Dynamical Systems and Control
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Lecture - 40
Feedback Control - II

Hello Viewers. Welcome to the lecture on Feedback Control. In this lecture, we will continue with the concept of feedback control which was discussed in the previous lecture. Now, I will demonstrate the computation of feedback control using an example. So first let us summarize the concept of feedback control which was seen in the last lecture.

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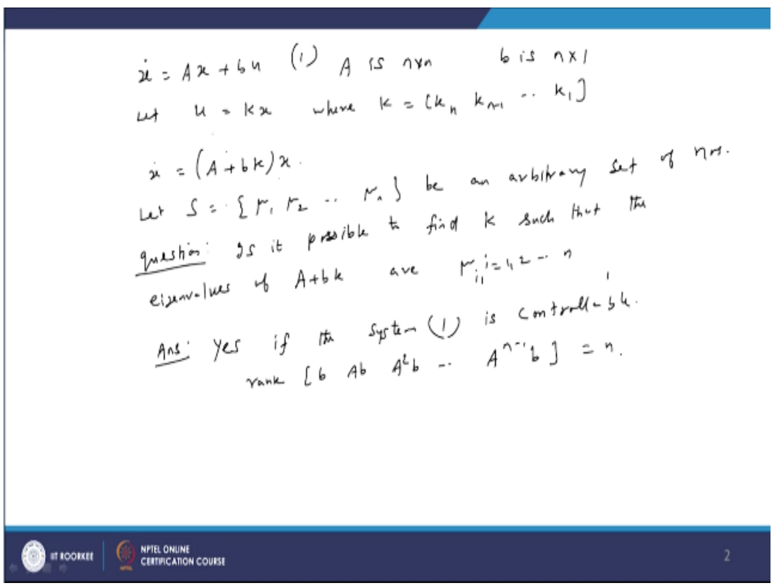

So let x dot = Ax + bu be a control system given to us where A is n cross n matrix and b is a column vector n cross 1 matrix. Now let the; okay. Let u the control is a feedback control of the form kx, where k is a unknown column; k is a unknown row vector. And so, when we substitute the control in this equation we will get Ax+b kx so that will finally become (A+bk) of x. So let us S be the set Mu 1, Mu 2 etcetera Mu n be an arbitrarily set of numbers, so n number are given arbitrarily.

So now the question is, is it possible to find the matrix k such that the eigenvalues of A+bk are exactly the set S, Mu i, where i=1, 2, 3 up to n. So is it possible to find a k in this manner. So the answer is we can find provided the system is controllable. So let us call this system equation as

1, is the system 1 is controllable, or the rank of b Ab A square b etcetera A-1b, if the rank is n then we can find a k like this. So if you are able to find how to find this k.

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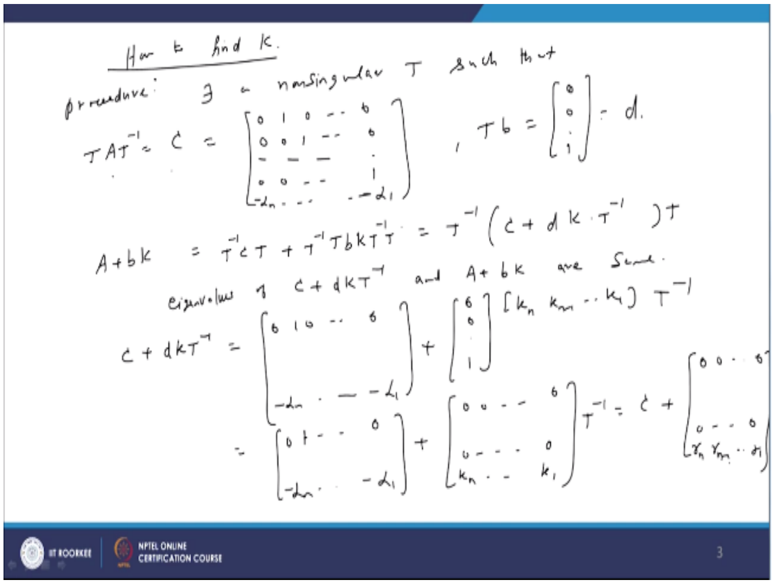

