Dynamical Systems and Control
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Lecture – 10
Fundamental Matrix- I

Hello friends, welcome to this lecture in this lecture we continue the decision which we have done in previous lecture. So far, we have seen that given a system like X dash = say AX.

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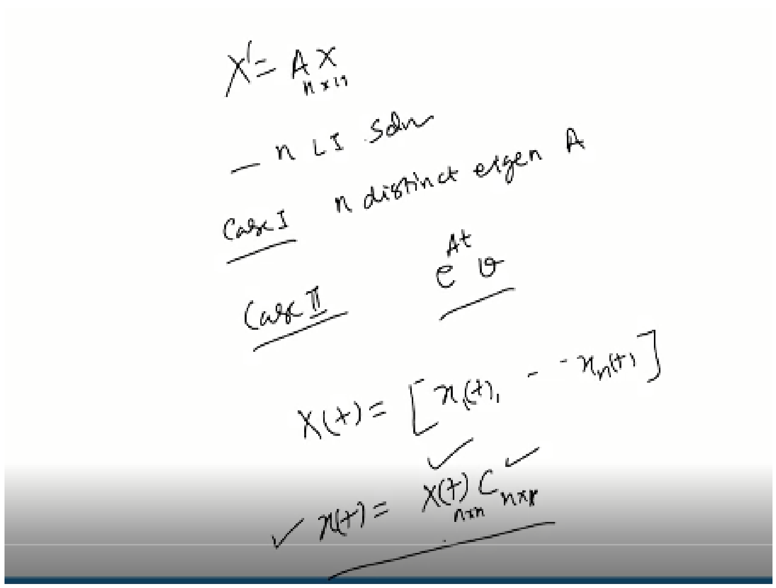

How to find out where is and n cross n matrix how do find out linearly independent eigen vector n linearly independent solution of this for this one case is case one is corresponding to the case when we have a n distinct eigen values of A. So, in this case we have seen that how to find out an linearly independent eigenvectors and hence n linearly independent solutions and in case 2 when we do not have n distinct eigen eigenvalues of A.

And then we have looked at the concept of e to the power At and with the help of e to the power At*v we have obtained the remaining the n linearly independent solution of X dash=AX and once we have n linearly independent solutions. Then we can form your fundamental matrix solution as say X1t to Xnt and we have seen one example where we have calculated fundamental matrix solution the reason or the benefit of calculating the fundamental matrix solution is that you can write down any general solution as X of t into some constant matrix C.

So, it means that this Xt is n*n and C is n*1 so it means that your general solution can be opted with the help of fundamental matrix solution. Xt as this and Xt=X of t =C where Xt is fundamental matrix solution is C is some constant factor and with this, we are able to find out a general solution. So, now our idea now our say aim is to find out the fundamental matrix solution X of t here and look at the properties of the fundamental matrix solution. So, now we observed here that the following Lemma.

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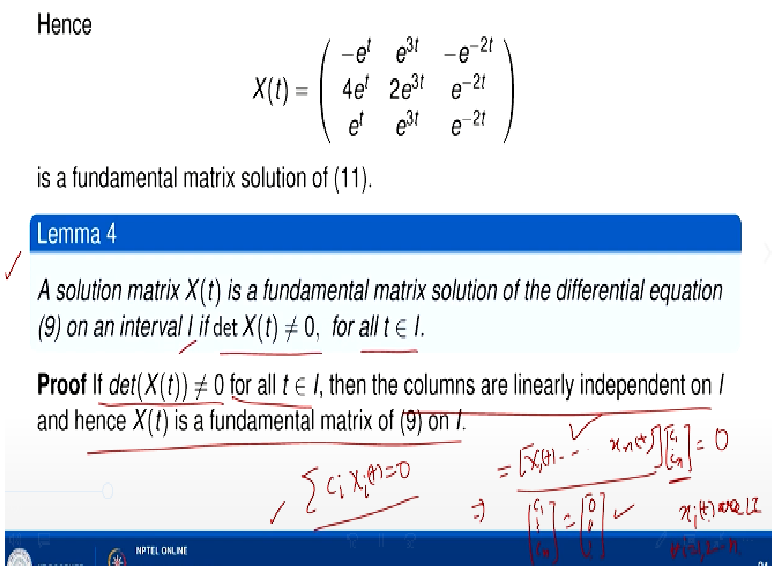

Lemma says that a solution matrix Xt is a fundamental matrix solution of the differential equation on an interval some I if determinant of X of t is not =0 for all t belongs to I. So, it means that any solution matrix Xt will be a fundamental matrix solution provided that determinant of Xt has to be non-zero for all t belongs to I. And if you look at the difference between solution matrix.

And fundamental matrix solution is that in solution matrix the columns are n solutions have your X dash=AX it may they may not be linearly independent. But in fundamental matrix solution the columns where vectors which are solutions of X dash=AX has to be linearly independent solutions of X dash= AX. So, here we say that if determinant of Xt is not 0 then we can say that a solution matrix is a fundamental matrix solutions.

So, we can prove it in this way that if determinant of Xt is non-zero for all t belongs to I then the columns are linearly independent on I and hence Xt is a fundamental matrix solution of 9 on I. So, this we have seen that that this the x1t2 Xnt and if say their determinant is 0 then we can simply say that if we write down C1 to Cn and then if we equate it to 0 then this implies that since determinant of this is non-zero.

