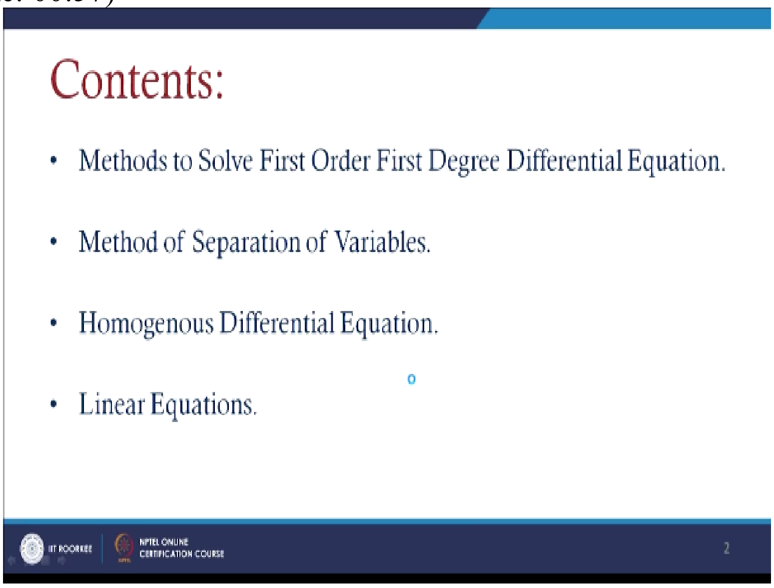
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
NPTEL ONLINE CERTIFICATION COURSE
MATHEMATICAL MODELING:
ANALYSIS AND APPLICATIONS
LECTURE- 12
SOLUTION OF FIRST ORDER FIRST DEGREE DIFFERENTIAL EQUATIONS
WITH
DR. AMEEYA KUMAR NAYAK
DEPARTMENT OF MATHEMATICS
INDUAN INSTITUTE OF TECHNOLOGY ROOKEE

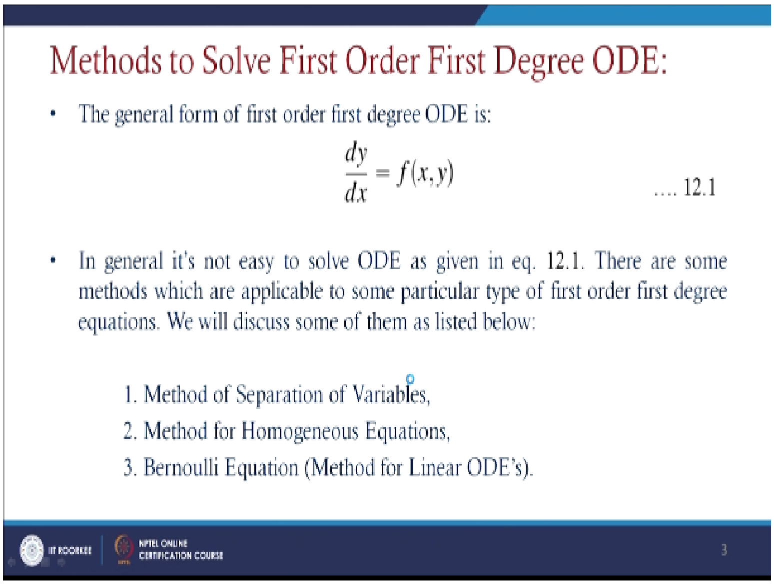
Welcome to the lecture stage of mathematical modelling analysis and applications in the last lecture we have discussed the difference between continuous and descript time levels and we have just explained the basics definition of like differential equations and patricianl differential equation and they linearity and non- linear properties and in this lecture we would discuss about general solution of this ordinal differential equations, so if we will just go for this solution of this ordinal differential equations.

