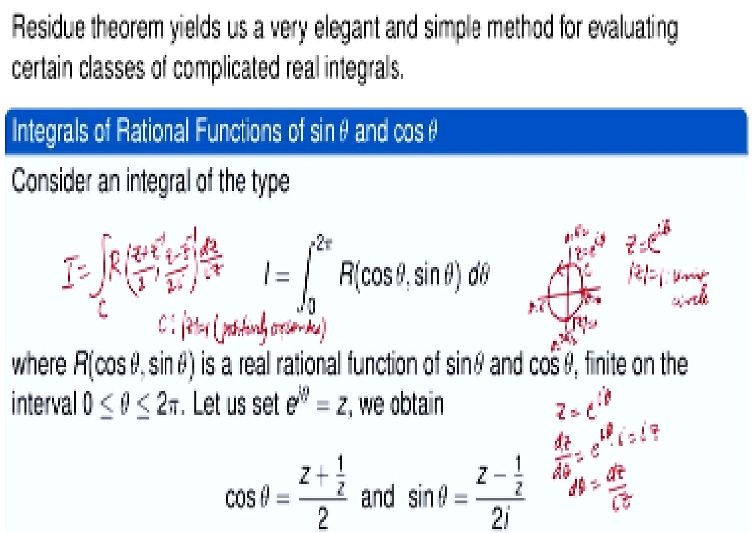
Advanced Engineering Mathematics

Prof. P. N. Agrawal
Department of Mathematics
Indian Institute of Technology- Roorkee

Lecture – 21
Evaluation of Real Integrals Using Residues - I

Hello friends welcome to my lecture on evaluation of real integrals using residues this is first lecture now residue theorem heals us a very elegant and simple method for evaluating certain clauses of complicated real integrals which are difficult to evaluate by using the methods which are given in the real calculus let us consider an integral of the type i = integral 0 2 Pi R cos theta sin theta D theta. Where R cos theta sin theta is the real rational function of sin theta and cos theta finite on the interval 0 < = theta <= 2Pi.

(Refer Slide Time: 01:07)



What we do we will convert this to the contour integral and then using the method of residue theorem we shall evaluate that contour integral to determine the value of the given integral so let us make a substitution let us put e to the power i theta = Z when e to the power i theta is = Z then 1/ Z is e to the power – i theta since we know that cos theta is e to the power i theta + e to the power – i theta /2 cos theta becomes Z +1/Z/2.

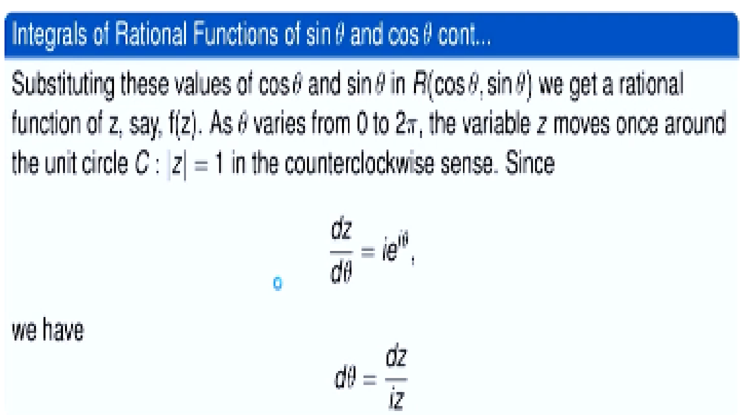
And sin theta is e to the power i theta - e to the power – i theta /2 i so sin theta becomes Z – 1/Z/2i let us replace the value of cos theta and sin theta in the expression for R cos theta sin theta then R cos theta sin theta becomes a rational function of Z and e to the power i theta = Z and when you differentiate Z = e to the power i theta but we get d Z/ d theta = e to the power i theta * i or we can say d theta / so this = i Z.

So, d theta = dZ/ iZ so for d theta we shall put here dZ / iZ and for cos theta we shall write RZ+1/Z/2 for sin theta we shall write Z - 1/Z/2i and when theta varies from 0 to 2Pi what we notice here Z = e to the power i theta means mod Z = 1 so let us draw the circle and mod Z = 1 so here is theta = 0 Z= e i theta is the argument of Z theta = 0 here theta is Pi/2 here theta is Pi here 3Pi/2 and here is 2Pi value come back.

