Dynamical Systems and Control
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Lecture - 30
Lyapunov Stability - II

Hello friends. Welcome to this lecture and in this lecture also we will discuss the Lyapunov stability for dynamical system and if you recall in previous lecture, we have discussed some say definition related to Lyapunov function V of x. We know that what is positive definite, negative definite, positive semi definite, negative semi definite and how to find out V star which is the derivative of V along the dynamical system x dash=f of x.

And we have considered one theorem which simply says that if you are able to find out a Lyapunov function V of x which is positive definite in a say spherical domain say sr sphere of radius r such that Vx is positive definite on s of r on an inside s of r and V star is<or=0. Then, the zero solution of x dash=f of x is stable here. In fact, if you look at geometrical way to look at this thing.

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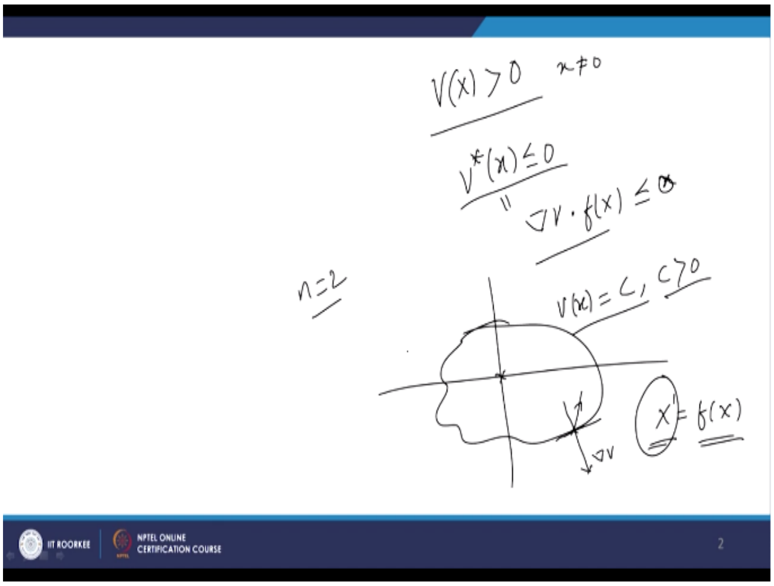

So here we simply say that since it is depending on V of x here. So V of x is negative definite means it is taking only say>0 for x non equal to 0 and it is 0 when it is x=0 and V star x is<or=0. Now V star x is basically what, it is basically del V dot f of x here. Now if you look at Vx>0, so it basically let us take n=2 now. So V of x is basically represents a kind of a surface and that surface include your 0 because when x is=0, then it will reduce to 0.

Because V of 0 is=0, so let us consider say V of x=some c here where c is certainly positive, why because Vx is positive definite. So it will take only the nonnegative value and it will take zero value only at the origin. So it means that here Vx=c where c is positive. So this will represent a surface which will contain origin here and if you look at this V star x<or=0 then this del V represent basically the normal vector pointing outside to this thing.

So at this point this del V is this, so del V is this. So del V is yeah del V. Now del V dot f of x is<or=0 means what, that the angle between the normal vector and f of x is more than 90 degree and<270 degree. So it means that the f of x which is a vector here, it must be say no way it is say outside in fact it is heading towards the inner of V of x=c. So it means that this f of x vector is say directing inside the domain Vx=c here.

Now what is this f of x, if you look at f of x it is nothing but x dash=f of x. So it means that f of x is nothing but the tangent vector of the x dash here. So it means that at this point this x dash represent this f of x represent the tangent vector defined at this particular point. So it means that the tangent vector is directing towards your interior of Vx=c. So it means that at no point on the boundary here, your tangent vector is directing outside or say outside or trying to take say outer region.

So it means at every point on the boundary, your tangent vector is always directing inside your Vx=c. So it means that solution if it start inside Vx=c then tangent vector always say direct towards the interior of Vx=c. So it means that it is always try to remain in Vx=c only. So it means that it is never going to be unbounded. So that is what we can say that geometrical way of looking the theorem number 1 here.

So now let us consider some more theorems based on Lyapunov function. So first theorem we have already discussed which says that if there exists a scalar function V of x which is positive definite on a sphere of radius r and V star x is<or=0. Then, the zero solution of x dash=f of x is stable solution, that is what we have proved in theorem number 3. Now let us consider one more theorem and this time we are not giving any proof of this.

For proof, we can say refer some very good book that is one book given by (()) (06:05) you can obtain the reference of this book in our reference list.

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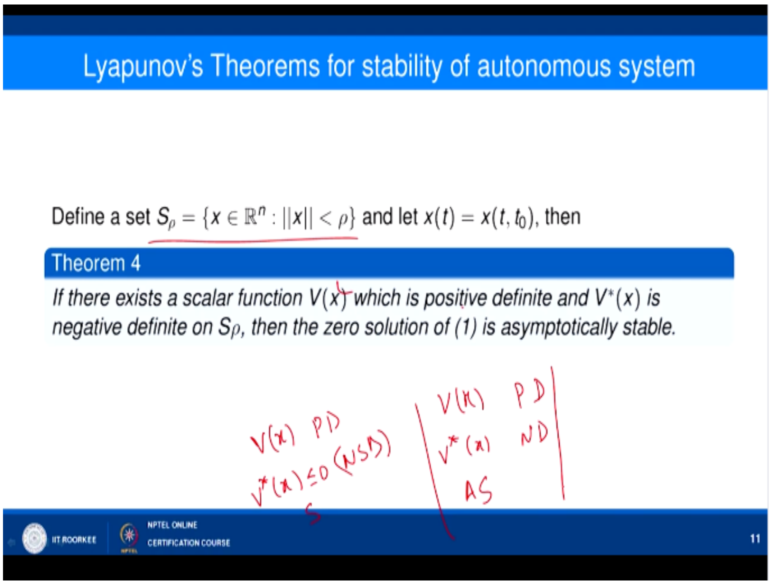

So this theorem number 4 says that if there exists a scalar function V of x which is positive definite and V star x is negative definite on S rho where S rho is defined as a sphere of a radius<epsilon. Then, the zero solution of 1 is asymptotically stable here. So this simply says that if Vx is there which is positive definite, Vx is positive definite and V star x is negative definite, then the zero solution is asymptotically stable.

So here it is asymptotically stable and earlier we have proved that Vx is positive definite and V star x is<or=0 means negative semi definite. Then, it is stable solution here and then we have one more, so here we are just stating this theorem number 4, we are not proving the theorem number 4.

