Advanced Engineering Mathematics
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Lecture - 43
Conditional Probability

Hello friends. Welcome to my lecture on conditional probability. So far we can see that the probability is without any condition or without prior knowledge of the occurrence of other events. Therefore, we call them unconditional probabilities. Now we shall consider the probabilities of events or simple events when the knowledge of the occurrence of some other event or simple events is available.

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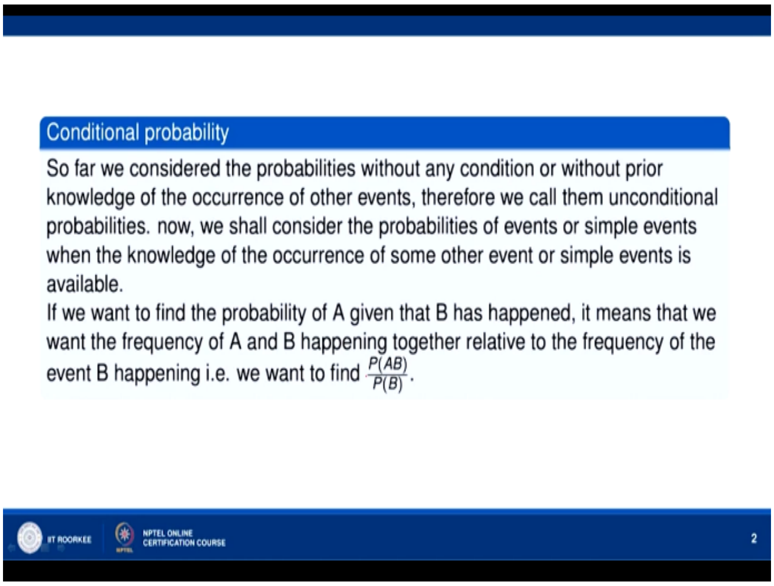

If we want to find the probability of A given that B has happened it means that we want the frequency of A and B happening together relative to the frequency of the event B happening that is we want to find P AB/PB.

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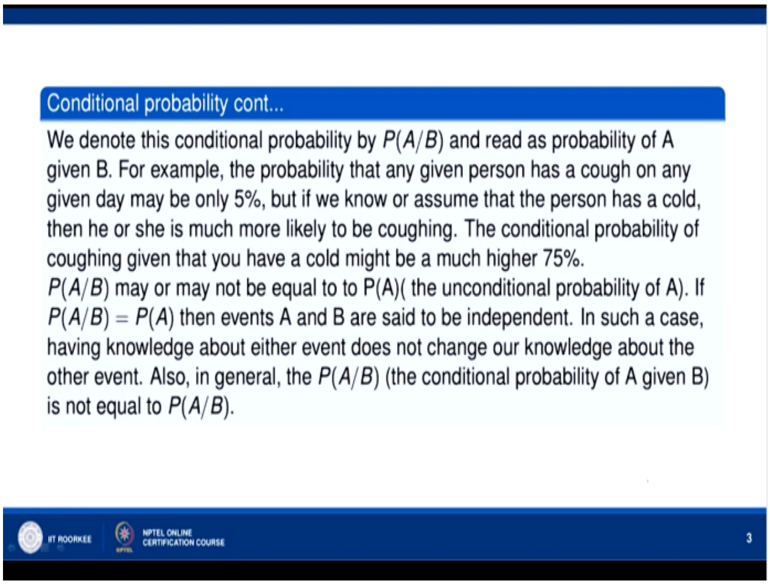

We denote this conditional probability by P A given B and read as probability of A given B. For example, the probability that any given person has a cough on any given day may be only 5%, but if we know or assume that the person has a cold, then he or she is much more likely to be coughing. The conditional probability of coughing given that you have a cold might be a much higher 75%.

Probability of A given B may or may not be equal to PA. PA is the unconditional probability of A. If P A/B=PA then events A and B are called independent. In such a case having knowledge about either event does not change our knowledge about the other event. Also in general, the probability of A given B the conditional probability of A given B is not equal to the probability of B given A okay. So let us now go to the formal definition.

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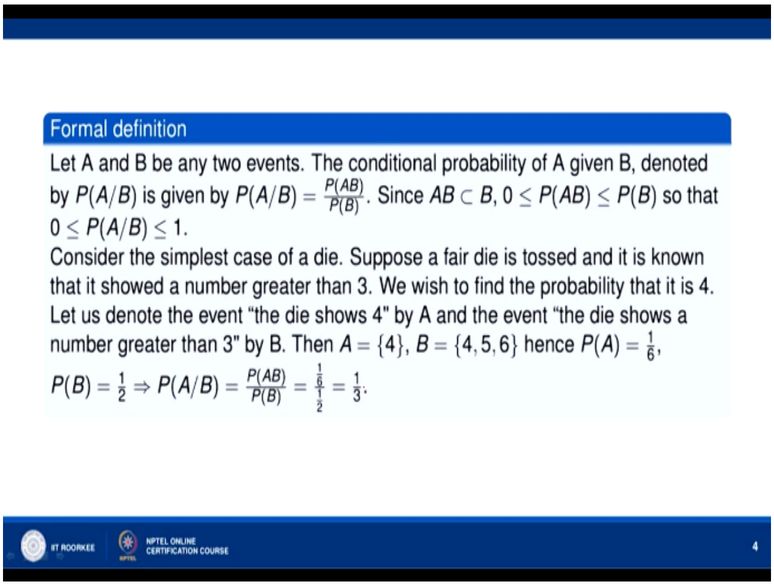

Let A and B be any two events. The conditional probability of A given B, denoted by P A/B is given by P A/B=P AB/PB. Now since A intersection B, A intersection B is a subset of B, so probability of A intersection B is<=probability of B because we have already shown that if A is subset of B then probability of A is<=probability of B. So 0 is<=P A intersection B<=PB and therefore P AB/PB is<=1.