So, when theta varies from 0 to 2 Pi we move along mod Z= 1 that is the unit circle in the anti clock by direction so when theta varies from 0 to 2 Pi we move along the unit circle mod Z = 1 in the anti clock by direction this unit circle let us denote by C so we shall write I = integral / C R Z + 1/Z to the power – 1/2 Z – Z to the power -1/2i and then d theta we shall write as dZ/iZ and hence then C is mod Z =1 positively oriented.

So, we convert the given integral i real integral i * contour integral and we value contour integral by means of residue theorem so we will get the value of i.

(Refer Slide Time: 04:40)



Now let us see how we go about it substituting the values of cos theta and sin theta we get the rational function of Z say f Z okay and as theta varies from 0 to 2Pi the variable z moves once around the unit circle mod Z = 1 in the counter clockwise d theta is dZ/iZ

(Refer Slide Time: 04:58)

Integrals of Rational Functions of $\sin\theta$ and $\cos\theta$ cont...

and hence the given integral takes the from

$$I = \int_C \frac{f(z)}{iz}\,dz,$$

the integral being taken in the counterclockwise sense around the unit circle C.

Example 1

Show that

$$I = \int_0^{2\pi} \frac{d\theta}{1-2p\cos\theta+p^2} = \frac{2\pi}{1-p^2}$$

where $0<p<1$.

So, when you put them we get I = integral /C fz *dz/iZ the integral along c is being taken in the anti clock direction and let us take a simple example suppose we have I = let me take it as I let I = integral 0 to 2Pi this is our 1 – 2p cos theta +p square we shall show that p lies between 0 and 1 the value of the integral is 2Pi over 1 – p square.

(Refer Slide Time: 05:29)

we have $I = \int_0^{2\pi} \frac{d\theta}{1-2p\cos\theta+p^2}$, $0<p<1$

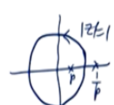
Let us set $z = e^{i\theta}$ C: $|z|=1$ (counterclockwise sense)

$\Rightarrow d\theta = \frac{dz}{iz}$

Then $\cos\theta = \frac{z+\frac{1}{z}}{2} = \frac{z^2+1}{2z}$

We get $I = \int_C \frac{\frac{dz}{iz}}{1-2p\left(\frac{z^2+1}{2z}\right)+p^2} = \frac{1}{i}\int_C \frac{dz}{z-p(z^2+1)+p^2z}$

$= \frac{1}{i}\int_C \frac{dz}{-pz^2+z(p^2+1)-p}$



$= -\frac{1}{ip}\int_C \frac{dz}{z^2-z\left(\frac{p^2+1}{p}\right)+1}$

$= -\frac{1}{ip}\int_C \frac{dz}{(z-p)(z-\frac{1}{p})}$

$z^2-(p+\frac{1}{p})z+1$
$= z^2-zp-\frac{1}{p}z+1$
$= z(z-p)-\frac{1}{p}(z-p)$
$= (z-p)(z-\frac{1}{p})$

Let $f(z) = \frac{1}{(z-p)(z-\frac{1}{p})}$

Then $f(z)$ has simple poles at $z = p, \frac{1}{p}$

We are given that $0<p<1$
$z=p$ lies inside $|z|=1$
while $z=\frac{1}{p}$ lies outside $|z|=1$

$\operatorname{Res}_{z=p} f(z) = \lim_{z\to p}(z-p)\frac{1}{(z-p)(z-\frac{1}{p})} = \left(p-\frac{1}{p}\right)^{-1}$

$I = -\frac{1}{ip}\times 2\pi i \frac{1}{\left(\frac{p^2-1}{p}\right)} = \frac{2\pi}{1-p^2}$

So, we have so let us define z = e to the power I theta so that d theta = dz/iz then cos theta we have seen cos theta is Z +1/Z/2 or we can say Z square +1/2z let us put the value we get I = now as theta varies from 2Pi and Z varies from z varies along with unit circle okay c here is mod Z = 1 in the counter clockwise sense and then we have d theta = dz/iZ/1 – 2p cos theta so 1 – 2p cos theta is z square+1/2Z+p square.