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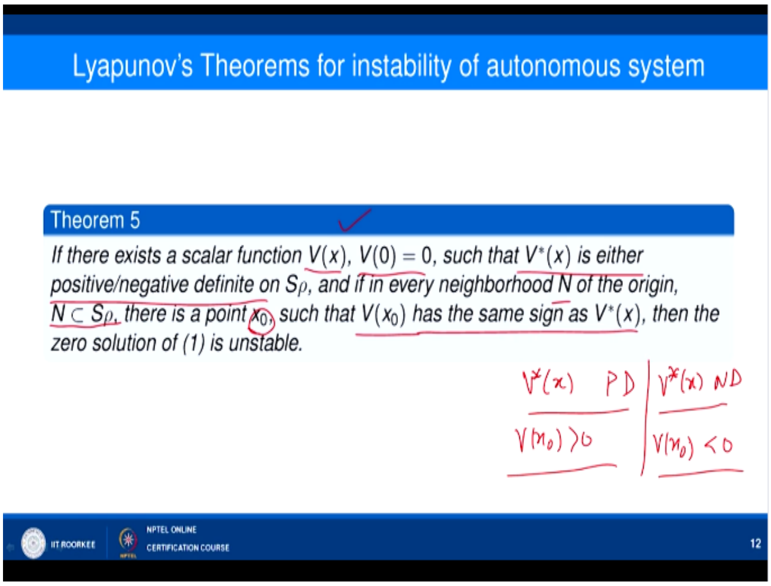

And in a same way we can define theorem number 5 which is given the instability of autonomous system here. So if there exists a scalar function V of x such that V of 0=0, here we are not requiring any say positive definiteness or negative definiteness of this scalar function such that V star x is either positive definite or negative definite on S rho and if every neighborhood N of the origin where N is contain in S of rho there is a point x0 such that V x0 has a same sign as V star x.

So if I assume that V star x is PD means positive definite, then we can find if there exists a point in the neighborhood of origin such that V x0 is positive, then zero solution is unstable and if we take V star x as negative definite and there exists a point x0 such that V x0 is<0 then also the zero solution of 1 is unstable. Please look at here, here the requirement that Vx is positive definite is removed here.

Here we just assume that Vx is a scalar function such that it is vanishing at 0 that is all. So everything here is given in terms of V star x. So V star x is positive definite and in a neighborhood of origin there exists a point such that V x0 is positive. Then, the zero solution is unstable and if V star x is negative definite and V x0 is<0 where x0 is a point in the neighborhood of the origin, then the zero solution is unstable solution here. So again I am not giving the proof of this.

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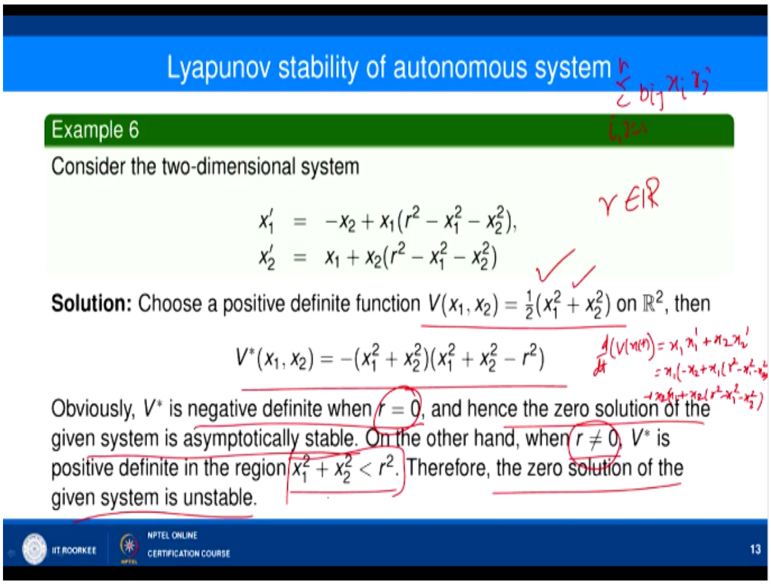

We are just discussing some example based on the given theorems here. So here consider the two-dimension system here x1 dash=-x2+x1 r square-x1 square-x2 square, x2 dash=x1+x2 r square-x1 square-x2 square. Now here r is some constant, r is some real constant here. So now here let us choose the positive definite function V x1, x2=1/2 x1 square+x2 square. So here most of the cases, we define our Lyapunov function, the function V x1, x2 as say a quadratic form.

And we have discussed the quadratic form is summation say bij xi xj and i, j=1 to n and this is a symmetric form. So here let us in this case when n=2 we simply define V x1, x2 is 1/2 x1 square+x2 square on r square. Then, we can calculate V star x1, x2 as –x1 square+x2 square x1 square+x2 square-r square. So this is not very difficult, so V star x1, x2 is nothing but d/dt of V of xt. So if you look at this is what V x1 that is x1 and x1 dash.

So x1 dash let me write it here+V x2 means dou V/dou x2 that is x2 here and x2 dash that is x2 dash. Now x1, x1 dash is already given as –x2+x1 r square-x1 square-x2 square+x2 and here it is x1+x2 r square-x1 square-x2 square and if you simplify you will get this V star x1, x2 as –of x1 square+x2 square x1 square+x2 square-r square. So you look at this – is because that we are simplifying this as we are writing this as x1 square+x2 square-r square okay.

V star x1, x2 we have already obtained here. Now we know that if V is our positive definite because it is taking the value 0 at only at the origin and in all other point it is taking only positive value and V star x1, x2 is negative definite if this r is 0 or r is<x1 square+x2 square here. So if x1 square+x2 square-r square is positive, then V star is negative definite and we can simply say that in particular case r=0.

The zero solution of the given system is asymptotically stable because it is negative definite. Is it okay? And on the other hand, when r is not equal to 0, V star is positive definite in the region when x1 square+x2 square is<square. So when x1 square+x2 square is<r square then this is negative at V star x1, x2 is positive definite. So V star x1, x2 is positive definite and there exists a point in the neighborhood of origin where it is taking the value positive.

In fact, if you look at here V x1, x2 so V x1, x2 is already positive definite and if V star x is also positive definite, then by the previous theorem here we simply say that your zero solution of 1 is unstable. So in case when r is not equal to 0 since r is fixed quantity and x1, x2 can be made smaller and smaller, then we can say that if r is nonzero we can take small enough x1, x2 such that this quantity is negative here.

So negative negative positive and V star can be made positive and hence we can say that zero solution of the given system is unstable when r is not equal to 0 here. So here we have obtained that the zero solution is stable, unstable only analyzing your function V of x1, x2. So here without finding the say a solution of this dynamical system we are predicting, we are telling that the zero solution is stable in this region and unstable in this region.

So here we say that if an r=0 we simply say that your zero solution asymptotically stable and if you look at the region x1 square+x2 square is<r square, then we simply say that zero solution is unstable. So here we are providing the region also in which your zero solution is unstable. So this is not possible when we use the approx. say linear approximation of a given weakly nonlinear system here. So this is very important say benefit of constructing the Lyapunov function.