So, this system has only retrieval solution that is C1 to Cn simply 0 and this implies that the new combination this is nothing but linear combination Ci Xi t=0. So, it means that linear combination has only a trivial solution so it means that that proves the linearly independent of these Xits. So, it means that if determinant of Xt is non-zero then the linear combination Ci Xi t can be written as the matrix X* this Ct Xt*C=0.

Now this determinant is non-zero so we take the inward inverse matrix of this and we can prove that C1 to Cn is nothing but 0 0 0 and hence we can show that these Xi t are linearly independent. Xit or LI for all I=1 2 and n so it means that so this is a very easy and important remark that a given solution matrix you need to check that if determinant is a non-zero for all t then we say simply say that it is a fundamental matrix solution.

And we can write a general solution with the help of fundamental matrix solution. Now here we need to observe one thing.

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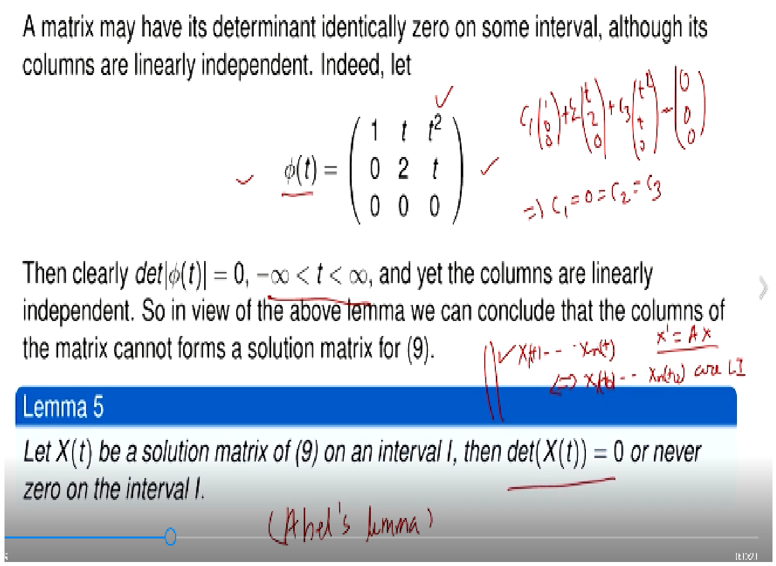

A matrix may have its determinant identically 0 on some interval although its columns are linearly independent for example look at this phi t phi t is 1 0 0 t 2 0 t square t, 0 here then clearly we can see that since it is an upper triangular matrix so determinant of this matrix is nothing but the product of a diagonal term that is 0. So, here this phi t is 0 for all t between - infinity to infinity.

But it is still you can say that your columns are linearly independent I cannot write the third column as linear combination of first two columns. So, it means that columns are linearly independent in fact if you form this C1*1 0 0 + C2 as t 2 0 and C3 as t square t 0=0 0 0 and then the only possibility I have is C1=C2=C3 that is 0. So, it means that columns are linearly independent but determinant is 0 for all t between - infinity to infinity.

So, it means that this simply says that that the columns of the matrix cannot form a solution matrix for the equation X dash= A of X so if the determinant of phi Xt is = 0 and columns are linearly independent it means that columns cannot form a matrix solution of X dash= AtX because for a solutions matrix if determinant of Xt is non-zero than it is a fundamental matrix solution.

And here we have already seen 1 example when a result if you remember that here X1t to Xnt which are solution of X dash= AX they are linearly independent if and only if that X1t0 to Xnt0 are linearly independent right. So, if these vectors are linearly independent then these vectors are also linearly independent and here we have shown that with the help of the existing and uniqueness around we have proved this result.

So, it means that if it is 0 at 1 point it has to be 0 for all point right. So, it means that for this here your columns are arbitrary functions for arbitrary function this result may not be true it means that the previous Lemma 4 may not be true for arbitrary functions but if the if the columns forms a solution matrix then of course it has to be true. In fact, we may have one more result which is a commonly known as Abels Lemma.

To prove the same thing that Xt be a solution matrix of 9 on an interval I then determinant is either 0 or never 0 on an interval. So, it means that once you have a solution matrix then determinant of Xt if it is 0 then it is 0 for the entire interval if it is non-zero then it is a non-zero for the entire interval and this result is known as Abels Lemma let us prove it and once we prove that then we say that a determinant of Xt is sufficient.

To say calculate at any given point in that interval so let us prove this small result.

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Proof Denote the columns of X by $x_1, x_2, \ldots, x_n$. Let x_j have components $(x_{1j}, x_{2j}, \ldots, x_{nj})$. Since x_j is a solution of (9) on I therefore can be written as

$$x'_{ij} = \sum_{k=1}^{n} a_{ik} x_{kj} \quad (i, j = 1, \ldots, n) \tag{12}$$

Now recall that the derivative of det X is a sum of n determinants:

$$(detX)' = \begin{pmatrix} x'_{11} & x'_{12} & \cdots & x'_{1j} & \cdots & x'_{1n} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{n1} & x_{n2} & & x_{nj} & & x_{nn} \end{pmatrix}$$

And we can prove this by showing that determinant of Xt satisfy a scale of differential equation and solving differential equation we can obtain our result. So, denote the columns x1 and let xj have component x1j to xnj. SO, please remember here that each xi your xj dash is basically a solution of AXj. Now if we look at the component vice relation that it is nothing but x1j to xnj =A is basically your matrix a11 a12 and so on.