So, this = let us simplify we have dZ let me take this 1 over i outside and then I will get here this 2zi I multiply this 2i cancel I will multiply this z here z-p*z square +1+p square z we get so this is 1/I times integral over c dz/-pz square and then we get +z times p square +1 – p this is what I get or I can write it as – 1 upon I *p inside the integral we have dz/-p we have taken common so z square – z times p square + 1/p we have here.

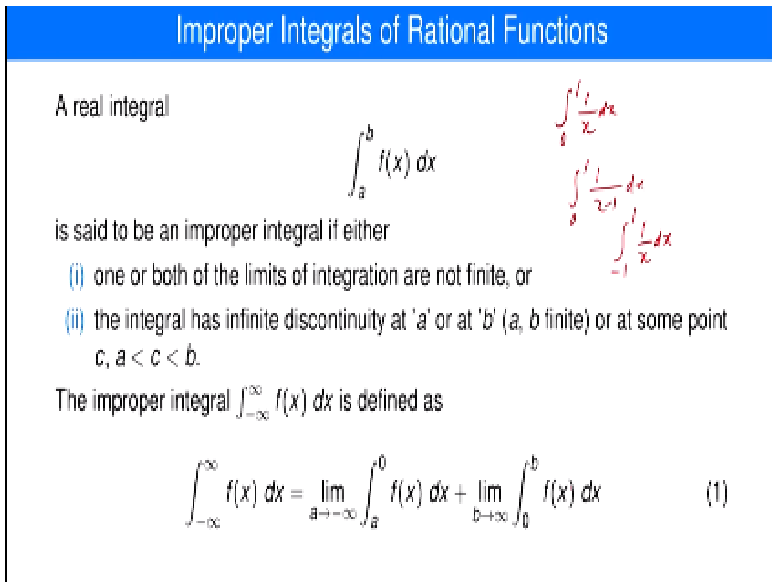
We get +1 we get now let us extract this expression so let us z square – p square+ 1/p I can write as p+1/p*z +1. I can write it as z square – z *p-1/p*z +1 or I can write it as z times z-p- 1/p *z-p so the factors are z-p*z-1/p so this will be – 1 upon ip integral over c dz/ z-pz-1/p now let us say let fz = 1/ z- p * z- 1/p then fz has simple poles at z = p and 1/p because you multiply fz /z-p and take the limit at z to p you get 1over -1/p.

Similarly, when you multiply fz /z – 1over p and take the limit at z is to 1/p you get 1 over 1/p-p so it has simple poles at z= p and 1 /p now let us see which pole lies inside mod z = 1 because our contour is mod z = 1 we are given that 0<p<1 so z=p lies inside mod z=1 while z=1/p lies outside mod z =1 so p lies here and 1/p lies here so we need to consider the similarities which lies inside the circle.

Because residue theorem says that the function fz analytic inside a non simple close curves except at finite number of isolated similarities inside c then the integral over c of z is 2Pi* some of residues at the similarities of object which lies inside c so let us find the residue of fz that z=p so this is limit z is to pz-p*fz that is 1over z-p* z-1/p this will cancel with this and we will get p – 1/p rest to the power -1.

Because we have 1 over z - 1/ p so we have I = -1/I *p and then we have to multiply the residue by 2Pi so the value of the integral is 2Pi 1/ p – 1/p that is p square -1/p so this I will cancel with this I and this p will cancel with this p here and we will get 2Pi times 1 /1-p square so this is the value of the integral 0 to 2Pi d theta /1 – 2p cos theta +p square 1 0< p<1 so this how we evaluate this contour this real integral using residue theorem.

(Refer Slide Time: 13:15)

