

**Advanced Engineering Mathematics**  
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**Lecture - 51**  
**Normal Distribution**


Hello friends. Welcome to my lecture on normal distribution. The normal distribution is also known as Gaussian distribution and it is very common continuous probability distribution.

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**Normal distribution**

The normal distribution, also known as Gaussian distribution is a very common continuous probability distribution. It is very important in statistics and is often used in the natural and social sciences to represent real valued random variables whose distributions are not known. A random variable with a Gaussian distribution is said to be normally distributed and is called a normal deviate.

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It is very important in the statistics and is often used in the natural and social sciences to represent real valued random variables whose distributions are not known. A random variable with a Gaussian distribution is said to be normally distributed and is called a normal variate.

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**Normal distribution cont...**

The probability density function of the normal distribution is given by

$$f(x/\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (1)$$

where  $\mu$  is the mean or expectation of the distribution (and also its median and mode) and  $\sigma$  is the standard deviation.

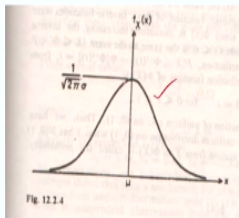


bell shaped

Fig. 12.3.4

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The probability density function of the normal distribution is given by  $f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$  where  $\mu$  is the mean or expectation of the distribution and also its median and mode okay because at the mean, the mean, median and mode okay you can see in this figure, they coincide and  $\sigma$  is the standard deviation.

This is the graph of the probability density function of the normal distribution okay. So this is the graph of the probability density function of the normal distribution. You can see that at  $x=\mu$  the function  $f(x)$  assumes its maximum value which is given by  $\frac{1}{\sigma \sqrt{2\pi}}$  and it is a bell-shaped curve okay. The shape of the curve is bell-shaped, so it is a bell-shaped curve okay.

Mean, median and mode coincide for this distribution okay,  $\mu$  is the mean and  $\sigma$  is the standard deviation. The maximum value occurs at  $x=\mu$  which is given by  $\frac{1}{\sigma \sqrt{2\pi}}$ .


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**Standard normal distribution**

The simplest case of a normal distribution is known as the standard normal distribution. In this special case,  $\mu = 0$  and  $\sigma = 1$ . It is described by the probability density function  $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ .

**Observation**

$\int_{-\infty}^{\infty} \phi(x) dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2} dx = 1$  and  $\sigma^2 = 1$ . This function is symmetric about  $x = 0$ , where it attains its max. value  $\frac{1}{\sqrt{2\pi}}$  and has inflection points at  $x = 1$  and  $-1$ .


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Now the simplest case of a normal distribution is known as the standard normal distribution.

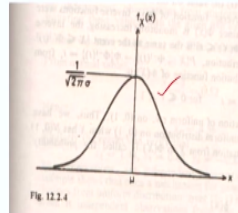
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### Normal distribution cont...

The probability density function of the normal distribution is given by

$$f(x/\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (1)$$

where  $\mu$  is the mean or expectation of the distribution (and also its median and mode) and  $\sigma$  is the standard deviation.



bell shaped  
 $z = \frac{x-\mu}{\sigma}$

Fig. 12.2.4

In this case, what we do is we put here  $z=x-\mu/\sigma$  okay. So when you put  $z=x-\mu/\sigma$  then what happens is the mean of the distribution, mean of the distribution for the standard normal distribution is  $=0$  okay,  $\mu$  becomes 0 and  $\sigma$  becomes  $=1$ . It is described by the probability density function  $1/\sqrt{2\pi} e^{-1/2 x^2}$ . So here if you take instead of  $x-\mu/\sigma$   $z$  then this becomes  $e$  to the power  $-1/2 z^2$ .

And the probability density function becomes  $1/\sqrt{2\pi} e^{-z^2/2}$  or we can also write it as  $1/\sqrt{2\pi} e^{-x^2/2}$ . Now this is the probability density function for the standard normal distribution can be seen from here.

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### Standard normal distribution

The simplest case of a normal distribution is known as the standard normal distribution. In this special case,  $\mu = 0$  and  $\sigma = 1$ . It is described by the probability density function  $\phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$ .  $\phi(x) \geq 0, \forall x$   $\int_{-\infty}^{\infty} \phi(x) dx = 1$   $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 1$   $\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx = 1$

### Observation

$\int_{-\infty}^{\infty} \phi(x) dx = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2} dx = 1$  and  $\sigma^2 = 1$ . This function is symmetric about  $x = 0$ , where it attains its max. value  $\frac{1}{\sqrt{2\pi}}$  and has inflection points at  $x = 1$  and  $-1$ .

We know that  $\int_0^{\infty} e^{-t} t^{-1/2} dt = \sqrt{\pi}$

$$\frac{1}{2} x^2 = t$$

$$\frac{1}{2} x dx = dt$$

$$dx = \frac{dt}{x} = \frac{dt}{\sqrt{2t}}$$

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2} dx = 2 \int_0^{\infty} e^{-\frac{1}{2}x^2} dx$$

$$= 2 \int_0^{\infty} e^{-t} \frac{dt}{\sqrt{2t}}$$

$$= \sqrt{2} \int_0^{\infty} t^{-1/2} e^{-t} dt = \sqrt{2} \int_0^{\infty} t^{-1/2} e^{-t} dt$$

This you can see that  $\phi(x) \geq 0$  for all  $x$  okay,  $\phi(x) \geq 0$  for all  $x$  which is the first condition for the probability density function. Second is that integral  $-\infty$  to  $\infty$   $\phi(x) dx = 1$

dx must be 1. So that we can see here  $\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2} dx$ . So let us integrate this. Integral over  $-\infty$  to  $\infty$   $e^{-\frac{1}{2}x^2} dx$ .

Since  $e^{-\frac{1}{2}x^2}$  is an even function, we can write it as 2 times  $\int_0^{\infty} e^{-\frac{1}{2}x^2} dx$ . Now let us take  $\frac{1}{2}x^2 = t$  okay. So then what we get  $\frac{1}{2} \cdot 2x dx = dt$  okay. So we get  $dx = \frac{dt}{x}$ ,  $x = \sqrt{2t}$  okay. So  $dt$  upon  $\sqrt{2t}$  okay, so let us put it here. Now when  $x = 0$ ,  $t$  is 0 and when  $x$  is  $\infty$ ,  $t$  is  $\infty$ , so the limits remain the same 0 to  $\infty$ .

We get  $e^{-t}$  and for  $dx$  we put  $\frac{dt}{\sqrt{2t}}$  okay. So what we get then? This  $\sqrt{2}$  we can cancel with here 2 and we get  $\sqrt{2}$  and integral 0 to  $\infty$   $t^{-1/2} e^{-t} dt$  okay. Now we can write the value of this integral by using gamma function. We know that integral 0 to  $\infty$   $e^{-t} t^{x-1} dt = \Gamma(x)$  okay. So here we have  $t^{-1/2}$ , we can write it as  $t^{1/2-1}$ .