A1n and so on an1 to ann and here we have x11 x1j x2j xnj and this is a derivative way so we can find out X dash ij=now look at the ith column ith column is ai say k and xkj right and that is what it is written here that X dash ij= summation k=1 to n aik Xkj for ij is running from 1 to n. So, here this is true for all i= 1 to n and j is for each solutions we have this result. So, it means that X dash ij= k=1 to n aik Xkj is true for all i from 1 to n and for all j from 1 to n.

Now look at the derivative of determinant of x and determinant of x derivative is nothing but the sum of n determinant. So, in first we differentiate the first row that is X dash 11 X dash 12 X dash 1j and X dash 1n and here. So, this is your first column X1 this is second X2 this is and so on. So. here we need to find out the devotee of determinant of X now let us for simplicity let write a determinant of X is same as determinant of X transpose.

So, let us write down the determinant of X transpose and find out the derivative here then derivative is also same. So, here I am assuming the first row is your X1 second row is X2 and last row is Xn. So, let us find out the derivative but have a derailment of X as derivative of X transpose and I am writing derivative of determinant of X transpose is nothing but in the first row.

You find out the derivative of X1 that is X11 dash X12 dash X1j dash and X1 dash n and keeping all n - 1 rows as it is +.

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$$
+\begin{vmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1n} \\ x'_{21} & x'_{22} & \cdots & x'_{2j} & \cdots & x'_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{n1} & x_{n2} & & x_{nj} & & x_{nn} \end{vmatrix} + \cdots
$$

$$
+\begin{vmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2j} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots & & \vdots \\ x'_{n1} & x'_{n2} & & x'_{nj} & & x'_{nn} \end{vmatrix}. \qquad (13)
$$

In second determinant you keep all rows as it is only the second row is differentiated right and in the last element last term we have the first and -1 rows as it is and the last rows is differentiated that is X dash n1 X dash n2 X dash nj and X dash nn. So, now this is the expression of determinant of X transpose derivative now use the equation number 12 that X dash ij =k=1 to n aik Xkj.

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Using (12) we get

$$
(\det X)' = \begin{vmatrix} \sum_{k=1}^{n} a_{1k}x_{k1} & \sum_{k=1}^{n} a_{1k}x_{k2} & \cdots & \cdots & \sum_{k=1}^{n} a_{1k}x_{kn} \\ x_{21} & x_{22} & \cdots & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & & \vdots \\ x_{n1} & x_{n2} & x_{nj} & & x_{nn} \end{vmatrix}
$$

$$
+\begin{vmatrix} x_{11} & x_{12} & \cdots & x_{1j} & \cdots & x_{1n} \\ \sum_{k=1}^{n} a_{2k}x_{k1} & \sum_{k=1}^{n} a_{2k}x_{k2} & \cdots & \cdots & \cdots & \sum_{k=1}^{n} a_{2k}x_{kn} \\ \vdots & \vdots & & \vdots & & \vdots \\ x_{n1} & x_{n2} & & x_{nj} & & x_{nn} \end{vmatrix}
$$

So, using this for each i we can write down that X11 dash can be written as k=1 to n a1k Xk1 similarly X12 dash can be written as k=1 to n alk Xk2 you look at this X11 dash and this is X12 dash and you look at this 1 a is written here 2 is written here and this is X1n dash. So, we have written the expression of X11 dash X12 dash X1n dash and similarly we can write down in second term and the expression for X21 dash X22 dash and so on.

And we have written the corresponding expression in terms of say a differential equation. So, using 12 we can change the first row as this second row and similarly nth row like this. Now here if you focus on the first row then you can simply say that i can simplify this from the first row we say subtract the second term multiply by a12 last term multiply by a1n and when you simply and when we subtract then determinant will be unchanged and it did in the first row you will get allX

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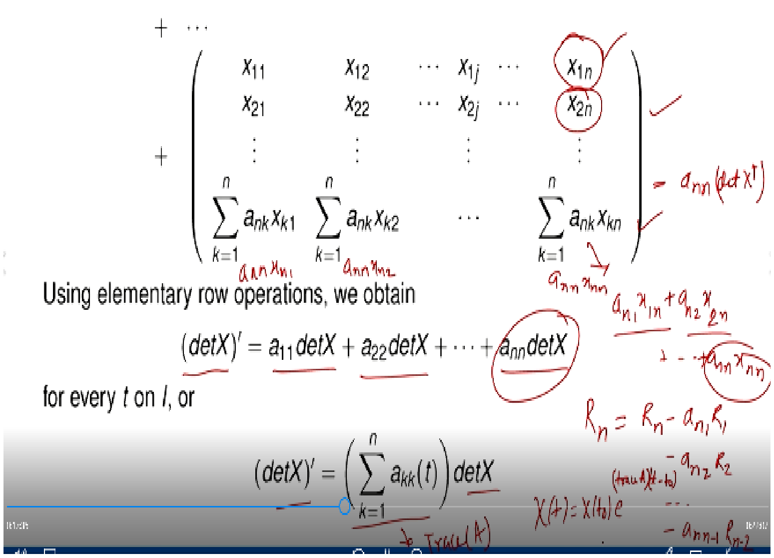

So, here you will get that all and the determinant of X and in second component we will get we will do the same thing and we will get a22 determinant of a a22 will be taken out from each term and we can write down this a second component as a22 determinant of X. Similarly, the last term can be written as ann determinant of X. You can look at it in this way when you simplify for k = 1 so let me write it here 1 term okay.

