/8 which is = 1/4. So expectation of x is 1/2, expectation of y is 1/4. Now let us find expectation of x square. So expectation of x square, x is taking value 0 and 1 okay. So 0 square means 0 multiplied by 1/2 + expected value of, sorry 1 we are getting values of x as 0 and 1.

So 1 square that is 1 multiplied by the corresponding probability that is 1/2 okay. So expected value of x square is 1/2, expected value of y square we can find, now y is taking value -1 and + 1, so -1 square is 1, 1 * the probability is 3/8 + 1 square means 1 multiplied by 5/8. So we get it as 8/8 that is = 1 okay. So we have got the values of expectation of x square, expectation of y square.

Now let us find the expectation of xy, so expectation of xy okay. So values of x are, x and y take values, x = 0, y = -1, so 0 – 1 okay, then 0 and 1 then x = 1, y = -1 so 1-1, x = 1, y = 1 so 1 1 okay. So expected value of xy is multiply the values of x and y with the joint probability that is joint probability mass function of xy. So x is 0, so 0 * -1 okay, x * y multiplied by 1/8 okay. Then 0 * 1 multiplied by the joint probability 3/8 then 1 * -1 multiplied by 2/8 and then 1 * 1 multiplied by 2/8 okay.

So how much is this, this is 0, this is 0 and here what we get -2/8, here we get 2/8 okay. So expectation of xy = 0 okay and thus what we get, thus we have rho = covariance of xy/sigma x sigma y. Covariance of xy is expected value of xy – Ex Ey/sigma x sigma y. Now this is expected value of xy = 0. So 0 –expectation of x, expectation of x is 1/2, expectation of y is 1/4 okay divided by sigma x.

Sigma x = square root Ex square – Ex whole square okay. Ex of x square we found = 1/2 – expectation of x = 1/2. So 1/2 square means 1/4, so this is 1/4, square root of 1/4 is 1/2. So we get 1/2 here okay. Now let us find sigma y okay. So sigma y = square root expectation of y square it is = 1, expectation of y = 1/4, so 1/4 whole square. So this is = square 15/4 okay. So we get here square 15/4 okay.

This cancels with this, this cancels with this and we get it as -1/root 15 okay. So rho = -1/root 15 which is = -0.2582.

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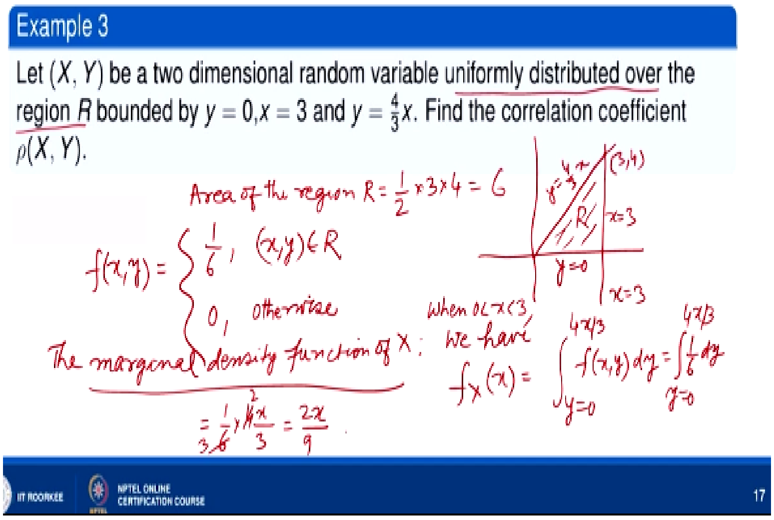

Now let us consider another problem, let xy be a 2 dimensional random variable uniformly distributed over the region R bounded by y = 0, x = 3, y = 4/3x okay. So this point of intersection is 3, 4. Okay now this is the region R okay, region R is bounded by y = 0, x = 3 and y = 4/3 * x. Now area of the region R = 1/2 * base because it is a triangle. So base = 3 * height.

Height is 4, so we get 6 okay. Now since the random variable is, since xy is 2 dimensional random variable which is uniformly distributed over the region R okay. So we have fxy = 1/R means 1/6, when xy belong to R, okay and 0 otherwise okay. We need to first find the marginal density functions okay. The marginal density function of x let us find first okay. So we have fxx = y varies from 0 to 4x/3 fxy * dy okay.