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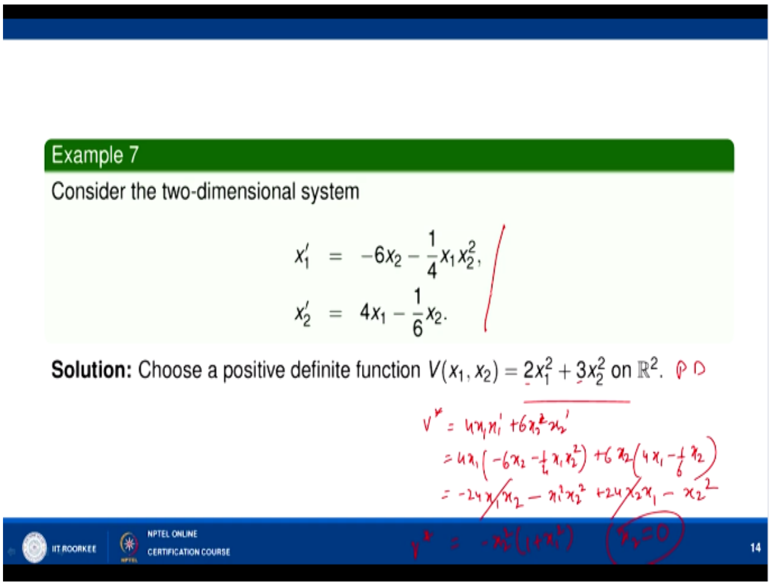

Now consider the two-dimensional system x1 dash=-6 x2-1/4 x1 x2 square, x2 dash=4 x1-1/6 x2 and again here we try to predict, try to find out the stability, unstability of zero solution. So here choose the positive definite function 2 x1 square+3 x2 square. Now this 2 and 3 will depend on the say coefficient available in x1 dash and x2 dash. You can start with A x1 square and B x2 square.

So let us find out V star here. So first of all you just look at here that V is a positive definite scalar function. Now look at the V star, V star is basically 4 x1 x1 dash+6 x2 x2 dash here, so it is x2 here. So it is 4 x1 x1 dash is basically what -6 x2-1/4 x1 x2 square+6 x2 it is 4 x1-1/6 x2 here. So if you simplify this is -24 x1 x2 –x1 square x2 square+24 x2 x1-x2 square. So if you look at here this will be canceled out and we will get what, -x2 square if you take it out then it is what 1+x1 square.

So if you look at V is positive definite and if you look at V star, then it is taking all the time nonnegative value. It can be zero when x2 is=0 here. So it means that when x2 is=0, x1 may be anything but still V star is 0. So it means that here V star is negative semi definite and V is a positive definite, so it means that zero solution is a stable solution here. So without finding the solution here, we simply say that zero solution is stable solution here okay.

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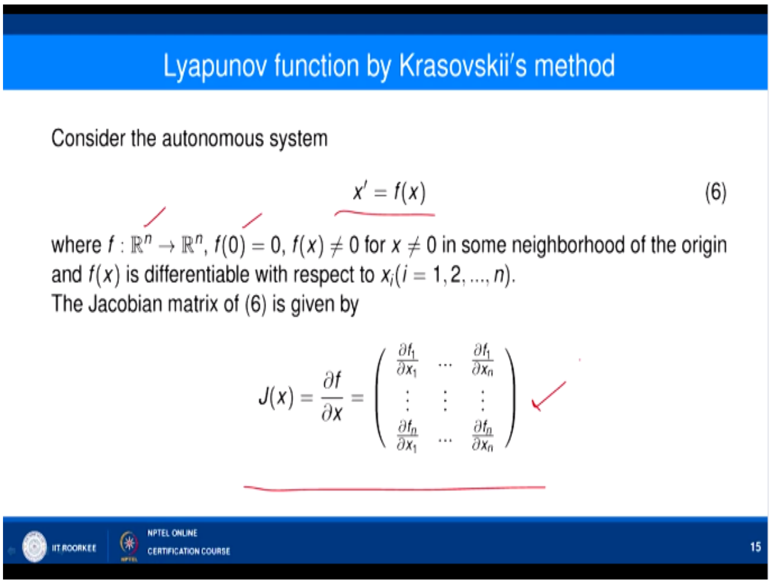

So here we have shown some example by constructing some suitable Lyapunov functions. We have predicted that zero solution is stable, unstable or asymptotically stable solution but the important part or the difficult part, most difficult part is how to construct Lyapunov function because construction of Lyapunov function is nowhere by no means it is so easy, so here the important part is how to construct Lyapunov function.

So here we are just considering some cases where we can construct Lyapunov function and there is some technique by which we can construct the Lyapunov function. So first method is given as say autonomous system say x dash=f of x and here we will show how to construct the Lyapunov function and we will try to show that under what condition this autonomous system is having zero solution asymptotically stable solution or stable solution or unstable solution.

So consider the autonomous system x dash=f of x where f is function from Rn to Rn f of 0=0 and f of x is nonzero when x is nonzero in some neighborhood of the origin. So it means that here we have assumed that zero is an isolated critical point because f of x is nonzero in a small neighborhood around the origin. So zero is an isolated critical point and we assume that f of x is differentiable with respect to xi.

So that existence and uniqueness maybe given, so only thing we have to check is the stability of zero solution. So we already know that the Jacobian matrix J of x is given by dou f/dou x and it is defined like this okay. So now with the help of Jacobian of f with respect to x, we can construct the Lyapunov function.

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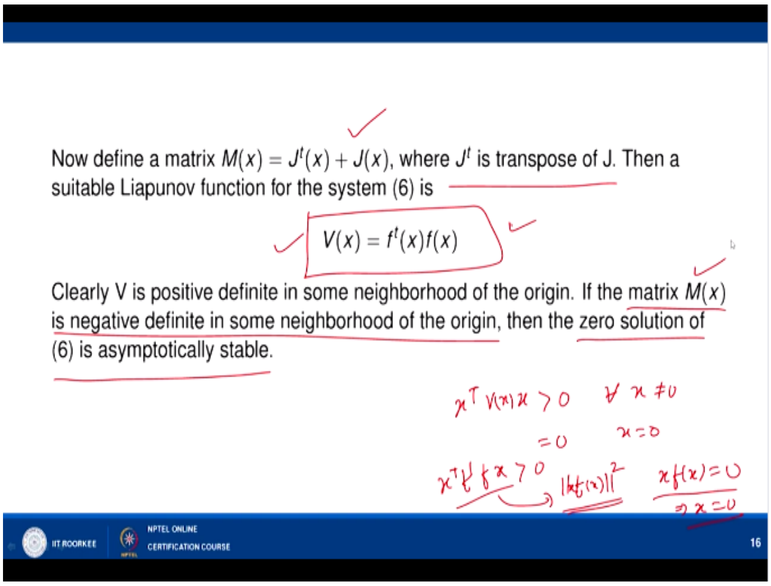

And the Lyapunov function is given as V of x=f transpose x f of x where f is given as this. So now how to check that it is the suitable Lyapunov function, for that we define a matrix M of x which is given as J transpose x+J of x where J transpose is the transpose of J and J is the Jacobian of f with respect to x and we claim that the suitable Lyapunov function for the system 6 is V of x=f transpose x f of x.