So, here this this is what it is written as an1 x1n+an2 x2n and so on ann xnn and now this x and an1 x1n is basically say a multiple of this component this component multiply by an1. Similarly, this is nothing but an2 multiply by this term. So, here what you can do you write Rn as Rn-an1

Rn-an2 R2 and so on -ann-1 Rn-1 when you do this then only this term left rest all terms are simply gone.

So, this is written us ann xn1 this is ann xn2 this is ann xnn. And so now if you take ann common from the last row then it is nothing but ann and the remaining is nothing but the expression of determinant of X transpose right and that is what i have written here and since determinant of X transpose is same as determinant of X we have written this as ann determinant of X. So, here we determinant of X dash=a11 determinant of X+a22 determinant of X and so on

And we can write this if we simply sum this then we have determinant of X dash= k=1 akk t determinant of x. If we look at this is nothing but the trace of the matrix A so trace of the matrix A and so it means that determinant of X dash = Trace of A*determinant of X and if you look at this as nothing but say a first order differential equation and we can find out the solution as this thing that X of t =X of t0*e to the power.

This exponential of this say Trace of AT A* this T yeah -t0 right and here you can simply see that t0 is any initial point in the interval i. So, if X t0 is 0 then Xt will be 0 for all t and if Xt0 is non-zero the Xt will be non0 for all t. So, this will give you that it will be 0 throughout or never 0 on the entire interval. So, it says that rather than checking that determinant of Xt is non0 for all t it is easy to check the determinant of Xt at a given point and it will remain a non-zero for all time taken.

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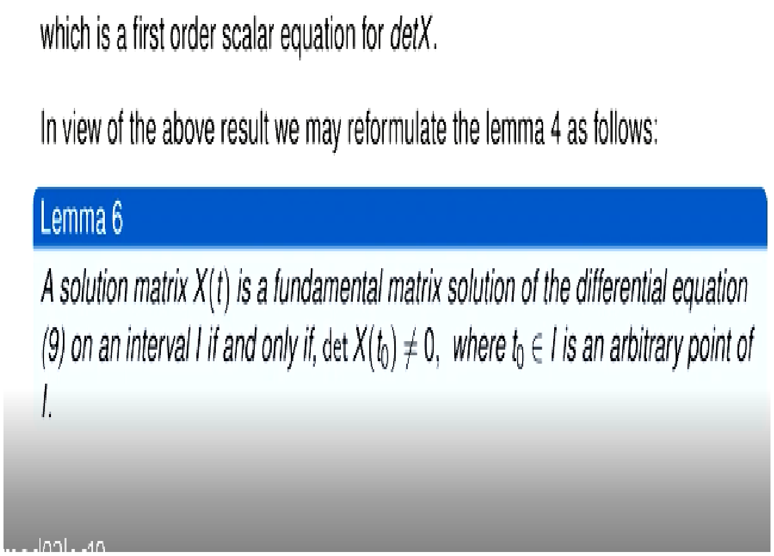

So, we can sum we can reformulate our previous Lemma and we write Lemma 4 as this the solution matrix Xt is a fundamental matrix solution and the differential equation on interval I if and only if determinant of X t0 is not = 0 where t0 is an arbitrary point of the interval I that we have shown here okay. So, we can write so now onward whenever we want to check that it is a fundamental matrix we will check only at one point otherwise not okay.

So, this is please try to understand this is true for solution matrix not for any matrix.

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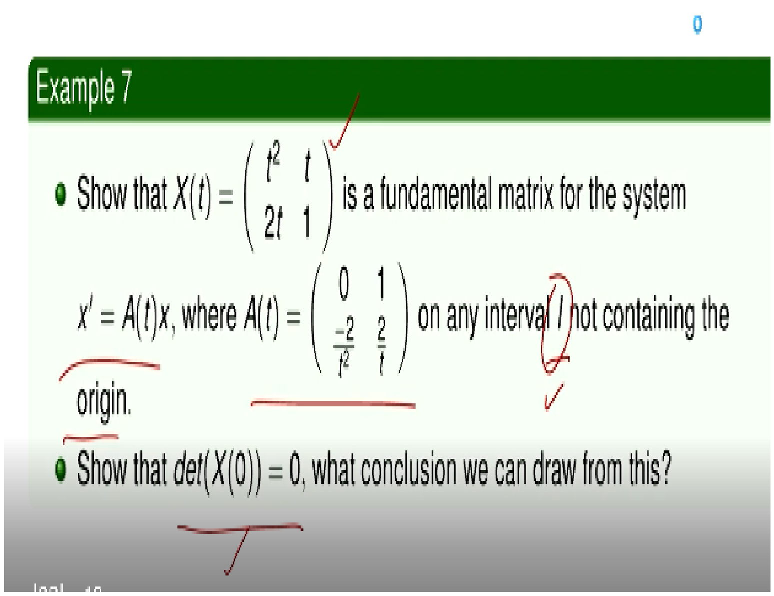

Okay now let us look at one example and we show that Xt=t square t 2t 1 is a fundamental matrix for the system x dash = Atx where At is this matrix 0 1 -2/t square 2.t on any interval which is not containing the origin the reason is that if it contained the t=0. That At may not be defined. So, to define the matrix At, we include; we exclude the point t = 0 from the interval I and in the second part.