This is the probability that x takes the value x. So this is = y varies from 0 to 4x/3, 1/6 dy. So this is = and here when 0 is < x, is < 3 okay. We have fxx = this, so this is = 1/6 4x/3, and this is = 2x/9. So fxx = 2x/9 when 0 is < x, <3 and otherwise it is 0. So we write it like this.

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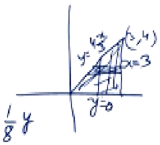
$$f_X(x) = \begin{cases} \frac{2x}{9}, & 0<x<3 \\ 0, & \text{otherwise} \end{cases}$$

Marginal density function of y



$$f_Y(y) = \int_{x=\frac{3y}{4}}^{3} f(x,y)\,dx = \int_{\frac{3y}{4}}^{3} \frac{1}{6}\,dx = \frac{1}{6}\left(3-\frac{3y}{4}\right) = \frac{1}{2}-\frac{1}{8}y$$

$$f_Y(y) = \begin{cases} \frac{1}{2}-\frac{1}{8}y, & 0<y<4 \\ 0, & \text{otherwise} \end{cases}$$

$$E(X) = \int_0^3 x f_X(x)\,dx = \int_0^3 x\left(\frac{2x}{9}\right)dx = \frac{2}{9}\left(\frac{x^3}{3}\right)_0^3 = \frac{2}{9}\times\frac{27}{3} = 2$$

$$E(Y) = \int_0^4 y f_Y(y)\,dy = \int_0^4 y\left(\frac{1}{2}-\frac{1}{8}y\right)dy = \left[\frac{1}{2}\left(\frac{y^2}{2}\right)-\frac{1}{8}\left(\frac{y^3}{3}\right)\right]_0^4 = 4-\frac{1}{8}\times\frac{4\times4\times4}{3} = \frac{4}{3}$$

$$E(XY) = \int_{x=0}^{3}\int_{y=0}^{4x/3} xy\,f(x,y)\,dy\,dx = \frac{1}{6}\int_0^3\int_0^{4x/3} xy\,dy\,dx = \frac{1}{6}\int_0^3 x\left(\frac{y^2}{2}\right)_0^{4x/3}dx$$

$$= \frac{1}{6}\int_0^3 x\,\frac{16x^2}{18}\,dx = \frac{8}{6\times9}\left(\frac{x^4}{4}\right)_0^3 = \frac{8}{6\times9}\times\frac{81}{4} = 3$$

fxx = 2x/9 when 0 is < x < 3 and 0 otherwise. Let us now find the marginal density function of y, fyy. So fyy = now this is the probability that y takes the value y. So x varies, we have this region, okay, this is y = 4x/3, this is x = 3 and this is y = 0. So x varies from 3y/4 to 3, so 3y/4 to 3 okay, and fxy dx. So this is 3y/4 to 3 1/6 dx okay. So 1/6 times 3-3y/4 okay, so this is 1/2 – 1/8y. Okay so fyy is given by 1/2 – 1/8y when 0 is < y < 4 okay y lies between 0 and 4, this is 3, 4 point and 0 otherwise.

Okay now let us find expected value of x, so expected value of x is x multiplied by it is probability density function and x varies from 0 to 3 okay, x varies from 0 to 3, so 0 to 3 x times fxx, fxx is 2x/9 dx. So this is 2x square/9 okay, so 2/9 integral of x square is x cube/3, so we put the limits and we get 2/9 * 3 cube, 3 cube means 27/3. So we cancel this and get expected value of xx2.

Now expected value of y, so integral over y fyy dy y varies from 0 to 4 and what we get is integral over 0 to 4 y times fyy is 1/2 – 1/8y dy. So this is = 1/2 y square/2 – 1/8 y cube/3. Let us put the limit and we get this is 4, 4 square/4. So we get 1/so we get 4 okay, -1/8, y cube is 4 cube, so 4 * 4 * 4/3 okay. So this will be 4-8/3. So this is 3 4s are 12, 12-8 so 4/3. So this is expected value of y.

Now expected value of xy, okay, so we take the joint probability mass function here. So for now let us see, we have to integrate over this area okay for the joint probability mass function, so y varies from 0 to 4 x/3 x varies from 0 to 3 and we have x * y, fxy dx dy, dy dx.

Fxy = we are given fxy = 1/6. So this is 1/6 times integral over 0 to 3, integral over 0 to 4 x/3 and we have x * y, dy dx.