So first thing we need to observe that it is a positive definite. In fact, positive definite means it is taking value positive for nonzero x here and for zero it is when x=0. So it is another alternative definition for positive definite where Vx is positive definite if x transpose Vx x is>0 for all x nonzero and it is taking the value 0 when x=0. So if you check look at here that x transpose V of x is basically f transpose f of x is>0.

So if we can simplify this is nothing but norm of x f of x is this is basically this quantity is nothing but norm of x f of x whole square and this is all the time nonnegative and we can say that this is 0 when x f of x=0. So it means that either f of x=0 or x=0. When f of x=0, we already seen that f of x cannot be 0 other than 0 origin. So it means that this implies that your x has to be 0 here right.

So it means that this V of x which is given as f transpose x f of x is a positive definite function in a neighborhood of the origin. Now if the matrix M of x is negative definite in some neighborhood of the origin, then the zero solution is asymptotically stable here. So here we talk about the negative definite of a matrix M.

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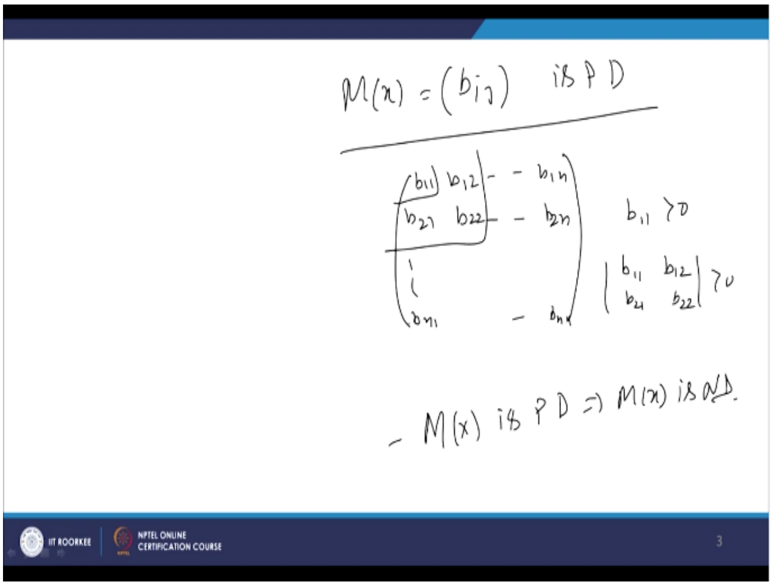

So here also we will discuss little bit that how given a matrix M of x how we say that matrix is negative definite, positive definite and negative semi definite. So here we simply say that M of x if we look at your bij, so we simply say that this matrix is positive definite, first let us define positive definite and negative definite is nothing but – of M is positive definite. So M of x is PD positive definite provided that the principal minor is all positive.

So what are principal minor, so if we look at b11 b12 up to b1n b21 b22 b2n and so on bn1 to bnn. So principal minor means this is positive, so it means that b11 is positive and second is this it means that b11 b12 b21 b22 this determinant is positive and so on. So all the principal minor if they are positive, then we say that M of x the matrix M of x is positive definite and it is negative definite if –of M of x is positive definite.

Then, we say that M of x is negative definite right. So here this is the positive definiteness and negative definiteness of a matrix. So we say that if the matrix M of x is a negative definite in some neighborhood of the origin, then the zero solution of 6 is asymptotically stable. Let us prove this statement here.

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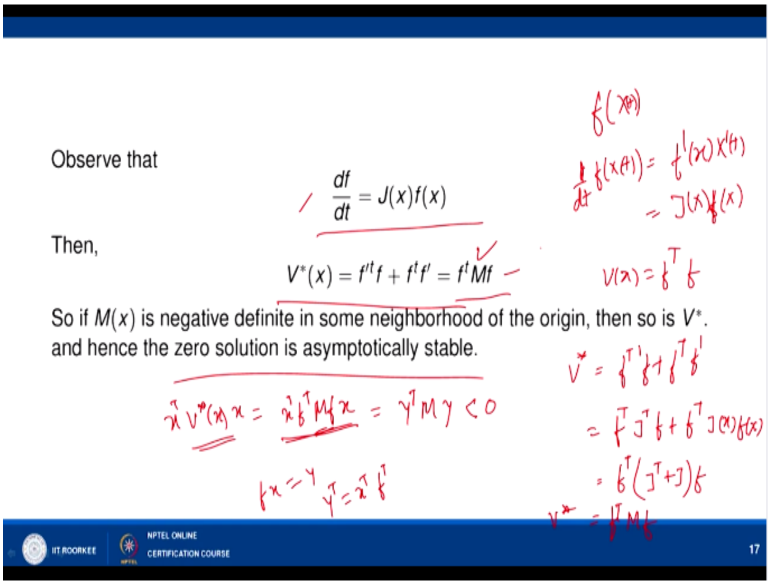

So for that we know that df/dt is=J of x f of x because if we look at f is a function of x and x is a function of t. So if you want to find out the derivative of this, then it is what, d/dt of f of x of t is basically your f dash x means and x dash t. Now f dash x is nothing but your Jacobian, so it is J of x your x dash t, x dash is nothing but f of x here. So df/dt is here –J of x f of x. So to find out V star x it is nothing but Vx, Vx is basically what, it is Vx is basically f transpose f here.

So V star basically it is f transpose dash f+f transpose f dash. Now f transpose dash is basically f dash right, f dash J transpose it is f transpose and it is already given here f transpose and J transpose f here+f transpose f dash is basically Jx f of x right. So we can write here f transpose and within bracket you can write J transpose+J and f M. So if you look at this is nothing but M so it is f transpose Mf so V star is nothing but f transpose M of f.

So if M is negative definite, then we can prove that V star is also negative definite and hence we can say that zero solution is asymptotically stable solution. How we say that if M is negative definite then V star is also negative definite, for that we simply look at x V star x here, it is x f transpose M f x here x transpose. So this I can write it if you look at this this I can write it x f M square here right.

So if M is negative definite, then this quantity is basically sorry this quantity is basically going to be negative for all x nonzero. So here we say that if M is negative definite, then x transpose f transpose M f fx basically here you can take f of x as y and similarly y transpose will be what, it is x transpose f transpose, so it is nothing but y transpose M y here and since M is negative definite then this quantity is<0.

So it means that V star x is negative definite and hence your Vx which we started with this f transpose f here this we have shown that it is positive definite and here V star which is given as f transpose M of f when M is negative definite we simply say that V star is negative definite, so it means that by theorem we simply say that zero solution is asymptotically stable solution.

