Dynamical Systems and Control
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Lecture – 51
Optimal Control - II

Dear students, welcome to this second lecture on optimal control, in the previous lecture we have seen the optimal control problem for fixed boundary problems and in this lecture, we will see various different types of boundary optimal control problems.

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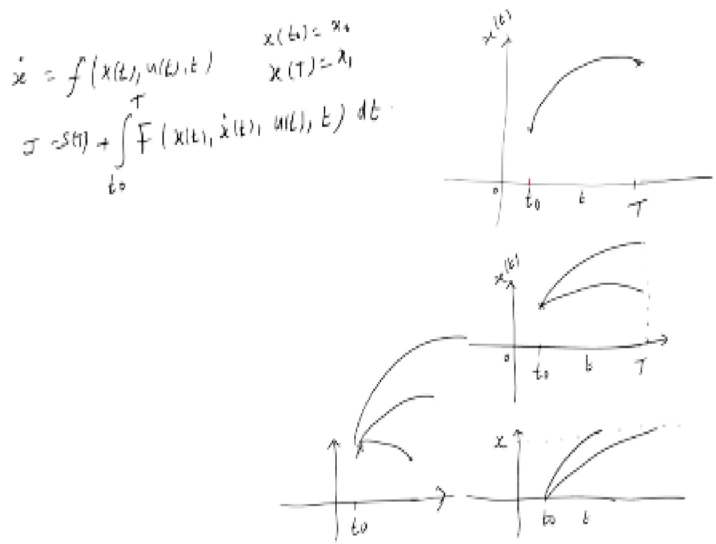

So, here the previous lecture we have seen the fixed boundary problem in this particular way that is initial time is t0, final time is capital T and initial condition is x0 and final condition is x1, all are; all of them are fixed, x of t0 is x0, x of capital T is x1, so all the values are fixed to one and the second type of problem which we will be consider is the t0 and capital T, both are fixed value but the initial condition is fixed, x0 is fixed but x1 can vary.

There is a; it is not a fixed value or fixed vector here, here we consider the state variable is a vector in Rn and the third type is the x1 is fixed, x0 and x1, both are fixed, initial condition and final condition are fixed but the time is not fixed, capital T can vary, the initial time is fixed, final time can vary and the other type is; if you consider the initial condition is fixed x of t0 = x0 is fixed both of them.

But the final time as well as the final condition both of them are not fixed, so it can be any function of this type, so various types of function can be considered neither the final time nor the final position, both of them are variables here in the fourth case. So, in all the cases we would like to find an optimal control for the control problem of this type x dot = x of t u of t, so we have a control system of the form.

We want to minimise a functional j, in the previous lecture, we have seen J is of the form, f of x of t and x0 of t u of t and t dt, so this expression but in addition to that we can also optimise expression of this form, S of t etc.

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Terminal Cost Problem:

Consider the system

$$\dot{x}(t) = f(x(t), u(t), t),$$

$x^*(t)$
$u^*(t)$

and the performance index as

$$J_1(u(t)) = S(x(T), T) + \int_{t_0}^{T} F(x(t), u(t), t)\,dt,$$

and given boundary conditions as

$$x(t_0) = x_0; \quad x(T) \text{ is free and } T \text{ is also free.}$$

where $x(t)$ and $u(t)$ are n and r dimensional state and control vectors.

The final a function, which is a function of the final time capital T and which can be minimised whether it is constant or variable, a function of S can be considered and the performance index or the cost functional J1 is defined like this which is to be minimised or maximised. So, if these values are fixed, x of t and capital T, both of them are fixed values, so at the optimum value of j1 with S or without S both will give the same control function u of t.

Because it will; S is going to be a constant only, if S is a variable, then the optimisation of this with S and without S may be different, will be different definitely, so for considering a cost functional of this type, it is required that the x of either the final position x of t or the final time t, they should be a variable or both of them are variables, so we consider the control system x dot = f of x of t u of t, t.

And the cost functional J1 = S of capital T, t + plus integral of this expression, x of t u of t and t, and the boundary conditions are given by x of t0 x0 + and x of t is free and t is also free and it will include all the cases, all the 4 cases, we can take x of t is fixed, then t is fixed and both of them are fixed, so all the 4 cases will be taken care by deriving a procedure for this general case.

