Dynamical Systems and Control
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Lecture – 22
Properties of Phase Orbits

Hello friends and welcome to this lecture and this lecture we will continue on study of basically geometrically steady of solutions of a lenient system. So, in previous lecture, we have discussed how we can obtain the orbit of the solution of the given system of differential equation. So, basically what we have done in previous class is this that.

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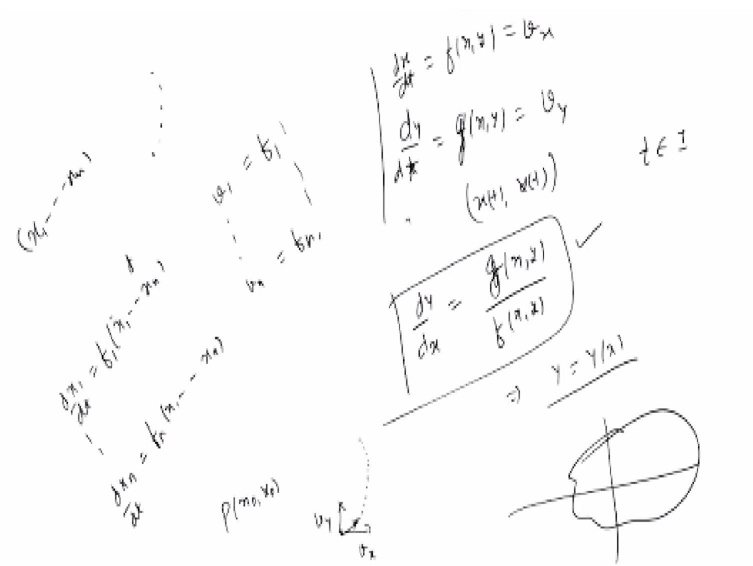

Given the system like dy/dx=f of xy sorry dx/dt= say f of x y and dy/dt=some gxy we can solution is given by x of t and y of t then they will trace a path for t belonging to some given interval I which we cannot obtain by dy/dx = here it is f of x sorry g of x,y/f of x,y and this comes out to be a differential equation in terms of y and x. And when you solve this it come out to be some function of y as a function of x.

And if you draw it will give you a relation between x and y and that will give you the face portrait of the system dx/dt = of f of xy and dy/dt=g of xy and the concept of arbitrary or trajectory can be generalized up to n dimension n dimensional system provided you had to use now the velocity system. Here if you look at this represent the velocity in the x direction and this will represent the velocity component in y direction.

And so given any point here if you know given any points say P as x0,y0 we can always find out the velocity component and along vx and along say vy. And we can find we can have the next point we can check and then again repeating this process we can find out the say orbit of a given system dx/dt=fxy and dy/dt=gxy. Then the similar thing we can generalize up to say nth order the only thing is that there in place of only 2 velocity component they are.

N velocity component is given as v1 to say vn here where v1 will give you the component like v1 say if your system is like this dx1/dt=f1 say x12 say xn up to dxn to dt=fn x1 to xn then your v1 is nothing but f1 and vn=f of n. So, it means that in that case in the nth dimensional thing at any given point you have a velocity component given as f1 to fn and using the velocity component f1 to fn.

We can further talk about the future say behaviour of the path traced by this solution x1 to xn here okay. So, this is something we have done in previous lecture now in this lecture we will just continue and we try to focus on the properties of the orbits. And how this these properties of orbits help us to find out a particular kind of a solution. We may talk some application of and the properties of orbits also.

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Properties of orbits

Consider the autonomous system

$$x' = f(x), \qquad (1)$$

where $x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$, and $f(x) = \begin{pmatrix} f_1(x_1, x_2, \cdots, x_n) \\ f_2(x_1, x_2, \cdots, x_n) \\ \vdots \\ f_n(x_1, x_2, \cdots, x_n) \end{pmatrix}_{n \times 1}$.

So, here we will continue our study so consider the autonomous system x dash = f of x the autonomous simply says that the independent variable t represent only in defining the derivative here dx/dt and it is not explicitly available in terms of f of x. So, here x is n*1 vector and f of x is given as n*1 cross vector where components of f of x is given as f1 x1to x2 xn f2 x1 to xn and fn x1to xn.

So, this is the system we are considering and it is an autonomous system then the first thing whenever we have some kind of a system the first thing we should worry about the existence of a solution and if we have the following condition.

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Theorem 1

Let each of the functions $f_1(x_1, ..., x_n)$,...,$f_n(x_1, ..., x_n)$ have continuous partial derivatives with respect to $x_1, ..., x_n$. Then the system (1) along with the initial condition $x(0) = x^0$ has one, and only one solution $x = x(t)$ passing through every point x^0 in $\mathbb{R}^n$.

Lemma 2

If $x = \phi(t)$ is a solution of (1), then $x = \phi(t + c)$ is also a solution of (1), where c is a real constant.

Remark 1

- *Lemma 2 is not true for the non-autonomous differential equations.*
- *We can easily verify the Lemma 2 for an autonomous differential equation.*

That let each of the function f1 to fn have continuous partial derivative with respect to f1 to fn then the system one along with the initial condition x0 x of 0 is = x0 has one and only one solution x=xt passing through every point x0 in Rn and if we if you recall this is the this is the existing and uniqueness condition discussed earlier also. And it is similar to your say a one variable case.