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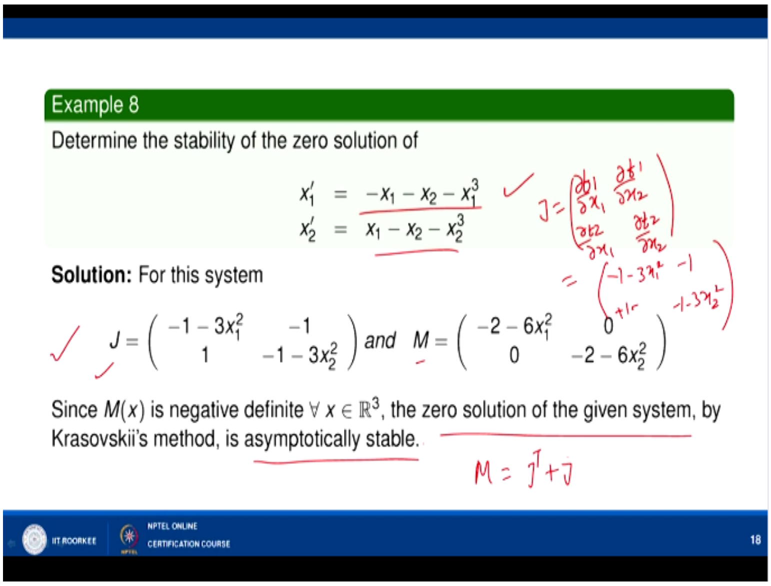

And let us take one example based on this. So determine the stability of the zero solution of x1 dash=-x1-x2-x1 cube and x2 dash=x1-x2-x2 cube. So we can find out Jacobian J here Jacobian, it is your f1, it is your f2. So Jacobian is basically dou f1/dou x1 dou f1/dou x2 dou f2/dou x1 dou f2/dou x2 so we can find out dou f1/dou x1 is basically -1 -3 x1 square, dou f1/dou x2-1 it is -1 –so it is dou x2/dou x1, so it is 1 only, so this is 1 here that is all, no other thing.

And here it is this is -1 -3 x2 square, so that is what we have written here J is -1 -3 x1 square-1 and 1-1-3 x2 square and we can find out M, M is nothing but J transpose+J and it is given as M=-2 -6 x1 square 0 0 -2 -6 x2 square. If you look at this is a diagonal matrix with negative entries on the diagonal so we can say that the eigenvalues of the matrix M has to be negative and there is another alternative definition of negative definite here.

That if a matrix has only negative say eigenvalues, then it will be a negative definite and here in this case since it is a diagonal matrix when the eigenvalues are nothing but the entries given on the diagonals. So it means that here M is a negative definite matrix and hence we can say that by the previous discussion we can say that the zero solution of the given system by Krasovskii method is asymptotically stable here.

Please look at here, it is a sufficient result. It simply says that if M is negative definite, then it is asymptotically stable then. Is it okay? So okay, so this gave an example based on the method discussed before. Now let us consider one more way we define construction of a Lyapunov function for linear system with constant coefficient.

Now consider the so earlier one is the nonlinear autonomous system and this one is a particular case of previous construction. So we simply say x dash=A of x where A is a n x n matrix over R whose entries are R.

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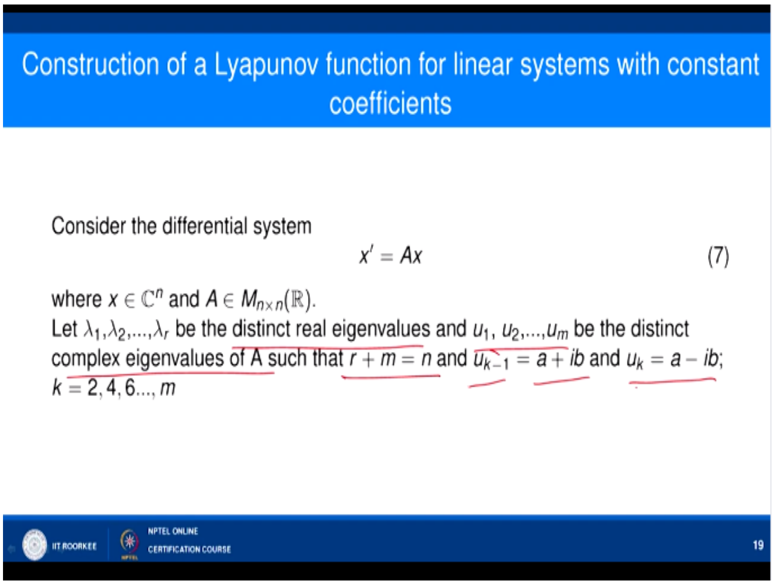

So let lambda 1 to lambda R be positive be the distinct real eigenvalues and u1 and um be the distinct complex eigenvalues of A such that r+m is=n. So here we can since this is all complex eigenvalues, so here we define uk-1 as a+ib and uk as a-ib, so it means that lambda 1 to lambda r real eigenvalues and u1 to um’s are complex eigenvalues and coming in pairs.

So we define uk-1 as a+ib and uk as a-ib for k between 2, 4, 6 and so on. So here a has eigenvalues this we have obtained here.

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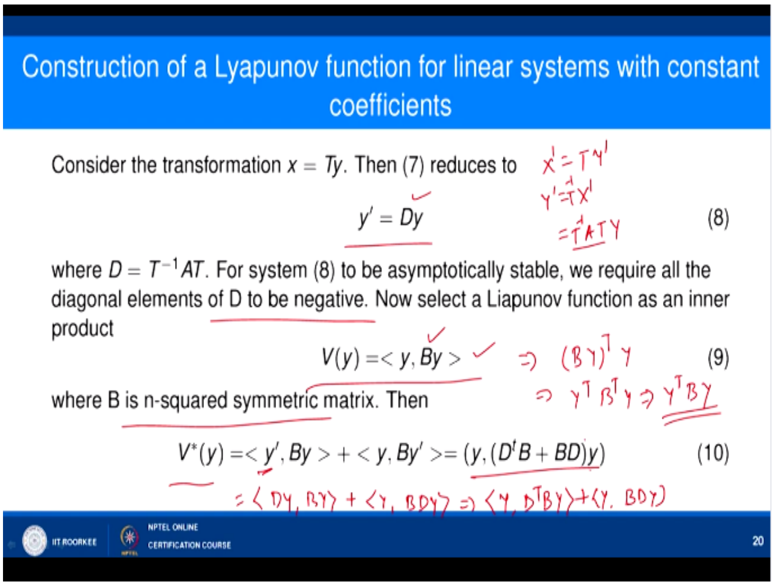

Now here we consider the transformation x=Ty here. So we can reduce x=Ty means you can say that x dash=Ax is now reduced as y dash=Dy. Here we can simply say that x dash=here it is T y dash here. So we can write y dash as T inverse so y dash=x dash, so T inverse x dash. Now x dash is basically what, T inverse x dash is nothing but A of T of y here. So I can write T inverse AT as D here and we can write this as y dash=D of y.

The idea is that if we choose the matrix T in a way such that this D represents a triangular system or diagonal system depending on the matrix A right. So here we can say that your D is T inverse AT and for system 8 to be asymptotically stable, we required all the diagonal element of D to be negative. So if we assume this y dash=Dy and if we take that all the diagonal element of D to be negative, then we simply say that the zero solution is asymptotically stable solution.