This fxy = 1/6 okay over region R. So we have 1/6 integral over 0 to 3 x and then we get y square/2 and we have the limits 0, 4 x/3 dx. So what we get is 1/6 0 to 3, x times y square means 16, x square/9. So 16 x square/9 * 2 that is 18 okay. y square/2 means 16, x square/9 * 2 that is x2/18 so we get here 2 8s are 16 and here we get 9 okay. This is dx okay. So we have 8/6 * 9 x cube, integral of x cube is x 4/4 0 to 3.

So we get 8/6 * 9 and then we have here 3 to the power 4 that is 81/4 okay. So 4 2s are 8 okay and 2 3s are 6 and then we can cancel 3 9s are 27, 27 will cancel okay. 3 27 here and 9 cancels 27/3 so we get expected value of xy as 3 okay. Now we need to find the expected value of x square, expected value of y square okay.

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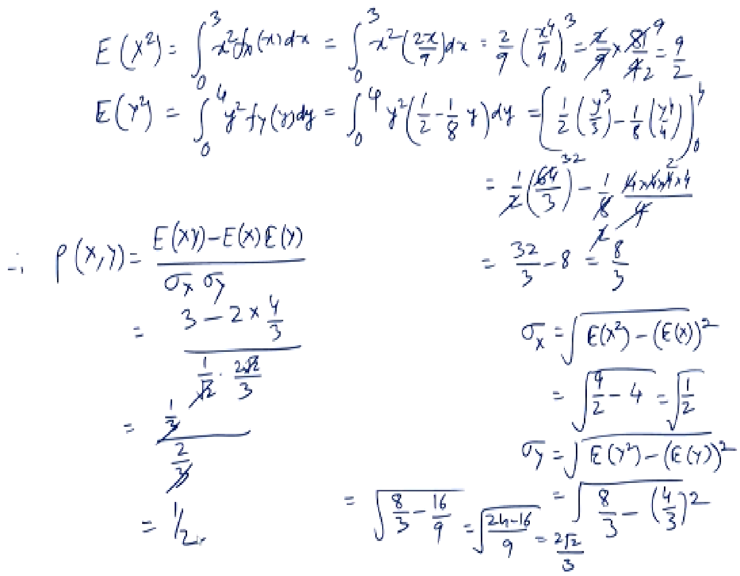

So expected value of x square is integral 0 to 3 x square * fxx dx okay and fxx we have found to be = 2x/9 over the integral 0 to 3. So this is 0 to 3, x square * 2x/9 dx. It comes out to be 2/9 integral of x cube is x4/4 0 to 3, we get 2/9 * 81/4 okay. So 9 9s are 81 and we get it 9/2. Similarly, we can find expected value of y square integral 0 to 4 y square fyy dy and it comes out to be integral 0 to 4, fyy is 1/2 – 1/8y.

So 1/2 -1/8 y dy and this is 1/2 y cube/3 – 1/8 y4/4 and we get the value as 1/2, 4 to the power 3, so 64/3 – 1/8, 4 to the power 4, so 4 * 4 * 4 * 4/4 okay, so this cancels and we get this cancels with this we get 2 this cancels with this we get 2 okay. So this is 8 and here we get

32. So 32/3 – 8 okay. So we get 8/3 okay and so rho xy = Exy which is Exy – Ex * Ey, this is covariance of xy okay.

Sigma x sigma y okay, so we found Exy = 3 okay and Ex = 2, Ey = 4/3 okay, so 3/2 * 4/3 / sigma x, sigma x = square root Ex square – Ex whole square. Ex square we found is = 9/2 and Ex we found to be = 2, so 2 square is 4, so we get, this is 1/2, so 1/2 square root and sigma y = square root Ey square – Ey whole square, Ey square we found to be 8/3, so 8/3 – Ey, Ey we found to be = 4/3.

So 4/3 whole square. So this is how much? 8/3 -16/9 okay, and this is lcm is 9, 24 -16 so we get 8/9 that is 2 root 2/3 okay. So we get here 1/ root 2 * 2 root 2/3 okay. So how much is that? 3 3s are 9, 9 –8, 1/3 so 1/3 /this cancels with this 2/3 and we get the value as 1/2 okay. So rho xy = 1/2 okay. So rho is = correlation coefficient = 1/2. So this is how we solve this problem. With that I would like to end my lecture. Thank you very much for your attention.