So this is  $\Gamma(1/2)$  okay. So this is  $\sqrt{2} \cdot \Gamma(1/2)$  okay. So what we get then okay thus we have found the value of this integral. Let us evaluate the integral over  $-\infty$  to  $\infty$   $\phi(x) dx$  now. So this will be  $\frac{1}{\sqrt{2\pi}}$  okay and here we get  $\sqrt{2} \cdot \Gamma(1/2)$  is  $\sqrt{\pi}$  okay, so  $\sqrt{2} \cdot \sqrt{\pi}$  okay. So this cancels with this and this cancels with this and we get 1 okay. Remember  $\Gamma(1/2) = \sqrt{\pi}$  okay.

So this integral over  $-\infty$  to  $\infty$   $\phi(x) dx = 1$  and therefore we can say that this  $\phi(x)$  is a probability density function when we replace  $\frac{x-\mu}{\sigma}$  which we have taken as  $x$  here okay. Now we can also see that for this probability density function okay, the standard normal distribution  $\mu = 0$ ,  $\sigma = 1$ .

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$$\begin{aligned}
 \mu &= E(\phi) = \int_{-\infty}^{\infty} x \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx & z = \frac{x-\mu}{\sigma} \\
 &= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} x e^{-\frac{1}{2}x^2} dx = 0, \text{ since } x e^{-\frac{1}{2}x^2} \text{ is an odd function of } x \\
 E(z^2) &= \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} dx & \frac{1}{2}x^2 = t \\
 &= \frac{1}{\sqrt{2\pi}} \cdot 2 \int_0^{\infty} x^2 e^{-\frac{1}{2}x^2} dx & x dx = dt \\
 &= \frac{2}{\sqrt{2\pi}} \int_0^{\infty} 2t e^{-t} \frac{dt}{\sqrt{2t}} = \frac{2\sqrt{2}}{\sqrt{\pi}} \int_0^{\infty} t^{1/2} e^{-t} dt \\
 \text{Now } Var(z) &= E(z^2) - (E(z))^2 = \frac{2}{\sqrt{\pi}} \int_0^{\infty} t^{1/2} e^{-t} dt = \frac{2}{\sqrt{\pi}} \cdot \frac{1}{2} \sqrt{\pi} = 1 \\
 &= 1 - 0 = 1 = \sigma^2
 \end{aligned}$$

So let us find mu okay. So mu is the expectation of the standard normal distribution phi okay. So we have integral over -infinity to infinity okay, we have taken okay so x times e to the power -1/2 x square dx and we also have 1/root 2 pi okay, 1/root 2 pi is the multiple of, so let me put it like this x times this okay. So x times the probability density function which is 1/root 2 pi e to the power -1/2 x square dx.

Now this is 1/root 2 pi okay. We have integral over -infinity to infinity x e to the power -1/2 x square dx okay. Now you can see that this is an odd function of x okay. This is an odd function of x, so integral over -infinity to infinity x e to the power -1/2 x square dx is 0 okay. Since x e to the power -1/2 x square is an odd function of x okay. Now let us also find the variance of this.

So we need to find E x square that is E z square, this is actually phi we are writing here the z=x-mu/sigma okay, z is the standard normal distribution, so we are writing E z square, this is actually E z, you can write E z here, expectation of z okay. So this expectation of z square let us find. This will be integral over -infinity to infinity, so x square we write x square okay e to the power -1/2 x square/root 2 pi okay.

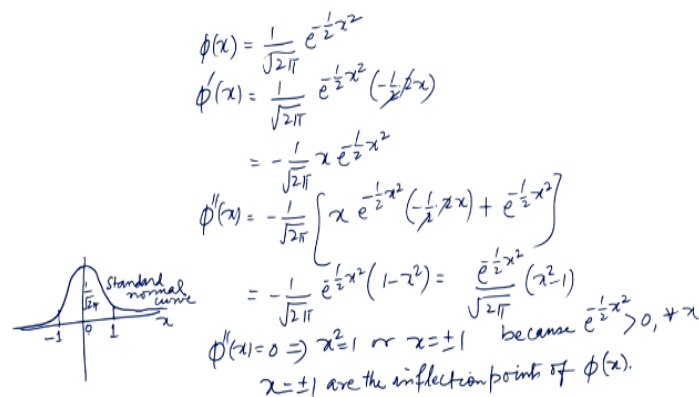
So now this is 1/root 2 pi x square e to power -1/2 square is an even function, so we can write it 2 times 0 to infinity x square e to the power -1/2 x square dx okay. Now let us put 1/2 x square=t as we have done earlier. So then this will be x dx=dt, so we shall get and the limits of integration will remain 0 infinity. So this is 2/root 2 pi 0 to infinity x square is=2t okay, x square is 2t e to the power -t and dx is=dt/x that is square root 2t okay.

So this is  $2/\sqrt{2\pi}$ ,  $\sqrt{2t}$  will cancel with  $2t$  and will give you  $\sqrt{2t}$ , so  $\sqrt{2}$  here integral 0 to infinity  $t$  to the power  $1/2$  okay,  $t$  to power  $1/2$  means  $t$  to power  $3/2-1$ . So this is like this we get okay. So this  $\sqrt{2}$  will cancel with  $2$  here,  $\sqrt{2}$  here and we get  $2/\sqrt{\pi}$  and this is  $\gamma 3/2$  okay. This can be evaluated from here 0 to infinity  $e$  to the power  $-t$   $t$  to the power  $x-1$   $dt = \gamma x$  okay.

So this is  $t$  to power  $3/2-1$ , so  $\gamma 3/2$  which is  $2/\sqrt{\pi} \cdot 1/2 \gamma 1/2$  okay. So this is  $2/\sqrt{\pi} \cdot 1/2 \sqrt{\pi}$  and this cancels with this, this cancels with this. So  $Ez^2$  is 1 okay. Now variance of  $z$  is  $Ez^2 - Ez^2$  whole square okay. This is  $Ez$ , so  $Ez^2$  whole square, so this is  $1 - Ez^2$  is 0 we have seen okay. So we get variance as 1. Variance is nothing but  $\sigma^2$  okay.

So  $\sigma^2 = 1$  okay. So here we can see that mean is 0 and  $\sigma^2 = 1$  or we can say  $\sigma = 1$ . Now this function you can see  $1/\sqrt{2\pi} e^{-1/2 x^2}$  is symmetric about  $x=0$  okay. We replace  $x$  by  $-x$  it does not change, so it is symmetric about  $x=0$  where it attains its maximum value. It attains its maximum value at  $x=0$  okay and it is  $1/\sqrt{2\pi}$ . It has inflection points at  $x=1$  and  $x=-1$ , so this we can see.