That if your component of nonlinear function fx satisfy this condition that it has continuous partial derivative with respect to the arguments x1 to xn then it satisfied the (()) (05:54) condition and we can say that it for any given x of 0 = x0. It will have a unique solution that is what this theorem 1 states. So, theorem 1 is the existence and uniqueness condition so it means that you provided that all these are having continuous partial derivatives.

We have a solution exists so existence is okay uniqueness is also okay. Now next result we want to consider is that that if x = phi t is a solution of 1 then x = phi t+c is also a solution of 1 means what?

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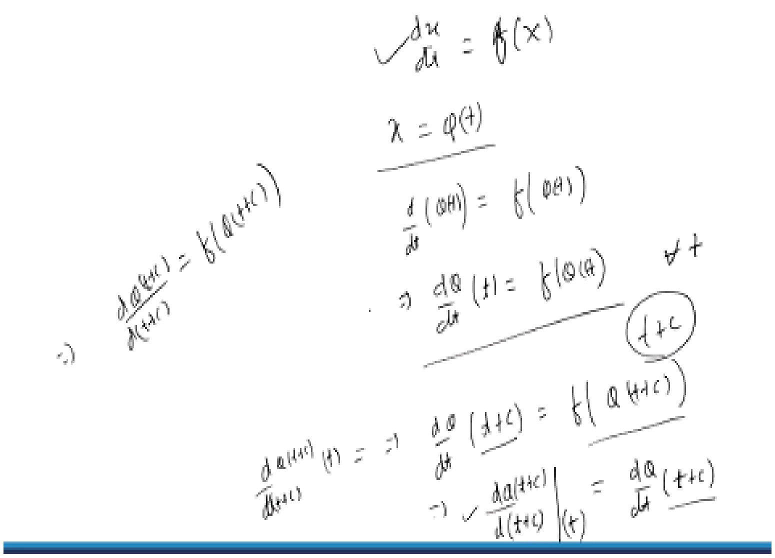

That you have this system dx/dt = this f of x here f of x here now our claim is that here the solution is given by x of phi of t here. Now we want to show that a solution means x=phi t solution means that if you differentiate d/ dt of phi t you should get f of phi of t or I can say that d phi dt evaluated at t is you can say that f of phi evaluated at t here right? So, this is true for all right so now our claim is that this relation is also true for t+c.

So, it means that if you look at since it is true for t and c is a fixed constant so I can write down this relation as d phi/dt evaluated at t+ c is given as f of phi t+c here. Right so it means that if it is true for all t and c is a constant then it is also true for t+c. So, it means that d phi/dt evaluated at t+c =f of phi t+c. Now I am saying that here this d phi/dt evaluated at t+c is same as saying that d phi t+c differentiation with respect to t+c.

Evaluated at t is same as saying that d phi/dt evaluated at t+c. So, it means that if you evaluate this that differentiation of phi t+c with respect to t+c. So, since it is true we can say that this is nothing but saying that d/dt+c phi t+c evaluated at t=this. So, it means that here we can say that d phi t+c/dt+c=f of phi t+c satisfy this system of equations. So, here that is what is contained of this Lemma if x=phi t there is a solution of 1.

Then x=phi t+c also a solution of 1 where c is a real constant so it means that given an autonomous system if phi t is a solution then phi t+c is also a solution. Now based on this Lemma we have the following two remarks that Lemma 2 is not true for the non-autonomous differential equation. It means that in place of this autonomous system.

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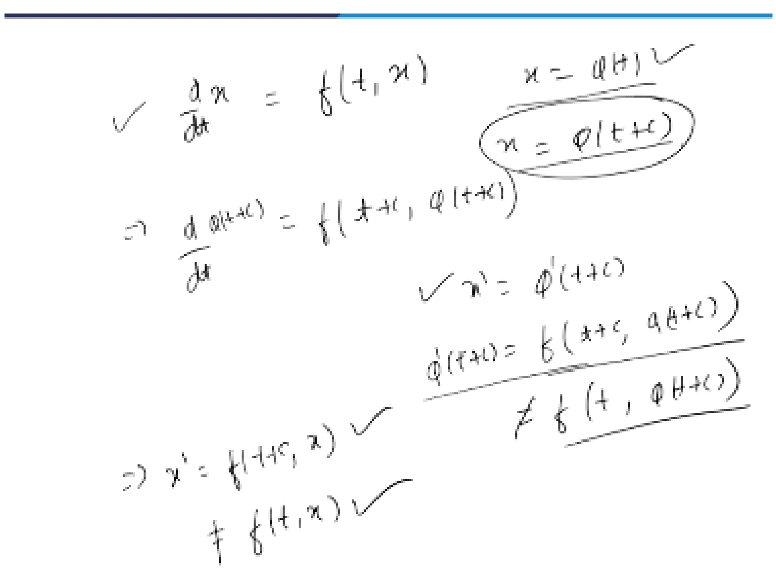

If we talk about the non-autonomous system it means d/dt of dx/dt=f of t,x then here if x=phi t is a solution then it may not be true that at x = phi t+c is also a solution because if you look at that if we replace t/t+c then it is what d/dt of phi t +c = f of t+c phi t+c here and in a same way we say that d/dt of phi t evaluated at phi t +c f of t x of means phi so we want to show here that the previous lemma is not true for non-autonomous system.