(Refer Slide Time: 00:57)



First we will just solve these first order first degree differential equations then the methods used for this lectures or like method of separations of variable and then this homogenous differential equation from linear equations,

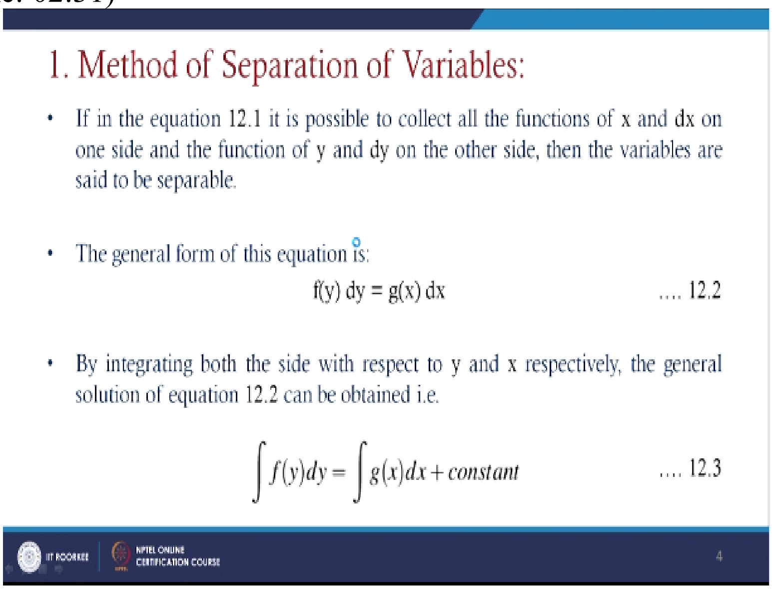
(Refer Slide Time: 01:15)



So this general form of any first order first degree ordinal differential equation is represented in the form of liked y/dx = f(x, y) which involves like 1 depended by (x, y) 1 in depended by (x) in general it is not it is to solve the ordinal differential equation is given in 12.

Since if it is involves like some non-linear factors of or some multiplied factors in terms of f(x, y)then it is not easy to solve ODE this solutions or if we cannot separate it out x and y in different forms then it is difficult to get this solutions there are some methods which are applicable to some particular type of first order first degree equations that will just discuss here and the methods are like a method of separations of variables methods of homogeneous equations and Bernoulli equation this is special class of equation we have just considered here and especially the Bernoulli equations are called also(method for linear ODE's) that can be used in a different engineering problems.

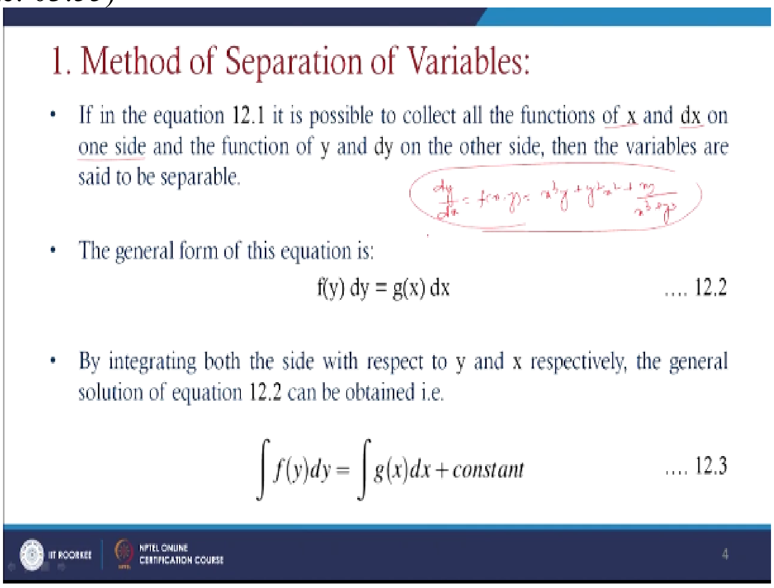
(Refer Slide Time: 02:31)