So 0 is<=probability of A given B<=1. Now for example, consider the simple case of a die okay. Suppose a fair die is tossed and it is known that it showed a number >3. We wish to find the probability that the number is 4. So let us denote the event the die shows 4 by A and the event the die shows a number >3 okay by B. Then, A is=set 4 and B is=the set consisting of 4, 5, and 6 because B is the event that the number is>3 okay.

So B is=4, 5, 6. Now therefore probability of A is=1/6 while probability of B is 1/2 because there are 3 elements here, 4, 5, 6 and the probability of B will be 3/6 so 1/2. Now probability of A given B is probability of AB/PB okay probability of AB. Now A is 4, B is 4, 5, 6, so A intersection B is the set consisting of 4 only and the probability of just 4 is 1/6. So probability of AB is 1/6/probability of B is 1/2 and therefore the probability that the number is 4 given that the number shows is>3 will be=1/3.

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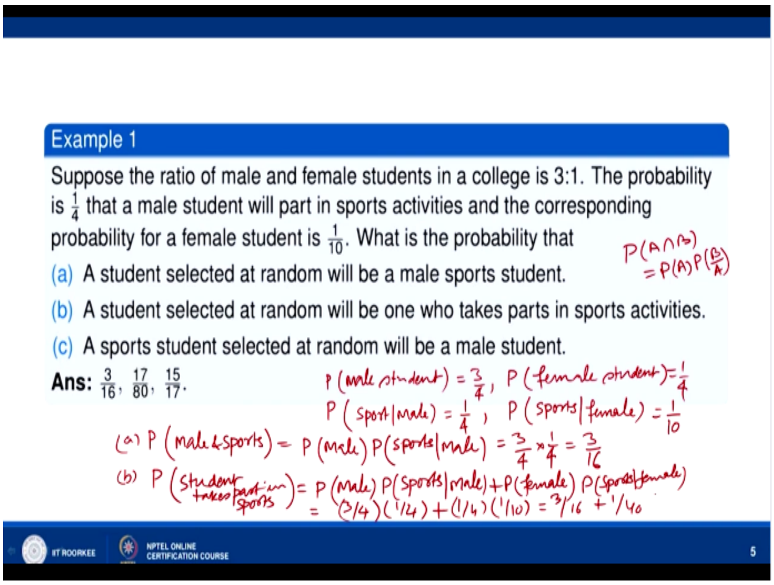

Now let us do some questions on conditional probability. Suppose the ratio of male and female students in a college is 3:1 okay. So probability of a male student is=3/4, probability of a female student will be 1/4. Now the probability is 4 that a male student will take part in sports activities okay. So probability that the male student takes part in sports activities, so sports and the student is male okay, this is=1/4.

While the same corresponding probability for a female student is 1/10 okay. Now what is the probability that a student selected at random will be a male sports student okay. So probability the student is male and play the sports, male and sports. So this will be equal to by multiplication theorem we know that probability of A intersection is=PA*P B/A okay. So probability of male*probability that a sports male okay.

So this will be=probability of male student is 3/4, probability that the male student takes part in the sports activities is 1/4. So the probability is 3/16. So this is part a. In part b, a student selected at random will be 1 who takes part in sports activities okay. So probability of a student takes part in sports okay. Now this will be=probability now this student could be a male as well as female.

So probability of male*probability of sports given that is a male+probability of female*probability of sports given that it is a female. So this is probability of male, this is 3/4*this one is 1/4 probability that the male student takes part in sports activities. So 1/4+probability that the student is a female 1/4*the female student takes part in the sports activities that is 1/10 okay, so this is 3/16+1/40 okay.

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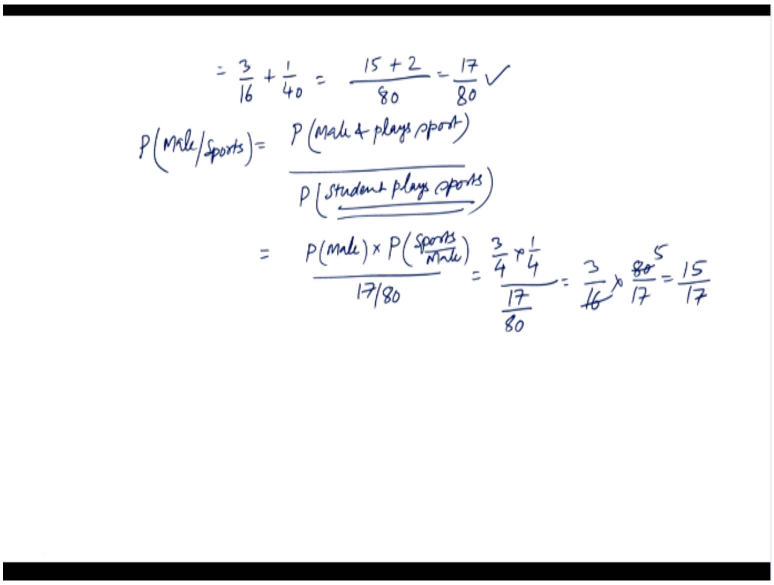

