Dynamical Systems and Control

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Lecture - 07
Solution of Linear Systems – I

Hello friends, welcome to this lecture. In this lecture we continue our study of dynamical system and control. So in previous class we have discussed Existences and Uniqueness for say non-autonomous system of linear equation and autonomous system of linear equation. Let us recall what we have done in previous class. So in previous class we have discussed is linear system x dash = A(t) x + f(t).

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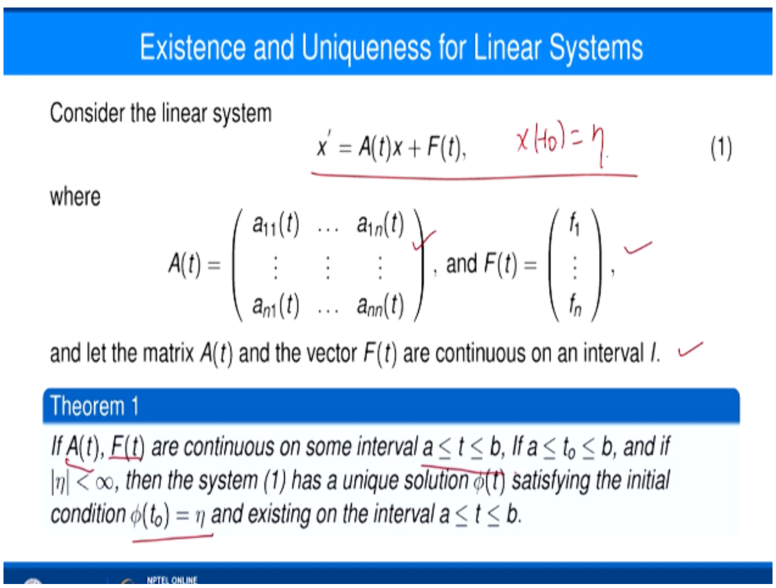

Where A(t) is given by this coefficient matrix a11(t) to a1n(t); an1(t) to ann(t). And F(t) is given by this f1 to fn is column vector And here we assume that A(t) and this vector function F(t) are continuous on a given interval. And we have discussed the following Existence and Uniqueness theorem and it says that if A(t) and F(t) are continuous n some close interval a to b and F(t0) is some initial point line in this interval a to b and if norm of theta is < infinity then this system 1 has a unique solution Phi(t) satisfying the initial condition Phi(t0)=theta.

So it means that if we assign a initial condition that Phi(t0)=theta then we have a solution, a unique solution of this initial value problem exist x = A(t)x + F(t) and x(t0)=theta, here it is x rather than x, so it is your x(t0)=theta. So this, and this solution exist on the entire interval a to b. And so this is the result for non-autonomous system of linear equation.

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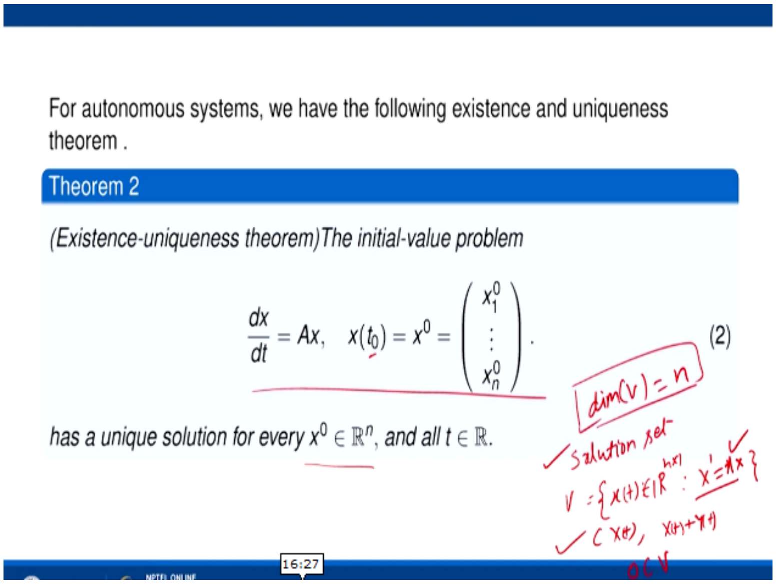

And similarly for autonomous system we have the following Existence and Uniqueness theorem. The initial value problem dx/dt = Ax, where A is independent of t and xt(0)= x0 where x0 is initial condition given at t=(t0) and it is like x1(0) to Xn(0). Then, this initial value problem as a unique solution for every x0 belong to R and for all t in R. So this we have discussed. So it means also we have shown apart from this Existence and Uniqueness theorem that once we have Existence and Uniqueness theorem then we try to find out this solution set.

So solution set is basically, solution set is defined as a set v such that it is basically x(t) belong to say R n1 such that x dash = Ax. So if we define this and we can check that that if x(t) is a solution then c of x(t) is also a solution and if xy; yt are two solution then x(t) + y(t) also be a solution or we can say that this set v from say vector space because 0 is already there because 0 belongs to v because x dash = Ax has a trivial solution.

So it means that this solution set from say vector sub-space and we have also checked that the dimension of this vector space is given as n, so it means that if we want to, so this is up to what we have discussed in previous class and we can say that once dimension of v is known to us then we can find out n linearly independent solution of x dash = Ax and any solution of x dash = Ax can be written as a linear combination of the L linearly independent solution.

So it means that this much we have done. But the problem is that once we have some solutions x dash = Ax then it is quite difficult, how do we know that this solution is actually a linearly independent solution. So here we discuss next result by which we can check that a given set of solutions is actually a linearly independent solution here. So let us consider the following theorem.

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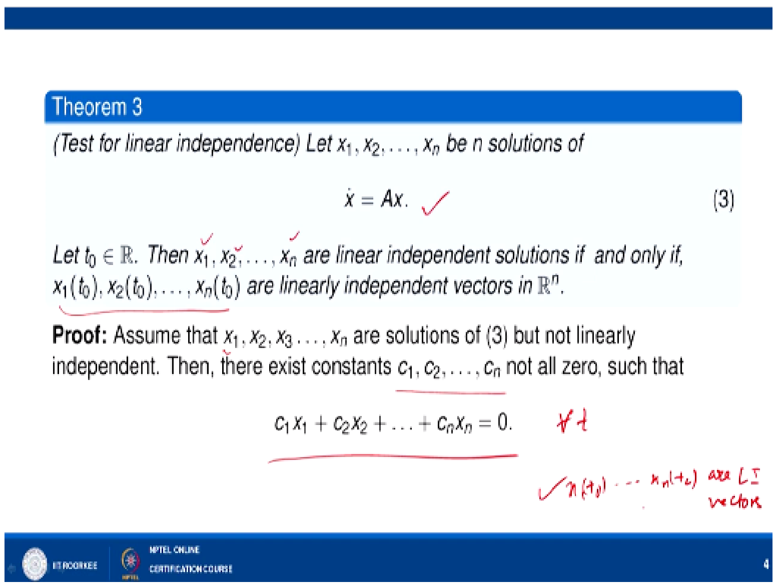

Which says that, let X1 to Xn be n solutions of x = Ax and let t0 is some real number in R. Then X1 to Xn are independently linear solutions if and only if the x1 t0 and x2 t0 and xn tn, xn t0 are linearly independent vectors in Rn. So please try to understand here this x1 to xn are n linearly, n solutions of a x dash = Ax so it means that these are functions basically. So the linearly independence of these functions can be obtained by linearly independence of these vectors. So these are constant vector in Rn and these are functions of t.