For example if we consider the non autonomous system like this dx/dt = f of t ,x and suppose x = phi t is a solution and here then there is no guarantee that x= phi t +c is also a solution of this y because consider x as phit+c. And we can find out the derivative here x dash = phi dash t+c then it will give you the value f of t + c,phi of t+c so if you look at this system. That is here this is what this system is x dash = fof t+c,x so this system is different from f of t from x .

So, it means that this phi t+ c satisfy this but phi t is satisfying x dash = ftx means that for non autonomous system if x = phi t is a solution then phi of t+c cannot be a solution until unless c = 0 here .So, it means that this lemma may not be true for the non autonomous case here and we can easily verify that the lemma 2 for an autonomous differential equation. For example if we have x dash = A of x this is 1 of the example of autonomous case.

So, here we can simply say that the solution is x= e to the power At * some constant then we can easily verify xof t+ let me use some other constant let us say c1 here then x of t + c = e to the power At +c here then it is what r to the power At e to the power Ac and c1 please mind here since At and Ac are say they commute each other. We can write e to the power At+c e to the power At * Ac * c1.

Now if you look at here since c is a constant then e to the power Ac*c1 is a constant vector so it means that x of t+c is nothing but e to the power At * sum constant value let us say y and we can say that this is also a solution of x dash = A of x because we have already seen that any solution of x dash is Ax can be written as say e to the power At times some vector say N cross 1 here. So, it means that this also be a solution of x dash = A of x so it means that lemma 2 is true we have shown that it is true for this particular problem of linear system.

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Existence and Uniqueness of Orbits

Property 1: Let each of the functions $f_1(x_1, \ldots, x_n), \ldots, f_n(x_1, \ldots, x_n)$ have continuous partial derivatives with respect to $x_1, \ldots, x_n$. Then there exists one, and only one orbit through every point x^0 in $\mathbb{R}^n$. In particular, if the orbits of two solutions $x = \phi(t)$ and $x = \psi(t)$ of (1) have one point in common, then they must be identical.

Now let us look at the properties of orbits qualitative properties of orbits here first property is this that let each of the function f1 to fn have continuous partial derivative with respect to the argument to x1 to xn then they exist 1 and only 1 orbit through every point it is not in Rn. That is basically the part of existence uniqueness. What is new in this property1 is the follows that in particular if the orbits of 2 solution x=Phi t and x =psi t have 1 point in common then they must be identical.

So, it simply says that 2 distict orbits cannot intersect if they intersect at 1 point, they has to be same. It means that if I have some orbits like this and another orbit is must be given by something which is not intersecting here. If it intersect here then it must be overlapping to each other it cannot happen like this that it is intersecting and still not the same. So, that we wanted to prove here that if we have 2 distinct orbits then they cannot intersect each other.

So, let us prove this result so the condition of f1 to fn it shows that for every point in Rn we have a unique solution and the orbit of that solution is given by the path traced by that solution so it means that let x0 belongs to Rn.

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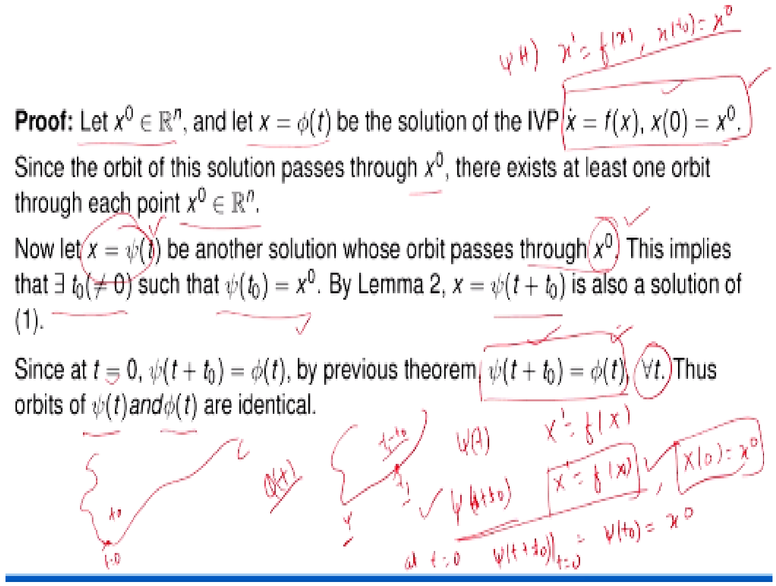

And x = phi t b the unique solution of the intial value problem that is xdash= f of x,x0 = x0. Since the orbit of this solution passes through x0 they exist atlest 1 orbit through each point x0 in Rn so it means that here x0 is the arbitrary point in Rn so you can simply say that for every point in Rn. We have a solution atleast 1 solution 1 orbit passing through that point because they exist a solution satisfying this initial condition

So, it means that for every point in Rn we have 1 orbit passing through this. Now let us say that we have another solution say Psi t whose orbit is also passing through x0 it means that we have 2 solutions Phi t and Psi t and their orbit is passing through this x0 a common point x0. So, this implies that there exists some t0 which is non-zero says that Psi t0 = x0 why because we know that if t0 is 0 then it is what it is nothing but Psi of 0 = x0.