We can easily see that if you look at the determinant of X of t this will be 0 at 0 and we try to see whether it will say create some kind of problem to the previous result. But let us assume that we have obtained the solution matrix as X of t as t square t 2t and 1 and here since the interval I is not containing.t=0. So, here the determinant of X0=0 will not create any problem to the previous result that is Xt is a fundamental matrix if and only determinant of Xt0 = 0.

Because t0 is an interior point of this interval I and here the 0 is not belonging to this interval. So, here the only remaining part we have to show is that we have to show that Xt= t square, t 2t and 1 is a fundamental matrix solution of the matrix of the differential equation X dash= Atx.

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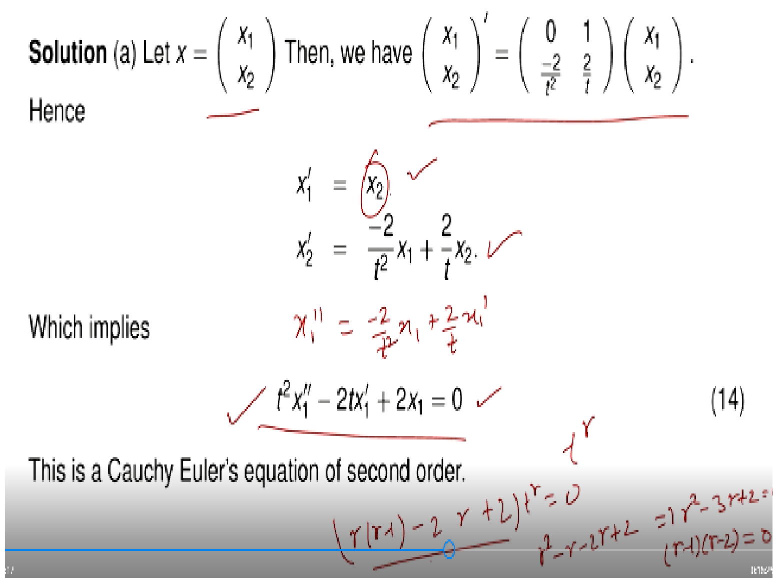

So, let us solve this and we write here that let x=x1 x2 and we can write down about a system of linear equations as in the following form X dash=Atx and since here we have not discussed and in fact it is very difficult to handle the non-autonomous case. So, here in this particular case we can solve this we can find out two linearly independent solution by converting this into a second to a second order a linear differential equation.

So, we are trying to convert this into a second order linear differential equation. So, for that if we look at x as x1 x2 then i can write x1 dash=x2 and x2 dash=-2/t square x1+2/t x2. So, using the value of x2 as x1 dash we can put it here and we can write down this has t square x1 double dash-2 t x1 dash+2x1=0. So, here we can write down the expression for x2 and we can write this as x1 double dash=-2/t square x1+2/t and x2 has x1 dash.

So, if you simplify you will get this t square x1 double dash -2 t x1 dash +2x1=0 and if you look at observe that this is nothing but a Cauchy Euler equation of the second order. So, in Cauchy Euler equation we assume that t to the power r is a solution of this where r you need to value r you need to find out.

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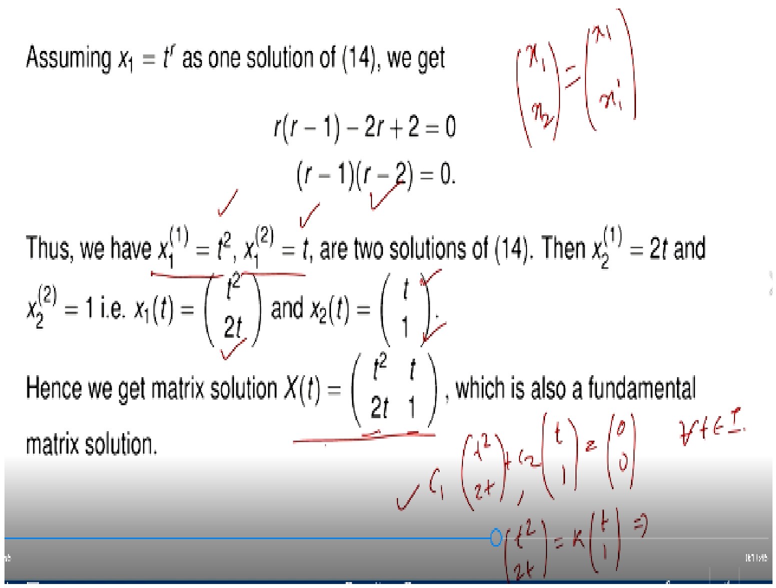