That is; for this we follow this procedure, the procedure is; earlier we studied that there exists because the system is construable, there exists a non-singular matrix T such that TAT inverse is the companion form 0 1 0 0, last is 1 and then - alpha n etcetera –alpha 1. And T*b is 0 0 1 this is the d matrix. So we can find a unique non-similar matrix T so that this happens is it not. So here can easily see that alpha 1, alpha 2, alpha n these are the coefficient of the characteristic equation; characteristic polynomial of the matrix C.

In other words, it is the coefficient of characteristic polynomial of matrix A because A and C are similar matrices, they will have the same eigenvalue so characteristic polynomial is the same for both of them, so the coefficient of characteristic is a last row of the matrix C. Now we are interested in A+bk, we want that the eigenvalues of A+bk are Mu 1 Mu 2 Mu n. So we can write this as A is T inverse, CT. And b is T inverse d, okay. So we can write this as T inverse Tb.

So this can be written as T inverse Tb, b can be written like this then we have k then T inverse T can be written like this, so that the final thing is we can take T inverse outside the left side and T can be taken out in the right side so the remaining things are C+Tb=d and then K*T inverse, so this expression is there. So it is easy to see here A+bk and C+dkT inverse, these are the similar matrices. So the eigenvalues of C+dkT inverse and A+bk are same.

So now if you calculate this; therefore, the last column last row which will appear in this matrix should be the characteristic coefficient of the characteristic polynomial of A+bk, so that is the meaning of this expression. Now if you calculate C+dkT inverse so C is 0 1 0 0 etc., –alpha n, -alpha 1 and d is 0 0 1 and k is kn kn-1 k1*T inverse. So this will appear like this, plus the product of this two it will be 0 0 etc., kn k1 * k inverse.

Further if we calculate we will get this expression to be the first matrix is c+, the second one is 0 0; first n-1 column, the last column we will multiply this row with this T inverse matrix we will get gamma n, gamma n-1, gamma 1.

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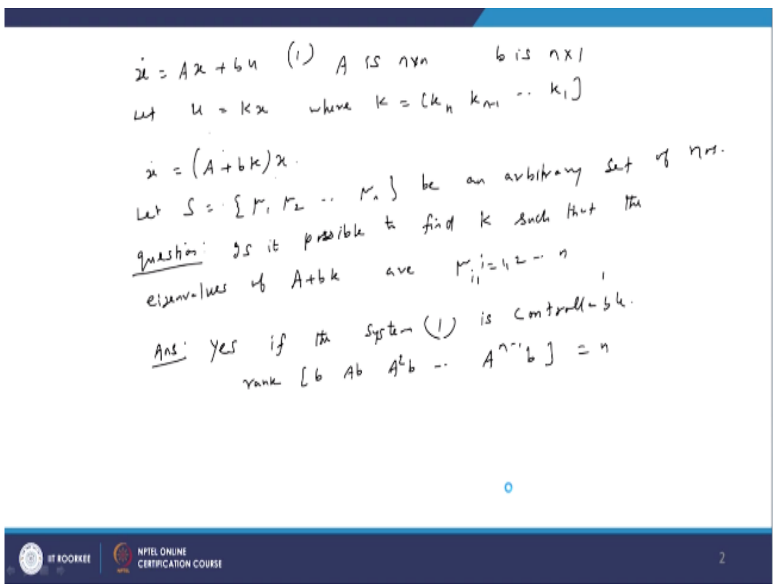

So adding these two matrices finally we will get the matrix to be 0 1 0 0; 0 0 1 then we will get –alpha n + gamma n and –alpha 1 + gamma 1. So this should be the coefficient of the polynomial. So the characteristic equation of A+bk, what is required is lambda – Mu 1; lambda –Mu 2; lambda –Mu n = 0. So this will be some polynomial like this + beta n=0. So what we have is the last column of this matrix; last row of this matrix should be –beta n, -beta n-1 etc.