So let me find how much it is, 3/16+1/40 so LCM will be 80 okay. So 16*5=80, so 5*3=15+2 okay so 17/80 okay. Now third part is a sports student selected at random will be a male student. So we are given that the student who has been selected is a sports student okay but we want the probability that the student is a male okay. So we have to find probability of a male given that he takes part in sports activities okay.

So this is probability that the student takes parts in sports activities, the student plays sports and then probability of male and play the sports. So probability of male and play the sports/probability student plays sports. So this part we have just now found, it is probability that a student plays sports is 17/80 okay and probability that the student is a male and plays sports is probability of being a male*probability that sports given that it is a male okay.

So this is probability of A intersection B okay. This is probability of A intersection B is probability of A*probability of B given A. So probability of A male, probability of male student is 3/4 and then probability of sports given that male 1/4 okay, so 3/4*1/4/17/80, so this is 3/16*80/17 and what we get 15, so 16*5 80 so we have 15/17 okay. So this is the probability of that a selected student is sports student. A sports student is selected and it is the male student.

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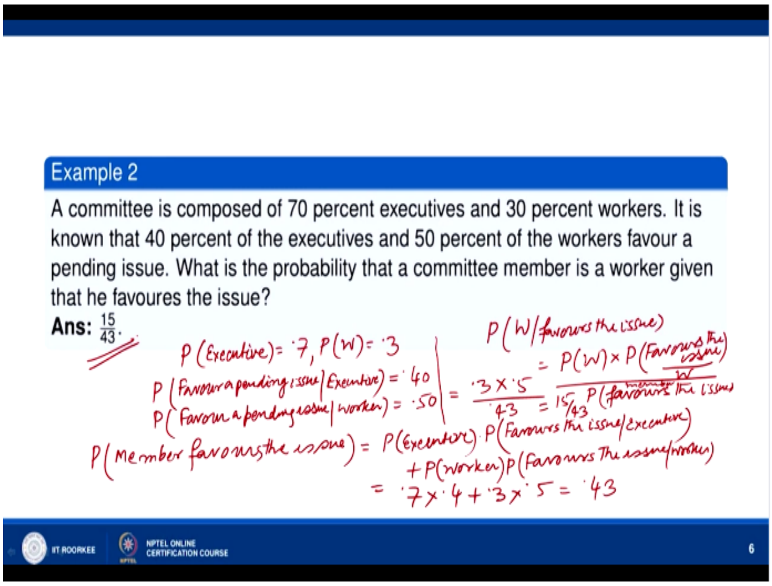

Now we go to second question. A committee is composed of 70% executives. So probability of an executive is 0.7, probability of a worker is 0.3 okay. If the member of the committee is executive, the probability if 0.7 okay. If the member is a worker, probability is 0.3 okay. It is known that 40% of the executives and 50% of the workers favour a pending issue. So probability favour a pending issue given that the member is executive okay=0.40 okay.

Probability that okay it is given that 40% of the executives and 50% of the workers favour pending issues. So probability that the member which is executive favours a pending issue is 0.40 and probability that the member is a worker and it favours a pending issue is 0.50. What is the probability that a committee member is a worker given that he favours the issue?

So probability of member favours the issue okay, a committee member favours the issue. It is that member could be executive as well as a worker. So probability of executive *probability of favours the issue given that it is executive okay+probability that the member is a worker*favours the issue given that it is a worker okay. So this will be=0.7*favours the issue 0.4+probability it is a worker 0.3*favours the issue 0.5.

So this is 0.28+0.15 means 0.43 okay. Now we are given that the members favours the issue, we have to find that the probability that the member is the worker okay. So probability that the member is the worker given that favours the issue okay. So this is=probability of w*probability that he favours the issue and is a worker/probability that member favours the issue okay.

So PW=0.3, P favours the issue and is a worker given that it is a worker 0.5 okay. So what we have? This will be=so this is=PW is 0.3*0.5/0.43, so this is 15/43, 0.15/0.43 is 15/43. So the answer is 15/43.