So how the linearly independence of functions can be obtained with the help of linearly independence of constant vectors in Rn. So that is a content of this theorem. And it says that, assume that x1 to xn are solution are solutions (3). So it means let us assume that x1 to xn are solutions but not linearly independent. So we assume that these are solutions but may not be linearly independent.

So if they are not linearly independent then the exits constant c1 to cn not all 0 such that this happens. So we have to say that exits constant c1 to cn not all 0 says that C1 X1 + C2 X2 +CnXn = 0. So here we are assuming that the solution to X1 to Xn are not linearly independent or we can say that x1 to xn are linearly independent. So we can find out, so it means that they exist some constant C1 to Cn not all 0 such that this linear combination equal to 0. Now this is true for all t, right. So if it is true for all t then in particular we evaluate at t=t0 also.

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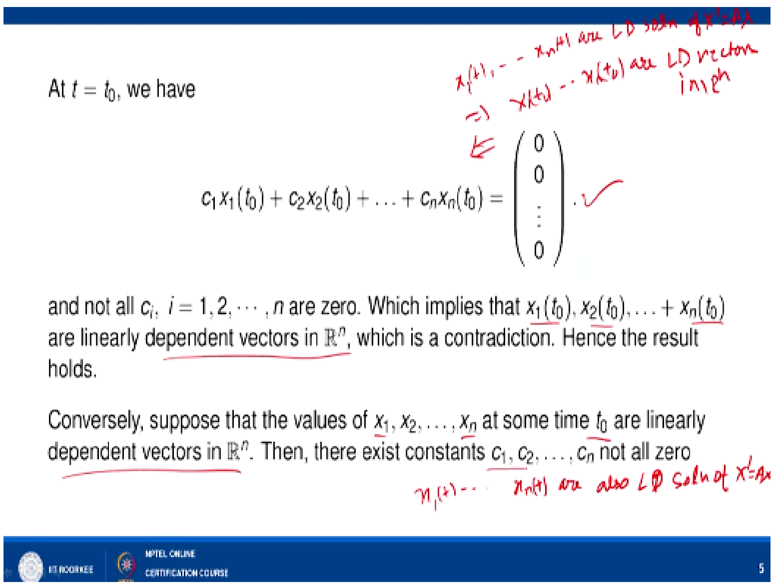

So, at t=t0 we have C1 X1(t0) + C2 X2(t0) and Cn Xn(t0) = 0, 0, 0. Now, so this implies what, that here not all Ci's are 0 and still this linear combination is equal to 0. So this implies that, that X1(t0), X2(t0) and Xn(t0) are linearly dependent vectors in Rn. But here we have already assumed that this X1t(0) to Xn(t0) are linearly independent. So here before proceeding this we are assuming that X1(t0) and Xn(t0) are Li vectors.

So here if we assume that these X1(t0) to Xn(t0) are LI vectors then we have a contradiction. It means that, the assumption that X1 to Xn are linearly dependent solution may not be correct. If we assume that X1(t0) to Xn(t0) are LI vectors. So here it means that if we assume that X1 to Xn are linearly dependent solution then X1(t0) to Xn(t0) has to be linearly dependent.

So this implies that if X1 to Xn are linearly independent then X1(t0) to Xn(t0) are also linearly independent vectors. So we have proved that, that if X1(t) to Xn(t) are solutions which are dependent linearly dependent solution then at t=t0 also X1(t0) to Xn(t0) are also linearly dependent vectors in Rn. So if solutions, so let me write it here that if X1(t) to Xn(t) are LD solutions of x dash = Ax then implies that X1(t0) to Xn(t0) are LD vectors in Rn, so that is what we have shown.

Now we want to show the reverse part of this. So it means, let us assume that the X1 to Xn at t0 are linearly dependent vectors in Rn. So we want to now go back to, so we want to show reverse thing. That let us X1 to Xn at t0 are linearly dependent vectors in Rn. Now we want to show that our claim is that X1(t) to Xn(t) are also linearly dependent solutions of x dash = Ax, that is what we want to show it here. So if we assume that, that X1(t0) to Xn(t0) are linearly dependent vectors in Rn it means that they exist constant C1 to Cn, not all 0.

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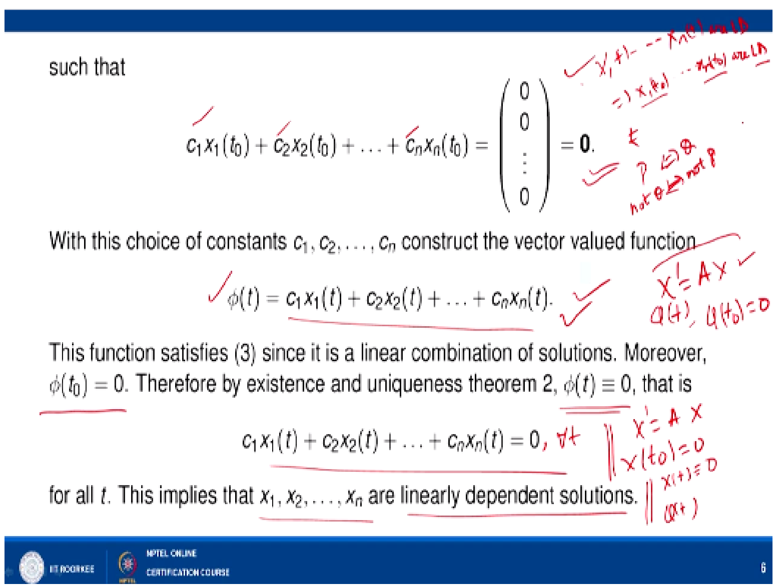

Such that C1 X1(t0) + C2 X2(t0) + Cn Xn(t0) = 0. So it means that linearly combination is equal to 0 and not all the Ci's are 0. So taking these combinations of Ci's let us form this function that is Phi(t) = C1 X1(t) + C2 X2(t) + Cn Xn(t). So once we have such a pair of constants C1 to Cn then with the help of these Ci's form a function like this C1 x1(t) + C2 C2 (t) + Cn Xn(t). Now here please observe that each Xi(t) is a solution of x dash = Ax.

So it means that each Xi is a solution of x dash = Ax. So in particular linear combination of Xi will also be a solution of x dash = Ax. So it means that this Phi(t) is also a solution of x dash = Ax because of the linear combination solution is again a solution. So; and not only this, we have Phi(t0)=0. Say t is at, if we evaluate Phi(t) at t=t0 then we have already seen that it is equal 0, so it means that your Phi(t) is a solution of this provided that Phi(t0)=0. Now we already know that, that this system x dash = Ax has a 0 solution.