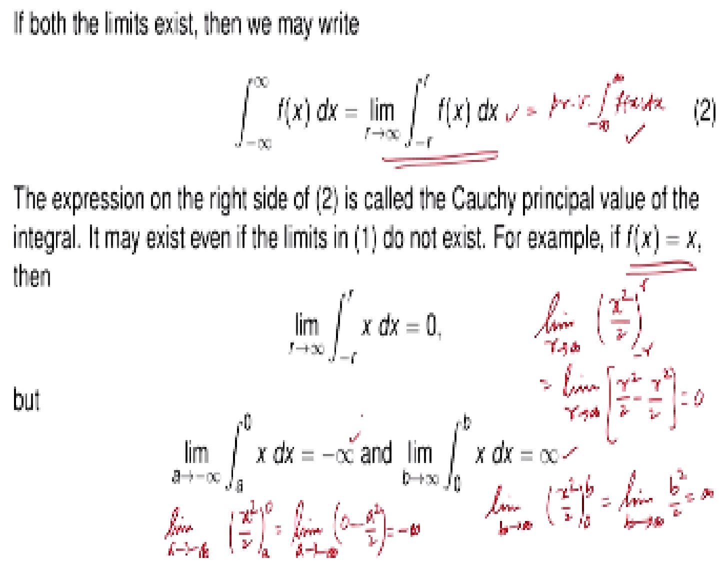


Now let us go to another kind of real integrals we will take now in to a count improper integrals rational functions so a real integral over a to b fxdx is called improper integral if either 1 or both the limits of the integration a and b are not finite are the integral has infinite discontinuity at a or at b say as example you can consider integral 0 to 1/ x dx then when x is near 01/x is not finite so 1/x as infinite discontinuity at x = 0.

Okay so are you can say suppose you take 0 to 1/x – 1 dx then at the upper limit this 1 / x - 1 has as infinite discontinuity while a and b are both finite so or you can consider this case suppose you take -1 to 1 1/x dx then at the interval of the integration -1 to 1 that is the point x =0 at which 1/x becomes infinite so there is infinite discontinuity at the point 0 which lies between -1 and 1 so in any of this situations the integral a to b fxdx is called improper integral.

Now integral – infinity to infinity fxdx is defined as - infinity to infinity fxdx = limit a tends to – infinity a to 0 fxdx + limit b tends to infinity 0 to b fxdx this is how we defined the improper integral/– infinity to infinity fxdx.

(Refer Slide Time: 15:04)



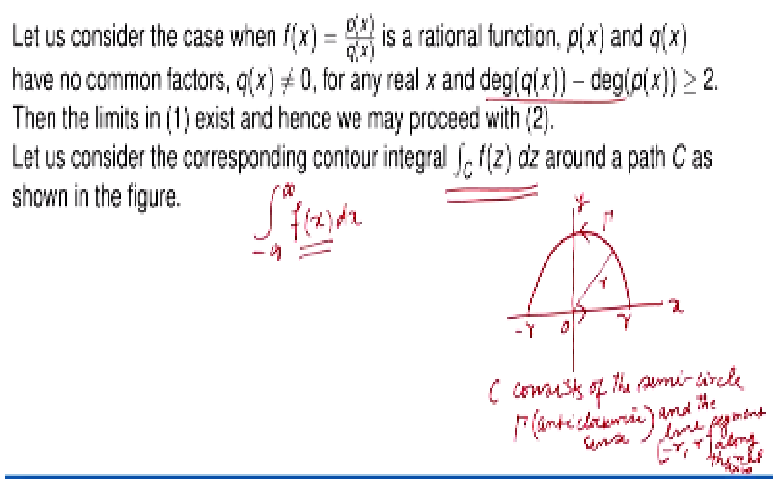
Now if both the limits exist here if this limit and this limit both these limits exist and then we write integral / - infinity to infinity fxdx as limit r tends to infinity –r to r fxdx now this expression on the right side is called the cauchy principal value of the integral this is cauchy principal value we denote it like this cauchy principal value of the integral - infinity to infinity fxdx.

Now this cauchy principal value of the integral - infinity to infinity may adjust even if the limit in 1 this limits do not exist. Let us see an example for example if you take fx =x then limit r tends to infinity cauchy principal value of the integral – infinity to infinity fx will be limit r tends to infinity –r to r fxdx that is x dx so what we will get limit r tends to infinity integral of x x square /2 and we have the limits of integration –r to r.

So, we have limit r times to infinity r square /2- r whole square /2 that is again r square /2 so this is 0 so the limit is also 0 and so this cauchy principal value of the integral – infinity to infinity fxdx exist while this limit as well as this limit both you can see do not exist limit a tends to infinity a to 0 x dx is – infinite because this will be a limit a tends to infinity – infinity x square / 2 and you put a to 0 so we get limit a tends to – infinity.

And when we put 0 we get 0-a square/2 so a goes to – infinity so a square goes to + infinity but there is a negative sign we get – infinity here and here you can see this is similarly limit b tends to infinity x square/2 0 to b so we get limit b tends to infinity b square/2 which is + infinity so this is infinity – infinity so both the limits do not exist which cauchy principal value exist so it can happen.