So this implies that –alpha i + gamma i that should be equal to –beta i, i=1 to up to n. Or we want to calculate this gamma i that is equal to alpha i – beta i. So –beta i, i=1 to n. And the previous slide we have seen the relation here, this expression we want the k matrix only. But what we have calculated is this expression. So the relation between k matrix and this gamma kn kn-1 k1 that is the last row. And this last row is gamma n etc.

So the relation between this two is k multiplied by T inverse is given by this gamma matrix. So from the previous page we know that kn kn-1 k1*T inverse that is gamma n, gamma n-1*gamma 1 up to that. So this implies the k matrix is gamma matrix multiplied by T. So if we use this k matrix as the feedback matrix here this one k, u=kx we will get Mu1 Mu2 Mu n as the eigenvalue of the matrix A+bk. So that is what we will see.

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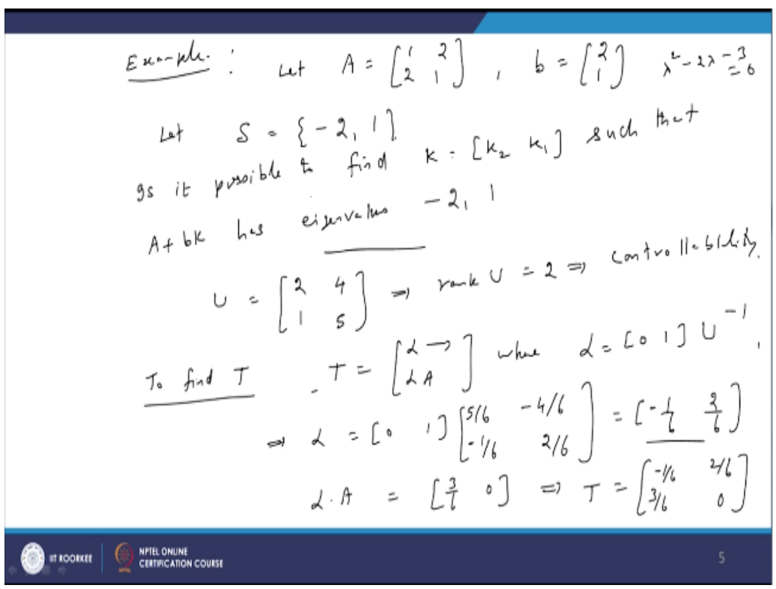

The computation as an example we will see here. So if you consider the matrix A to B say 1 2; 2 1 and the matrix B as 2 1, okay. And let S be the matrix, the set -2 and 1. So the question is, so is it possible to find a matrix k which is called K2 K1 such that A+bk has eigenvalues -2 and 1. The arbitrarily given set are -2 1 here. So is it possible to find k? Now we can first check whether the system is controllable or not.

So the system is controllable, so the set U, the matrix U is, b is 2 1 and Ab is 4 and 5 if you compute this one. So this implies the rank of U is 2 therefore it implies the controllability. So according to the principle which we have seen earlier we have to find the matrix T. So we have to find the matrix T first, so T is, the first row is alpha, the second row is alpha*A, where the companion form we have studied that the alpha matrix can be computed by using 0 1 and U inverse, okay.

So this implies alpha = 0 1, U is this matrix and U inverse if you calculate that is 5/6, -4/6 and -1/6, 2/6. So if you multiply this two we will get the alpha value to be -1/6 and 2/6 as the alpha value. The second row of T is alpha*A. So this is alpha and A matrix is there, if you calculate this one we will get 3/6 and 0, okay. So this implies the T matrix is -1/6 and 2/6, 3/6, 0. So T matrix is obtained.