So that we already know right, so what we do not know is that how to construct the Lyapunov function in this case. So here the Lyapunov function we are defining as V of y as y, in a product of y and By where this B is still unknown and we simply say that only condition on B we are putting as that it is a n-squared symmetric matrix. So it is a first of all if it is symmetric matrix then we can simply say that B of y is basically positive definite function.

Basically, it is nothing but By transpose y here so it is y transpose B transpose y here and it is nothing but y transpose B of y here. So B of y is nothing but y transpose B of y is given here where B is a symmetric matrix. So Vy is given as y transpose B of y here. So we can say that By is this. Now look at the B star y, so B star y is inner product of y dash By+inner product of y of By dash. So we can write this as y D transpose B+BD y here.

Because we are just assuming y dash=so this is what this is Dy, By+y, BD y here. So this I can write it here, this I can look at here, so it is y and D transpose By+y BD y. So if you use the inner product properties then it is nothing but this. If you do not want to use inner product, you can simply use this and we can still find out this is nothing but the condition here which is obtained here okay, so V star y is given by this.

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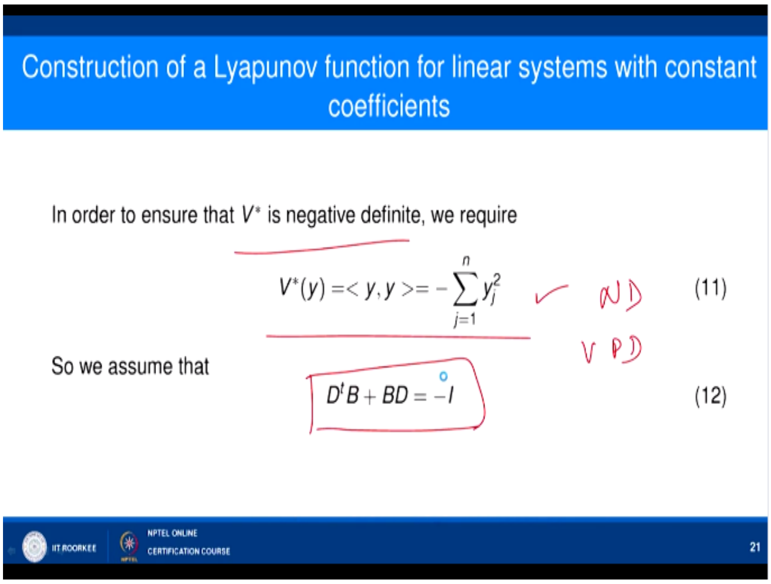

Now we want to find out whether it is asymptotically stable, so for that we need to show that V star is negative definite and for that we require that V star y is something like inner product of y, y it is written as –j=1 to n yj square here. So this can be achieved if we assume that D transpose B+BD is=-I if you look at here, if this is –I then this is nothing but y inner product with –of y.

And hence we can say that it is –I, - summation j=1 to n yj square and hence it is a negative definite. So V by definition is positive definite and V star is negative definite and it means that the zero solution is going to be asymptotically stable solution and so we assume that B is a symmetric matrix of size n such that D transpose B+BD is=-I here.

Now D is already given to you, D is nothing but T inverse AT where T is some transformation which we choose in a way and then we can solve this and we can get our B here. Now once your B is given we can find out the Lyapunov function like this.

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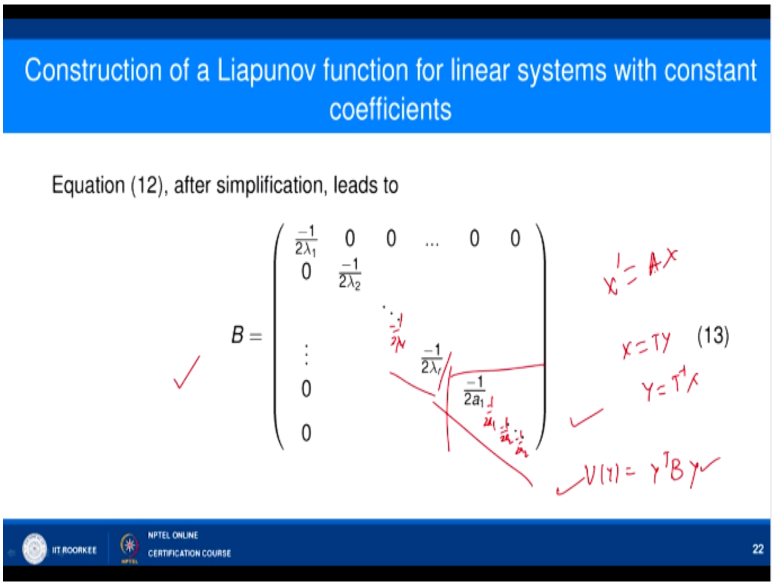

And we say that if you solve this, then we can say that equation 12 after simplification leads as B should be it is -1/2 lambda 1 only look at the diagonal terms, -1/2 lambda 1, -1/2 lambda 2 and up to -1/2 lambda r and then we have yeah it is already given here and then we have the say diagonal entries corresponding to complex terms. So it is -1/2 a1 and again -1/2 a1 and then -1/2 a2 and again -1/2 a2.

So here all are distinct. Here since they are coming in a pair, so we are writing twice times 2 times -1/2 ai's bracket okay. So here your B is coming out to be this, so once we have B then we can define Vy as y transpose By and if you use the expression y=so x=Ty if you use this where T is invertible matrix that T inverse x then we can find out the Lyapunov function for the original system x dash=A of x here okay.

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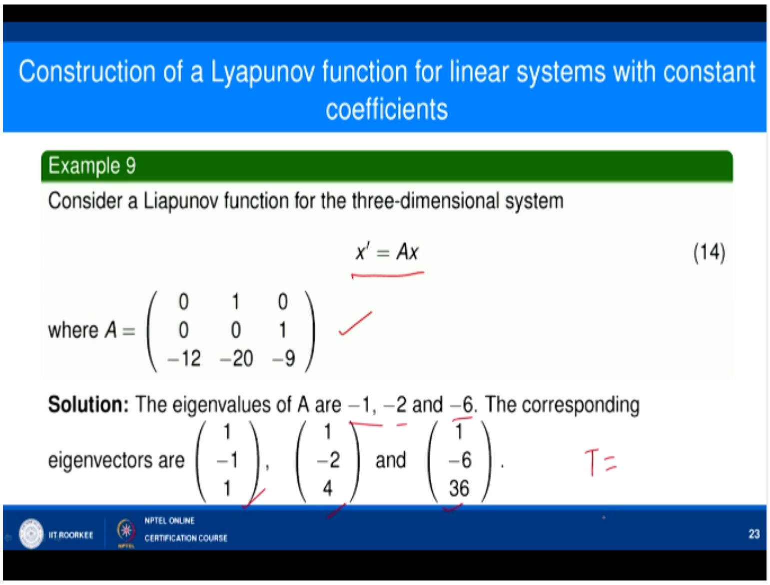

So let us have one example based on this. So consider Lyapunov function for three-dimensional system x dash=A of x where A is given as this matrix 0 1 0 0 0 1 -12 -20 -9. So eigenvalues you can find out -1, -2, -6. So we simply say that here zero solution is asymptotically stable solution, let us try to prove that we have done, we can conclude with the help of theory which we have discussed earlier.