And it also satisfies the same initial value problem but we know that every initial value problem has a unique solution provided f satisfy the given condition. It means that here t0 has to be different from 0 so it means that there exists a t0 such that Psi of t0 = x0.So, it means that we have a solution x = Psi t on a point t0 which is different from 0 and Psi of t0 = x0. Now if Psi of t0 = x0 then consider the function Psi t + t0.

Now since Psi t is a solution of this system x dash = f of x and x of t0 = x of 0. So, we simply say that Psi t is a solution of this then Psi t + t0 is also a solution of this initial value problem. So, now Psi t + t0 is a solution of system x dash = f of x and x of 0 = x0 because when you put t = 0 what you will get Psi of t0 and Psi of t0 is nothing but x0 so it means that first thing we know that if Psi t is a solution then Psi t + t0 is also a solution.

Now Psi t + t0 is a solution of what ? Psi t is a solution of x dash = f of x then we know that for any constant Psi t + C is also a solution of x dash = f of x so it means that Psi t + t0 is a solution of x dash = f of x.Now we say that what is the initial condition satisfy by Psi t + t0so if you put t as 0 then you will get Psi of t0 and that is the value Psi t0 is x0. It means that Psi t + t0 is a solution of x dash = f of x with the condition that Psi of 0 + t0 = x of 0.

So, it means that Psi t + t0 is a solution of the initial value problem given by this that is x dash = f of x and x of 0 = x0.So, it means that Psi t + t0 and Phi t both are the solution of the same initial value problem that is x dash = f of x and x of 0 = x0 and by existence and uniqueness it must be same. So, it means that Psi t + t0 and Phi t has to be ideally = each other for every t so it means that orbits of Psi t and Phi t are identical.

It means what we have assumed that now let x = Psi t be another solution whose orbit pass through x0. So let us say that another orbit corresponding to a new solution a different solution x = Psi t and which also passes through this x0. So, it means that the orbit of Psi t and the orbit of the Phi t are intersecting at the point x0. Now what it means since the orbit of Psi t is passing through x0 it means that there exist a t0 which is non-zero.

And Psi of t0=x0 so it means that the solution Psi t is passing through this point x0 at some time t0 here where Phi t is passing through this x0 at t = 0 or we can say that it is initiating from x0 point itself but Psi t is reaching at that point at some point t0 which is > 0 . Now we already know that since Psi t is a solution of x dash = f of x and t0 is just a fix value so we can say that Psi t + t0 is also a solution of x dash = f of x.

So, it means that Psi t + t0 is a solution of this differential equation x dash = f of x the only thing we want to see that this what initial condition it will satisfy so since at t = 0 Psi of t + t0 is nothing but Psi of t0 it is = x of 0. So, Psi t + t0 is a solution of this with the condition that x of 0

= x0. So, it means that Psi t + t0 is a solution of this differential equation and satisfying the initial condition x of 0 = x0.

But this simply says that solution Phi t and Psi t + t0 both satisfy the same initial value problem that is x dash= f of x with initial condition x0 = x0 and since f satisfy the initial conditions then by the existence and uniqueness theorem Phi t and Psi t + t0 has to be ideally equal so Psi t + t0 = Phi of t for all t. It means that the orbit traced by Phi of t is also traced by Psi t + t0. If we say that t is belonging to say some interval and it is passing through this.

Then this Psi t + t0 will trace the same path only thing is that now it is delayed by say the time t0 so this is Phi t and it is starting at t = 0 and we have a path like this. Then your Psi is starting somewhere here and reaching at this point at t0 and after this it will trace the same path because Psi of t + t0 = Phi of t. So, it means that the path traced by Phi of t is traced by Psi t also the only thing is that it is delayed by the time t0.

So, it means that the orbit of Psi and orbit of Phi must be same if we consider all t here or I can say that if we consider the orbit of Psi. For example like this then you can simply say at this point if Psi is reaching at point say t1 then Phi will reach at this point at t1 – t0 . It means that the same path is achieved by Psi at time t1 and by Phi at time t1 – t0. So, it means that every point on this orbit is achieved by both Psi and Phi. The only difference is the time taken by Phi or Psi for Phi it is t1 + t0 and for Psi it is t1 here.

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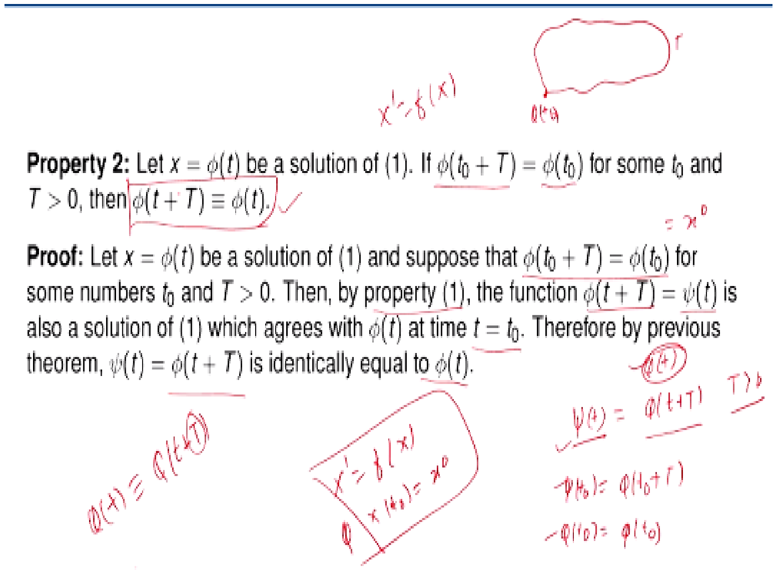

