Dynamical Systems and Control
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Lecture - 31
Introduction to Control Systems - I

Hello viewers. Welcome to the course on dynamical systems and control. In this introductory lecture, I will speak about systems, in particular about dynamical systems and control systems with some examples. To start with let us see what is a system.

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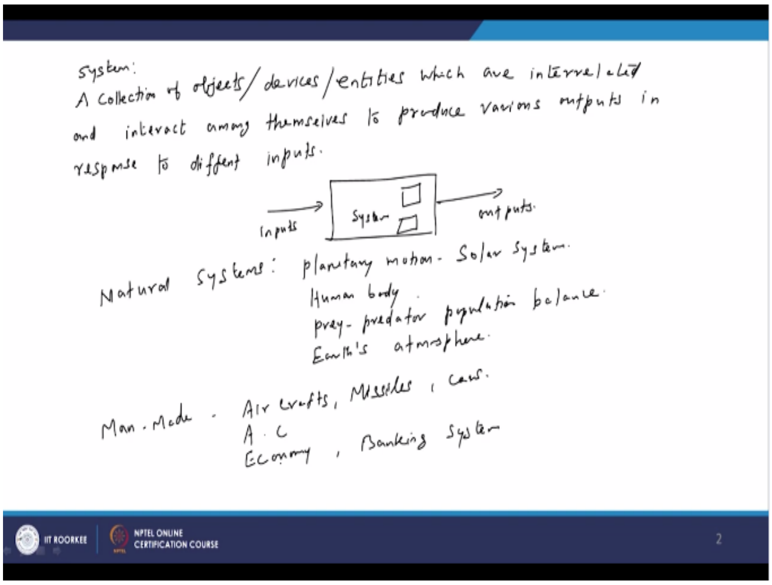

A system is a collection of objects or devices or entities which are interrelated or connected and interact among themselves to produce various outputs in response to different inputs. So we have a system and there is an input-output relation. So there may be several inputs and several outputs of the system and they will be subsystems. The objects or devices which form the entire system are called subsystems.

So in our everyday life, we will be able to see several things or several phenomenon's which can be fit into this particular description of systems. So many systems are natural systems which we can observe in our everyday life and many systems are artificial or manmade systems which we use in our everyday life various devices. So for example the planetary motion, we call it as the solar system.

So it is a system with several subsystems which are the planets, sun etc and they are governed by the gravitational force, gravitational pulls among themselves. The output of the system or the positional velocity of the planets at each instant of time that is of our interest. Similarly, the human body, it is a system with several organs as subsystems and any action by a human body can be called as a output in response to the input which is given by the brain.

So there is an input-output for a human body which is a natural system. Similarly, the prey-predator population balance. Nature is balancing the prey-predator population by the food chain in the natural system and you can say the earth’s atmosphere is a huge natural system which controls the seasons and climates, etc and these are the natural systems.

And for manmade system we have several examples like the aircrafts, missiles, cars, etc and various devices which are moving through mechanical or electrical energy. So the output or the outputs which are of interest are the positional velocity of these moving objects at each instant of time and we are interested always in controlling these devices for a particular purpose.

And the air conditioner which controls the room temperature is another device is a control system where the output is the temperature of the building and the input is the cool or hot air pumped from the device and the economy of a country or a banking system. So these are some manmade devices which is following the system principle. So we have several such manmade systems whether the systems are natural system or manmade system, they are governed by certain laws or rules and regulations pertained to that particular system.

So based on these laws or rules and regulations and based on the interaction among the subsystems and the interaction of the environment on the system or the effect of the environment on the system such as the disturbances, measurement errors and friction, force, etc one can find certain relation between the inputs and the outputs of the system. So these mathematical relations or inequalities are called the mathematical models of the system.

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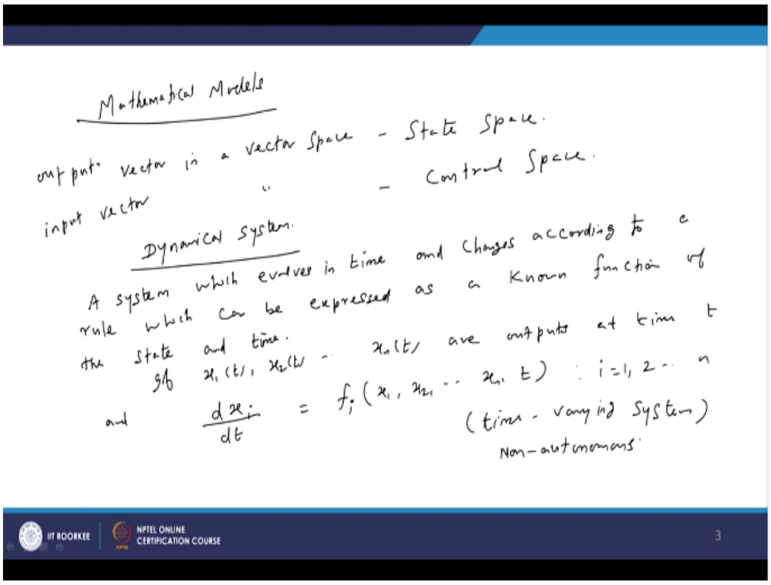