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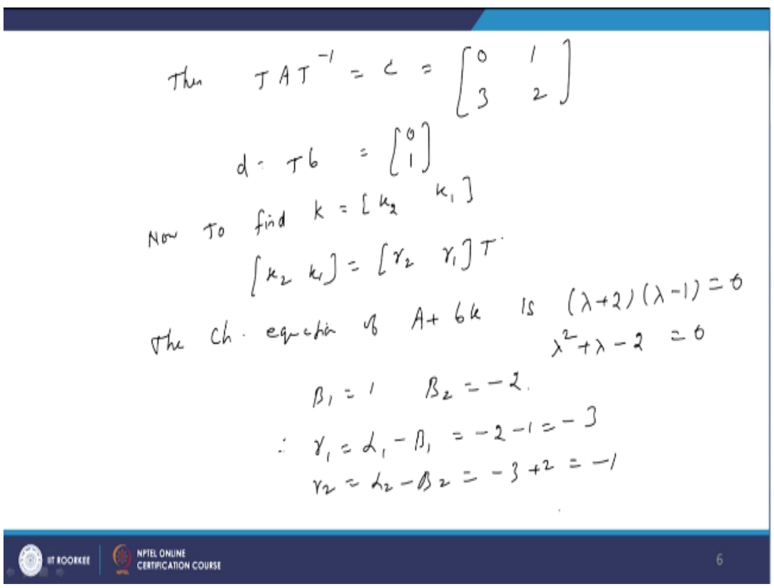

Then it is clear that TAT inverse will be equal to C. So here C is nothing but the companion form. And the last row of this will be having the coefficient of the characteristic polynomial of the matrix A, okay. So we will get the matrix C to be; because if you see A the eigenvalues of A matrix can be calculated and from there we can calculate or directly we can calculate the characteristic polynomial that is lambda square -2, lambda -3 = 0.

So the last row of C will be 3 and 2 is the C matrix and Tb we can easily check it will come out to be 0 1 here, okay. So we got the T matrix and C d matrix etcetera all these are available. Now to find the K matrix which K2 K1, we have to follow the procedure that, we have to follow that

K2 K1 it is equal to gamma 2, gamma 1 multiplied by the T matrix. K matrix = gamma*T, gamma 2, gamma 1 * T matrix.

And gamma values are obtained by alpha i-beta i. Now we have to see what should be the value of beta i. Beta i are calculated by the characteristic equation of A+bk, it is given by because we require the set -2 and 1 are the eigenvalues, so the characteristic equation can be calculated as lambda+2*lambda-1=0, so this implies lambda square + lambda-2=0. So beta 1 is 1 here and beta 2 is -2.

Therefore, we get the value gamma 1 is alpha 1 – beta 1 which is equal to; alpha 1 is here -2 and beta 1 is 1 here so the value is 3, gamma 2 is alpha 2-beta 2 alpha 2 is -3 and –beta 2 is +2=-1. So we got the value of gamma and gamma multiplied by T matrix.

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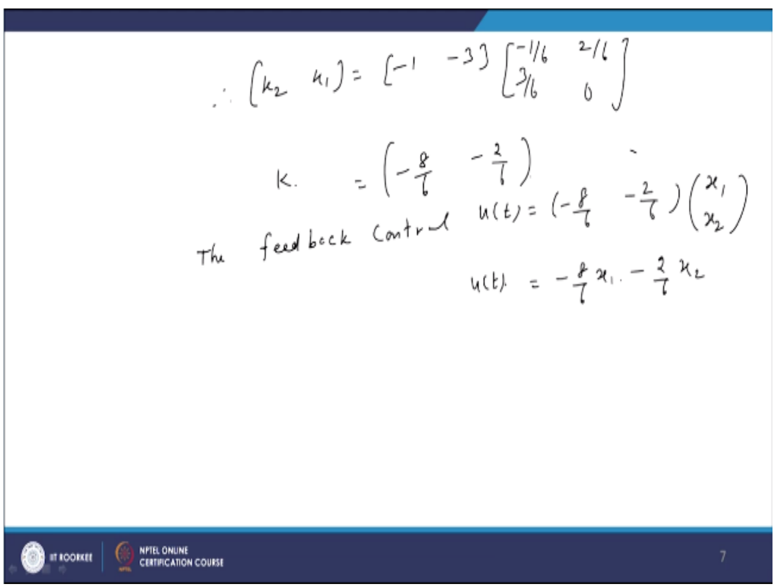