Now let us re-verify by constructing the Lyapunov function. So corresponding eigenvector we can find out and once your eigenvectors are there then you can consider your T matrix as the matrix of eigenvectors.

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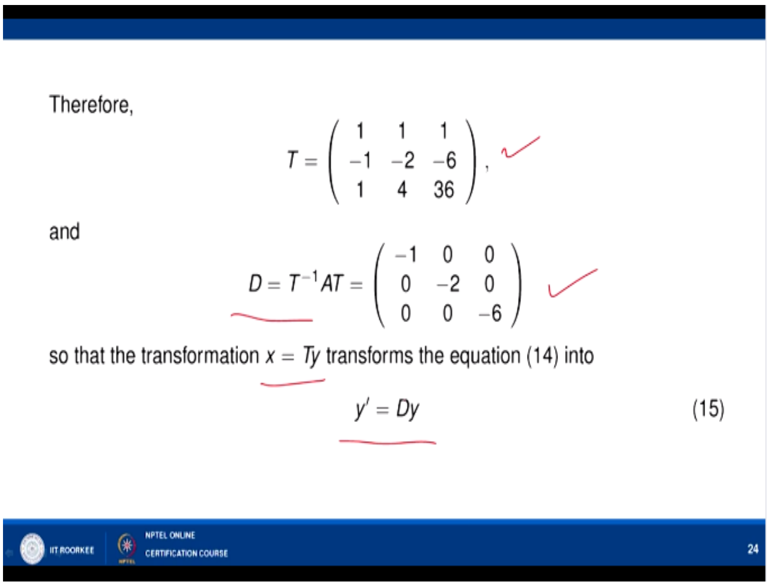

So we write T as 1 -1 1 1 -2 4 1 -6 36 and we can write D as T inverse AT that is the diagonal form corresponding to matrix A. So T is=-1 0 0 0 -2 0 0 0 -6 and we can find out x=Ty to transfer our system as y dash=Dy. Now D is of this form.

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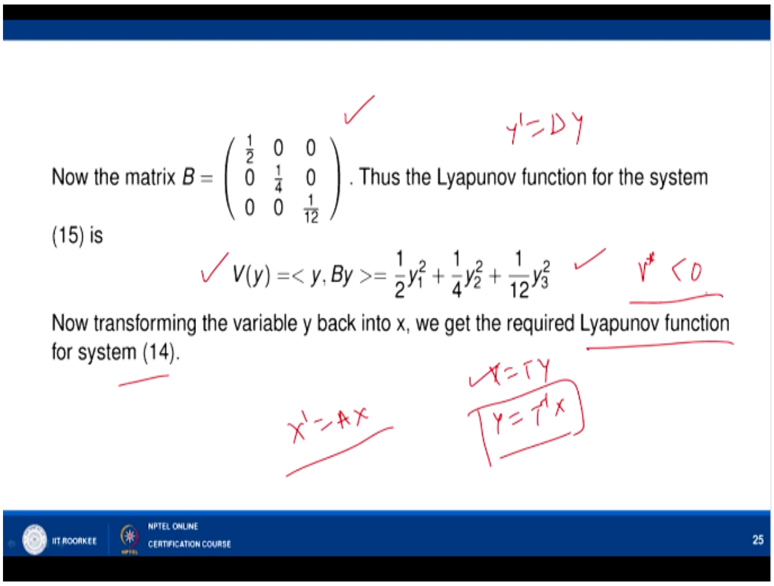

So with the help of D, we can find out the matrix B that is I think the solution of B transpose I think it is given in D transpose B+BD=-I. So D is already given, we can simply plug in the value and you can find out the expression for B here. So once we have this, then we can find out B and it is given as 1/2 0 0 0 1/4 0 0 0 1/12 and then the Lyapunov function for the system y dash=Dy is given as inner product of y By or By transpose y,

And you can simply say it is nothing but 1/2 y1 square+1/4 y2 square+1/12 y3 square and you can see that these are this Vy is positive definite because it is taking value 0 only on the origin y1, y2, y3. Now we already have this relation that x=Ty. So here y=T inverse x, so T is invertible, so we can use this and we can have we can obtain the Lyapunov function for the system x dash=Ax by putting x=sorry by putting y=T inverse x here.

And we can get the Lyapunov function for x dash=A of x here okay and here by the way we have already obtained B in a way such that B star is negative definite. So it means that solution is going to be asymptotically stable solution.

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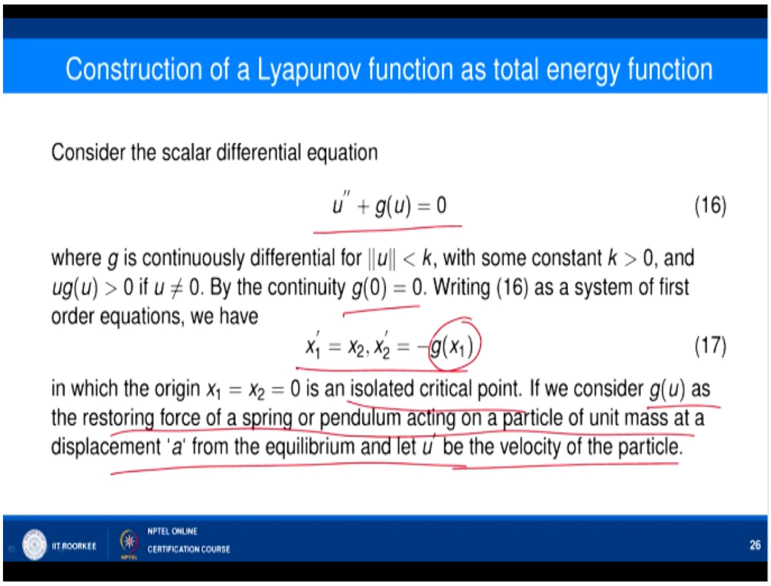

So now let us look at one more kind of a method to construct the Lyapunov function and here we construct the Lyapunov function as total energy function. So for that let us consider the scalar differential equation u double dash+g of u=0 here where g is continuously differentiable function for norm of u is<k where k is some constant positive and u of g of u is positive if u is nonzero.

Now by the continuity g of 0=0 here and if we write we can write down the system 16 as x1 dash=x2 and x2 dash=-g of x1. So we can write down the scalar differential equation as a system like this and which x1 and x2=0 is an isolated critical point and if we consider g of u as a restoring force of a spring or pendulum acting on a particle of unit mass at a displacement a from the equilibrium and let u dash be the velocity of the particle.