So mathematical models of the system, so some mathematical models may reflect the reality close to the actual happenings, some mathematical models may not be reflecting the reality perfectly or even close to this thing but how to obtain a good mathematical model, it depends on the quality of the observation made on a particular system.

And making use of those observations and interpreting these observations to form a mathematical equation on the system behavior. So we will see some examples of the mathematical models. So for any system the outputs are considered as components of a vector in a vector space, so we call that vector space as the state space of the system that is called the output vector.

The output vector in a vector space that is called the state space of the system and similarly the input vector in a vector space that is called the control space of the system. So for any given system, we have inputs and outputs and they are considered as elements of a state space and the inputs are considered as the elements of a control space. We will define what you mean by a dynamical system.

So a dynamical system is a system which evolves in time and changes according to a rule or which can be expressed as a known function of the state and the time. So if you know the current state and a function of these current state and time and if the system evolves according to this function, then we call it as a dynamical system.

So for example, if x1 t, x2 t, xn t if these are outputs of the system at time t and if the derivatives dxi/dt if it is a function of these variables x1, x2, xn as well as the time variability for i is=1, 2, 3 up to n, then we say that this is a dynamical system, mathematical model of a dynamical system.

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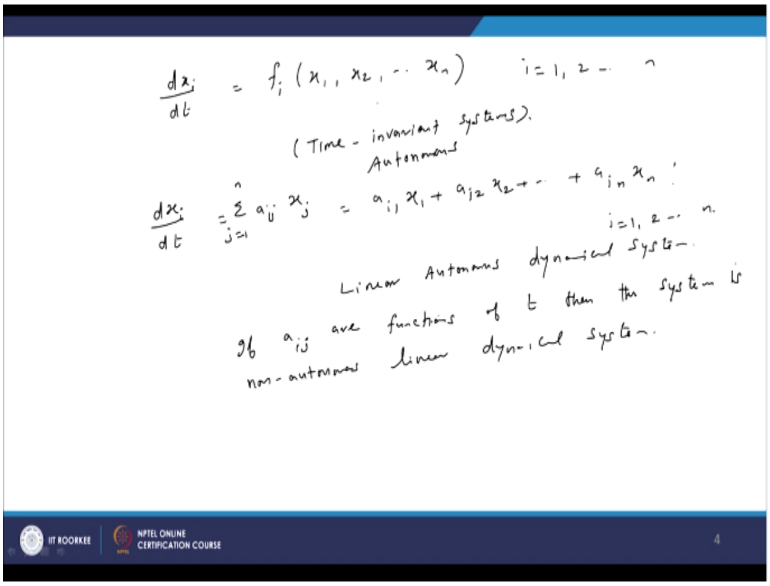

So if the function f is a nonlinear function, then it is a nonlinear dynamical system dxi/dt is fi of x1, x2, xn i is=1, 2, 3 up to n. So here the variable t does not appear explicitly, the functions x1, x2, xn are functions of time but t does not appear explicitly so such systems are called time-invariant system and the previous equation so this is t is appearing explicitly so it is called time-variant system, it is also called non-autonomous.

And the time-invariant system can also be called the autonomous system. Now if this function fi if they are linear expressions, then it is called a linear system. So we are familiar with the system of differential equation of the form aij xj summation j=1 to n. So this system this is ai 1, x1+ai 2 x2, etc ai n xn i is=1, 2, 3 to n. So this system of differential equation, it is a linear autonomous system, linear autonomous dynamical system.

If aij these are functions of time, then it is non-autonomous system, linear dynamical so we have this linear autonomous and non-autonomous dynamical system. So here we note that in dynamical system only output variables are available, there is no input variable in the equation itself. So here the current position or the current output itself is acting as input for the evolution of the system in the future time.