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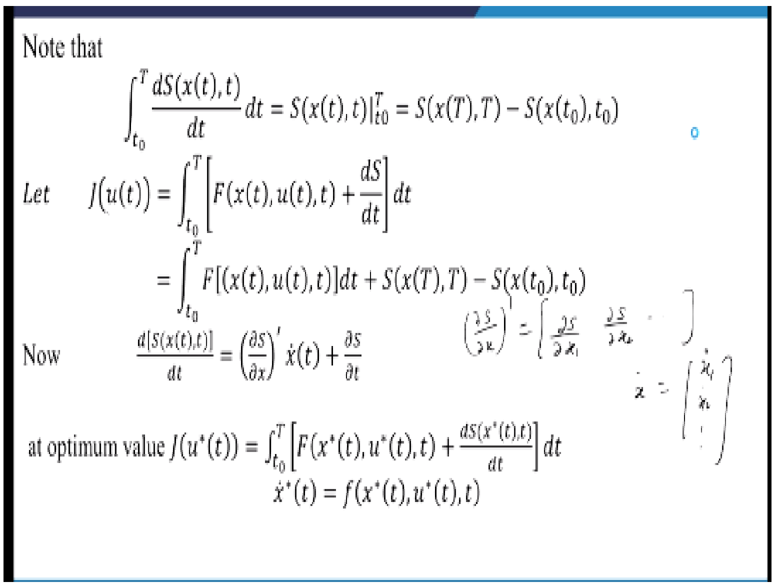

Now, dS/dt integral is given by this expression, it is well known, S at final position, – S at the initial position of this and if you substitute, because here is J1 is defined as S at capital T is there, so using this expression, S at capital T is given by integral dS/dt - S at the initial condition, so we substitute that expression, we will get the expression, J as integral here already f is there, in the place of S at capital T, we will replace integral dS/dt - + S at the initial condition.

So, we can substitute the expression here, so consider the equation, integral t0 to capital T dS/dt = S at the final time – S at the initial time, so that is given, so let us take a new functional J, J of u given by integral t0 to capital T capital F + dS/dt and integral dS/dt from the previous step, if we substitute, we will get J = integral t0 to capital T f of x, u, t dt + replacing dS/dt integral as S at final time - S at the initial time.

So, now if we compare this J1 and J, only extra term is here - S at the final position otherwise both of them are the same and S at the final; S at the initial point t0, it is a constant, so by minimising J, whatever control function we will get the same control function we will get by minimising J1, so instead of solving this problem J x dot = f and J; minimising J with this performance index, we can do the minimisation problem with the J; J of u of S, this expression as the performance index, which will be easy to handle.

So, now dS/dt because S is a; if you take S as a function of x, t; x of t, t, then it is del S/ del x * dx/dt + del S/ del t, here we note that S is; x is a vector, so the dash indicates that it is the transpose of the vector, del S/ del x, del S/ del x means it is del S/del x1, del S/ del x2, these are the component of the vector; the column vector and when we put the transpose, it is the row vector, del S/ del x1, del S/ x2 etc.

So, this vector is del S/ dx transpose, is this one and x dot, it means it is the vector as we know x1 dot x2 dot etc. so and del S/ dt is a scalar function, now at the optimum value, we say that the solution of the problem; optimal solution of the problem is the solution x star of t is the optimum state and u star of t is the optimum control, so at this 2 functions, if we evaluate J, so we are saying that J u star is given by wherever x and u is there, we will replace it with x star and u star in the expression J.

And the equation x dot is replaced by x dot star = f of x star u star t that means the given equation is; we are substituting just the optimal state, x star in the given equation, so we get this equation and J is given by this at the optimum value.

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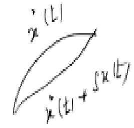
Consider the variations $x(t) = x^*(t) + \delta x(t)$; $u(t) = u^* + \delta u(t)$



Then the state equation and the performance index become

$$\dot{x}^*(t) + \delta\dot{x}(t) = f(\dot{x}^*(t) + \delta\dot{x}(t), u^*(t) + \delta u(t), t)$$

$$J(u(t)) = \int_{t_0}^{T+\delta T}\left[F(x^*(t) + \delta x(t), u^*(t) + \delta u(t), t) + \frac{dS}{dt}\right]dt$$

$T, \quad T + \delta T$

Now, at any other function that is in the neighbourhood of the optimum x star, we take x star + del x star, if this is our x star function then by adding x star of t + del x star of t, we get a new function, depending on the initial and final condition, if they are all fixed it will be like this, if they are not, if one is a variable, then we may get some other expression where there will be variation at the end position also.