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The image shows a handwritten derivation of the standard normal distribution function  $\phi(x)$  and its first and second derivatives. On the left, a small graph of the standard normal curve is shown, centered at 0, with the peak labeled  $1/\sqrt{2\pi}$  and the x-axis marked at -1, 0, and 1. The text "Standard normal curve" is written above the graph.

$$\begin{aligned}\phi(x) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \\ \phi'(x) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \left(-\frac{1}{2} \cdot 2x\right) \\ &= -\frac{1}{\sqrt{2\pi}} x e^{-\frac{1}{2}x^2} \\ \phi''(x) &= -\frac{1}{\sqrt{2\pi}} \left[ x e^{-\frac{1}{2}x^2} \left(-\frac{1}{2} \cdot 2x\right) + e^{-\frac{1}{2}x^2} \right] \\ &= -\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} (1 - x^2) = \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} (x^2 - 1) \\ \phi''(x) &= 0 \Rightarrow x^2 = 1 \text{ or } x = \pm 1 \quad \text{because } e^{-\frac{1}{2}x^2} > 0, \forall x \\ x &= \pm 1 \text{ are the inflection points of } \phi(x).\end{aligned}$$

We have  $\phi(x) = 1/\sqrt{2\pi} e^{-1/2 x^2}$  okay. So let us find its first derivative  $\phi'(x)$ . So  $1/\sqrt{2\pi}$ , we have  $e$  to the power  $-1/2 x^2 \cdot -1/2 \cdot 2x$ . So we get  $-1/\sqrt{2\pi} x e^{-1/2 x^2}$  okay. Now let us find  $\phi''(x)$ . So  $-1/\sqrt{2\pi}$  and

we have  $x^2 e^{-\frac{1}{2}x^2}$  okay + derivative of  $x$  that is 1. So  $e^{-\frac{1}{2}x^2}$  to the power  $-\frac{1}{2}x^2$  okay.

Now what do we get here? So this is  $-\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$  we can write outside and we have  $1-x^2$  here okay or we can say  $e^{-\frac{1}{2}x^2}$  okay. So  $\phi''(x)=0$  implies that  $x^2=1$  okay or  $x=\pm 1$  because  $e^{-\frac{1}{2}x^2}$  is never 0 okay, it is never 0, it is in fact  $>0$  okay for all  $x$  okay. So if we have a differential function  $\phi(x)$  and it turns out that  $\phi''(x)=0$ , then the points where  $\phi''(x)=0$  gives us the inflection points.

What are the inflection points? Inflection points are those points on the curve where the concavity of the curve changes okay. So you can see how it changes. So  $x=\pm 1$  are the inflection points okay of  $\phi(x)$ . So if you draw the graph of  $\phi(x)$ , it will look like something this okay. This is your  $x$ -axis, this is 0,  $x=0$  at  $x=0$  it assumes maximum value  $1/\sqrt{2\pi}$  and then it drops down okay and at  $x=\pm 1$ .

So this is the point  $x=1$  okay where the concavity of the curve changes. It was downward earlier, now it becomes upward okay. So let me write again. So okay so here okay the concavity of the curve changes. This is -1 and this is 1, so concavity of the curve changes and this is  $1/\sqrt{2\pi}$ . This is your standard normal curve we call it, standard normal curve okay. So you can see it is symmetric about the  $y$ -axis. Now let us go further.

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The probability of the variate  $X$  lying between  $x_1$  and  $x_2$  is given by the area under the curve for (1) from  $x_1$  to  $x_2$ , i.e.

$$P(x_1 < X \leq x_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx. \quad (2)$$

The integral

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} dv \quad (3)$$

is called the distribution function. If we set  $(v - \mu)/\sigma = u$ , then  $dv = \sigma du$  and the limit of the integration in (3) are from  $u = -\infty$  to  $u = (x - \mu)/\sigma$ . Hence equation (3) becomes

The probability of the variate  $X$  the normal variate okay, the probability of the normal variate  $X$  lying between  $x_1$  and  $x_2$  is given by the area under the curve for 1 under this curve okay under the curve for 1.

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Normal distribution cont...

The probability density function of the normal distribution is given by

$$f(x/\mu, \sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad (1)$$

where  $\mu$  is the mean or expectation of the distribution (and also its median and mode) and  $\sigma$  is the standard deviation.

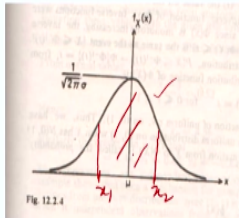


Fig. 12.2.4

Say  $x_1$  is here,  $x_2$  is here okay, then the area under this curve from  $x_1$  to  $x_2$  okay, this area okay. So this area will give us the probability of the normal variate from  $x_1$  to  $x_2$ . So that probability is given by integral over  $x_1$  to  $x_2$   $1/\sigma \sqrt{2\pi} \int_{x_1}^{x_2} e^{-\frac{(x-\mu)^2}{2\sigma^2}} dx$  okay. The integral  $F(x)$  okay  $F(x) = 1/\sigma \sqrt{2\pi} \int_{-\infty}^x e^{-\frac{(v-\mu)^2}{2\sigma^2}} dv$  then gives us the cumulative distribution function of  $x$ .

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The probability of the variate  $X$  lying between  $x_1$  and  $x_2$  is given by the area under the curve for (1) from  $x_1$  to  $x_2$ , i.e.

$$P(x_1 < X \leq x_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{x_1}^{x_2} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx. \quad (2)$$

The integral

$$F(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^x e^{-\frac{1}{2}\left(\frac{v-\mu}{\sigma}\right)^2} dv = P(X \leq x) = \int_{-\infty}^x f_X(v) dv \quad (3)$$

is called the distribution function. If we set  $(v - \mu)/\sigma = u$ , then  $dv = \sigma du$  and the limit of the integration in (3) are from  $u = -\infty$  to  $u = (x - \mu)/\sigma$ . Hence equation (3) becomes



This is nothing but you can see probability that  $X \leq x$  okay, probability that  $x \leq x$  is what? It is given by integral over  $-\infty$  to  $x$  probability density function of  $x$  okay. This we know. So its probability density function here is  $1/\sigma \sqrt{2\pi}$  okay. Instead of  $x$  we are using the variable  $v$ , so we can write variable  $v$  here okay. So integral over  $-\infty$  to  $x$   $e^{-v^2/2\sigma^2}$   $dv$  and this is  $1/\sigma \sqrt{2\pi}$  integral over  $-\infty$  to  $x$   $e^{-v^2/2\sigma^2}$   $dv$  okay.

This is called as the distribution function or cumulative distribution function for the normal variate okay. Now if we put  $v-\mu/\sigma=u$  here okay,  $v-\mu/\sigma=u$  if you put then you can see  $dv/\sigma=du$ , so  $dv=\sigma du$  and the limits of the integration change from  $-\infty$  to  $x$  to  $-\infty$  to  $u=x-\mu/\sigma$  okay.

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$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{(x-\mu)/\sigma} e^{-u^2/2} du = \Phi\left(\frac{x-\mu}{\sigma}\right) = \Phi(z), \text{ say } \quad (4)$$

From (2) and (3), we obtain

$$P(x_1 < X \leq x_2) = F(x_2) - F(x_1)$$

Hence

$$P(x_1 < X \leq x_2) = \Phi\left(\frac{x_2 - \mu}{\sigma}\right) - \Phi\left(\frac{x_1 - \mu}{\sigma}\right) = \Phi(z_2) - \Phi(z_1) \quad (5)$$

The integral in (3) cannot be evaluated by elementary methods. But its representation in (4), has been tabulated

Hence, equation 3 becomes  $F(x) = 1/\sqrt{2\pi} \int_{-\infty}^{x-\mu/\sigma} e^{-u^2/2} du$  and this is a function of  $x-\mu/\sigma$ , so we can write it as  $\Phi(x-\mu/\sigma)$  and this  $\Phi(x-\mu/\sigma)$  then becomes  $\Phi(z)$  where  $z$  is the standard normal variable okay,  $z=x-\mu/\sigma$  as we have discussed earlier,  $z=x-\mu/\sigma$ . So  $x-\mu/\sigma$  is replaced by  $z$ . Now from 2 and 3 okay from this and this okay let us see what happens.