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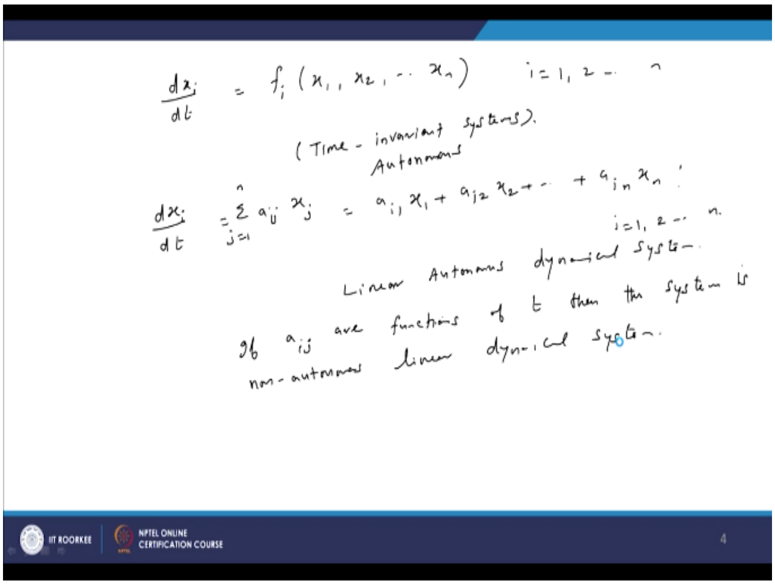

Now the system of the form dxi/dt=fi of x1 x2, etc u1, etc un of t, t i is=1, 2, 3 up to n. So this system where the inputs as well as input ui as well as the output xi are available in the equation, they are called control systems. So here if you denote x of t, the vector x1 of t, x2 of t, etc xn of t, it belongs to the vector space Rn for each value of t. So Rn is called the state space and the output vector sorry input vector u1 t, u2 t, etc un of t it belongs to Rn for each fixed value of t, so that is called the control space of the control system.

So we can find in various models mathematical models which can fall in any of this category of whether dynamical system or control system or they can be a nonlinear or linear, so various variety of systems we can encounter depending on the requirement of a real life situation, how much of information we require on real life problem we can model the system accordingly.

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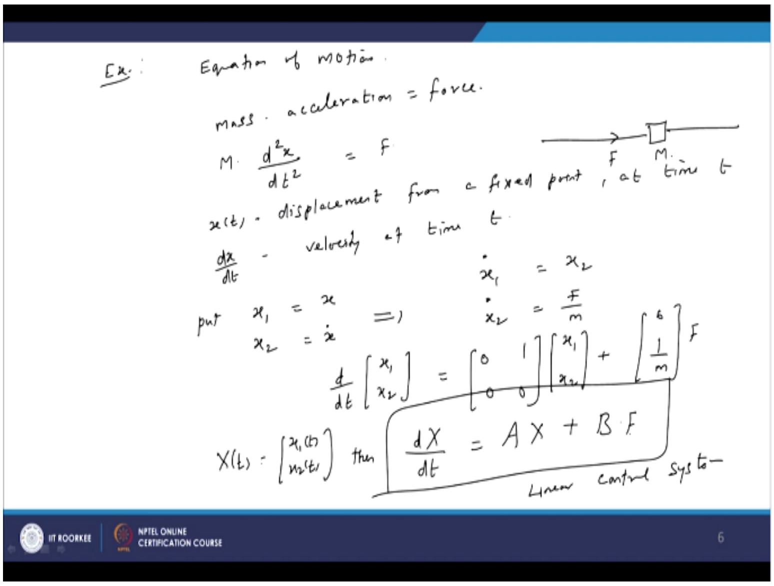

So for example if we consider various simple and familiar models, already we might have come across various equations of this type for example the equation of motion is a familiar one. So the straight line motion of a particle under a force F, if m is the mass of the object then by Newton's law of motion, we have mass*acceleration=the force.

So this can be written as mass*d square x/dt square that is=F where x of t is the displacement from a fixed point at time t and dx/dt represent the velocity. So it is a very familiar equation but here it is not in the form of a dynamical system. We can convert it into the standard form of dynamical system by replacing substituting x1=x and x2=x dot. The position is called x1 and velocity is called x2 at each instant of time.

Then, this will imply that x1 dot is x2 the derivative, x2 dot is d square x/dt square which is=F/m. So this equation can be written in the form d/dt of the derivative x1, x2 in the form 0 1 0 0*x1 x2+0 1/m*force F here. So if you call the output vector x of t as x1 t, x2 t then this equation can be written as dx/dt the left hand side you can call this as the matrix A*the vector x+B is this matrix and the force F here.

So it is the standard form of a linear control system where F is the control that is the input to the system and x is the output to the system and A and B are matrices, so they are linear operators, so it is called a linear control system and the constant m, m is the parameter of the system.