(Refer Slide Time: 17:40)



Now let us consider the case when fx is a rational function that is px fx of the form px/ qx where px and qx have no common factors qx does not varies on the real x among qx is not 0 for any real number x and degree of qx is at least 2 units higher than the degree of px that is degree of qx – degree of px is >=2 when we assume this condition the degree of px is >= degree of px +2 then the limits in 1 exist.

And hence we may proceed with 2 these limits exist this limit as well as this limit exist and therefore we can proceed with this definition – infinity to infinity fxdx = limit r tends to infinity –r to r fxdx now if it happens that this limit r do not exist then we will need to write that we are finding the cauchy principal value when we write integral – infinity to infinity fxdx from this definition.

In the case where either this 2 either 1 of these 2 are both of them do not exist then we need to write that we are writing and finding the cauchy principal value of the integral - infinity to infinity but here we in this with this conditions it follow that both the limits in 1 exist and so cauchy principal value of the integral is same as the integral over - infinity to infinity fx dx so let us now consider the corresponding contour integral.

We have integral/– infinity to infinity fxdx which we need to determine we are assuming that px is the rational function that is this is the quotient of 2 polynomial functions the polynomial in the numerator and the polynomial in the denominator are such that the degree of the denominator is at least 2 units are then the degree of the numerator the denominator polynomial does not - for any real value x.

And then we shall be able to find the value of this integral by using the corresponding contour integral so let us consider the corresponding contour integral over c of fzdz around a path c as shown in this figure now we have to see the figure so we will consider this path we have a semicircle of radius r the centre at the origin so we move along the semicircle this semicircle let us take as gamma.

So, this c actually consist of contour c consist of the semicircle gamma in the anti clock wise direction and the line segment – r to r along the real axis so this is my contour c so what we will do we will take r to be whole of z all the similarities of the function fz which occur in the upper half plane and then come inside the circle inside the contour c so let fz since now fx is a rational function.

We are writing the corresponding function fz so we are replacing x /z that means that fz = pz/qz so fz = pz/qz is the rational function of z so fz is the rational function therefore the wherever qz will be 0 suppose qz is 0 z = z1z2 z3 and so on zn then fz will have poles at those points.

(Refer Slide Time: 22:08)

Since $f(z)$ is a rational function, it has finitely many poles in the upper half of the z-plane.

Let us choose r to be so large that C encloses all these poles. By the residue theorem

$$\int_C f(z)\,dz = \int_\Gamma f(z)\,dz + \int_{-r}^{r} f(x)\,dx = 2\pi i \sum Res\, f(z) \qquad (3)$$

where the sum $\sum Res\, f(z)$ consists of all residues of $f(z)$ at its poles in the upper half plane.

From (3) we have

$$\int_{-r}^{r} f(x)\,dx = 2\pi i \sum Res\, f(z) - \int_\Gamma f(z)\,dz$$

So, fz have finite number of poles in the so out of these poles finite number of them will lie in the upper half plane and finite number of them if at all will lie in the lower half plane so since fz is the rational function it has finitely many poles in the upper half of the z plane let us choose r to be so large let us choose this r radius of the semi circle r gamma to be so large that all these similarities of fz which lie in the upper half plane lie inside the contour c.

So, c encloses all those poles by the residue theorem then the integral over c fz dz be can write as integral/ c fzdz will be consisting of 2 parts integral/ gamma + integral along the line segment – r to r when we move along the line segment – r to r since we are moving along the real axis z = x by = 0 that becomes = x and therefore integral/ c fz dz = integral/ gamma fzdz + integral/ - r to r z becomes x so we write fx dx.

Now we have taken r to be sole r that seeing closer r all the poles in the upper half plane therefore by residue theorem integral over c fzdz will be 2Pi * sigma residue of fz sigma residue of fz is the consists of all residues of fz at its poles in the upper half plane now we have assumed that qx is not 0 for any real x qz is not equal to 0 on the real x is z = 2x so fz not have any pole on the real axis.

Now let us write my integral/ – r to r fxdx = 2Pi i sigma res of fz – integral/ gamma fzdz we can write like this.