So, assume that x1= t to the power r is one solution then we can put it here as a solution here so we will get what r*r-1 -2tr 2r +2* t to the power=r= now t to the power r is non-zero for this t0 in the interval which is not containing 0. So, it means that this has to be 0 and if you simplify this r square -r-2r+2 and this is r2-3r+2=0 and we can simply say that this is r-1*r-2=0. So, we can find out the roots of this equation which is known as Indicial equation.

and roots are written as r=1 and r=2. So it means the solution of this Cauchy Euler equation is given us this t square and second solution is given as t. Now corresponding to r = 1 we have x12 as t and corresponding to r=2 we have solution like this but we need to find out the solution on the system not for this Cauchy Euler equation.SO, here solution of the system is x1 x2 so x1 x2 is basically y and sorry let me use it here it is x1 is okay x1 x2 x1 dash.

So, it is x1 here let me write it here so it is x1 ad it is x1 dash so x1 we already know if we take x1 as t square then your x2 is going to be 2t. And if you take x1t as t then your x2 is going to be 1 so it means that two linearly independent solutions are t square 2t as one solution another solution is t and 1. And you can see that these are linearly independent solution and how we can check you write C1 + C2 = 0 0 and this is true for all t in this interval I.

So, in particular take some value of t in this interval and you can verify that C1 and C2 are linearly independent. Or you can simply say that i cannot write t square 2t as combination of as Cauchy multiple of t*1. If you look at this let me simplify this and let us say that if C1 or C2 are non-zero at least one of them is non-zero then i can divide by that. So, I can write t square 2t as constant multiple of t1.

But you can see that this cannot be true because t square cannot be written as constant multiple of t. So, it means that here C1 and C2 has to be 0 so it means that this solution t square 2t and t1 both are linearly independent solution and we can find out the matrix solution xt as t square 2t and t 1 and since we have already seen that these are linearly independent so we can call this as a fundamental matrix solution.

So, in case of term a non-autonomous system we have very limited option to find out the n linearly independent solution. But in this case it is boiled down to very simple problem that is Cauchy Euler solving a Cauchy Euler equation and this case we can find out the fundamental matrix solution as follows.

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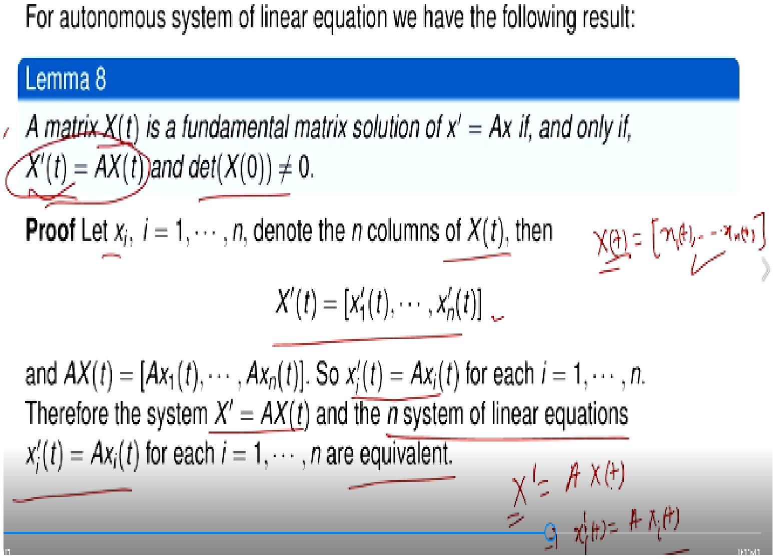

Let us move further and let us look at that for autonomous system of linear equation we have the following results. Basically now we are trying to characterize the fundamental matrix solutions so this Lemma say is that a matrix Xt is a fundamental matrix solution of x dash=Ax if and only if x dash t=AXt and determinant of X0 is not=0. So, here in case of a nonautonomous we had to look at the interval but in case of autonomous we can work for the entire t nr.

So, here we need not to mention any interval here so here we simply write and we want to show that any matrix Xt is a fundamental matrix solution of x dash= Ax provided it satisfies the two properties one is that it is satisfying to X dash t=Axt and second is the determinant of X0 is non-zero. So let us prove this similar results so let xi denote the n columns of Xt it means that your Xt is written as x1t and so on xnt then we can find out the derivative as x1 dash t xn dash t.

Please observe here finding the derivative of a matrix and finding the derivative of determinant are two different things and derivative of determinant you had to write this as a sum of n determinant and in each sum you are finding the derivative of one row. But in case of a matrix a derivative of the matrix is nothing but derivative of each row at a time. So, here x1 dash is = x1 dash t to xn dash t.

Now Axt is given as you can write Axt as AXt= Ax1t as a first column Axn t as a nth column and these two matrix are equal. So, we can see that xi dash=Axit so it means that the system X dash=AXt and this n system of linear equations xi dash=Axi t are equivalent. So, it means that saying that X dash=AXt where Xt is this equivalent to show that each column xi t satisfy A is a solution of this system exi dash t=Axit.

So, it means that the first condition is nothing but to show that the columns of the matrix Xt is a solutions of x dash t =AXt. Now so it means that first condition is nothing but to show that it is a solution matrix now second is what second we want to show that it is nothing but showing that columns are linearly independent.