Idea is that if g x1 is basically a force which will try to reduce the displacement to the initial place means it is trying to say make our system in an original position. So it is a restoring force basically and u dash is the velocity of the particle.

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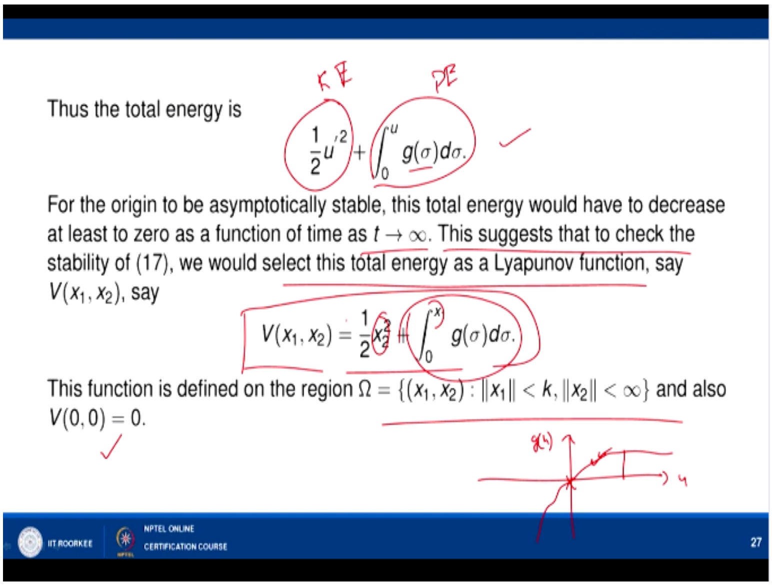

Then, the total energy we can say that it is what it is basically 1/2 u dash square. Basically, it is nothing but the kinetic energy. Kinetic energy is basically what it is 1/2 mass if we look at m as 1 then it is 1/2 of V square. So 1/2 m V square, so it is a kinetic energy of the system here and if you look at the second part is basically g of sigma is basically the restoring force and if we calculate the restoring force between 0 to u, we can say that it is nothing but potential energy.

So we can say that 0 to u g of sigma d sigma, it is a potential energy and the first one is the kinetic energy. So it is a kinetic energy due to velocity, so it means that the total energy is given as 1/2 u dash square+0 to u g sigma d sigma and for the origin to be asymptotically stable, this total energy would have to decrease at least to 0 as a function of time as t tending to infinity.

Idea is to that your system is going to a stable solution, going to be stable position if the total energy is reducing and it will be same it will remain in a stable position if the total energy is not increasing, it will remain means in a minimum value then it will not change its stability behaviour. So idea is that if origin is asymptotically stable solution then this total energy is tending towards 0 here.

So this suggest that to check the stability of 17, we will select this total energy as a Lyapunov function and we write Lyapunov function as V of x1, x2=1/2 x2 square+0 to x1 g of sigma d sigma and this function is defined between this. Omega is all those x1, x2 such that norm of x1 is<k and norm of x2 is<infinity and we can easily check that V of 0, 0 is=0 here and here we can easily check that V x1, x2 is say nonnegative.

In fact, it is taking V x1, x2 is a positive definite function. Here we are putting the condition that ug u is>0 here. So if you put ug u is a positive, so it is your behaviour of ug u is like this. So it is your u and it is of g of u. So it means that if u we take between here to here between 0 to x1, then it will take only the positive value. So this integral is taking only the positive value, so if it is positive, it is positive.

Then, we say that V x1, x2 is all the time positive because negative it is vanishing only on the point when x1=0 and x2=0. So it is vanishing only on the origin and in all other point it is taking only the positive value. So here we simply say that V x1, x2 is given as this which is considered to be as total energy function here. Now this also suggests that in general we take Lyapunov function as the sum of the squares or say quadratic functions here.

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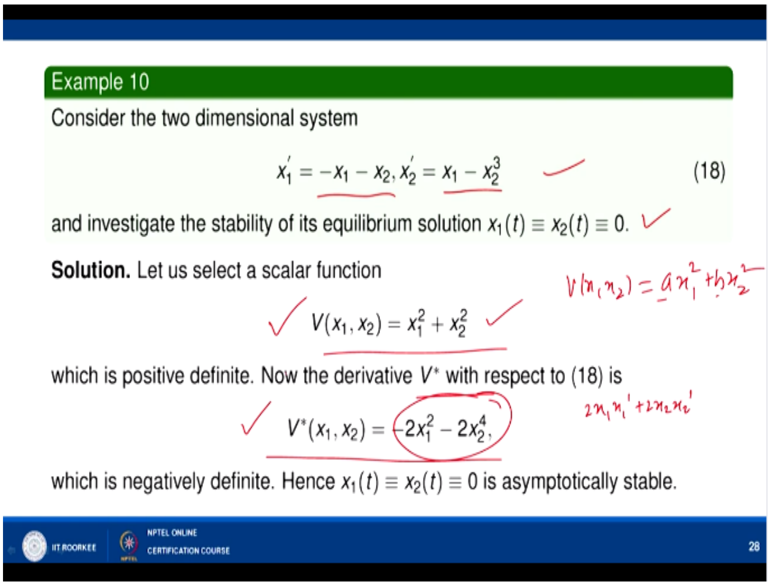

Anyway now let us take one example based on this. So consider the two-dimensional system x1 dash=-x1 –x2 x2 dash=x1-x2 cube and investigate the stability of its equilibrium solution zero solution. Now let us select a scalar function x1 square+x2 square in fact most of the time, we consider x1, x2 as some a x1 square+b x2 square and then we fix the value a and b dependent on the say requirement.

So which is first of all this is a positive definite and if you look at the derivative of V along the system 18, then it is what, 2 x1, x1 dash+2 x2 x2 dash and put the value of x1 dash and x2 dash and you can say that V star x1, x2 is given as -2 x1 square-2 x2 power 4 and if you look at this is nothing but negative definite here. This will be zero only when x1 and x2 both are zero and hence we can say that we have a Lyapunov function which is positive definite and whose derivative is negative definite and hence the zero solution is asymptotically stable solution here.

So with this we end our discussion and we say that Lyapunov direct method of finding the analysis based on the Lyapunov function. So the main difficulty part is to construct the Lyapunov function but once we have the Lyapunov function, suitable Lyapunov function then we can directly say the stability, unstability and asymptotically stability of zero solution of a given system without finding the other solution of the dynamical system.

And this is equally valid by saying that we can check stability of any equilibrium solution of dynamical system because we can always translate our dynamical system so that your equilibrium solution is at as a zero solution. This we have already discussed, how we say change, how we shift our origin so that a nonzero critical point will take a zero critical point of a given nonlinear system here.

So here this Lyapunov stability we have discussed that if we have suitable Lyapunov function then the zero solution of given dynamical system is stable, asymptotically stable or unstable under certain condition, so with this we stop and that is all for this class here. So thank you very much for listening us. Thank you.