So, depending on the 4 types of problem, we will get x star and x star + del x star function, similarly u star and del u function, so this can be added, so now substituting this function x star + del x in the given equation, this the state equation, the first one; x dot = f of x, so we get that = f of x star + del x and u star + del u, here this star should be cancelled, f is a function of the x and u, so it should be x start + del x and u star + del u.

J of u = integral t0 to t + del t at the say, t is the final position and t + del t is the; del t is the variation in the final position but if t is a fixed value then del t is 0, so here for the case; first case where t is fixed, the del t will be 0 but this is a general procedure for all the 4 cases, so we take a general variation for the final position t, t0 to capital T + del t and at the any function x of t in the neighbourhood of the optimum control, optimum state we substitute those values here, + dS/dt.

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Lagrange Multiplier: Introducing the Lagrange multiplier vector $\lambda(t)$ and the augmented performance index at the optimal conditions is defined as

$$J_a(u^*(t)) = \int_{t_0}^{T}\Bigg[F(x^*(t),u^*(t),t) + \left(\frac{\partial S}{\partial x}\right)'\dot{x}^*(t) + \left(\frac{\partial S}{\partial t}\right) + (\lambda(t))'\{f(x^*(t),u^*(t),t) - \dot{x}^*(t)\}\Bigg]dt$$

In the place of the functional J, we are substituting; first we substitute x of t and u of t and subtract from the original value at x star and u start, so now we take the augmented functional, J suffix a at the optimal control u start, so that is given by integral t0 to capital T f at x star u star and everything is calculated at the optimal state x star, all this terms are calculator at x star.

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and at any other perturbed condition as

$$J_a(u(t) = \int_{t_0}^{T+\delta T}\Bigg[F(x^*(t)+\delta x(t),u^*(t)+\delta u(t),t) + \left(\frac{\partial S}{\partial x}\right)'(\dot{x}^*(t)+\delta\dot{x}(t)) + \left(\frac{\partial S}{\partial t}\right) + (\lambda(t))'(f(x^*(t)+\delta\dot{x}(t),u^*t)+\delta u(t),t) - \{\dot{x}^*(t)+\delta\dot{x}(t)\})\Bigg]dt$$

And in the neighbourhood of x star, next we substitute Ja at any u of t that is u star is the optimum control and u of t is in the neighbourhood of optimum, so we substitute in the place of x, we substitute x star + del x and u star + del u etc. so, wherever x star was there, we just add the variational function del x and del u and calculate the same expression for the Ja function.

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Define the Lagrangian function at optimal condition as

$$L = L((x^*(t), \dot{x}^*(t), u^*(t)), \lambda(t), t)$$
$$= F(x^*(t), u^*(t), t) + \left(\frac{\partial S}{\partial x}\right)' [\dot{x}^*(t)] + \left(\frac{\partial S}{\partial t}\right)$$
$$+(\lambda(t))'\{f(x^*(t), u^*(t), t) - \dot{x}^*(t)\}$$

$$L^{\delta} = F(x^*(t) + \delta x(t), u^*(t) + \delta u(t), t) + \left(\frac{\partial S}{\partial x}\right)' (\dot{x}^*(t) + \delta \dot{x}(t)) + \left(\frac{\partial S}{\partial t}\right)$$
$$+(\lambda(t))'(f(x^*(t) + \delta \dot{x}(t), u^*t) + \delta u(t), t) - \{\dot{x}^*(t) + \delta \dot{x}(t)\})$$

So, now we define the Lagrangian as the function which is present inside the integral, the entire integral for Ja u star, the function is capital F + del S / del x transpose * x dot etc. so this function we call it as L and the incremental function which is evaluated in the neighbourhood function that the whole bracket we call it as L delta, super fix delta, so now we find the variation between Ja at u star and Ja at u.