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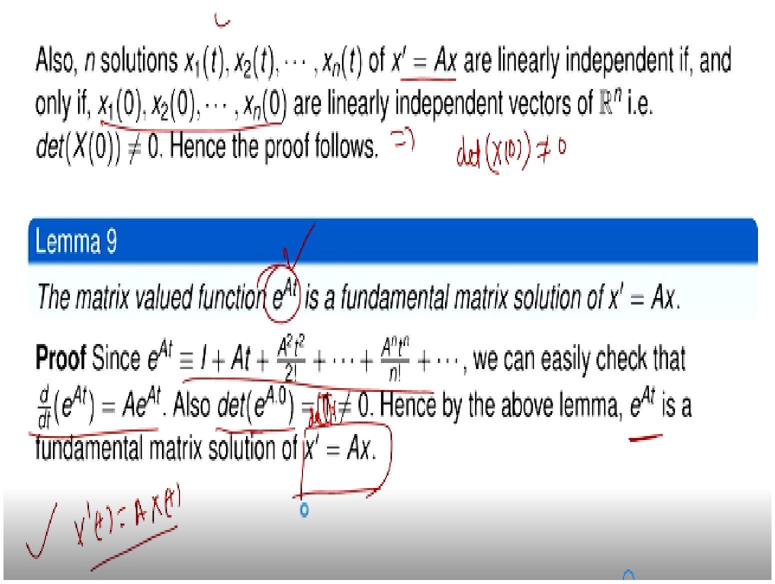

So, here n solution of x1t of xn t of x dash=Ax are linearly independent if and only if x10 if the solution evaluated at any given point are linearly independent vectors that we have already proved and it can be proved further with the help of Abels Lemma so it means that you can evaluate at t= 0 and you can see that that x10 x20 xn0 are linearly independent and enhance this implies that determinant of X at 0 is non-zero right.

So, this shows that this Xt any matrix Xt is a fundamental matrix solution here the size of Xt is n*n. So, any matrix of size n*n is a fundamental matrix solution of x dash=Ax provided that it satisfies this equation x dash t=Axt and determinant of x=non-zero. So, please take a look at the difference between this X dash t=Axt and x dash t=Axt. Please look at the difference here it is n*n and it is n*n and it is n*n and here it is n*1 n*n n*1.

So, here it is a matrix differential equation and here we have a system of a differential equation so this is the only difference okay. So, fundamental matrix will satisfy this matrix equation matrix differential equation now with the help of this now we try to prove that matrix valued function e to the power At is a fundamental matrix solution of x dash=Ax. So, to prove this we simply observed two things the first thing is that e to the power At.

Since it is written as this then we can easily check that derivative of e to the power t is nothing but A e to the power At. So, it means that it satisfies this equation X dash t= A of Xt now second thing we need to prove that determinant of X0 so determinant of A*0 is nothing but Identity. SO, determinant of identity is nothing but one which is non-zero. So, hence we can show that e to the power At satisfy the condition given in the previous Lemma.

That is e to the power At is a solution of x dash t=Axt and determinant of eAt evaluated at t=0 is non-zero. So, hence we can say that e to the power At is a fundamental matrix solution of X dash a=A of x. So, we have proved that the matrix value function e to the power At is a fundamental matrix solution of x dash=Ax and as we have pointed out earlier also that there may be more than one fundamental matrix solution.

So, now we try to see that if they are a more than one fundamental matrix solution what are the relation between these fundamental matrix solution and that is the content of the next Lemma.

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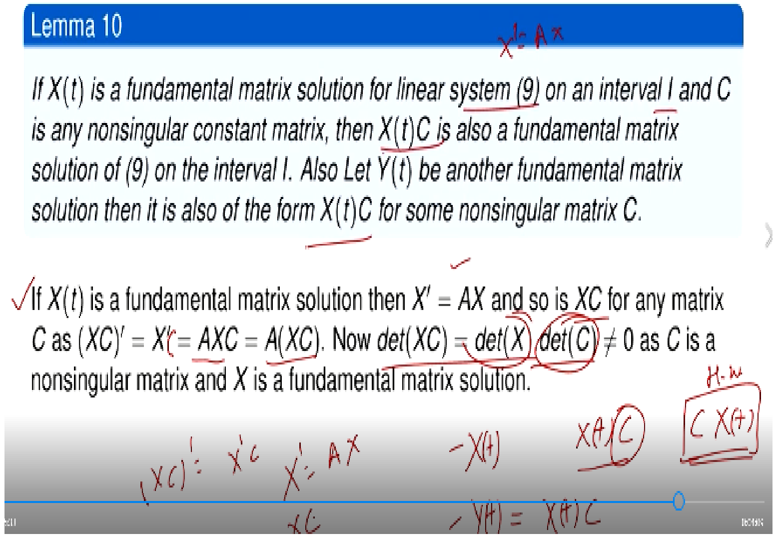

It says that If Xt is a fundamental matrix solution for linear system X dash=Ax on some interval I and C is any non-singular constant matrix then Xt*C is also a fundamental matrix so it means that given a fundamental matrix you can find out another fundamental matrix by multiplying by non-singular constant matrix C. So, it means that once we know one we have infinitely many also in a reverse conversely.