Property 2: Let $x = \phi(t)$ be a solution of (1). If $\phi(t_0 + T) = \phi(t_0)$ for some t_0 and $T > 0$, then $\phi(t + T) \equiv \phi(t)$.

Proof: Let $x = \phi(t)$ be a solution of (1) and suppose that $\phi(t_0 + T) = \phi(t_0)$ for some numbers t_0 and $T > 0$. Then, by property (1), the function $\phi(t + T) = \psi(t)$ is also a solution of (1) which agrees with $\phi(t)$ at time $t = t_0$. Therefore by previous theorem, $\psi(t) = \phi(t + T)$ is identically equal to $\phi(t)$.

Now let us consider the next important property that let x = Phi t be a solution of 1 that is x dash = f of x and if Phi t0 + t = Phi of t0. It is starting point Phi of t0 after say time t it is coming back again to this point for some t0 and T > 0 then it says that Phi of t + T = Phi of t. So, if after sometime T it is again coming to the same point then we say Phi of t + T = Phi of t or we can say that the interpretation of this mathematical equation is to show that.

In this case Phi t is a periodic solution with a period T if T is the smallest number having this property. So, let us have a proof of this we simply say that let x = Phi t be a solution of 1 and suppose that Phi t0 + t = Phi of t0, let us call this as x0. For some number t0 then by property 1 the Phi t + T = Psi of t let us denote this by Psi of t. Then Psi of t is again Phi of t + T we already know that Phi t is a solution then Phi t + T is also a solution by lemma 2.

Where T > 0 is some constant it means that Psi t another solution and Phi t is another solution so Phi t and Psi t both are 2 solutions and they agree at point t = t0. So Psi of t0 = Phi of t0+ T and what is the value of Phi of t0it is your Phi of t0 and both are equal it means that Psi t0 and Phi t0. Both are the solution of x dash = f of x and x of t0 = x0. So, it means that Psi t and Phi t both are the solution of this initial value problem that is x dash = f of x and x of t0 = x0.

So, it means that by existence and uniqueness condition both the solution has to be ideally same so it means that this Phi t + T is ideally = Phi of t. So, your Phi of t = Phi of t + T means that it will again come back to the initial condition where it started after say time T. Here we can simply say that solution x = Phi t be a periodic solution with a period T if the T is smallest such number satisfying this condition.

And it is very useful to check that in what condition your solution is a periodic solution so we will use this property 2 to determine whether a system has a periodic solution or not.

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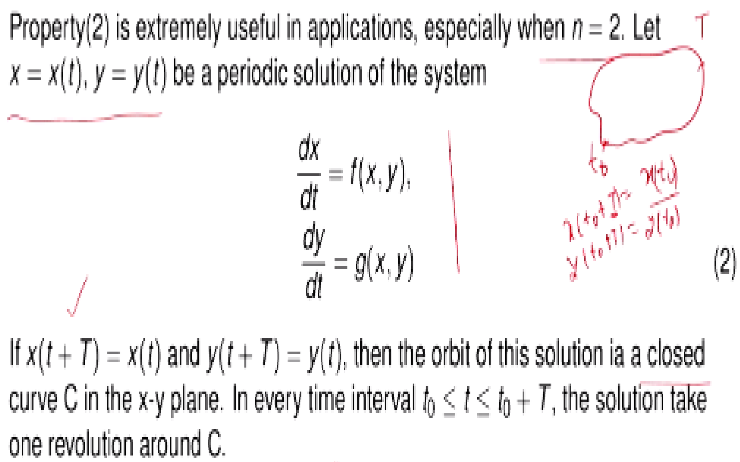

So, let us see that how we can utilize this property 2 to check whether a given system has a periodic solution or not. So we can simply say that property 2 is extremely useful in application especially when n = 2. Let x = x t and y = y t be a periodic solution of dx by dt = f of xy and dy by dt = g of xy here let us prove in both way we simply say if this condition hold that if x of t + T = x of t and y of t + T = y of t.

Then the orbit of solution is a closed curve C because the starting point is the final point. If they start at t0 then we know that x of t0 + T = x of t0, so after a time T it will again come back to the initial point where they have started. Similarly y of t0 + T = y of t0 so if they start at t0 then after the time T it will come back to the same point. So this curve the orbit of the solution xtyt is a closed orbit a closed curve basically in the xy plane.

And it will take time T to reach back to the point where it started so it is a periodic solution. Periodic solution implies that we have a closed curve in the xy plane or we say that periodic solution has closed orbit in xy plane. Next we want to show that if we have a close orbit and it does not contain any stationary point then that will indicate that we have a periodic solution.