So 0 solution is what, 0 is already a solution such that 0 solution at any point is again a 0 vector. So it means that by Existence and Uniqueness theorem it can have only a unique solution. So it means that, that x dash = Ax with the condition that x(t0)=0, it has it can have only a unique solution but if you look at we have two solution given as x(t) ideally equal to 0 and Phi(t) given as this. So it means that, that these solutions has to be at in the equal to same. So it means that your Phi(t) has to be ideally equal to 0.

So it means that this linear combination that is C1 X1(t) + C2 X2(t) + Cn Xn(t) has to be ideally equal to 0 for all t. So it means that, that these Ci's are not all 0 so it; but Ci's are not all 0 but it still this linear combination is equal to 0 proves that X1 to Xn are linearly dependent solution. So it means that what we have proved here that in first part we have shown that X1(t) to Xn(t) are LD. This implies that X1(t0) to Xn(t0) are LD and in a reverse way we have shown that if X1(t) to Xn(t) are LD vectors in Rn then X1(t) to Xn(t) are LD solutions x dash = Ax.

So it means that this X1(t) to Xn(t) are LD solutions of x dash = Ax if and only if X1(t0) to Xn(t0) are LD vectors in Rn. And we can say that not that this implies that P if and only if Q. So we can also show that not Q if and only if not 8. So it means, so negotiation of this we can say that, that if X1(t) to Xn(t) are LI solutions of x dash = Ax if and only if X1(t0) to Xn(t0) are LI vectors in Rn. So with the help of this we have shown that, the solutions X1 to Xn the n solutions of this.

And we can easily check the linearly independent of these vectors if we can evaluate at any given point t0 and if the vectors X1(t0) to Xn(t0) are linearly independent then we are done. So it means that here you need not to check for all t rather than you can check at one single point t0. And if it come out to be linearly interdependent vectors we can say that the our solutions are also linearly independent solutions.

So here the problem of checking the linearly independent functions now reduced to checking the linearly independent vectors in Rn because it is quite difficult to check that given set of functions are linearly independent functions.

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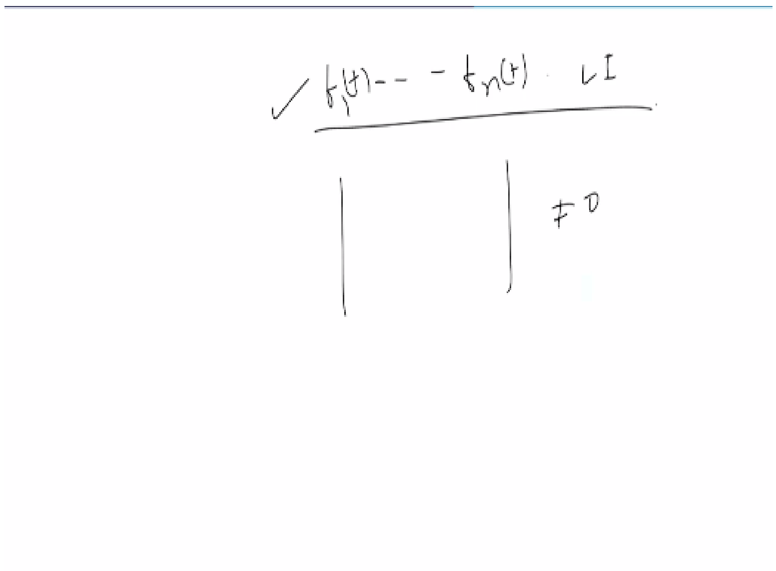

So what I am trying to say is that given any say function f1(t) to say fn(t) it is quite difficult to check whether these are LI or LD. So it is quite difficult because here we cannot say that we can write, we can have a determinant if it is coming out to be non-zero then it is LI it may not be to at all. Or we can say that there are say LI functions for which determinant is 0 but still we cannot say anything whether it is LD or not.

So here we simply say that the problem of finding the LI, linearly independence of functions is now reduced to the problem of checking linearly independence of vectors in Rn and checking LI linearly independence of vectors is relatively easy then finding the independence of functions. Now here in the particular case when all these fi's are solutions of x dash = Ax this is quite relax.

And we can say that, if we simply calculate the X1 to Xn at any given point t0, if they are coming out to be linearly independent so we can say that our solutions a function X1 to Xn's are linearly independent functions and they are solution of x dash = Ax. So now it means that we can easily check that given set of solutions are linearly independent or not.

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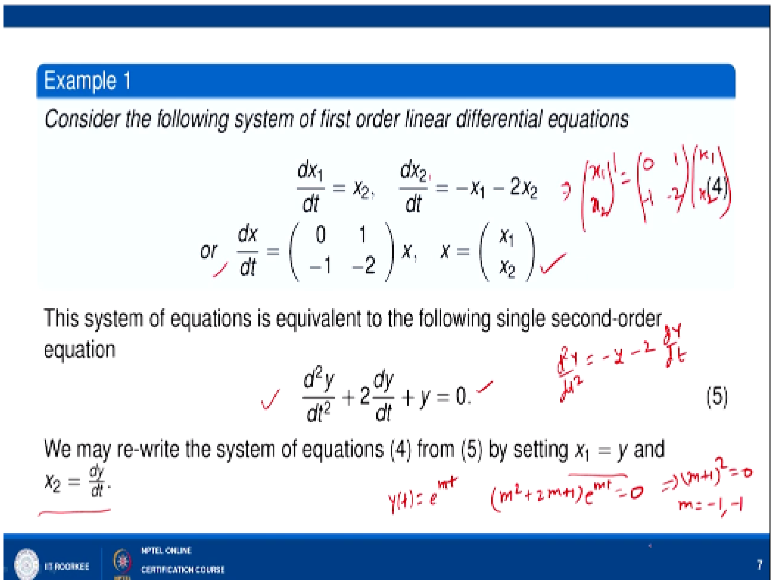

Now, let us take one simple example and proceed. So here consider the following system of first order linear differential equations that is dx1/dt = x2 and dx2/dt = -x1 – 2x2. So we can easily write down this as a system of linear equation, we can write down x1, x2 dash = x1 dash = x2 so it is 0 1, let me writ it here x1 x2. So x1 dash = x2 and x2 dash = -x1 so -1 and here it is -2. So we can write down dx/dt = 0 1 -1 -2 where x is this vector x1, x2.

And here if we simply that if you can look at carefully then this system of linear equation can be reduced to high order differential equation cost of coefficient. So here we can simply say that if dx1/dt = x2 and dx2/dt = -x1 – 2x2. So here we simply differentiate this. And we can write down this as d, so it is what you can simply write say x1=y you can simply write and x2=dy/dt. If you write x2 as dy/dt then dx2/dt will be what it is simply d2y/dt square equal to –x1 –x1 is –y; -2, x2 we are writing dy/dt and it is this.

So we can simply write this equation, the system of linear equation is reduced to this single second order equation that is d2y/dt square + 2dy/dt + y=0 and we can simply solve this equation. And this can be solved using the method we have already say assume that this can be solved our solution is given by e to the power –t and t e to the power –t. So it is not very difficult.