We can show that if Yt be another fundamental matrix solution then it is also a can be written as the form of Xt*C for some non-singular matrix C. So, idea is what that if we have Xt then XtC will also be a fundamental matrix and if we have some other matrix Yt fundamental matrix solution then we can find out a suitable non-singular matrix says that Yt can be written as Xt*C right.

So, it means that any two fundamental matrix are unique up to a multiplication of a known singular matrix C. Now here please observe we are writing Xt*C we are not writing C*Xt you may observe that this may not give you the fundamental matrix solution. So, this I am leaving into as a homework for you. So, now we will focus on this that if Xt is a fundamental let us try to prove this Lemma 10.

So, if Xt is a fundamental matrix solution then X dash= Ax it will satisfy this X dash= A of X then we can verify that X of C will also satisfy the same matrix solution matrix equation that is

XC dash if you calculate it is nothing but what X of C dash=X dash C because C is not C is a constant matrix so we can write X C dash we can write it as C and X dash we already know that it is A of X.

So, we can write AXC and this is nothing but A times X of C so it means that XC will also satisfy the system X dash=A of X. Now we can check with so it means that first thing is obvious that it is a solution matrix so that X of C is also solution matrix now second thing we have to show that determinant of X of C is non0. So, determinant of X of C is nothing but determinant of X*determinant of C.

Now determinant of X is a non0 because X is it a fundamental matrix solution and determinant of C is known 0 because C is a non-singular matrix which we have taken as non-singular matrix. So, determinant of Xt is also non-zero so it means that XC will also act as a fundamental matrix solution. So, we can say that if X is a fundamental solution and C is a non-singular matrix then X*C is also a fundamental matrix solution.

So, this is the first part now we try to grow the second part that any fundamental matrix solution can be written as Xt*C where C is some suitable non-singular matrix C.

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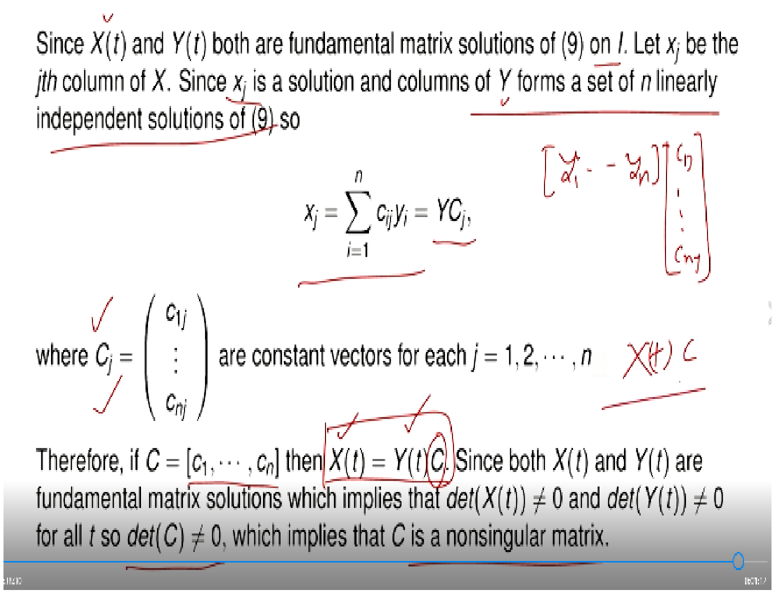

So, now let us prove this now since Xt and Yt both are fundamental matrix solution on I and if you look at the Xt is fundamental means determinant of Xt is non-zero and determinant of Yt is also non-zero. So, it means that columns of Xt and columns of Yt forms n linearly independent solutions and hence they will generate they will act as a basis of the solution space of X dash=Ax. So, it means that any solution can be written as a columns of Y.

And any solution can be written as a linear combination of columns of X. So, using this let us find out the matrix C which we wanted to know. So, let xj is a solution where xj is a column of a jth column of the fundamental matrix Xt. So, xj is a solution and columns of Y forms a set of n linearly independent solution of 9. Or we can say the columns of Y forms a basis of solution a solution set of X dash=Ax.

So, we can write xj as linear combination of yi. So, I can write xj=i=1 to n Cij yi and we can write this as y of Cj where y is your this y1 to yn and this Cj is nothing but C1j to Cnj okay where Cj is given as C1j to Cnj so it means that we can write the X of t as Y of t=C where C is the column C is the matrix whose columns are these Cjs. So, it means that i can write X of t as Yt*C here now Xt is fundamental.

So, determinant of Xt is on 0 Yt is fundamental so determinant of Yt is non-zero. So, we can show that determinant of C has to be non-zero. So, it means that determinant of C is non-zero so it means that C is a non-singular matrix and any fundamental matrix Yt can be written as in this form Xt=C where C you can obtain with the help of Xt and Yt. Is it okay so this is that any two fundamental matrix solutions are related like this that Yt can be it as this?

And this is a very important observation and with the help of this observation we will we will try to obtain the expression of e to the power At and once you know e to the power At you can write down any solution in the form of e to the power At. So, but that will do in next class so here will we stop and we will continue in next lecture from this onward. Thank you very much for listening, thank you.