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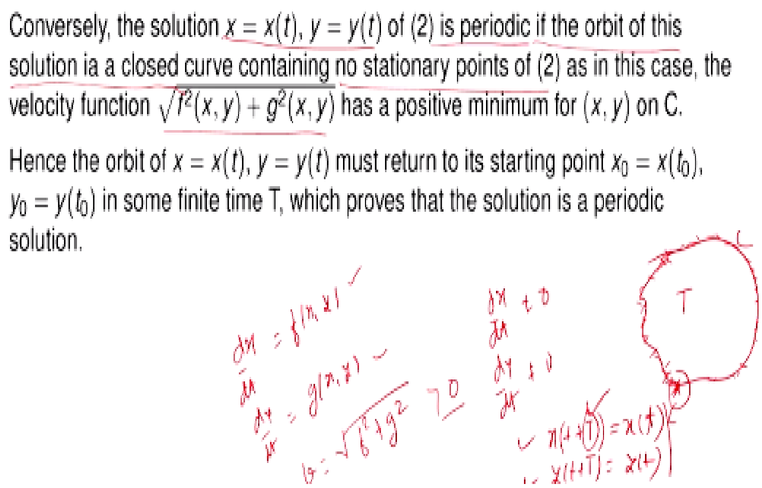

For example if we have x = xt and y = yt be a solution and we claim that this is periodic if the orbit of the solution is a closed curve containing no stationary point of 2. Stationary point of 2 means where dx by dt and dy by dt both are 0. We have a orbit like this and at no point on this dx by dt and dy by dt both are 0 so first of all, if both are non-zero then we simply say that dx by dt = f of x y and dy by dt = g of x y.

So, it means that no point is f and g is non-zero so we can look at the velocity component is f and g and velocity I can give it as under root f square+ g square so this has at least some positive value at all point on the orbit say c so at every point on this orbit c this v has to be non-zero basically it is >0 now we simply say that hence the orbit of x =x t y – yt if they start from this and at any point the velocity is never 0.

So, it means that it goes on proceeding and after sometimes say it will again come back to this why because the orbit is a close curve os it means initial point is same as final point and at any point on the orbit c we do not have say 0 velocity so it means that every point we have a velocity whether it is less or more but it will keep on moving on this curve and ultimately at some time say call it T.

It will come back again to this so it means that x of t +T = x of t similarly y of t +T = y of t here so it means that the solution satisfy the condition implies that solution is a periodic solution so what we have shown here. That if we have an orbit which is close orbit containing no equilibrium point than it will start at some point and after some point. It will have and it will again come back to the initial point.

Where they have started it means that they exist time capital T such that x of t + T = x of t and y of t + T = y of t implying that our solution is a periodic solution with the periodic t where t is the minimum of all such constant T here. So, we try to check periodicity of period whether a given solution is periodic or not provided that it has a close orbit and contain no stationary point so let us consider 1 example based on this.

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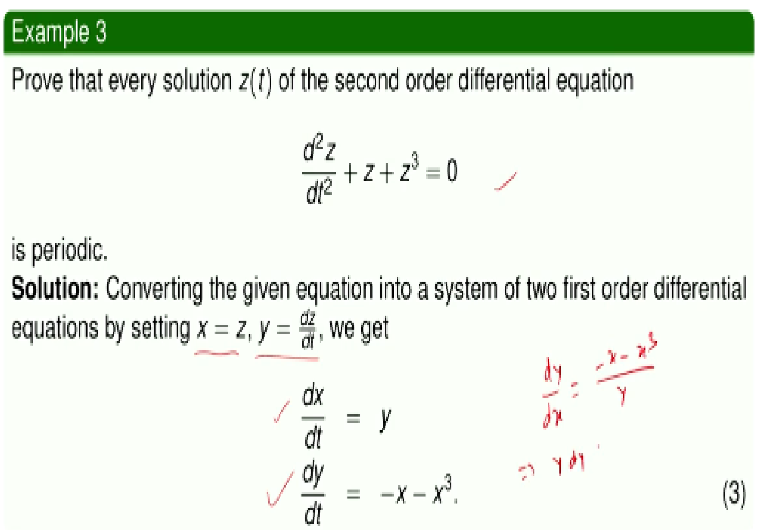

So, prove that every solution t of the 2 nd order differential equation d2z /dt square +z+z cube = 0 is periodic.So, first think we need to check that this must have a close orbit so 1 st think we need to find out the orbit and then we have to make sure that the orbit does not contain any equilibrium point so let us convert in to a system of 2 1st order differential equation. Let us call this let us assume that x=z and y = dz/dt so we can say that dx/dt = y and dy/dt =-x-xcube.

Then we can find out the orbit by solving dy/dx= -x-xcube/y or we can say that ydy+xdx+x cube d of x = 0 when you integrate this it is y square /2+x square /2+x to the power 4/4 = sum constant and let us call it as c square so it means that orbit is given by this.