From 2 and 3, we obtain that probability that  $x_1 < X \leq x_2$  is then  $F(x_2) - F(x_1)$  because  $F(x)$  is the cumulative distribution function okay. It gives the probability that  $x \leq X$ , so  $F(x_2) - F(x_1)$  will give us the probability that  $x_1 < X \leq x_2$ . So if you put the value of  $x_2$  here, then  $F(x_2)$  becomes  $\Phi(x_2-\mu/\sigma)$ , so probability that  $x_1 < X \leq x_2$  can be now written as  $\Phi(x_2-\mu/\sigma) - \Phi(x_1-\mu/\sigma)$ .

This can then be written as  $\Phi(z_2) - \Phi(z_1)$  which will be nothing but the area under the standard normal curve okay, standard normal curve, this is standard normal curve  $\phi(z)$  okay from  $z_1$  to  $z_2$  okay,  $\Phi(z_2) - \Phi(z_1)$  okay. This is the area under the graph of  $\phi(z)$  from  $z_1$  to  $z_2$ . So the integral in this  $\phi(z)$  represents the area, remember  $\phi(z)$  represents the area under the graph of the density function okay.

Density function is this. Let me write density function, density function is  $\frac{1}{\sqrt{2\pi}} e^{-z^2/2}$  okay. So this is the graph of  $\frac{1}{\sqrt{2\pi}} e^{-z^2/2}$  okay and  $\phi(z)$  represents the area  $\phi(z)$  is the area from  $-\infty$  to  $z$  okay  $\frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz$  okay. So  $\Phi(z_2) - \Phi(z_1)$  gives this area okay under the standard normal curve okay.

The integral in 3, now integral here cannot be evaluated by elementary methods okay, this integral, you can see this integral cannot be evaluated by the elementary methods that we know in integral calculus. So it has been numerically evaluated and it is available in the form of a table. Now it is given in all standard engineering mathematics books at the end as an appendix. So you can refer to that table to determine the value of this integral okay.

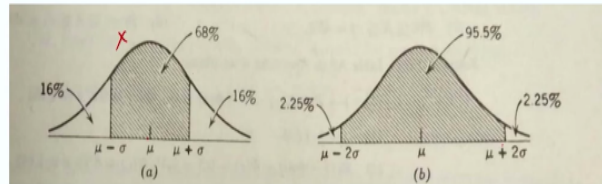
Now this integral has been evaluated for the standard normal curve okay for this curve okay,  $\frac{1}{\sqrt{2\pi}} e^{-z^2/2}$  that means this  $\phi(z)$  has been evaluated okay. This area under the graph of the function  $\frac{1}{\sqrt{2\pi}} e^{-z^2/2}$  has been evaluated for any given  $z$ . You can take any given  $z$  here when the area under the graph has been evaluated. So that we can use, we shall see just now when we do examples how to use that table. So we will evaluate the probability using that table okay for the standard normal curve.

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and hence from the table of  $\Phi$ , we have

$$\begin{aligned} (a) \quad & P(\mu - \sigma < X \leq \mu + \sigma) \approx 68\% \quad \checkmark \quad P(-1 < z \leq 1) \quad \checkmark \\ (b) \quad & P(\mu - 2\sigma < X \leq \mu + 2\sigma) \approx 95.5\% \quad \checkmark \quad P(-2 < z \leq 2) \quad \checkmark \\ (c) \quad & P(\mu - 3\sigma < X \leq \mu + 3\sigma) \approx 99.7\% \quad \checkmark \quad P(-3 < z \leq 3) \quad \checkmark \quad (6) \end{aligned}$$

$\frac{x - \mu}{\sigma} = z$



Now from the table of phi okay, from the table of phi it follows that the area or you can say the probability when  $x$  lies between  $\mu - \sigma$  to  $\mu + \sigma$  okay is 68%, this is say this is your mean okay, mean of the normal variate  $x$  okay, normal variate  $x$ , so on either side when you go to by a distance  $\sigma$ , so then from  $\mu - \sigma$  to  $\mu + \sigma$  68% of the area lies under the graph of the density function of  $x$  okay.

That means two-third of the values of  $x$  lie in the region between  $\mu - \sigma$  to  $\mu + \sigma$  approximately and then from  $\mu - 2\sigma$  to  $\mu + 2\sigma$  95.5% values of  $x$  lie and when you take  $\mu - 3\sigma$  to  $\mu + 3\sigma$  99.7% of the values lie that means about a quarter percent remain outside this region which lies between  $\mu - 3\sigma$  to  $\mu + 3\sigma$ . If you convert this to standard normal variable  $z$  then what it means?

You put  $\frac{x - \mu}{\sigma} = z$  okay so this is same as probability that  $-1 < z < 1$  okay and this means probability that  $-2 < z < 2$  okay and this means probability that  $-3 < z < 3$ . So from the table of phi from the standard normal curve, we have calculated the area which lies between  $z = -1$  to  $z = +1$  that area is 68%, so that area is 68% for the normal variate  $x$  here. We have converted this  $z$  to this  $x$  here okay.

So from these areas we have concluded about the values of  $x$  that lie in those ranges  $\mu - \sigma$  to  $\mu + \sigma$  it is 68%,  $\mu - 2\sigma$  to  $\mu + 2\sigma$  it is 95.5% and  $\mu - 3\sigma$  to  $\mu + 3\sigma$  it is 99.7% okay.

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From (6), it follows that a large number of observed values of the random normal variable  $X$  are distributed as follows:

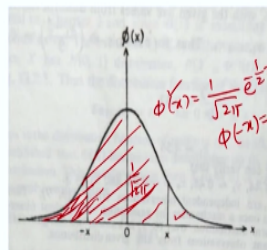
- (a) Nearly two third of values lie within  $\sigma$  of the mean.
- (b) About 95% lie within  $2\sigma$  of the mean.
- (c) Almost all values (except about a quarter percent) lie within  $\mu - 3\sigma$  and  $\mu + 3\sigma$ .

Now from these inequalities, from these values it follows that a large number of observed values of the random normal variate  $X$  are distributed as follows. Nearly two third of values lie within sigma of the mean, about 95% lie within 2 sigma of the mean, almost all values except about a quarter percent okay 0.25%. If you leave 0.25%, all other values lie between  $\mu - 3\sigma$  to  $\mu + 3\sigma$  okay.