(Refer Slide Time: 24:10)

Now we shall show that $\lim_{r\to\infty} \int_\Gamma f(z)\, dz = 0$. By our assumption, let

$$f(z) = \frac{a_n z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_0}{b_m z^m + b_{m-1} z^{m-1} + \ldots + b_1 z + b_0}, \quad a_n \neq 0,\ b_m \neq 0$$

where $m - n = d \geq 2$ and d is an integer.
Now,

$$f(z) = \left(\frac{z^n}{z^m}\right) \frac{a_n + \dfrac{a_{n-1}}{z} + \ldots + \dfrac{a_1}{z^{n-1}} + \dfrac{a_0}{z^n}}{b_m + \dfrac{b_{m-1}}{z} + \ldots + \dfrac{b_1}{z^{m-1}} + \dfrac{b_0}{z^m}}$$

And let us now show that this integral/gamma fzdz as r becomes sufficient infinitely large this tends to 0 so we can show this that limit r tends to infinity integral/ gamma fzdz = 0 by our assumption let us write pz as polynomial in z of degree n so fz = an z to the power n an-1z to the power n-1 a1z a0 and qz the denominator polynomial. We are taking to be of the degree m so bmz to the power m bm-1 z to the power m -1 and b1zb0 an is not 0 bm is not 0.

Because this is the polynomial of the degree n and this polynomial of the degree m now difference between the degrees of the numerator and denominator is this and n is the degree of the denominator and m is the degree of the numerator the degree of the denominator exceeds by at least 2 units then the degree of the numerator so m-n if it is = d then d > r=2 and d is an integer and now let us write fz in the following form.

We can take 2 power n outside the numerator z to the power m outside in the denominator then we have this function a n+n-1 upon z a 1/ z at 2 power –n +a0/z to the power n similarly we have in the denominator bm + bm-1/z and so on b1/zm-1+b0/z to the power m now let us take mod of fz.

(Refer Slide Time: 25:43)

then for sufficiently large r_0 such that $|z| = r > r_0$, we have

$$|f(z)| \leq \left(\frac{|z|^n}{|z|^m}\right) \frac{|a_n| + \frac{|a_{n-1}|}{|z|} + \ldots + \frac{|a_1|}{|z|^{n-1}} + \frac{|a_0|}{|z|^n}}{|b_m| + \frac{|b_{m-1}|}{|z|} + \ldots + \frac{|b_1|}{|z|^{m-1}} + \frac{|b_0|}{|z|^m}}$$

$$= r^{n-m} \left\{ \frac{|a_n| + \frac{|a_{n-1}|}{r} + \ldots + \frac{|a_1|}{r^{n-1}} + \frac{|a_0|}{r^n}}{|b_m| + \frac{|b_{m-1}|}{r} + \ldots + \frac{|b_1|}{r^{m-1}} + \frac{|b_0|}{r^m}} \right\} < \frac{M}{r^d}, \quad d = m - n$$

where M is any number greater than $\left|\frac{a_n}{b_m}\right|$.

So, then if you take r to be sufficiently large for sufficiently large r0 such that mod of z = r> r 0 we will have these mod of fz <=mod of z to the power n/mod of fz by triangular equality we have mod of an + mod of an -1 / mod of z mod of a1 / mod of z to the power n-1 mod of a0/ mod of z to the power n then mod of bm – mod of bm -1/mod of z – mod of b1/ r to the power m -1 – mod b / mod of z to the power m.

So, mod of z = to r because we are evaluating the maximum value of mod of fz along the curve c so we put mod of z = r so we get this expression when r becomes sufficient large this quotient is this one this expression tends to mod of an / mod of bm and therefore if you take a number M to be greater than mod of an /bm then we can say that this mod of fz is strictly greater than m over r to the power d is m – m-n.

So, mod of fz is < m/ r to the power d where M is any number greater than mod of an / mod of bm and this r 0 is sufficiently large so that this tends to mod of an / mod of bm.