K2 K1 that is equal to gamma 2, that is -1 -3 * T matrix -1/6, 2/6, 3/6, 0 so by multiplying this two we will get the value k matrix is gamma 2 gamma 1 * T. So we will get the value – that is +1/6 -8/6 and -2/6 that is the value of the matrix k. So the feedback control u(t) is given by -8/6 -2/6 * the Xn X2 are the solution of the problem. The given control system, so that is nothing but -8/6 x1, -2/6 x2 is the feedback control of the problem.

And substituting this control; for finding the control, we have to find x1, x2 but in real life situation at each instant of time we can measure the x1 and x2 value the position, the coefficient; the current position of the dynamical system can be measured at each instance of time. So depending on those values the control should be applied on the system, so that it proceeds the control is working on the system.

And it will drive the system as we desired, because the eigenvalue are selected for a particular purpose. In this case just to demonstrate the problem we have taken -2 and 1. So it may not serve any physical purpose of any particular problem. But normally this procedure is applied for stabilizing a system. If you have a control system x dot = Ax+bu to make the system stable the eigenvalues of the A+bk matrix it should be negative that is; for asymptotically stable solution we need that the; the eigenvalue of the matrix A+bk all of them should be having negative real part.

So if we select the set S in such a way that all the eigenvalues have negative real part and then find the feedback control then we are sure that the system becomes asymptotically stable, so that is the purpose of this particular result. That is the main purpose for stabilizing a control system.

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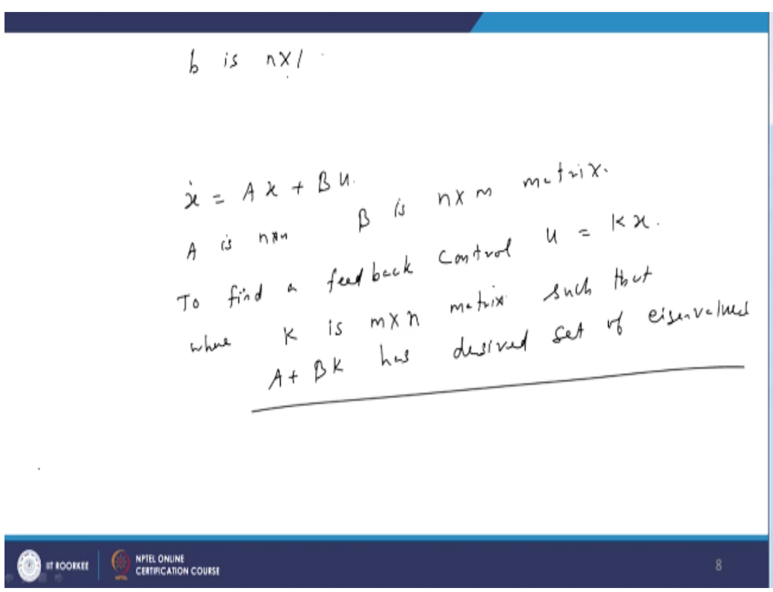

So in this lecture we have seen a demonstration of how to find the feedback control for the system where A is a n cross n matrix but b is a column n cross 1 matrix. But the general case is we have to do a similar work for the general system where A is n cross n and B is a n cross m matrix. So in this case how to find a feedback control, so here we want to find a feedback control u=k times x same procedure but only difference is k is here.

So we want to find a feedback control u=kx in such a way that A+B*k has desired eigenvalues. So this is the; so far we have seen the method of finding the feedback control for the system in which the matrix b is n cross 1, that is a column vector matrix. And the next lecture we will consider the system X dot = Ax+bu in which b is a general n cross m matrix. So we want to find a feedback control u=kx, where k is a n cross m matrix so that A+Bk has desired set of eigenvalues. Okay, thank you.