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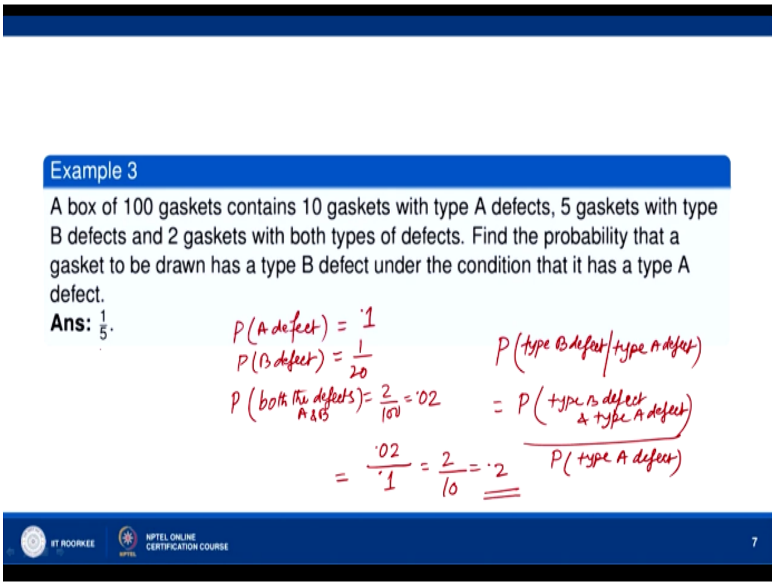

Now a box of 100 gaskets contains 10 gaskets with type A defects okay. So probability of gasket having A defect is=10/100 means 0.1 okay. Probability of a gasket having B defect is=5/100 means 1/20. Probability of a gasket having both types of defect okay, so probability both the defects, probability of a gasket having both the defects A and B is=2/100 means 0.02 okay.

And the probability that a gasket to be drawn has a type B defect under the condition that it has a type A defect okay. So it is given that we have to find the probability that the type B defect given that it has a type A defect. So by conditional probability we need to get the probability that the gasket should have type B defect intersection type A defect so and type A defect both the defects/the probability that it has type A defect okay.

So probability that a gasket has both A and B defect is 2/100 okay, so 0.02/this is type A defect means 0.1 okay, so we have 2/10 okay. So this is=0.2 that is 20% okay. So the probability is 0.2 or 1/5 okay.

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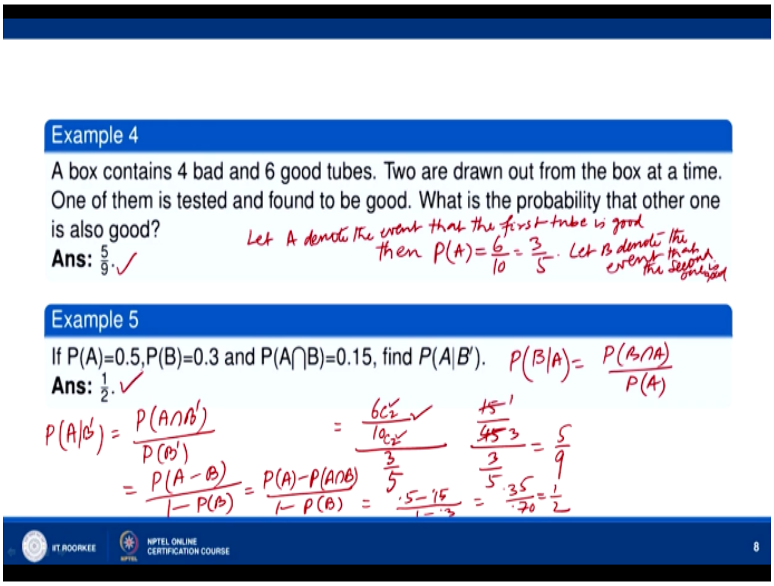

Now a box contains 4 bad and 6 good tubes. Two are drawn out from the box at a time. One of them is tested and found to be good. What is the probability that the other one is also good okay? So let A denotes the event that the first tube is good. Then, PA=6 are good okay 6 are good, so we have 6/10 or we can say 3/5 okay and then let we denote the event that the second one is also good okay.

Then, probability of B given A okay B given A is=probability of A intersection B, B intersection A we can say/PA okay. Now PB intersection A is what 10c2 no 6c2/10c2 okay/3/5 okay. We draw two tubes from the box okay which are both good okay. So 6c2/10c2 will be the probability. So 6c2 is 6*5/2 that is 15/10*9/2 that is 45 okay. So this is 15 okay 6c2 is 15 and this is 10c2 is 45/3/5, so how much is this?

This is 1/3 okay, so we have 5/9 okay. So this is the answer. Now if PA=0.5, PB=0.3, P A intersection B=0.15. Then, we want to find P A intersection B dash sorry P A given B dash that means we want the probability of P A intersection B dash/PB dash. P A intersection B dash is P A-B/PB dash is 1/PB. P A-B is=PA-P A intersection B/1-PB. PA is=0.5, P A intersection B is 0.15/1-PB is 1-0.3. So this is 0.35/0.70 means it is 1/2 okay. So we get this probability okay.