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$$J_a(u^*(t)) = \int_{t_0}^{T} L dt$$
$$J_a(u(t)) = \int_{t_0}^{T+\delta T} L^{\delta} dt = \int_{t_0}^{T} L^{\delta} dt + \int_{T}^{T+\delta T} L^{\delta} dt$$

Using Taylor series, and retaining the linear terms, we have

$$\int_{T}^{T+\delta T} L^{\delta} dt = L^{\delta}(T) \cdot \delta T$$
$$\approx \left\{L + \left(\frac{\partial L}{\partial x}\right)\delta \dot{x}(t) + \left(\frac{\partial L}{\partial \dot{x}}\right)\delta \dot{x}(t) + \left(\frac{\partial L}{\partial u}\right)\delta u(t)\right\}_T \delta T$$
$$\approx L|_{at\,T} \delta T$$

So, we just denote it as Ja at u star as integral L dt, Ja at u as this expression. Because the; there is a variation in capital T also, the limit; upper limit also will vary, t + del t and L delta * this expression, so this can be split into t0 to capital T and capital T to t + delta t of this expression and using the Taylor series expansion for L delta, then we will get the expression like this, the second term integral t t0 t + del t of L delta is given by the value is approximately given by this expression using the mean value theorem that this calculating L delta at the final position t multiplied by the length of this interval that is del t.

So that is approximate this value because it is a very small variation del t and we get this value for approximate value. Now, using Taylor series for L delta, we get the L delta will; as we have defined here, L delta is this expression, so if you write the Taylor series for the function capital F and the remaining things and collecting the first order terms only at omitting all the second order terms, we get this expression and so L delta is this one, okay.

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$$\Delta J = J_a(u(t)) - J_a(U^*(t))$$

$$\int_{t_0}^{T} (L^\delta - L)\,dt + L|_{at\,T}\,\delta T$$

$$\delta J = \int_{t_0}^{T} \left(\left(\frac{\partial L}{\partial x}\right)' \delta x\,(t) + \left(\frac{\partial L}{\partial \dot{x}}\right)' \delta \dot{x}(t) + \left(\frac{\partial L}{\partial u}\right)' \delta u(t) \right) dt + L_{at\,T}\,\delta T$$

Considering the $\delta \dot{x}(t)$ term in the first variations and integrating by parts, we get

$$\int_{t_0}^{T} \left(\frac{\partial L}{\partial \dot{x}}\right)' \delta \dot{x}(t)\,dt = \int_{t_0}^{T} \left(\frac{\partial L}{\partial \dot{x}}\right)' \frac{d(\delta x)}{dt}\,dt$$

$$= \left[\left(\frac{\partial L}{\partial \dot{x}}\right)' \delta x(t) \right]_{t_0}^{T} - \int_{t_0}^{T} \frac{d}{dt} \left\{ \left(\frac{\partial L}{\partial \dot{x}}\right)' \delta x(t) \right\} dt$$

Now, the variation del J is Ja calculated at u - Ja calculated at the optimal control and that is nothing but integral t0 to capital T L delta - L * dt, this L is not there, L * dt +; we have seen the previous slide, the value is L at capital T * del t, so we get this expression, so the variation del J is given by this expression + L at capital T * del t, now we take the second term in this integral, del L/ del x dot transpose * del x dot.

So, del x dot can be written as d/dt of del x, sorry this dot to be removed, so d/dt of del x is this expression, now we integrate by parts this 2 terms, we get del L/ del x dot * del x evaluated at t0 and capital T – d/dt of del L/del x dot * del x * dt, so this expression we get.

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Also since $x(t_0)$ is specified $\delta\dot{x}(t_0) = 0$. Hence

$$\delta J = dt\int_{t_0}^{T}\left\{\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right)\right\}'\delta x(t)\,dt + \int_{t_0}^{T}\left(\frac{\partial L}{\partial u}\right)\delta u\,(t)dt$$

$$+L/_{at\,T}\,\delta T + \left[\left(\frac{\partial L}{\partial \dot{x}}\right)'\delta\dot{x}(t)\right]_T$$

$x(t)$

$\delta u(t)$ is arbitrary

$\delta J = 0$

Now, substitute this term instead of the second term, then the variation del J is obtained by this formula that is integral del L/ del x – d/dt del L/ del x dot, its transpose into del x dt + del L/ del u that is also there in the expression here, del L/ del u * del u, we have taken the first term, for the second term we have taken this integral d/dt of del L/ del x dot, del L/ del u is also there, so this 3 terms + del L at capital T * del t that term is there.