Here we just assume that yt=e to the power m t is a solution and we can simply say that it is what you can simply write m square + 2m + 1* e to the power m t = 0 or e to the power m t is not cannot be 0 so this implies that m+1 whole square = 0 so m=-1 and -1 repeated we can consider.

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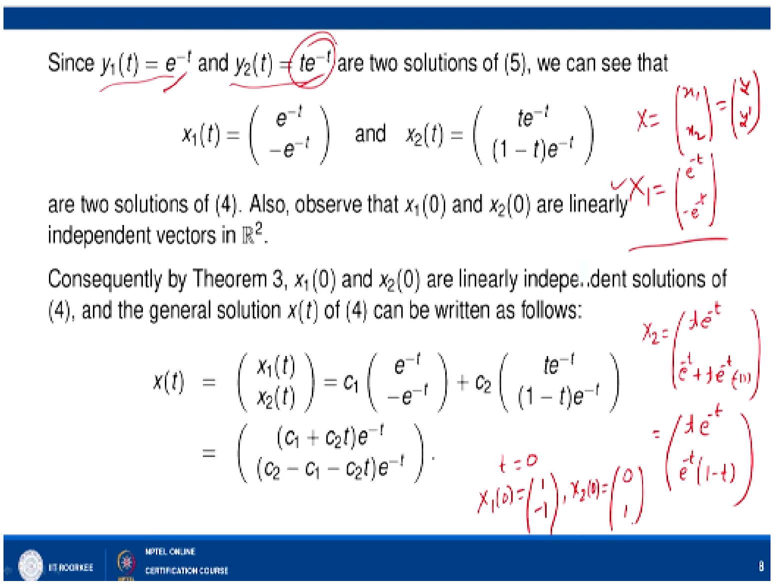

And we can check that our solutions y1(t) = e to the power –t is one solution and second solution you can find out using variation of parameter method or the alternative method available to find out the second linearly independent solution. So here in this case we can say that y1(t)= e to the power –t is one solution and y2(t)=e to the power te to the power –t is the second solution. Now with the help of these two solutions.

Now let us find out two solution of the system of linear equation here. Now here let us take the solution is basically what, here our solution is basically x1 and x2 and x1 we have assume it is y and it is y dash. So we can find out one solution x1 as you take y as a e to the power –t. So if you take y as e to the power t then it is e to the power –t and thereby it is this thing. So t means that x1 is one solution of this system given as equation number 4.

Now if we take y as this solution t to the power –t then your second solution is given as t e to the power –t and thereby this and this you can find out the e to the power –t + t e to the power –t and -1 will come out, so we can simply say this is nothing but t to the power –t, e to the power –t you can take it out and it is 1-t here. So it means that first solution is given by e to the power –t –e to the power –t, a second solution is given by t e to the power –t, e to the power –t into -1-t.

And, so we have two solutions and now we want to check that these two solutions are linearly independent or not. So for that you can take any point and you check linearly independence of the corresponding vectors. So here for the simplicity I am taking t=0 then your x1 0 = what it is 1-1 and x2 0 is what, it is your say 0 and this is what, you write it 1. So it means that you can check that 1-1 and 0 and 1 are say linearly independent. That you can check very easily.

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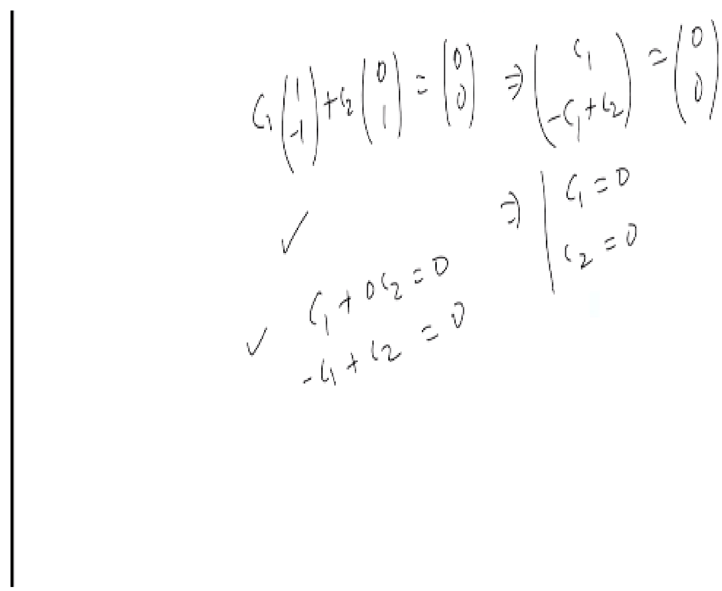

You write C1*1-1 + C2*0 1 and you simply write it, it is 0 0 and we will not, that this implies what, this implies at C1 and this is C1-C1+C2 and this equal to 0 0. So if you check that we have C1=0 and C2=0 the only solution. So this system of linear equation that is C1+0C2 = 0 and –C1+C2=0. This system of linear equation has only trivial solution that is C1=0 and C2=0, so this implies that the solution we have obtained X1(t) and X2(t) are linearly independent solutions, linearly independent solution of x dash = Ax.

So once we have linearly independent solution then we can write down the general solution using these x1 and x2. So we can write down the solution x(t) as say that is general solution I am writing as C1 times X1(t)+ C2 times X2(t). So X1(t) is given by e to the power –t – e to the power –t and X2(t) is given by te to the power –t and 1-t e to the power –t.

So general solution you can write it down C1(t) to C2(t) * e to the power –t and C2-C1-C2(t) e to the power –t. So general solution you can obtain using this. So it means that here in this particular example we have shown that how to find out a solutions of the system of linearly and how to write down, how to check that the obtained solutions are LI solutions and we can write down the general solutions once we have the basis of one basis of your solutions.

So here general solution is given by this. Now what happen, here we have solved by converting the system into this single second order equation and we know how to solve the single second order equation. But in general it may not be say may not be convertibles system of differential equation into a single second order equation then how to find out the solutions n linearly independence solution of x dash = Ax, that is what we wanted to know now.

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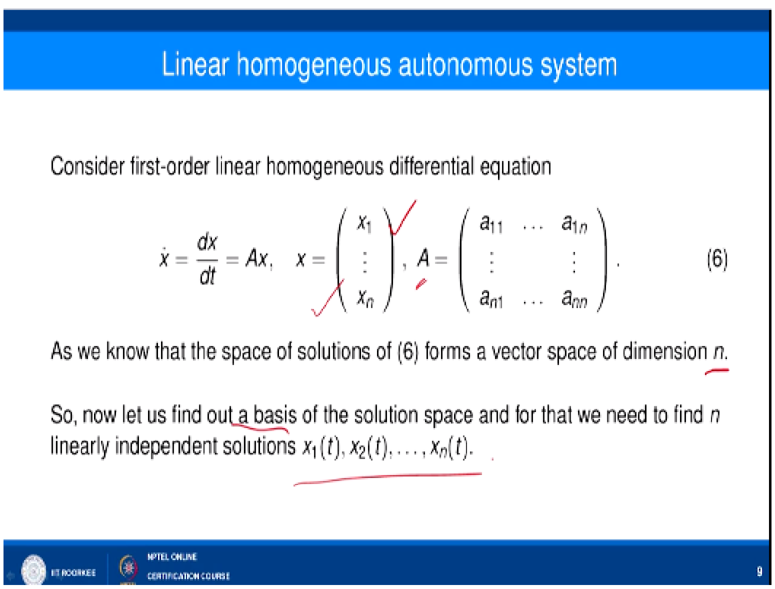

So now let us proceed further and we say that consider first order linear homogenous equation, x dash = dx/dt = Ax, where x is this n cross 1 vector and A is given by eij. Here this A is a constant matrix and X is a of course functions in functions, okay. So we know that the space of solution (6) forms a vector space of dimension n, that we have already discussed.