(Refer Slide Time: 27:19)

Hence

$$\left|\int_\Gamma f(z)\,dz\right| < \frac{M}{r^d}\pi r$$

$$= \frac{M\pi}{r^{d-1}}, \ \forall r > r_0$$

≥ 2

Thus, as $r \to \infty$,

$$\int_\Gamma f(z)\,dz \to 0,$$

and so

$$\int_{-\infty}^{\infty} f(x)\,dx = 2\pi i \sum Res\, f(z).$$

Now, hence what we can say by cosign equality mod of integral/gamma of fzdz will be < the maximum value of mod of fz that is M /r power d * length of the semi circle gamma and length of semicircle gamma is Pi r because gamma is semicircle of radius r so Pi r * by M/r to the power d so this I can write as M pi/r to the power d-1 now d is >=2 so mod of integral /gamma fzdz is < M Pi/r to the power d -1 test to 0 as r test to infinity.

When r is 0 is very large then this quantity can be read smaller than any epsilon r route as epsilon so we can say that as r goes to infinity integral / gamma fzdz tends to 0.

(Refer Slide Time: 28:17)

Since $f(z)$ is a rational function, it has finitely many poles in the upper half of the z-plane.

Let us choose r to be so large that C encloses all these poles. By the residue theorem

$$\int_C f(z)\,dz = \int_\Gamma f(z)\,dz + \int_{-r}^{r} f(x)\,dx = 2\pi i \sum Res\, f(z) \qquad (3)$$

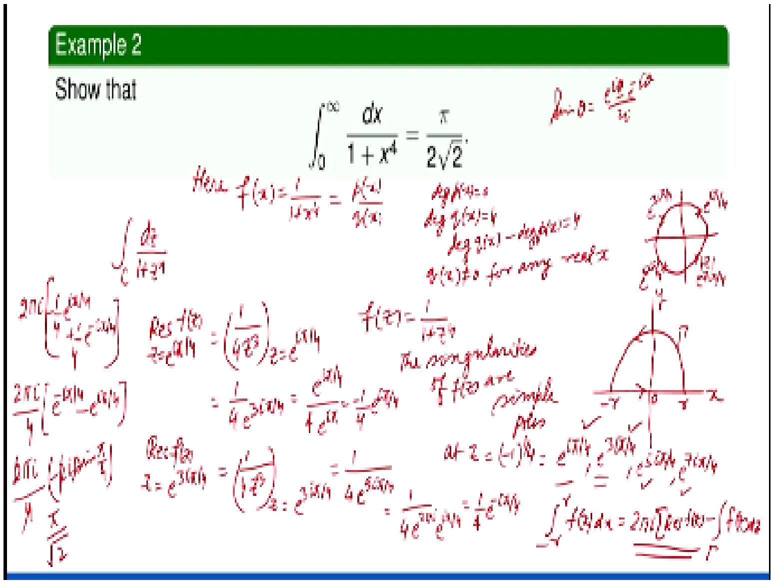
where the sum $\sum Res\, f(z)$ consists of all residues of $f(z)$ at its poles in the upper half plane.

From (3) we have

$$\int_{-r}^{r} f(x)\,dx = 2\pi i \sum Res\, f(z) - \int_\Gamma f(z)\,dz$$

And hence integral / - infinity to infinity fxdx is 2Pi i * sigma residue of fz so this is the article.

(Refer Slide Time: 28:29)



And now let us see an example on this suppose we have the function 1 / 1+x to the power 4 which we want to integrate/ 0 to infinity so here fx = 1/1+ to the power 4 you can see this is px/qx the degree of px is 0 because it is constant polynomial and degree of qx is 4 so degree of – degree of px = 4 which is obviously > r =2 this is>2 so the degree of the denominator exceeds the degree of the numerator by at least 2 units.

And that condition is fulfilled moreover qx is never 0 for any real x 1+x to the power 4 does not have any real route that condition is also fulfilled so px and qx are polynomials were the degree of the denominator is at least 2 units have the degree of the numerator qx does not vanish for any real x so the conditions for the function fx are fulfilled and therefore we can consider the corresponding contour integral over c fzdz that is dz/1+z to the power 4.

So, let us consider contour c from –r to +r now let us find the similarities of fz =1/1+z to the power 4 we have discuss the similarities of 1/1+z to the power4 earlier these are simple poles at z = -1 to the power 1/4 and we have seen earlier by demoivres theorem that similarities occur at e iPi/4 e rest to the power 3iPi/4 e rest to the power 5iPi/4 e rest to the power 7iPi/4 now out of these 4 simple poles.