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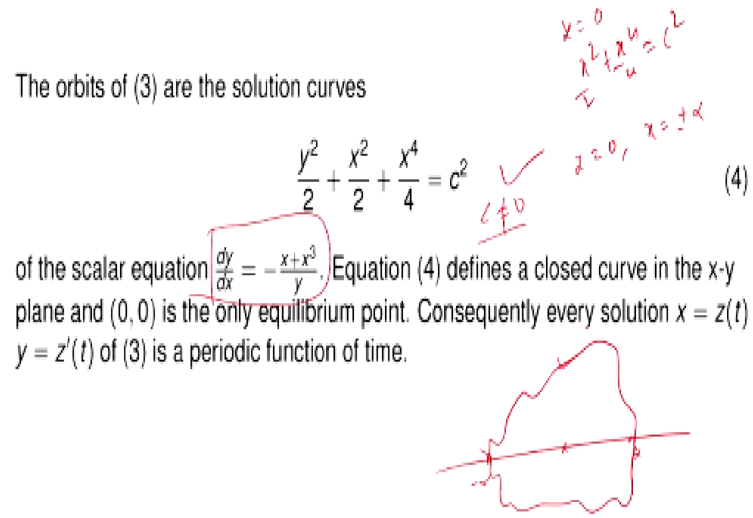

So, that is we get this y square/2+ x square/2 + x to the power/4 = c square which we obtain by solving this differential equation. Now we say that this is a close curve infact we can simplify we can say that let us put y = 0 then we have equation x square/2+x to the power 4/4= c square and we can check that we can obtain 2 real root of this equation so it means that for y = 0 we can find out some real root +- alpha which satisfy this equation.

It means that if it is started from this – alpha and alpha so it will behave something like this. And since it is symmetric around say line this y = 0 then it will again trace the same thing so it will form a close curve in the xy frame and now we simply look at here the equation y = -x-x cube what are the critical point of this the critical point of this be y = 0 and x 1+ x square = 0 so here the critical point is only the 0.

And here you look at 0 0 will be on this orbit provided that c = 0 so when c=0 this is nothing. But the origin so it means that if c is != 0 then this orbit is a close orbit and containing no equilibrium solution it means that this will represent a periodic solution for c0 = 0 here so that here we have prove that the solution that t = every non 36:52 solution that you have taken as a 2nd order differential equation d2z/dt square+z+z cube = 0 is a periodic solution.

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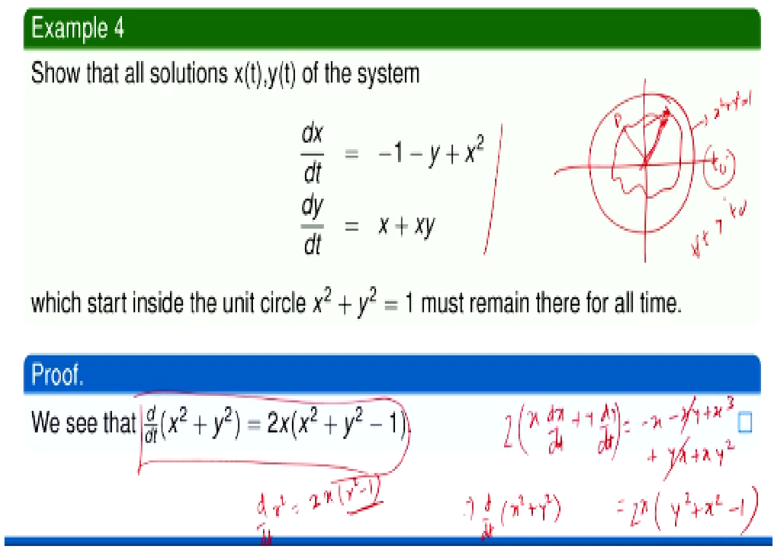

Now let us move to next problem and here we want to show that all solution xt yt of the system dx/dt= -1-y+x square dy/dt = x+xy which start inside the unit circle we have a unit circle here must remain there for all time so we want to show that if we have intial point here and if it start which si a solution here of this system will remain there for all the time so for that just calculate this d/dt of x square+y square.

How we can calculate you can calculate xdx/dt+ydy/dt what you will get this is –x-xy+x cube +yx+xy square so what you will get here so here we simplify this yx will cancel out and we can say that you can take out this x here so what you will get y square + x square -1 we will get so you will get xdx/dt +dy/dt =x*y2+x square- 1 and if you multiply 2 here then this is nothing but d/dt of square + y square.

So, we can say that we have calculated d/dt of x square + y square = 2x x square+ y square -1 so now if we look at that if we take at point here it means that the distance between this to origin is <1 this is what x square+y square -1=1so if you take any point here then for this x square+ y certain <1 the distance from the origin is certainly <1 so it means that the radius let me write it here can be consider as d/dt of r square = 2xr square- 1.

If the distance are is< 1 r square-1 is negative it means that d/dt of r square is a decreasing function r square the distance from a origin is a decreasing function it means that the velocity the distance between this point and this point decrease as t is bigger than intial point let us take t = t0 so it means that as t is > t0 the distance from this point to origin must be < t0 so it means that it is an orbit.

And you look at this point where the distance from this to origin is <1 than this orbit must be at any point if you look at any point say p on the orbit the distance has to be <1. Because of this that d/dt of r square = 2xr square -1 and this r square is<1 so r square -1 is negative so r square is a decreasing function with respect to t. And if at t0 it is somewhere inside unit circle it will always remain inside for all time t >t0 is it okay?