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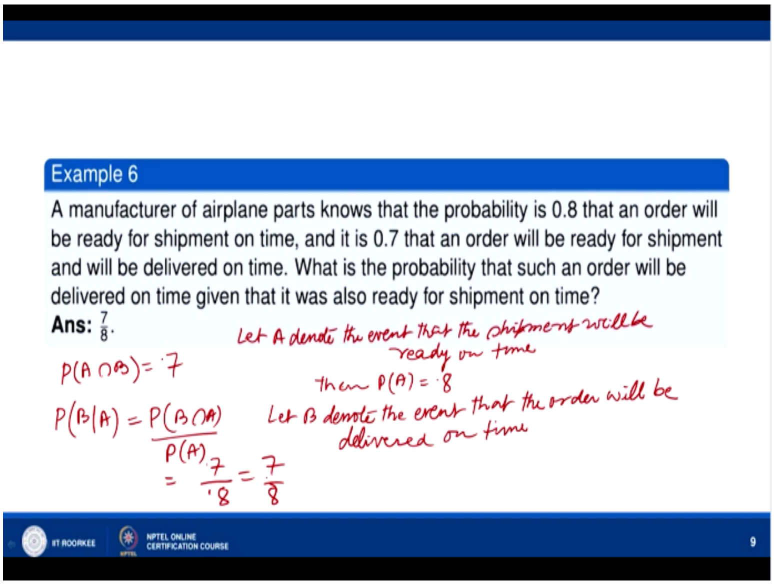

Now we go to sixth question. A manufacturer of airplane parts knows that the probability is 0.8 that an order will be ready for shipment on time. So let A denotes the event that the shipment will be ready on time. Then, PA is=0.8 okay. Let us denote B by the event that it will be delivered on time okay. Let B denote the event that the order will be delivered on time okay. So then what we want?

What is the probability? 0.7 is the probability that an order will be ready for shipment and will be delivered on time okay. So P A intersection B is given. P A intersection B is=0.7 okay. What is the probability that such an order will be delivered on time given that it was ready for shipment okay. So we want probability of B okay, probability of B given A. So this will be=probability of B intersection A/probability of A. So this is 0.7/0.8, so this is 7/8.

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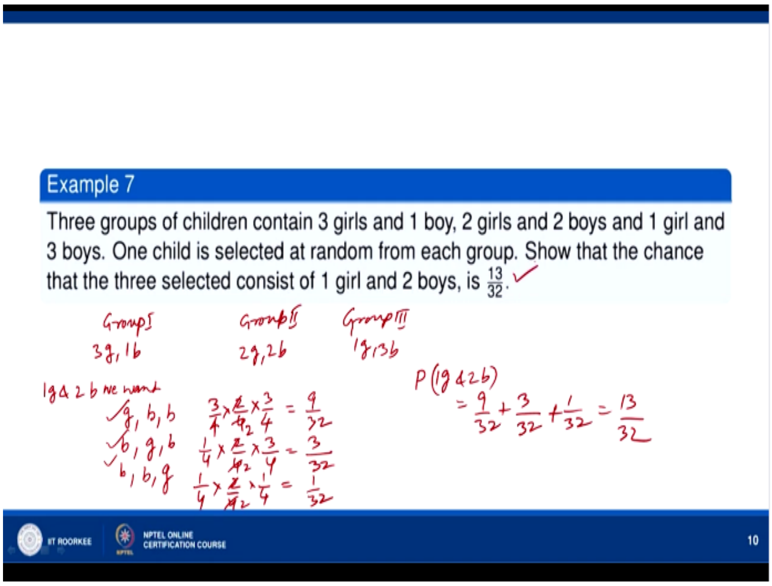

Now three groups of children contain 3 girls and 1 boy, 2 girls and 2 boys and 1 girl and 3 boys, so let us say group I, group II, group III. Here there are 3 girls and 1 boy okay, g means girl, b means boy, 2 girls and 2 boys and here 1 girl and 3 boys okay. One child is selected at random from each group so that the chance that the 3 selected consist of 1 girl and 2 boys okay, so we want 1 girl and 2 boys okay.

So 1 girl and 2 boys we want, so how many possible combinations are there from group I okay. Girl from group I, so I will write in the order okay, so girl from group I, boy from group II and boy from group III this is one combination. Then, boy from group I, girl from group II and then boy from group III this is second combination. Third can be boy from group I, boy from group II and girl from group III okay.

Now having a girl from group I, so the probability of having a girl from group I is 3/4, having a boy from group II is 2/4, having a boy from group III is 3/4. Here having a boy from group I is 1/4, having a girl from group II is 2/4, having a boy from group III is 3/4 and here having a boy from group I is 1/4, having a boy from group II is 2/4, having a girl from group III is 1/4 okay. So this is how much?

This is we can cancel out, so this is 2 here. So this will be 9/32. Here what we will get? So 3/32 and how much here we get 1/32 okay. So there are 3 possibilities, 1, 2, 3, okay. So probability of having a girl and two boys is=9/32+3/32+1/32 is 13/32 and we get the answer okay.

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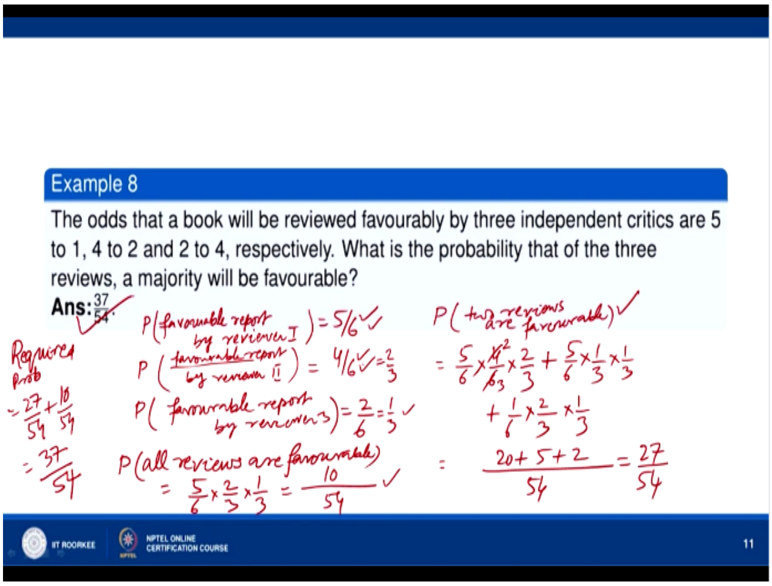