Now we want to find out a basis, a basis of the solution space and for that we need to find out n linearly independent solutions x1(t) to xn(t). We have seen that in particular case when a is a kind of a is simpler matrix we can convert this system into a single first order differential equation and we can solve that. But in general it may not be true. So in general in; for general matrix a how to find out n linearly independent solutions that is the matter of this next few slides.

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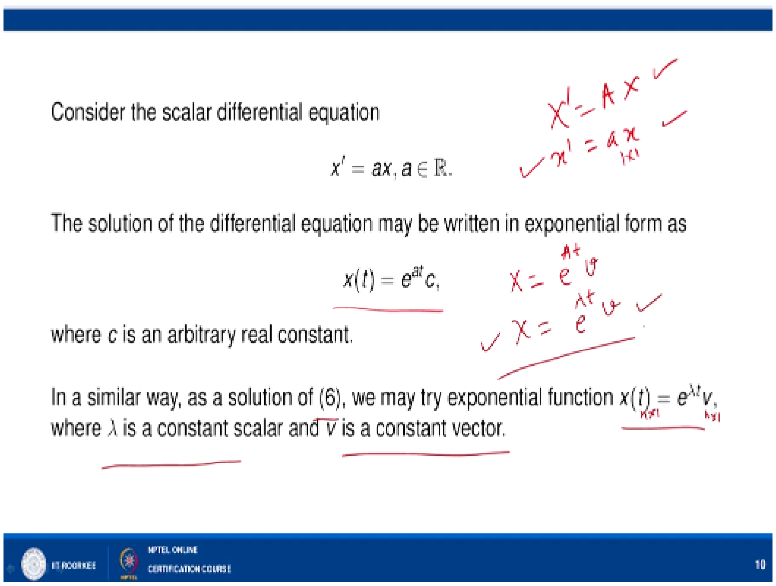

So here, let us take the idea of scalar differential equation x dash = ax, where is a is some constant in real value constant. Now we know that the solution of this x dash = ax, why we are considering this, because this x dash = ax and x dash = ax, this is a vector value and this is a scalar value, the only difference is that here this can be written as one cross one matrix. So here we already know the solution of this scalar value the x dash = ax is given by x(t) = e to the power at * c, where c is some real constant.

The solution of this system, this scalar value differential equation is given x(t)= e to the power at c. Now from this can we get some idea to find out the solution of this, one idea is that can we write this similar kind of instruction like let us e to the power at x. Another way is can we write down the solution sorry this should be written as in a different manner. Here we simply write that, can we consider this candidate x=e to the power e at*some kind of v as a candidate of solution.

Or another way is can we consider this that x=e to the power some constant into v as a solution. So let us try this first, because it is a quite simpler one. In a similar way as a solution of (6) we may try exponential function that is x(t)=e to the power lambda t v as a solution where lambda is a some constant and v is a constant vector. So v is a constant vector means m cross one size, so this will be n cross one vector and we try to see whether this will be a solution of x dash = ax or not. So if we say that let us first try this and if it will not work then we will try this.

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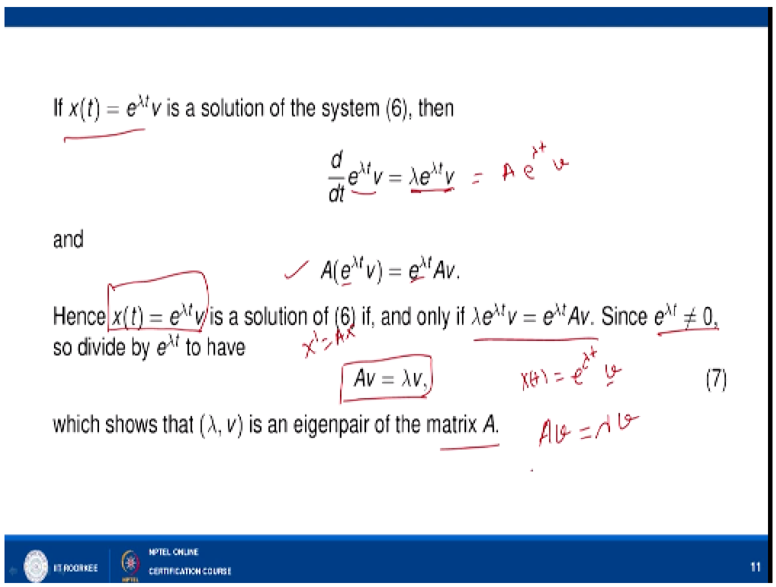

So here let us assume that x = e to the power lambda t v is a solution of the system (6), then if is a system it must satisfy the differential system of linear differential equation, so differentiate it. So if you differentiate it then we will get what, d/dt of e to the power lambda t v is nothing but lambda e to the power lambda t*v and this should be equal to your a of this e to the power lambda t*v.

So this is; it means that a e into e to the power lambda t v = e to the power lambda into a v. Now e to the power lambda t is simply a constant which we can take it out, so it means that if x(t)= e to the power lambda t v is a solution, if and only if this condition hold, that is lambda e to the power lambda t v = e to the power lambda t a v. Now e to the power lambda t is nearby 0 so we can simply write that x(t) e= e to the power lambda t v is a solution of (6), 6 means x dash = ax provided that we have the following relation a v = lambda v.

So it means that, that this x(t) e = e to the power lambda t v is a solution if this lambda m v forms an Eigen pair of the matrix A. It means that the following relation Av=lambda v holds. So we have a proper definition of Eigen vectors.