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### Z-Scores

The z-score of a value from a random variable is the number of standard deviation away from the mean. By calculating the area under the bell curve, a z-score provides the probability of a random variable with this distribution having a value less than this score.



$$z = \frac{x - \mu}{\sigma}$$

$$\Phi(x) = \int_{-\infty}^x \phi(x) dx$$

$$\Phi(-x) = 1 - \Phi(x)$$

$$\int_{-\infty}^{-x} \phi(x) dx = 1 - \int_{-\infty}^x \phi(x) dx$$

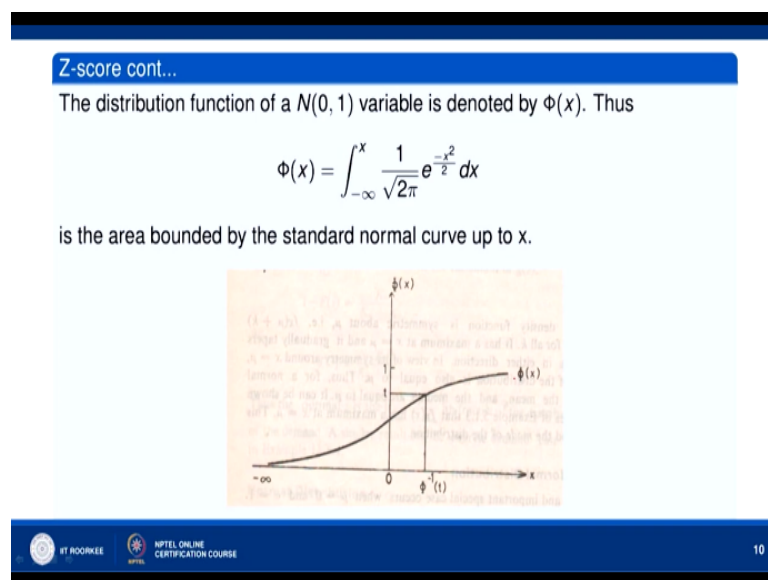
The z-score of a value okay  $z$  is  $x - \mu / \sigma$  we call it  $z$  square. So  $z$  square of a value from a random variable is the number of standard deviation away from the mean. You can see from the definition,  $z$  is nothing but the number of standard deviation away from the mean. By calculating the area under the bell curve, this is bell curve okay. This is the curve for  $z$  okay standard normal curve.

We have written here  $\phi(x)$  for  $\Phi(x)$  okay. You can see at  $x=0$ , it assumes the maximum value  $1/\sqrt{2\pi}$ . So this  $\phi(x)$  here is  $1/\sqrt{2\pi} e^{-x^2/2}$  okay. So this is  $1/\sqrt{2\pi}$ . You can see that  $\phi(-x) = \phi(x)$  okay, so it is symmetric okay. It is symmetric with respect to y-axis and the area under capital  $\Phi(x)$  okay gives the area under the graph of  $\phi(x)$  okay – infinity to  $x$   $\phi(x)$ , note the difference between this  $\phi$  and this  $\Phi$  okay.

This is capital  $\Phi$  okay. It is the area under the graph of this probability density function  $\phi$ . This is probability density function  $\phi(x)$  = this probability density function. So under the graph of  $\phi(x)$  from  $-\infty$  to  $x$ , so this area okay. Suppose  $x$  is positive okay, if  $x$  is positive then you can see because of the symmetry of  $\phi(x)$  about the y axis, the area under the graph of  $\phi(x)$  from  $-\infty$  to  $-x$  okay.

$\Phi(-x) = 1 - \Phi(x)$  okay. If you want this area okay, to obtain this area okay, what we do, the total area under the curve is  $=1$ , from 1 we subtract this entire area okay. So then we will get this area. So this area integral over  $-\infty$  to  $-x$  okay,  $\phi(x) dx$  okay area under the graph of  $\phi(x)$  from  $-\infty$  to  $-x$  can be obtained by subtracting the area under the graph of  $\phi(x)$  from  $-\infty$  to  $x$  okay because of symmetry okay.

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So now the distribution function, this is capital  $\Phi(x)$  okay like which I have talked about here this capital  $\Phi(x)$  okay. So this is capital  $\Phi(x)$ , capital  $\Phi(x)$  is the cumulative distribution function of a small  $\phi(x)$  okay. It is denoted by given by  $-\infty$  to  $x$   $1/\sqrt{2\pi} e^{-x^2/2} dx$  and is the area bounded by the standard normal curve up to  $x$  okay. This is the graph of standard normal distribution okay, this  $\phi(x)$  okay, this is the graph okay.

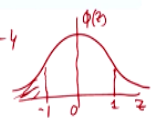
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**Example 1**

If the lifetime  $X$  of a certain kind of automobile battery is normally distributed with a mean of 4 years and a standard deviation of 1 year, and the manufacturer wishes to guarantee the battery for 3 years, what percentage of the batteries will he have to replace under the guarantee?

**Ans:** 15.87%.

$\mu=4, \sigma=1$

$$P(X < 3) = 1 - P(X \geq 3)$$
$$P(X < 3) = P(Z < -1) = \Phi(-1) = 1 - \Phi(1)$$
$$Z = \frac{X - \mu}{\sigma}$$
$$Z = \frac{X - 4}{1} = X - 4$$
$$\frac{X < 3}{Z < -1}$$


The figure shows a standard normal distribution curve with the area to the left of  $z = -1$  shaded. The curve is labeled  $\phi(z)$  and the shaded area is labeled  $Z < -1$ .

Now what we do? Let us go to first example. If the lifetime  $X$  of a certain kind of automobile battery is normally distributed with mean of 4 years. So  $\mu$  is given to be 4 years okay, standard deviation  $\sigma$  is given to be 1 year okay and the manufacturer wishes to guarantee the battery for 3 years. That means he says that the battery will at least work for 3 years okay. What percentage of the battery will he have to replace under the guarantee?

That means we want to find the percentage of those batteries which will not last for 3 years okay. So probability that  $X$  is  $< 3$  okay, this is what we want okay and probability  $X < 3$  is  $1 -$  probability that  $X$  is  $\geq 3$  okay. Now we will change this probability  $X < 3$ , we will change this variable  $X$  to  $z$  okay, so  $z = (x - \mu) / \sigma$  okay, so  $\mu$  is  $= 4$ , so  $z$  will be  $= (x - 4) / 1$  okay, so  $z = x - 4$  okay. We want the probability that  $x$  is  $< 3$  okay.

Probability that  $x < 3$  means okay, this  $x$  is  $< 3$  means  $z$  is  $< -1$  okay because  $x$  is  $< 3$ , so this  $z$  is  $< 3 - 4$  so  $z$  is  $< -1$ . So this is  $=$  probability that  $z$  is  $< -1$  okay. Now let us look at the graph okay, graph of the standard normal curve. This is standard normal curve okay 0 and  $z$   $=$  this is  $z$  okay, this is  $\phi(z)$  okay the probability density function  $\phi(z)$ . So this is  $z = -1$  here okay. As I said the area under the graph of  $\phi(z)$  okay from  $-\infty$  to  $-1$  can be found by subtracting the area from  $-\infty$  to 1 okay,  $-\infty$  to 1 from 1.

So this is  $= 1 - \Phi(1)$ . Let me write like that in terms of the capital  $\Phi$  function, so  $1 - \Phi(1)$  okay. This is  $= 1 - \Phi(1)$  okay. Now let us go to the table. See the value of  $\Phi(1)$  from there.

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Table A6  
Normal Distribution  
Values of the Standard Normal Distribution

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This is the table, you can see in this table when  $z$  is 1,  $\Phi$  is 8413 okay, this is the value of  $z$  and this is the value of capital  $\Phi$   $z$  okay. So when  $z$  is 1, this is 8413 okay, so let us put that value there.

(Refer Slide Time: 32:21)

**Example 1**

If the lifetime  $X$  of a certain kind of automobile battery is normally distributed with a mean of 4 years and a standard deviation of 1 year, and the manufacturer wishes to guarantee the battery for 3 years, what percentage of the batteries will he have to replace under the guarantee?