And this expression del L/ del x dot * del x evaluated from the t0 to capital T, so this; so, now it is evaluated only at capital T because here we see that there is no variation at 0, when we take t = t0, the variation of the x at t0 is 0 because the initial position is same for all the functions which we consider, there is no change in the initial condition, so only at the upper bound; upper boundary value T, we evaluate this.

So, this term is del L/ del x dot. transpose * del x dot t evaluated at capital T, so this is our variation; first variation. Now, for the optimal control, the first variation should be = 0, del J should be = 0 that gives the necessary condition for the optimal control.

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First we chose $\lambda(t) = \lambda^*(t)$ which is at our disposal such that the coefficient of the dependent variation $\delta x(t)$ is zero. Thus, we have the Euler-Lagrange equation

$$\left(\frac{\partial L}{\partial x}\right) - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0 \qquad (A)$$

Where the partial derivatives are evaluated at the optimal function. Next, since the independent control variation $\delta u(t)$ should be zero, that is

$$\left(\frac{\partial L}{\partial u}\right) = 0 \qquad (B)$$

at the optimal condition, we get

$$\delta J = L|_{at\,T}\,\delta T + \left[\left(\frac{\partial L}{\partial \dot{x}}\right)' \delta \dot{x}(t)\right]_T = 0 \quad (C)$$

as boundary condition for optimum solutions.

And that can be obtained by the following. further we see that the lambda value we have to choose, it is at our disposal, so it can be selected in such a way that the first bracket turns out to be 0, we select the lambda value in this expression, del L/ del x – d/dt, if you see here the expression l; L is having the lambda value here, lambda dash * all this thing, so first we select the lambda in such a way that this expression vanishes; del L/ del x – d/ dt of del L/ del x dot = 0 by choosing suitable value of lambda.

So, we get the first equation as the Euler Lagrange equation and then we can see that there is no variation in because the variation we are considering only in the function, the x of t, the control function is not affected by the variation x of t, so the this is arbitrary, here del u of t is arbitrary, so here del u; del L/ del u can be selected to be 0 that is our second condition we get and finally, after equating the first term, it is already 0.

The second term is also 0, only the remaining terms are del J = L at capital T del t + this expression that should be = 0, so we get the condition; boundary condition as del J = this = 0, so with this 3; A, B, C if you solve a given optimal control problem, we will be able to get a solution x star and u start, so which may turn out to be the optimal required control because its only necessary condition.

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Consider the Hamiltonian

$$H^* = F(x^*(t),u^*(t),t) + (\lambda(t))^* f(x^*(t),u^*(t),t)$$

Then

$$\mathcal{L}^* = \mathcal{L}(x^*(t),\dot{x}^*(t),u^*(t),(\lambda(t))^*,t)$$
$$= H^*(x^*(t),u^*(t),(\lambda(t))^*,t)$$
$$+\left(\frac{\partial S}{\partial x}\right)' \dot{x}^*(t) + \left(\frac{\partial S}{\partial t}\right) - (\lambda(t))^*\dot{x}^*(t)$$

Now the conditions (A), (B) and (C) in terms of the Hamiltonian can be written as follows. Eq (B) $\frac{\partial L}{\partial u} = 0$ implies $\frac{\partial H}{\partial u} = 0$ ... (D)

and (A) implies

$$\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0 \Rightarrow \left(\frac{\partial H}{\partial x}\right) = -\dot{\lambda}(t) \qquad (E)$$

$$\frac{\partial L}{\partial x} = \frac{\partial H}{\partial x} + \frac{\partial^2 S}{\partial x^2}\dot{x} + \frac{\partial^2 S}{\partial x \partial t} \quad \text{and} \quad \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{d}{dt}\left(\frac{\partial S}{\partial x} - \lambda\right) = \frac{\partial^2 S}{\partial x^2}\dot{x} + \frac{\partial^2 S}{\partial x \partial t} - \dot{\lambda}$$

So, the second variation should be considered to asset whether the solution obtained are actually optimum or not, okay, so but in most of the practical problem, this solution turns out to be the optimal control problem that we will see some examples we will see later, so now we will introduce the notation Hamiltonian, introduce the function H which is called the Hamiltonian which is say defined by capital F + lambda.