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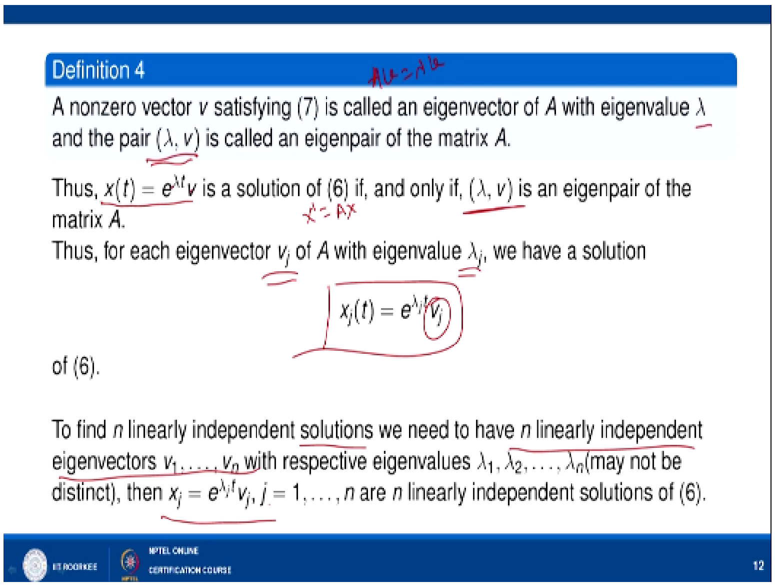

We simply say that a non-zero vector v satisfying Av=lambda v is called an Eigenvector of A with Eigenvalue lambda. And this pair lambda and v is called an eigenpair of the matrix here. So what we have discussed here is a following that this candidate e to the power lambda t * v is a solution of x dash = Ax if and only if the pair lambda v, this lambda and this v is an eigenpair of the matrix here. So it means that if lambda v is an eigenpair then we can write x(t) as e to the power lambda t v as a solution of this.

So it means that for each eigenvector corresponding to eigenvalue lambda j this candidate at Xj(t) = e to the power lambda jt vj is a candidate of a solution. So it means that xj(t) = e to the power lambda jt vj is a solution of (6) that is x dash = Ax. Now we need to find out n linearly independent solution so it means that we need to find out n linearly independent eigenvectors v1 to vn. And corresponding to some eigenvalue say lambda 1 to lambda n. So idea is to find out n linearly independent eigenvectors.

And once we have n linearly independent eigenvector then we can write down our solution xj = e to the power lambda jt vj = vj where j is from 1 to n are n linearly independent solution of (6). So it means that the problem of finding n linearly independent solution is now reduced to finding n linearly independent eigenvectors v1 to vn. So that is what we wanted to now find out.

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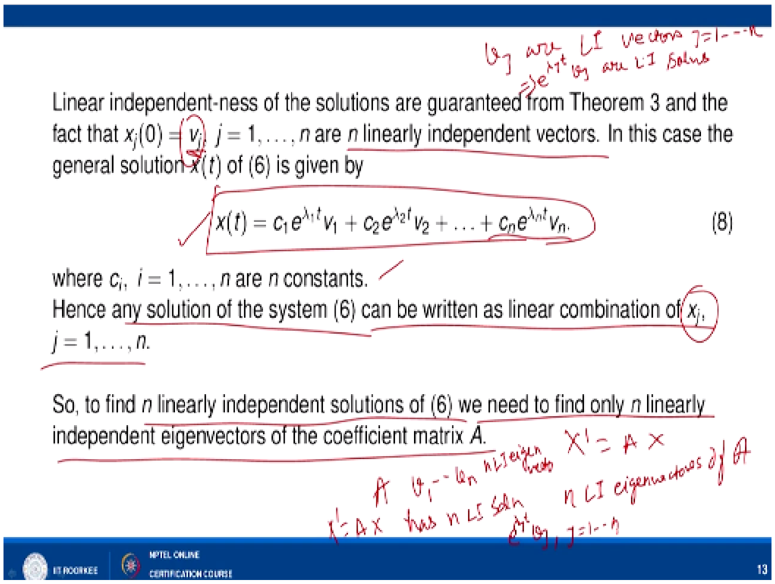

Now, once we, now we want to check whether these are linearly independent or not, so that can be check very easily because since these are solutions. So Xjt are solutions, now we need to check the values at some points. So let us check the values at 0 so xj(0) is vj. So it means that the linearly independence of the solution is depending on the linearly dependence of vj. So if vj are n linearly independent vectors then the solutions are also n linearly independent solutions.

So it means that in this case, so it means that if we vj are linearly independent LI vectors for j=n then we can write that e to the power lambda jt vj are also LI solutions, right so this implies. And once we have LI solutions, any LI solutions then we can write down the general solution as follows x(t)=C1 e to the power lambda 1 t v1 + C2 to the power lambda 2 t v2 and so on. So this is your solution, right where Ci's are some constants and real constant.

And hence; by this discussion we can say that any solution of this system (6) can be written as linear combination of xj and for this to find out this xj n linearly independent solution of (6) we need to find only n linearly independent eigenvectors of the coefficient matrix A. So it means that if you want to find out n linearly independent solution of this we need to find out n linearly independent eigenvectors of A.

So it means that given a matrix A once we have say, v1 to vn, n LI eigenvectors then x dash = Ax has n LI solutions and it is given as e to the power lambda jt vj for j=1 to n. So our now, we will focus on how to find out n linearly independent eigenvectors. But finding n linearly independent eigenvectors may not be easy task, and we have only one result which says that under a special condition. It may have, we have a guarantee that we have n linearly independent eigenvectors and that is the following result.

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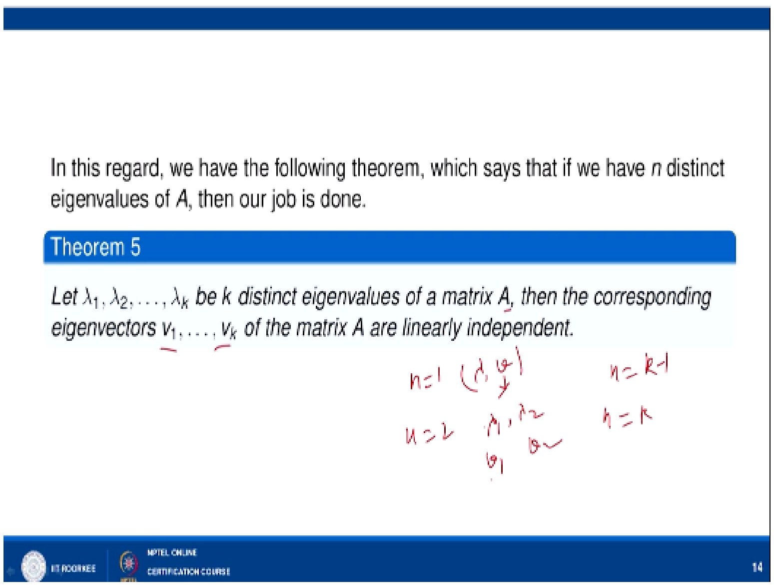

That let lambda 1 to lambda k be k distinct eigenvalues of a matrix A, then the corresponding eigenvectors v1 to vk are linearly independent. So it means that the crux of this theorem is that corresponding to distinct eigenvalues we have linearly independent eigenvectors. So this can be proved using mathematical indexation. So let us; I am giving you the info that n=1 it is quite obvious because for n=1 we have lambda v and eigenpair and by definition of eigenvalue this has to be a nonzero vector.

So it has to be linearly independent. Now for n=2 let us say that we have lambda 1 and lambda 2 r2 eigenvalues corresponding to which lambda 1 and we have v1 as an eigenvector corresponding to lambda 2 we have v2 as an eigenvector. And we want to show that v1 and v2 are LI vectors.