**Ans:** 15.87%.

$\mu = 4, \sigma = 1$

$$P(X < 3) = 1 - P(X \geq 3)$$

$$P(X < 3) = P(Z < -1) = \Phi(-1)$$

$$= 1 - \Phi(1) = 1 - 0.8413$$

$$= 0.1587 = 15.87\%$$

$z = \frac{x - \mu}{\sigma}$

$$z = \frac{3 - 4}{1} = -1$$

$z < -1$

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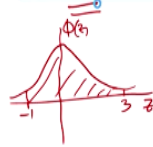
So this is  $1 - 0.8413$  okay, so this is 1587 okay. So this means that it is 15.87% okay. So he will have to replace 15.87% of the batteries because they will not last for 3 years or more okay. So this is how we will use the area under the graph of the standard normal curve to solve the problems.

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### Example 2

Specification for a certain job call for bolts with a diameter of  $0.280 \pm 0.002$  cm. If the diameters of the bolts made by some manufacturer are normally distributed with  $\mu = 0.279$  and  $\sigma = 0.001$ , what percentage of these bolts will meet specifications?

Ans: 84%



$$\begin{aligned} P(278 \leq X \leq 282) &= P(-1 \leq Z \leq 3) \\ &= \Phi(3) - \Phi(-1) \\ &= \Phi(3) - (1 - \Phi(1)) \\ &= \Phi(3) + \Phi(1) - 1 = .9987 + .8413 - 1 = .8400 = 84\% \end{aligned}$$
$$z = \frac{x - \mu}{\sigma} = \frac{278 - 279}{.001} = -1$$
$$z = \frac{x - \mu}{\sigma} = \frac{282 - 279}{.001} = 3$$

Now let us go to a specification for a certain job call for bolts with a diameter of  $0.280 \pm 0.002$  centimeter. If the diameters of the bolts made by some manufacturer are normally distributed with  $\mu = 0.279$  and  $\sigma = 0.001$ , what percentage of these bolts will meet specifications? So specifications are that the diameter of the bolt should lie between this range okay.

So let us see  $X$  should lie between  $0.280 - 0.002$ . Let us find the range of values, so we subtract first  $0.002$  and get here  $278$  okay. So  $0.278$  okay and then when we add  $0.002$  to  $0.280$ , we get  $0.282$ . So we want the percentage of bolts which will meet the specifications that means we want the probability that the  $X$  denotes the diameter of the bolts. So diameter if it lies in the range  $0.278$  to  $0.282$ , then it is acceptable okay, it needs the specification.

So now let us calculate the corresponding values of  $z$ . So  $z = x - \mu / \sigma$  okay. So we get  $z = 0.278 - \mu = 0.279 / 0.001$ . So this is  $-1$  okay and  $z = x - \mu / \sigma$  when  $x = 0.282$ , so we get  $0.279 / 0.001$ . So this is  $0.003 / 0.001$ , so this is  $3$  okay. So this is = probability that  $z$  lies between  $-1$  to  $3$  okay. Now let us look at the graph okay again. So suppose this is graph, so  $-1$  is here okay, this is your  $z$ -axis okay, this is your  $\phi(z)$  function okay, probability density function  $\phi(z)$ .

Now this probability is we have to find, so  $-1$  to  $3$  we want okay this is  $3$  here, this area we want  $-1$  to  $3$  okay. This means what? We find the area under the graph of the curve from  $-\infty$  to  $3$  and subtract the area from  $-\infty$  to  $-1$ . So this is  $\Phi(3) - \Phi(-1)$  okay and  $\Phi(-1)$  can be found by subtracting the value of  $\Phi$  at  $1$  from  $1$  okay. So  $\Phi(3) - 1 + \Phi(1)$  okay, so what



we get  $\Phi(3) + \Phi(1) - 1$  okay. Now let us see the value of  $\Phi(3)$  and  $\Phi(1)$  and then we will add them and subtract 1.