Now we go to this question. The odds that a book will be reviewed favourably by 3 independent critics are 5 to 1, 4 to 2 and 2 to 4 okay. So probability that the review will be in favour okay is 5 to 1 means 5/6 okay, so favourable report by reviewer 1 has the probability 5/6, probability of a favourable report by reviewer 2 okay. So I am taking the same thing favourable report by reviewer 2.

So this is 4 to 2 that means 4/6 and similarly probability of a favourable report by reviewer 3 is=2/6 okay. Now what is the probability that of the 3 reviews okay and majority will be favourable. So probability that 2 reviews are favourable. Let us first find this probability. Probability that 2 reviews are favourable, so this is let us say this is favourable, this is favourable and this is not favourable, so 5/6*4/6, this is not favourable.

So this will be this is 1/3 okay, so this means 2/3 okay. So this is favourable, this is favourable, this is not favourable+now this is favourable 5/6, this is not favourable, this is 2/3. So this is not favourable means 1/3 and this is favourable means 1/3. Then, suppose this is not favourable so 1/6, this is favourable 2/3 and this is also 1/3 okay. So we have so this is 2/3 okay. So this is 54 okay, 54 5*2=10, 10*2=20, this is 5 here okay, this is 2 here.

We have here 2*2=4, 4*5=20, then we have here 5, here we have, so this is 27 okay. Then, all 3 give the favourable report, all reviews are favourable. So this will be=5/6+2/3+1/3, so this will be=54. No, not 54, we have to multiply, 5/6*2/3*1/3. So we have 5*2=10/54 okay, so either this or this okay. If it happens the majority will be favourable okay. So required probability is=27/54+10/54. This gives us 37/54, so this answer okay.

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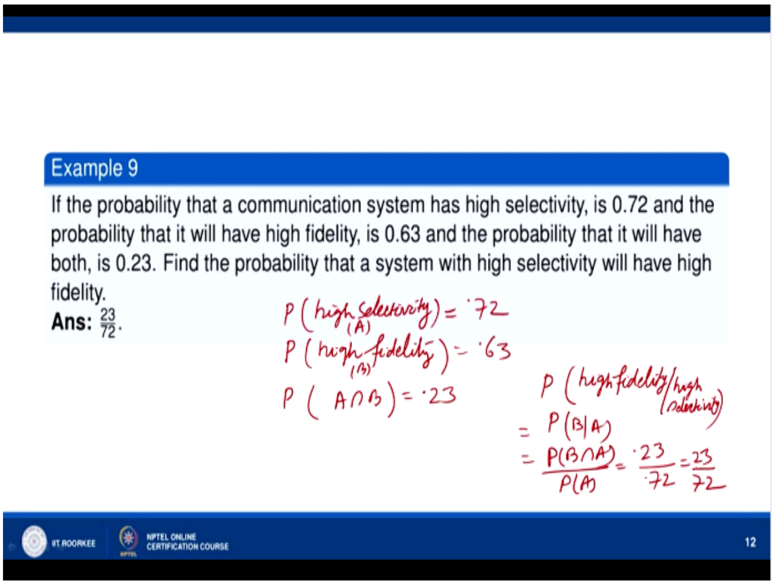

Now if the probability that a communication system has high selectivity is 0.72 okay, so high selectivity, this is 0.72 and the probability that it will have high fidelity, this is 0.63 and the probability that it will have both okay. So let us say this is A event, this is B then probability of A intersection B is=0.23 okay. Now find the probability that a system with high selectivity will have high fidelity.

So probability of high fidelity given that it has high selectivity, so this means that we want the probability of B given A which is=probability of B intersection A/probability of A. So this is=0.23/probability of A that is 0.72 and we get 23/72 okay.

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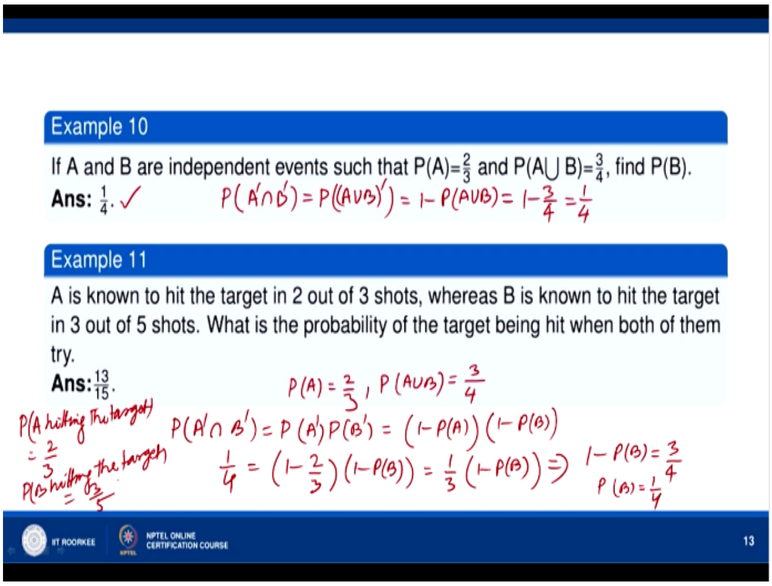