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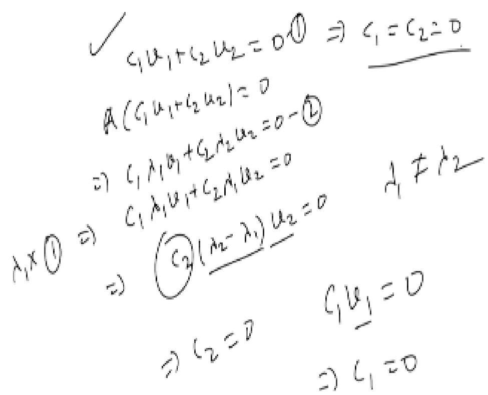

So here we want to check that c1 v1 + c2 v2 = 0; we want to show that this has only a trivial solution that is C1=C2=0. For that, let us some apply matrix A on this, so A of c1 v1 + c2 v2 = 0 so this implies at c1 lambda 1 v1 because A v1 is lambda 1 v1 + c2 A v2 is lambda 2 v2 = 0. So we have relation c1 lambda 1 v1 + c2 lambda 2 v2 = 0. Now here we have this equation number 1, this is equation number 2.

So multiply the equation number 1 by say lambda 1 so we have this 1 implies this, so lambda 1 times this will give you c1 lambda 1 v1 + c2 lambda 1 v2 = 0 so this implies that if you subtract these two then we have c2 lambda 2 – lambda 1 * v2 = 0. Now here lambda 2 – lambda 1 is nonzero because here we have assumed that lambda 1 is not to lambda 2. And v2 is nonzero eigenvector. So this implies that f it is nonzero eigenvector then the only possibility left out is that c2 has to be 0.

Now if c2 is 0 we can put it in value here in 1 and we can have c1 v1 = 0. And v1 is nonzero vector so this implies at c1 has to be 0. So it means that this c1 v1 + c2 v2 = 0 has the only solution that is C1=C2=0. So it means that for n=2 if lambda 1 is not equal to lambda 2 then v1 and v2 has to be linearly independent eigenvectors. And now you can prove; you assume for n=k-1 and you can prove for n=k using the similar idea.

So this I am leaving it to you. And we simply say that, let us summarize it and we simply say that, let lambda 1 to lambda k be k distinct eigenvalues of matrix A then the corresponding eigenvectors are also linearly independent vectors. So it means that if we take this theorem and the previous thing then we say that if you want to find out n linearly independent eigenvectors then it is quite easy that if we have n distinct eigenvalues of A then we can have n linearly independent eigenvectors.

So in this case finding n linearly independence solution is quite easy when we have n distinct eigenvalues of the matrix A.

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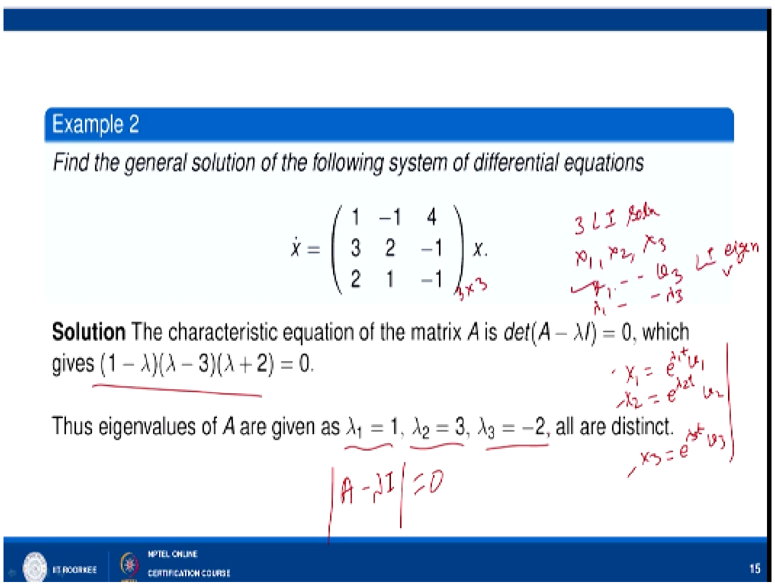

Okay, so let us take one example based on this theorem 5. Example is this, find the general solution of the following system of differential equations x dash = Ax where A is given as 1 -1 4; 3 2 -1; 2 1 -1. So as we have pointed out the system, the general solution can be written as, once we have since it is 3*3 matrix we need to find out 3 LI solutions say X1, X2 and X3. So if we have three LI solutions then we can write down the general solution.

And also we have described that, that if we can find out say v1 to v3 LI eigenvectors then corresponding to lambda 1 to lambda 3 then the solution e to the power lambda 1 t v1 can be treated as x1, x2 is e to the power lambda 2 t v2 and x3 is written as e to the power lambda 3 t v3. Then, if v1 to v3 are LI eigenvectors then x1 to x3 are the LI solutions of this system. So once we have solutions we can write down the general solution.

So idea is to find out LI eigenvectors v1 to v3. So for that let us simplify, let us first find out the eigenvalues of this. So here taking the liberty that finding the eigenvalues is quite easy and you can do that by writing A-lambda I determinant of this = 0. So as a root of A-lambda I=0 we can have this characteristic equation that is 1-lambda times lambda -3 * lambda +2=0. So here I am leaving it to you that so that characteristic equation of this matrix A is given by this.

So once we have this then we can check that lambda 1 = 1, lambda 2 = 3, a lambda 3 = -2 or 3. eigenvalue of this matrix is A. Now here we can see that they are distinct. So it means that now if they are all distinct so we have this assurance that the corresponding eigenvector are also linearly independent eigenvector. So let us find out these linearly independent eigenvectors.

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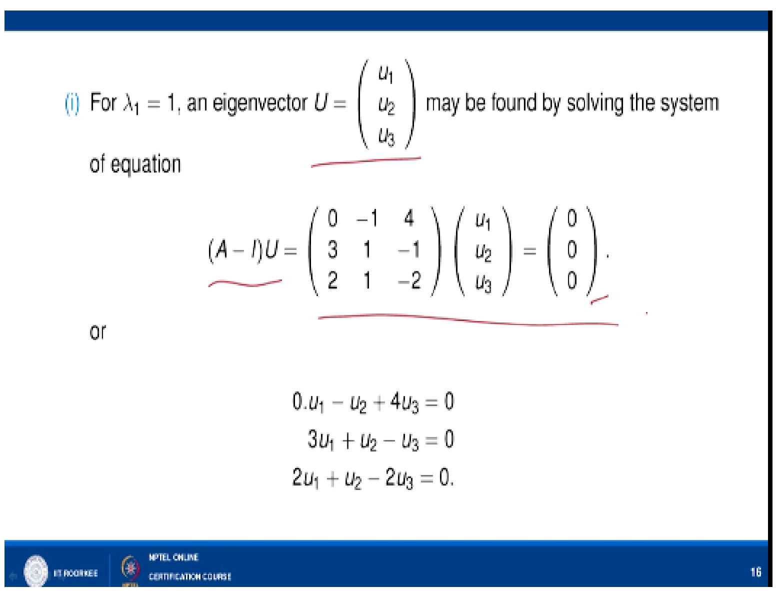

So corresponding to lambda 1 = 1, let us eigen; assume that eigenvector is of this form u1 u2 u3 and we can find out by solving this system of linear equation (A-I)U=0. So here again I am putting lambda 1=1 and just writing –I*U so we have this system of linear equation and this is to this. And here again I am assuming that we can easily solve this system of linear equation.