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Table A8 Normal Distribution Values of the distribution function $\Phi(z)$ (cf. (4), Sec. 23.8) $\Phi(-z) = 1 - \Phi(z)$ , $\Phi(0) = 0.5000$											
$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$	$z$	$\Phi(z)$
0.0	0.5000	0.1	0.5398	0.2	0.5793	0.3	0.6179	0.4	0.6554	0.5	0.6915
0.6	0.7257	0.7	0.7580	0.8	0.7858	0.9	0.8123	1.0	0.8379	1.1	0.8625
1.2	0.8849	1.3	0.9032	1.4	0.9192	1.5	0.9332	1.6	0.9452	1.7	0.9554
1.8	0.9641	1.9	0.9713	2.0	0.9772	2.1	0.9817	2.2	0.9854	2.3	0.9884
2.4	0.9906	2.5	0.9920	2.6	0.9929	2.7	0.9936	2.8	0.9941	2.9	0.9945
3.0	0.9949	3.1	0.9952	3.2	0.9954	3.3	0.9956	3.4	0.9957	3.5	0.9959
3.6	0.9960	3.7	0.9961	3.8	0.9962	3.9	0.9963	4.0	0.9964	4.1	0.9965
4.2	0.9966	4.3	0.9967	4.4	0.9968	4.5	0.9969	4.6	0.9970	4.7	0.9971
4.8	0.9972	4.9	0.9973	5.0	0.9974	5.1	0.9975	5.2	0.9976	5.3	0.9977
5.4	0.9978	5.5	0.9979	5.6	0.9980	5.7	0.9981	5.8	0.9982	5.9	0.9983
6.0	0.9984	6.1	0.9985	6.2	0.9986	6.3	0.9987	6.4	0.9988	6.5	0.9989
6.6	0.9990	6.7	0.9991	6.8	0.9992	6.9	0.9993	7.0	0.9994	7.1	0.9995
7.2	0.9996	7.3	0.9997	7.4	0.9998	7.5	0.9999	7.6	0.9999	7.7	0.9999
7.8	0.9999	7.9	0.9999	8.0	0.9999	8.1	0.9999	8.2	0.9999	8.3	0.9999
8.4	0.9999	8.5	0.9999	8.6	0.9999	8.7	0.9999	8.8	0.9999	8.9	0.9999
9.0	0.9999	9.1	0.9999	9.2	0.9999	9.3	0.9999	9.4	0.9999	9.5	0.9999
9.6	0.9999	9.7	0.9999	9.8	0.9999	9.9	0.9999	10.0	0.9999	10.1	0.9999
10.2	0.9999	10.3	0.9999	10.4	0.9999	10.5	0.9999	10.6	0.9999	10.7	0.9999
10.8	0.9999	10.9	0.9999	11.0	0.9999	11.1	0.9999	11.2	0.9999	11.3	0.9999
11.4	0.9999	11.5	0.9999	11.6	0.9999	11.7	0.9999	11.8	0.9999	11.9	0.9999
12.0	0.9999	12.1	0.9999	12.2	0.9999	12.3	0.9999	12.4	0.9999	12.5	0.9999
12.6	0.9999	12.7	0.9999	12.8	0.9999	12.9	0.9999	13.0	0.9999	13.1	0.9999
13.2	0.9999	13.3	0.9999	13.4	0.9999	13.5	0.9999	13.6	0.9999	13.7	0.9999
13.8	0.9999	13.9	0.9999	14.0	0.9999	14.1	0.9999	14.2	0.9999	14.3	0.9999
14.4	0.9999	14.5	0.9999	14.6	0.9999	14.7	0.9999	14.8	0.9999	14.9	0.9999
15.0	0.9999	15.1	0.9999	15.2	0.9999	15.3	0.9999	15.4	0.9999	15.5	0.9999
15.6	0.9999	15.7	0.9999	15.8	0.9999	15.9	0.9999	16.0	0.9999	16.1	0.9999
16.2	0.9999	16.3	0.9999	16.4	0.9999	16.5	0.9999	16.6	0.9999	16.7	0.9999
16.8	0.9999	16.9	0.9999	17.0	0.9999	17.1	0.9999	17.2	0.9999	17.3	0.9999
17.4	0.9999	17.5	0.9999	17.6	0.9999	17.7	0.9999	17.8	0.9999	17.9	0.9999
18.0	0.9999	18.1	0.9999	18.2	0.9999	18.3	0.9999	18.4	0.9999	18.5	0.9999
18.6	0.9999	18.7	0.9999	18.8	0.9999	18.9	0.9999	19.0	0.9999	19.1	0.9999
19.2	0.9999	19.3	0.9999	19.4	0.9999	19.5	0.9999	19.6	0.9999	19.7	0.9999
19.8	0.9999	19.9	0.9999	20.0	0.9999	20.1	0.9999	20.2	0.9999	20.3	0.9999
20.4	0.9999	20.5	0.9999	20.6	0.9999	20.7	0.9999	20.8	0.9999	20.9	0.9999
21.0	0.9999	21.1	0.9999	21.2	0.9999	21.3	0.9999	21.4	0.9999	21.5	0.9999
21.6	0.9999	21.7	0.9999	21.8	0.9999	21.9	0.9999	22.0	0.9999	22.1	0.9999
22.2	0.9999	22.3	0.9999	22.4	0.9999	22.5	0.9999	22.6	0.9999	22.7	0.9999
22.8	0.9999	22.9	0.9999	23.0	0.9999	23.1	0.9999	23.2	0.9999	23.3	0.9999
23.4	0.9999	23.5	0.9999	23.6	0.9999	23.7	0.9999	23.8	0.9999	23.9	0.9999
24.0	0.9999	24.1	0.9999	24.2	0.9999	24.3	0.9999	24.4	0.9999	24.5	0.9999
24.6	0.9999	24.7	0.9999	24.8	0.9999	24.9	0.9999	25.0	0.9999	25.1	0.9999
25.2	0.9999	25.3	0.9999	25.4	0.9999	25.5	0.9999	25.6	0.9999	25.7	0.9999
25.8	0.9999	25.9	0.9999	26.0	0.9999	26.1	0.9999	26.2	0.9999	26.3	0.9999
26.4	0.9999	26.5	0.9999	26.6	0.9999	26.7	0.9999	26.8	0.9999	26.9	0.9999
27.0	0.9999	27.1	0.9999	27.2	0.9999	27.3	0.9999	27.4	0.9999	27.5	0.9999
27.6	0.9999	27.7	0.9999	27.8	0.9999	27.9	0.9999	28.0	0.9999	28.1	0.9999
28.2	0.9999	28.3	0.9999	28.4	0.9999	28.5	0.9999	28.6	0.9999	28.7	0.9999
28.8	0.9999	28.9	0.9999	29.0	0.9999	29.1	0.9999	29.2	0.9999	29.3	0.9999
29.4	0.9999	29.5	0.9999	29.6	0.9999	29.7	0.9999	29.8	0.9999	29.9	0.9999
30.0	0.9999	30.1	0.9999	30.2	0.9999	30.3	0.9999	30.4	0.9999	30.5	0.9999
30.6	0.9999	30.7	0.9999	30.8	0.9999	30.9	0.9999	31.0	0.9999	31.1	0.9999
31.2	0.9999	31.3	0.9999	31.4	0.9999	31.5	0.9999	31.6	0.9999	31.7	0.9999
31.8	0.9999	31.9	0.9999	32.0	0.9999	32.1	0.9999	32.2	0.9999	32.3	0.9999
32.4	0.9999	32.5	0.9999	32.6	0.9999	32.7	0.9999	32.8	0.9999	32.9	0.9999
33.0	0.9999	33.1	0.9999	33.2	0.9999	33.3	0.9999	33.4	0.9999	33.5	0.9999
33.6	0.9999	33.7	0.9999	33.8	0.9999	33.9	0.9999	34.0	0.9999	34.1	0.9999
34.2	0.9999	34.3	0.9999	34.4	0.9999	34.5	0.9999	34.6	0.9999	34.7	0.9999
34.8	0.9999	34.9	0.9999	35.0	0.9999	35.1	0.9999	35.2	0.9999	35.3	0.9999
35.4	0.9999	35.5	0.9999	35.6	0.9999	35.7	0.9999	35.8	0.9999	35.9	0.9999
36.0	0.9999	36.1	0.9999	36.2	0.9999	36.3	0.9999	36.4	0.9999	36.5	0.9999
36.6	0.9999	36.7	0.9999	36.8	0.9999	36.9	0.9999	37.0	0.9999	37.1	0.9999
37.2	0.9999	37.3	0.9999	37.4	0.9999	37.5	0.9999	37.6	0.9999	37.7	0.9999
37.8	0.9999	37.9	0.9999	38.0	0.9999	38.1	0.9999	38.2	0.9999	38.3	0.9999
38.4	0.9999	38.5	0.9999	38.6	0.9999	38.7	0.9999	38.8	0.9999	38.9	0.9999
39.0	0.9999	39.1	0.9999	39.2	0.9999	39.3	0.9999	39.4	0.9999	39.5	0.9999
39.6	0.9999	39.7	0.9999	39.8	0.9999	39.9	0.9999	40.0	0.9999	40.1	0.9999
40.2	0.9999	40.3	0.9999	40.4	0.9999	40.5	0.9999	40.6	0.9999	40.7	0.9999
40.8	0.9999	40.9	0.9999	41.0	0.9999	41.1	0.9999	41.2	0.9999	41.3	0.9999
41.4	0.9999	41.5	0.9999	41.6	0.9999	41.7	0.9999	41.8	0.9999	41.9	0.9999
42.0	0.9999	42.1	0.9999	42.2	0.9999	42.3	0.9999	42.4	0.9999	42.5	0.9999
42.6	0.9999	42.7	0.9999	42.8	0.9999	42.9	0.9999	43.0	0.9999	43.1	0.9999
43.2	0.9999	43.3	0.9999	43.4	0.9999	43.5	0.9999	43.6	0.9999	43.7	0.9999
43.8	0.9999	43.9	0.9999	44.0	0.9999	44.1	0.9999	44.2	0.9999	44.3	0.9999
44.4	0.9999	44.5	0.9999	44.6	0.9999	44.7	0.9999	44.8	0.9999	44.9	0.9999
45.0	0.9999	45.1	0.9999	45.2	0.9999	45.3	0.9999	45.4	0.9999	45.5	0.9999
45.6	0.9999	45.7	0.9999	45.8	0.9999	45.9	0.9999	46.0	0.9999	46.1	0.9999
46.2	0.9999	46.3	0.9999	46.4	0.9999	46.5	0.9999	46.6	0.9999	46.7	0.9999
46.8	0.9999	46.9	0.9999	47.0	0.9999	47.1	0.9999	47.2	0.9999	47.3	0.9999
47.4	0.9999	47.5	0.9999	47.6	0.9999	47.7	0.9999	47.8	0.9999	47.9	0.9999
48.0	0.9999	48.1	0.9999	48.2	0.9999	48.3	0.9999	48.4	0.9999	48.5	0.9999
48.6	0.9999	48.7	0.9999	48.8	0.9999	48.9	0.9999	49.0	0.9999	49.1	0.9999
49.2	0.9999	49.3	0.9999	49.4	0.9999	49.5	0.9999	49.6	0.9999	49.7	0.9999
49.8	0.9999	49.9	0.9999	50.0	0.9999	50.1	0.9999	50.2	0.9999	50.3	0.9999
50.4	0.9999	50.5	0.9999	50.6	0.9999	50.7	0.9999	50.8	0.9999	50.9	0.9999
51.0	0.9999	51.1	0.9999	51.2	0.9999	51.3	0.9999	51.4	0.9999	51.5	0.9999
51.6	0.9999	51.7	0.9999	51.8	0.9999	51.9	0.9999	52.0	0.9999	52.1	0.9999
52.2	0.9999	52.3	0.9999	52.4	0.9999	52.5	0.9999	52.6	0.9999	52.7	0.9999
52.8	0.9999	52.9	0.9999	53.0	0.9999	53.1	0.9999	53.2	0.9999	53.3	0.9999
53.4	0.9999	53.5	0.9999	53.6	0.9999	53.7	0.9999	53.8	0.9999	53.9	0.9999
54.0	0.9999	54.1	0.9999	54.2	0.9999	54.3	0.9999	54.4	0.9999	54.5	0.9999
54.6	0.9999	54.7	0.9999	54.8	0.9999	54.9	0.9999	55.0	0.9999	55.1	0.9999
55.2	0.9999	55.3	0.9999	55.4	0.9999	55.5	0.9999	55.6	0.9999	55.7	0.9999
55.8	0.9999	55.9	0.9999	56.0	0.9999	56.1	0.9999	56.2	0.9999	56.3	0.9999
56.4	0.9999	56.5	0.9999	56.6	0.9999	56.7	0.9999	56.8	0.9999	56.9	0.9999
57.0	0.9999	57.1	0.9999	57.2	0.9999	57.3	0.9999	57.4	0.9999	57.5	0.9999