Now if A and B are independent events such that probability of A is 2/3, probability of A union B is 3/4. Let us find probability of B okay. So probability of A dash intersection B dash okay, probability of A dash intersection B dash is what, probability of A union B complement okay, this is=1-probaility of A union B and this is 1-3/4, so we get 1/4. We have got the probability of A dash intersection B dash.

Now probability of we are given the probability of A okay, probability of A is 2/3 okay. Probability of A union B okay is 3/4. Now probability of A intersection B okay, we need to have the probability of we have found the probability of A dash intersection B dash. A and B are independent events, so A dash and B dash are also independent events okay. So probability of A dash intersection B dash okay is=P A dash intersection P B dash okay.

It can be shown that if A and B are independent events, then A dash and B dash are also independent events. So probability of A dash intersection B dash is PA dash intersection PB dash*PB dash. So that is PA dash means 1-PA*1-PB okay. So what we will get here, probability of A dash intersection B dash is 1/4, 1/4=1-we need to find PB okay. So 1/4=1-2/3*1-PB, this is 1/3*1-PB.

And this gives you what, 1-PB=3/4. So PB=1/4 okay. So we get this 1/4. Now A is known to hit the target in 2 out 3 shots okay. So probability of A hitting the target, it is =2/3. Probability of B hitting the target, it is 3/5 okay. What is the probability of the target being hit when both of them try okay? So target will be hit if one of the two try hits the target okay.

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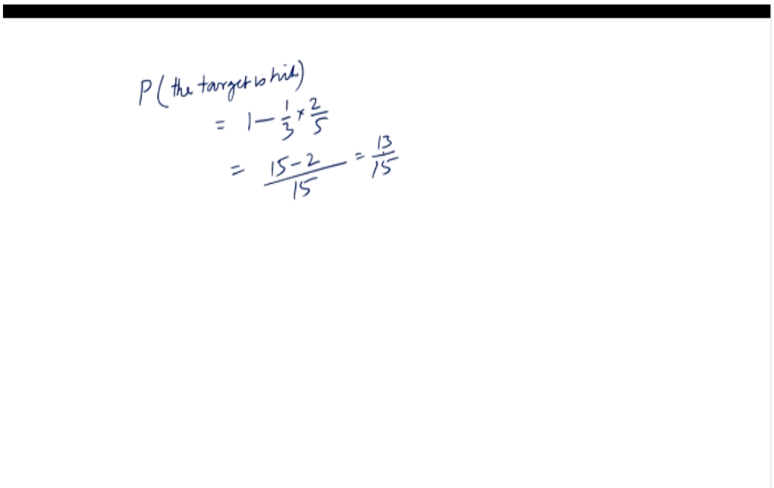

Probability of the target is hit okay. The probability that the target is hit is=1-probability that no one hits the target okay. So that will give you 2/3 means A does not hit the target it is 1/3 and B does not hits the target is 2/5, so 1/3*2/5. So this is 13/15 okay.

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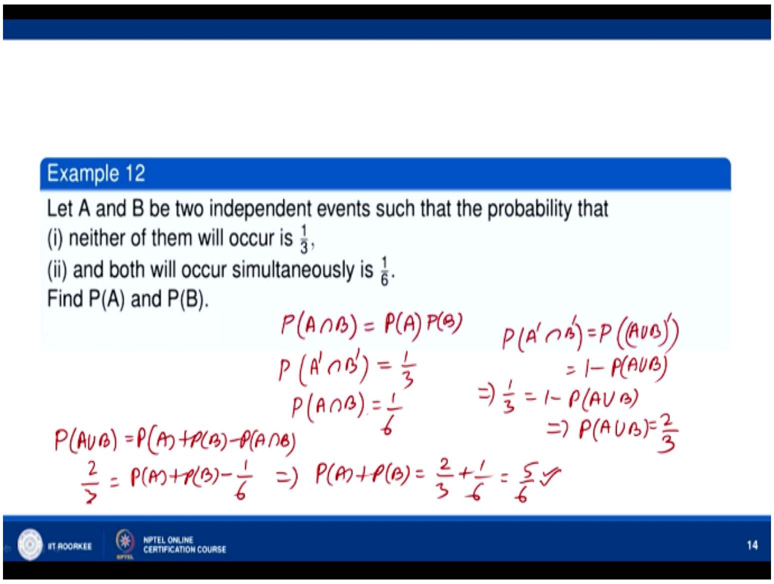

Now we go to this question. Let A and B be two independent events. So P A intersection B is=PA intersection PB okay. P A intersection B is=PA*PB because A and B are independent events such that the probability that neither of them will occur. P A dash intersection B dash neither A occurs nor B occurs is=1/3 okay and both of them occur simultaneously means A intersection B probability of A intersection B is 1/6 okay.