If we observe earlier the Lagrangian in function, L is defined as capital F + all this terms, now what we do is; we just take those terms in which x dot is not there, the derivative of x, so we take Hamiltonian as capital F function and del S/ del t function there is no x dot and here lambda multiplied by the small f function, so we just omit the x dot terms, so that is called the Hamiltonian, okay.

So, if you define H in this way, then we will get L as the remaining terms to be added, L star = H star function Hamiltonian + all the terms which contains the x dot term, so we write this expression, I think del S/del t is also included in this one, it is not included in the H, Hamiltonian function, okay. So, now we can easily see that; already we have derived the necessary condition for the optimal control.

One can solve the problem using this expression, if the problem is quite straightforward, simple expressions are there but a convenient expression is using the Hamiltonian function, so after introducing the Hamiltonian function, we can convert all this equation A, B, C in terms of H, so if we take the first one, Hamiltonian can be written in this form, okay, so here; so, now we have shown that the conditions A, B, C are the necessary condition for the optimal control of the given system.

Now, we are interested in converting this conditions A, B, C in terms of the Hamiltonian function, so we can see that the condition B that is del L/ del u = 0, directly we can see that it is same as del H/ del u = 0 from the expression of L and similarly, the condition A that is del L/ del x – d/dt of del L/ del x dot = 0 that is condition A, so now we replace it with the H function, so del L/ del x.

If you see the expression L, so del L/ del x = here, del L/ del x will be del H/ del x here and then the derivative del square S/ del x square and del square S/ del x del t that will come and here there is no x involved, so this will be; the derivative will be 0, so we will get this expression, del square S / del x square * x dot, okay, so this expression we will get. Now, the second term is d/dt of del / del x dot.

So, if you substitute here, del d/dt of del L/ del x dot is d/ dt of del L/ del x dot will give del S/ del x and - lambda term will come and when we differentiate it, we will get del square S/ the total derivative d/dt of del S/ del x is given by del square S/ del x square * dx/dt – del square S/ del x del t – d lambda/ dt, so this term, so if you subtract this 2 in the; if you substitute in the condition A, we will directly get this expression.

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$$\left(\frac{\partial L}{\partial \lambda}\right) = 0 \text{ implies } \left(\frac{\partial H}{\partial \lambda}\right) = \dot{x}^*(t) \qquad (F)$$

$$\delta J = L|_{at\,T}\,\delta T + \left[\left(\frac{\partial L}{\partial \dot{x}}\right)' \delta \dot{x}(t)\right]_T = 0 \text{ implies}$$

$$\left(H + \frac{\partial S}{\partial t}\right)_T \delta T + \left(\frac{\partial S}{\partial x} - \lambda^*(t)\right)_T \delta x_T = 0 \qquad (G)$$

(To be proved)

Del H/ del x = - d lambda/ dt that is the condition E; second condition, del L/ del lambda = 0 that is same as del H/ del lambda = x dot from the expression of L, we can verify, further we get the boundary condition as this thing, as we have seen earlier the boundary condition is C, so to replace that this is a condition C and which is equivalent to this expression, wherever L is there, if you substitute the H function as given in this equation.

We get this expression, H + del S / del t evaluated at capital T * del t + del S/ del x - lambda star evaluated at capital T * del xt, okay = 0, so this result we will prove in the next lecture that is because this needs a proof separately and which will be used for various cases, the 4 cases which discussed for the optimal control, this is the key boundary condition for various this thing. Now, we can easily see here after proving this result, we can see that if there is no change in the final condition.

If capital T is fixed, then del t is 0 and similarly, if the final position is also fixed, then del xt is 0 but if one of them is having a variation, we will get a suitable boundary condition for solving a system of equation. So, for finding the optimal control in terms of Hamiltonian, we have the 4 conditions, D, E, F and G, where G is the boundary condition for the solution of the optimal control.