Okay 2 simple poles simple pole e to the power iPi/4 e to the power 3iPi/4 they lie in the upper half plane while these 2 lie in the lower half plane you can see we had this unit circle so 1 similarity iPi/4 here e to the power 3iPi/4 is here e to the power 5iPi/4 is here and e to the power 7iPi/4 is here so only this similarity we need to consider because we need to consider all these similarities of z that lie in the upper half plane.

So, let us find the residue of fz = e to the power iPi/4 so this we have 1/1+z to the power 4 we shall apply the formula for the residue in the case of the simple pole pz0/2piz0 fz is pz/qz and fz has the simple pole at the z=z0 then we have the formula for the residue in the case of simple pole as pz0/2Piz0 so we will have here 1/4q at z= e to the power iPi/4 so this will be 1/4 e to the power 3iPi/4.

I can multiply e to the power ipi/4 and so but I will get e to the power i Pi/4/4 e to the power i Pi so this is e to the i Pi – 1/4 e to the power iPi/4 and similarly residue of z fz = e to the power 3iPi/4 we can find this is= again 1/4zq z =e to the power 3iPi/4 so this is 1/4 e to the power 9iPi/4 which I can write as e to the power iPi/4 so this is 1/4 e to the power 2Pi i * e to the power iPi/4 e power 2Pi i is 1 and I can write 1/4 e to the power –iPi/4.

Now we have integral /-r to r fzdz= 2Pi i sigma residue of z- integral over gamma fzdz and this is the formula so 2Pi i * sum of residues is how much 2Pi i* sum of residue is -1/4 e to the power iPi/4 and then +1/4 e to the power –iPi/4 so I simply finding this one first so this is 2Pii/4 and we have e to the power –iPi/4- e to the power iPi/4 we know that sin theta is e to the power i theta – e to the power – i theta /2i.

So, this is 2Pi i/4* -2i sin Pi/4 so this 2 and this 2 cancel with 4 here I square is -1 what we will get Pi sign Pi/4 so this is Pi/root 2 now as r goes to infinity this will go to this is integral/ fxdx not z dz now r goes to infinity, we tend to integral /- infinity to infinity.

(Refer Slide Time: 36:01)

As $r \to \infty$
we have

$$\int_{-\infty}^{\infty} f(x)\,dx = \frac{\pi}{\sqrt{2}} - \lim_{r \to \infty} \int_{\Gamma} f(z)\,dz$$

$$f(z) = \frac{1}{z^4+1}$$

$$|f(z)| \le \frac{1}{|z|^4-1} = \frac{1}{r^4-1}$$

$$\left| \int_{\Gamma} f(z)\,dz \right| \le \frac{1}{r^4-1}\,\pi r \to 0 \text{ as } r \to \infty$$

And hence $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^4}\,dx = \frac{\pi}{\sqrt{2}}$

$$2\int_{0}^{\infty} \frac{1}{1+x^4}\,dx = \frac{\pi}{\sqrt{2}} \Rightarrow \int_{0}^{\infty} \frac{1}{1+x^4}\,dx = \frac{\pi}{2\sqrt{2}}$$

So, as r goes to infinity we have integral/– infinity to infinity xdx= we found the value as Pi/root 2 – limit r tends to infinity integral/ gamma fzdz we can show that this integral goes to 0 where r goes to infinity so for that we have fz = 1/z to the power 4+1 the mod of fz is < =1/mod of z to the power 4 -1 and we have to estimate along with gamma along gamma z = mod of z =r so 1/ r to the power 4 -1 so this 1/ r to the power 4 -1.

So, we can say that integral /gamma fzdz mod of this <=1/ r to the power 4-1* length of gamma which is Pi*r so Pi*r to the power 4-1 now this clearly goes to 0 as r goes to infinity so this integral which tends to 0 and hence integral / - infinity to infinity 1/1+x to the power 4 dx = Pi /root2 now 1/1+x to the power 4 is the given function so I can write 2 times 0 to infinity 1/1+x to the power 4 dx=Pi/root 2 which gives us.

The value of the integral 0 to infinity1/1+x to the power 4 dx =Pi/2 root2 so this is how we can evaluate this improper integra in the next lecture we can consider certain other classes of improper integrals and we shall see how we can apply the cauchy residue theorem to determine the; those real integrals which are not easy to evaluate by using the methods of the calculus integral calculus which we know okay so thank you very much for your attention.