We have to find PA and PB okay. P A dash intersection B dash gives you probability of A union B dash okay which is=1-P A union B okay. So P A dash intersection B dash is=1/3=1-P A intersection B. So this gives you P A intersection B=2/3 P A union B sorry okay, P A union B=2/3. Now P A union B is=PA+PB-P A intersection B okay. P A union B is 2/3=PA+PB-P A intersection B is 1/6 okay.

So this gives you PA+PB=2/3+1/6. So this will be 4/6+1/6 means 5/6 okay. Now we know the value of PA+PB and we also know the value of P A intersection B that is PA*PB okay. From that we can easily find the values of PA and PB.

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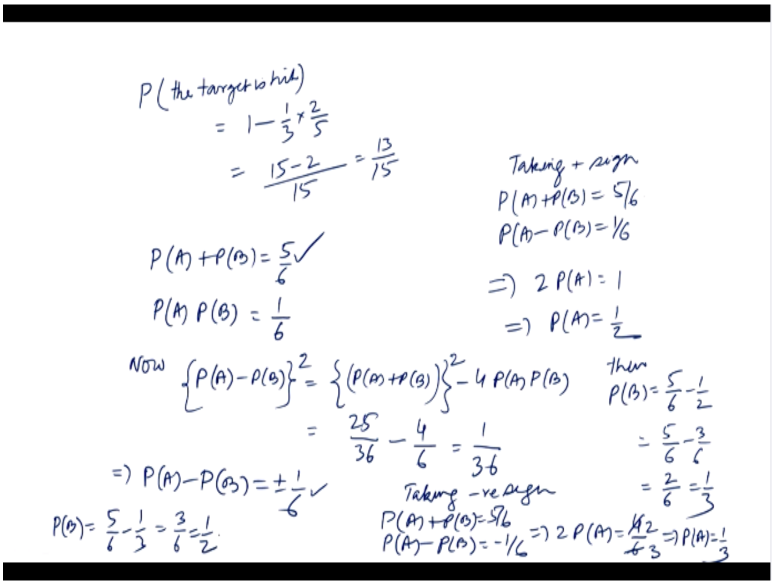

So let us see we can do this. PA+PB=5/6 okay. PA*PB okay this is PA*PB=1/6. So this is=1/6. So now PA-PB whole square=PA+PB whole square-4 PA PB okay. So this will be=25/36 okay 5/6 whole square-this will be 4/6 okay 4*1/6=4/6 okay. So this will be=25-24 so 1/36 okay. So this gives you PA-PB=+-1/6 okay, so we know the value of PA+PB and PA-PB okay. So taking positive sign PA+PB is=5/6, PA-PB=1/6 okay.

We are taking positive sign here. This gives you 2 PA you can add these equations. So we get 5/6 1/6 that means 6/6 so that is 1. So PA=1/2 okay. PA=1/2 and PB will be=5/6-1/2, so we get 5/6-3/6. So this means 2/6 means 1/3 okay. So PA=1/2, PB=1/3. If you take negative sign, taking negative sign we get PA+PB=5/6 and PA-PB=-1/6.

So what we do, we would again add this give you 2 PA=5/6-1/6, so 4/6 okay. So this means 2/3 okay. This gives you PA=1/3 and when PA is 1/3, PB will be=5/6-1/3 okay. This means 2/6, so 3/6 means 1/2 okay.

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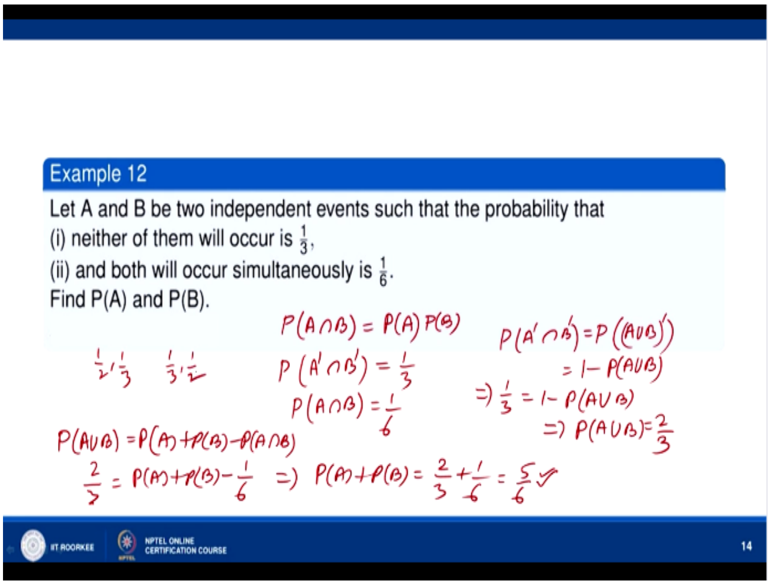

So the answers are 1/2, 1/3 1/3, 1/2 okay. With that I would like to end my lecture. Thank you very much for your attention.