So if you just go for method of seperation of variables, so that equation we have just written that is the form of dy/dx= f(xy) it is not possible collect all the functions of x and dx on one side, since as I have told that if you just write equation that is in the form of (dy/dx = f(x, y) suppose this f(x, y) term it involves like x and qy + y^2 x x^2 + all other terms may be it is multiplied are like x y/ x x_2 +y_2 then it is not easy to separate this x or y in a complete sentences and this solution of y and d, y if it is cannot be separate in a complete sentences of variables are we cannot use like a separation of your technique to this solution.

So that's why the restriction is at it is possible to collect all the functions of x and dx on one side it and the function of y and d, y on the other side, then the variables are set to be separate work and in this class of problems we cannot say that we can separate this variables that is dx side we can just take all the x variables and y side we cannot take the y variables, so that's why

(Refer Slide Time: 03:55)

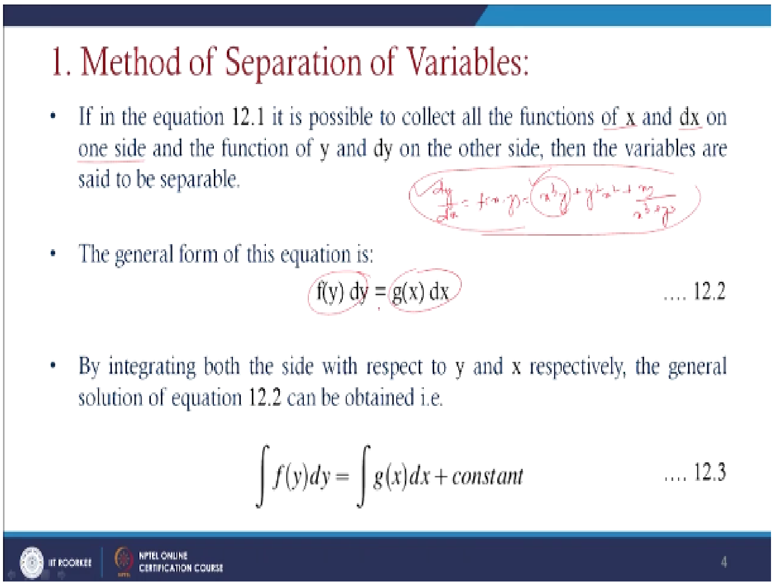


If it can be separated then we can use variable that is a method of separation variables and this general form of this equation is like f(y d, y = x dx), so always this methods cannot be applied as I have told for this generalistic equations, so many times you cannot just find the difficulty of the problem that you cannot have a solution.

Since why we have considering such differential equations is that, so in many engineering problems or many like practical application problems are not certain phenemenouls can just find that many terms it has been multiplied in an non- linear sense it is very difficult to often this solution in a complete sense or in extents solution their but we are just approaching or we are just starting process here that why we are just considering the simplify methods here.

So first multiply method of separation of variables were this variables can be separated in a complete form of both the sides, so if we have just separated this variables of y and x

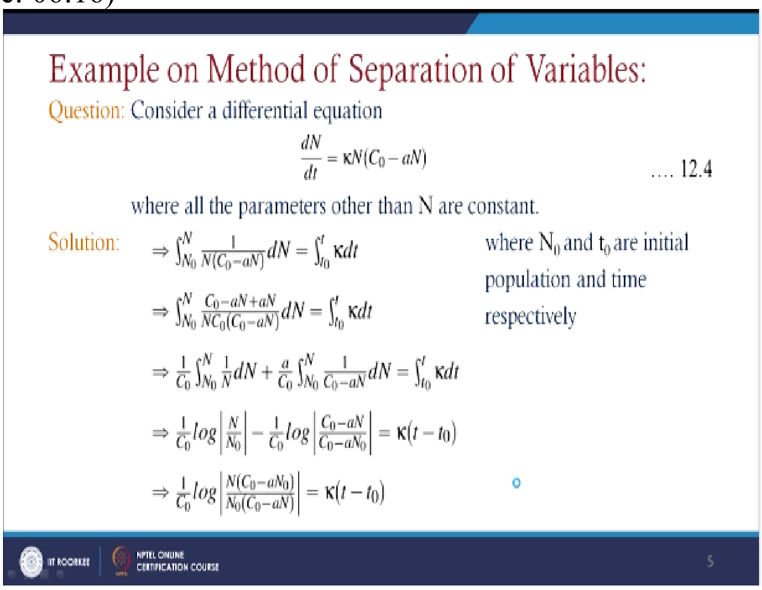
(Refer Slide Tim: 05:09)