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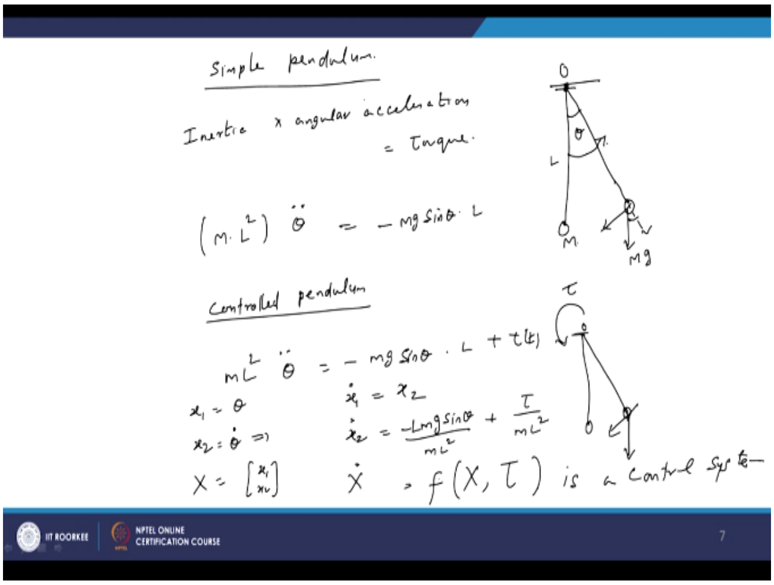

So if you consider another simple model that is the simple pendulum. So the simple pendulum model is let us consider a pendulum of length L and m is attached at the end of the pendulum and it is swinging slowly under the gravitational force. Then, the force acting downwards is mass*gravity and if you resolve this into the two sides, the angle is theta at each instant of time.

So we can use the Newton's equation that is inertia*angular acceleration=torque. So because it is a rotational motion we have this second law of motion. So the inertia calculation is the mass*L square is the inertia*the angular acceleration is theta is the angular angle, theta dot is angular velocity and theta double dot is the angular acceleration. The torque acting on the body is if you resolve in this direction, it is mg cos theta and this is mg sin theta*the length is the torque acting at this joint of the particle and the torque is acting in the opposite direction.

Positive direction is this one the anti-clockwise direction, the torque is acting in the negative direction. So the torque balance equation is the equation of motion of the simple pendulum. Now if you consider the controlled pendulum. Other than the natural force, there is no other force in the simple pendulum but the controlled one we have the force same thing here but if you apply a torque at the joint O here.

So here we have to add in the right hand side, the torque we have add an extra torque that is the only difference. So the controlled pendulum equation is mg sin theta*L+the torque applied on the joint here so it may be a constant or function of t. So this represents the system. Now if you convert it into the dynamical or control system form, we can call x1=theta, x2=theta dot.

So this will imply x1 dot=x2, x2 dot=theta double dot that will be –mg L/this mL square+tau/mL square. So we can call this as if x is the vector x1, x2, we will get dx/dt capital X=a function the nonlinear function of this quantity x1, x2 is written as x and tau is the torque the control acting on this body okay, so is a control system. Here the parameters are L and m, mass and the length of the pendulum.

So this example simple pendulum as well as the controlled pendulum they are the nonlinear systems. Earlier, we have seen the equation of motion, it is a linear system.

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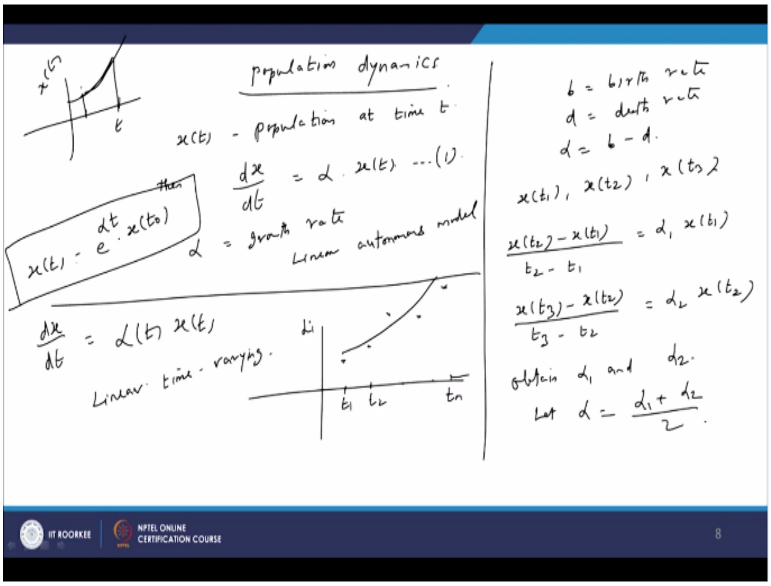

We have seen some very simple models so far. In the next lecture, we will be seeing some more models which maybe dynamical systems or control system, linear, nonlinear, etc and reflects more practical situations. Thank you.