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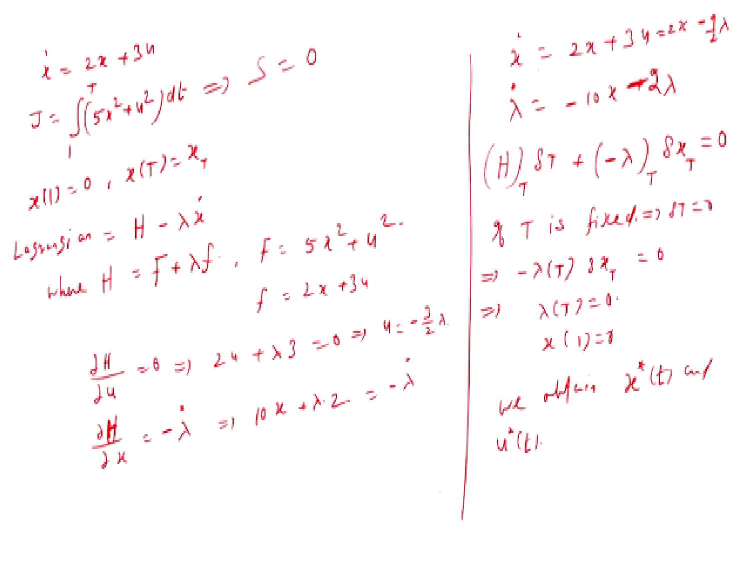

Now, we will see an example illustrating this procedure, so let us consider a equation x dot = 2x + 3u and the cost functional is given by 1 to capital T 5x square + u square * dt and the initial condition x of 1 is 0 and final condition x of t = x of xt, so there can be different cases t can be fixed and x suffix t may be a variable similarly, x of xt may be fixed and t is a variable or both of them are variables and or both of them are fixed 4 cases.

So, in all this cases, how to find an optimal control for the given equations, so here the Lagrangian is given by the Hamiltonian H - lambda times x dot, if you compared the equation L star = H + all this, so if you note that the we have started with the cost functional J as S of xt, t, capital T + etc. but in this particular example, the first term S is not there, so it is = 0, only the remaining integral is there.

Here, we get, this implies S = S function is 0, so wherever S comes, we take to 0 function therefore, the Lagrangian = H - lambda x dot, where H is given by the function F + lambda times small f, and where capital F is the integral, integrand 5x square + u square and small f function is 2x + 3u, so these are the equation, now we directly make use of the four conditions for the necessary condition for finding the optimal control.

So, del H/ del u = 0, so if we take directly del H/ del u that is 2u + lambda times small f, so derivative is 3 = 0, so this implies u in terms of lambda -3/2 lambda and the second is del H/ del x = - lambda dot, the second equation, so del H/ del x will give 10x + lambda times 2 that is = - lambda dot, the second equation and the other equation is already given in the question itself, so we have got x dot = 2x +3u and lambda dot = -10 x + -2 lambda.

So, in the place of u, we can substitute -3 / 2 lambda, so it is 2x – 9/ 2 lambda, so this 2 equation and for the boundary condition, we make use of this equation, H; S is 0, therefore H evaluated at capital T * del t and - lambda evaluated at capital T * del xt, it is H function evaluated at lambda * del t + - lambda function evaluated at capital T * the variation of the final position = 0, now if the t is fixed, then del t = 0.

So, only this will be the boundary condition, so that implies - lambda of capital T * del xt = 0 but the variation is arbitrary, so this implies lambda of t = 0 and already we have given some initial condition x of 1 = 0, so these are the condition, we have this 2 equations, x dot = this, lambda dot = this equation with this 2 boundary conditions, we can solve the system of equation and obtain the optimal control u.

And then after substituting the optimal control in the same equation, we will get the optimal state value, so we obtain x star of t and u star of t, because first we solve in terms of lambda, lambda and x, then by substituting the lambda value, we get here u = -3 / 2 lambda, so we get the optimal control then substitute the optimal control in the first equation, we get the x star also, so we can solve the equation in this manner.

So, same procedure can be adopted for the if xt is fixed and t is a variable, we can evaluate in a similar manner, so a different examples we will demonstrate in the next lecture that is optimal control 3 and we will prove this result also, the boundary condition result in the next lecture, okay, thank you.