So, this simply says that all the solution xt yt of the system which is start inside the unit circle must remain there for all time t.

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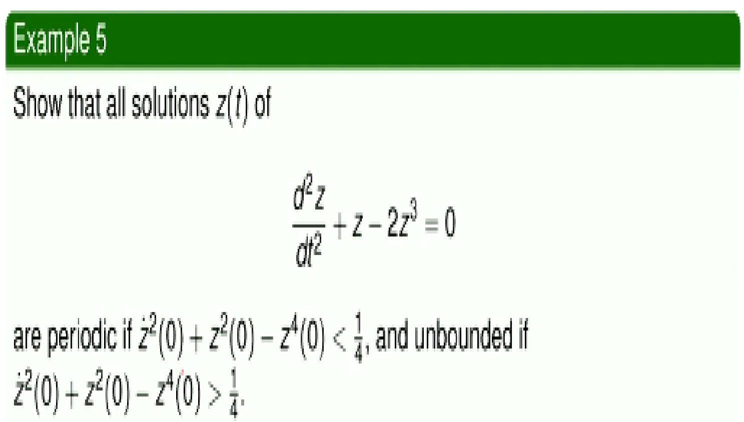

Now 1 more example let us consider show that all solution t of this d square z/dt square+z-2 z cube =0 are periodic if z square 0+ z square 0- z to the power 4 0 is < 1/4 and unbounded if we have this condition hold. So, we want to get this as we have done for in the example number 3 here.So, periodic means the orbit of this is a close curve containing no stationary point .

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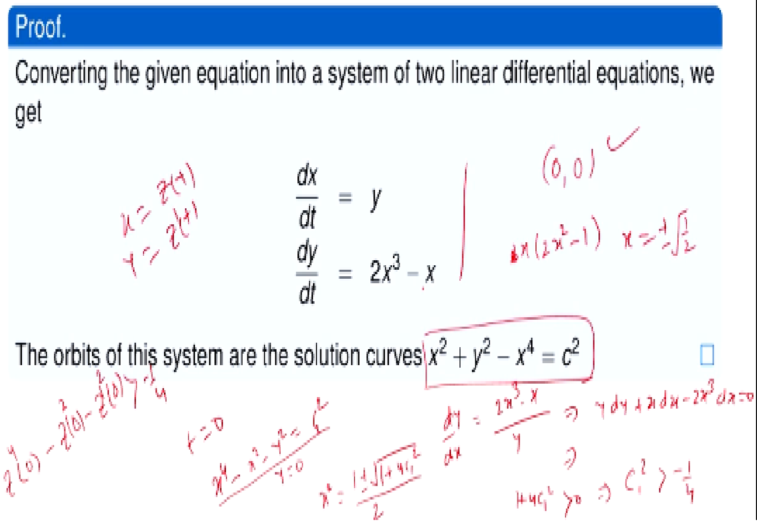

So, 1st convert in to a system of linear differential equations so, dx/dt = y and dy/dt = 2xcube –x so only critical point is given by 0 0 and is there any other critical point. Let me look at x*2x square -1 so they are more critical point x= +-1/2 1/root 2 so here we have critical points 0 0 and x as +-2 here now once we have critical points then. We try to find out the orbit of the system so orbit of the system we can find out by solving dy/dx = 2xcube –x/y.

And it is nothing but ydy+xdx-2xcube dx = 0 and when you solve this you will get this that is x square + y square - x to the power 4 = c square now again you have to look at that we want that it should be a close curve. So, we need to find out the condition here if I look at this x to the power 4 – x square – y square = c square let me use some c1 square. Here so let us say that this will define a close curve if we are able to get y =0.

If you are able to get 2 real root of x here so let us say that this is a quadratic equation in terms of x square so put y = 0 what you will get you will get this as x square as 1+--this 1+4 this is – c1 square. So, this is 4c1 square/2 here so x square you will get 1+= under root 1+4c1 square/2 means that you will get 2 real root provided that this quantity 1+4c1 square 1+4 c1 square is say positive.

Because if it is not positive then it will give you an image dilute. Hence it will not give you a closed orbit here. So, the condition that 1+4c1 square is >0 so first you need to find out the value of c1 square so c1 square value is already given here so it is you can simply say that c1 square is>-1/4 here. So, c1 square you just obtain from this so what is the value of c1 square you can get it x square what is x here x is your z say t and y is your z dash.

So, it is true for t = 0 also so it means that evaluate your c1 square at t=0 you will get this z of 0power 4-z0 square –z dash 0 square and that has to be > -1/4 to get a periodic solution and if it is not periodic and it does not contain any critical point then your velocity is 0. That is all the time positive so it means that your solution keep on moving on your orbit and it is never be unbounded solution right in that case we do not we have a periodic solution.

And if the orbit is not containing any say equilibrium solution. Then the solution will be unbounded and if you simplify this is nothing but the condition which we are going to write it here so it means that the solution of this are periodic if z dash square + z square – z to the power 4 <1/4 and t =0 here or otherwise. It is unbounded here so with this we stop here and will continue our study in next lecture. Thank you very much.