resistors okay. Then, we want the probability that X lies between 98 and 102 okay. So this is=probability that z lies between the  $z=x-\mu/\sigma$ , so z will have the values  $98-100/2$ , so that is=-1 okay.

And when X is 102, it is +1,  $102-100/2$ , so +1, so z lies between -1 and +1 okay. So now what we will do, let us see okay. So this is your -1 okay. This is +1 okay. We want the area under the standard normal curve from -1 to +1 okay. So what we will do? We will find the area under the standard normal curve from  $-\infty$  to 1 and from that we subtract the area under the standard normal curve from  $-\infty$  to -1.

So this is  $\Phi(1)-\Phi(-1)$  okay and  $\Phi(-1)$  we know is  $1-\Phi(1)$ . So  $\Phi(1)-1+\Phi(1)$  okay. So this 2 times  $\Phi(1)-1$  okay and we have seen  $\Phi(1)=0.8413$ , so we can use this value here. So 2 times  $0.8413-1$  okay, so this is  $1.6826-1$  okay. So this is 0.6826 meaning that it is 68.26% okay. So 68.26% resistors will have resistance between 98 ohms and 102 ohms okay.

(Refer Slide Time: 41:12)

**Example 4**

Sacks of grain packed by an automatic loading machine have an average weight of 114 lbs. It is discovered that 10 percent of the bags are over 116 lbs. Find the standard deviation?

**Ans:** 1.56

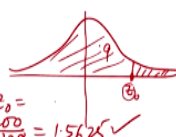
$\mu = 114 \text{ lbs}$   
 $\sigma = ?$


$z = \frac{x - \mu}{\sigma} = \frac{116 - 114}{\sigma} = \frac{2}{\sigma}$

$P(X > 116) = 0.1$  given

$P(Z > \frac{2}{\sigma}) = 0.1$

or  $P(Z \leq \frac{2}{\sigma}) = 0.9 \Rightarrow Z_0 = \frac{2}{\sigma} = 1.28 \Rightarrow \sigma = \frac{2}{1.28} = \frac{200}{128} = 1.5625 \checkmark$





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Now we go to this last question. Sacks of grain packed by an automatic loading machine have an average weight of 114 pounds. So here we are given  $\mu$ ,  $\mu=114$  pounds. It is discovered that 10% of the bags are over 116 pounds. Find the standard deviation. We have to find  $\sigma$  okay; we have to find  $\sigma$  here. We know that  $z$  is  $x-\mu/\sigma$  okay. We are given that probability that  $x$  is more than 116 pounds okay is=0.1 okay, this is given to us.

The 10% of the bags are over 116 pounds,  $x$  denotes the weight of the bag, so probability that  $x$  is  $>116$  is=0.1 which is given to us. So  $z$  will be=let us see the corresponding value of  $z$ , so

$z$  will be  $=116-114/\sigma$ . So  $2/\sigma$  okay, so probability that  $z$  is  $>2/\sigma$  is  $=0.1$  okay. Now we have to see from the table that for what value of  $z$  we get the probability to be  $0.1$ , you see let us look at this okay.

Suppose this  $0.1$  is the area okay, so suppose this value of  $z$  okay and  $z$  is  $>2/\sigma$ . So this area is given to us okay. We want to find the value of  $z$  for which when  $z$  is  $>2/\sigma$  okay  $z$  is  $>z_0$  suppose this is  $z_0$  okay. So when  $z$  is  $>z_0$ , the area becomes  $0.1$ , so that means that the area to the left of  $z_0$  okay. Suppose  $2/\sigma$  is  $z_0$  okay, so we want this area, this area is  $0.9$  okay. So let us find the under the graph of standard normal curve for what value of  $z_0$  okay we have the area  $0.9$  okay.

So let us go this one. So here you can see. This is I think  $1.26$ ,  $1.26$  gives you the area close to  $0.9$  okay. So we have let us take, so  $z_0$  is  $1.26$  okay. So then this is or you can say probability that  $z$  is  $\leq z_0$  okay,  $z_0$  is  $=2/\sigma$  okay is  $=0.9$ . So then we obtain  $z_0=1.2$  sigma, so  $z_0=2/\sigma$  implies that  $2/\sigma=1.26$  or  $\sigma=2/1.26$ , so how much is that? This is  $200/126$  okay.

So what we have? And then we have  $1.28$ , so  $2/1.28$  okay, so we get okay so  $1.5625$  okay. Let us put the exact value okay. So what we get is  $1.5625$ . So the standard deviation is  $1.5625$  okay. So with this I would like to end my lecture. Thank you very much for your attention.