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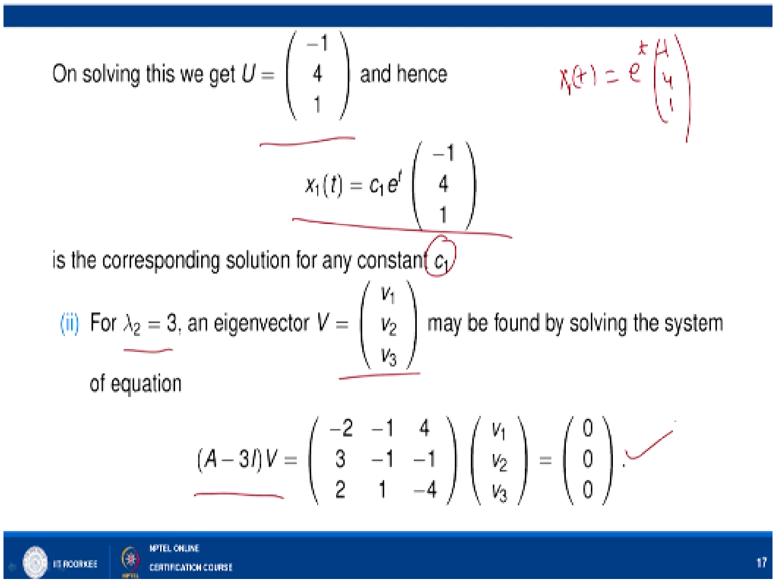

And we can have solution as -1 4 1 as a solution. So once we have eigenvectors and solutions can be written as C1 e to the power lambda 1 t so solution is what x1(t)=e to the power lambda 1 is 1 here; e to the power t*v1, so v1 is -1 4 1. So the solution x1(t) as written as e to the power t -1 4 1, so that is what we have written. And C1 may be considered as a constant C1. So if x1(t) is a solution then any cost multiple will also be a solution of this.

Now let us look at for lambda 3=3. Let us assume that the eigenvector is given by v1 v2 v3 and we can find out v1 v2 v3 by solving the system of linear equation A-3I*v=0.

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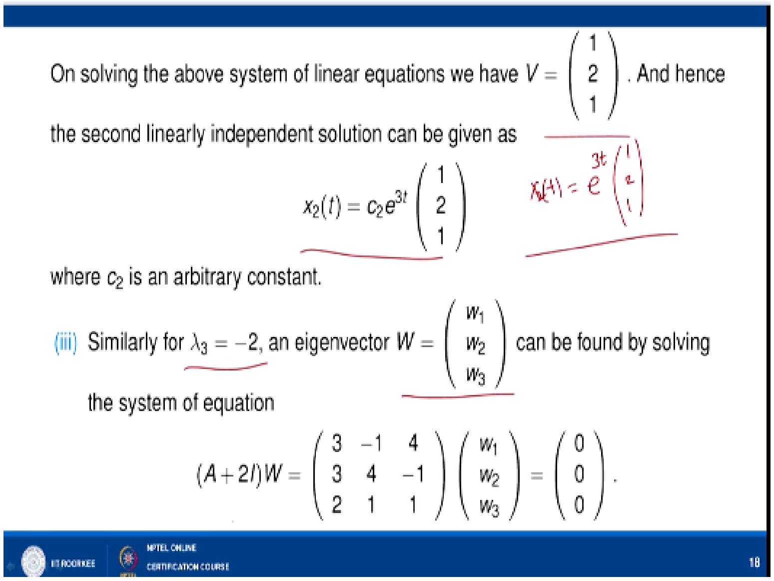

And after simplification we have the solution given as 1 2 1 and solution is given by x2(t) e = e to the power lambda 2 is 3t * 1 2 1. So this is a solution and any constant multiple of this will also be a solution. So x2(t) can be written as c2* e to the power of 3t 1 2 1. And similarly for lambda 3 = -2, let us assume eigenvector is taking this W1 W2 W3 kind of format. And this we can find out by solving this system of linear equation (A+2I)W=0. Now A is known to you, you can find out (A+2I)W and you can find out this.

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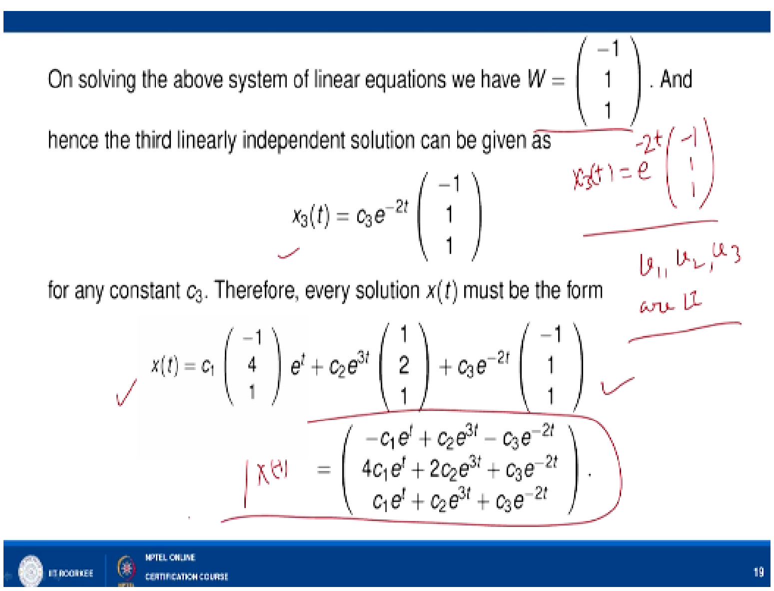

As the solution is given by -1 1 and 1. So t means that the solution is given by X3(t)=e to the power lambda 3 is – 2t * -1 1 1. So here solution x3(t) is given to us. We can write down the general solution x3(t)=c3 e to the power -2t -1 1 1. Now, we already know that v1 v2 v3 are LI.

How we know? We know by the result but you can still check whether you are getting a LI solution or not, so you can check that -1 4 1; 1 2 1 and -1 1 1 is basically are LI eigenvectors.

So once they are LI eigenvectors then the general solution can be written as linearly combination of x1(t) and x2(t) and x3(t). So C1 -1 4 1 + C1 e to the power 3t 1 2 1 + C3 e to the power -2t -1 1 1. Then general solution is given by x(t)= this, the following thing will serve as a general solution of this system. So it means that if we have distinct eigenvalue then we have linearly independent eigenvectors and we can find out the n linearly independence solutions. And; but it may not always be true.

So we will discuss more cases of this and how we can find out n linearly independence solution in say coming lectures. So here we stop, we will continue this study in next lecture also. Thank you very much for listening us, thank you.