Independently then we can integrate both the sides with respect to their variables then we can just have a general solutions this means since f, y d, y is involving here so you can just integrate this side here and z, x d, x that can be integrated that's side here and then we can have like aividrate constant and in we will just integrate this difference equations then we will have aividrate constant.

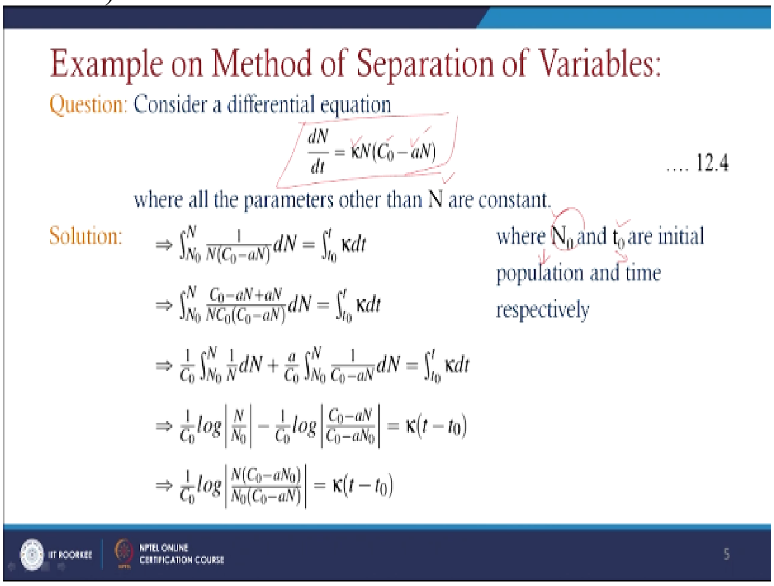
So that's why the complete solutions if we will just integrate both the sides to respect to y and x and then the general solution can be written as integrations off (d y = to integration g , x + the aividrate constant , so this constant values that can be define on initial conditions are boundary conditions, so whatever they conditions describe to us accordingly we can determined this constant, so for this separation of variables we have just considered a particular example of difference equations.

(Refer Slide Time: 06:16)



Or differential equations that has a d n/d t = k n (c0_ n $_a$ where all parameters is just see here other than N are constant here that is c_0 are constant and we have just specified some of conditions that is n_0 are t_0 are the initial levels for like it may be it is populations or time or any parameters you can just assume for best differential equation but for this differential equation we have just considered here N_0 is a population level suppose P_0 is the time level and the initial level.

(Refer Slide Time: 06:59)



So whenever we will just go for this solution of this defferential equation here then as we have discussed first we have to separate this N variable terms to the left hand side and T variable terms to the right hand side, so that's why we have just represented this equation that is in the form of like dn/ n c_0 _ aN since if you just see here all the terms involved here and variables here and right hand side just we can keep it is credit here.

Than we have just integrated this one which has just written S ,since N_0 is a initial or starting here and this is just going of to like any aivitry and realistic here and, so we have written it like orbitraion and T_0 is an initial time level here and T is a iniate time level here, so that's why we have just written integration N_0 to N 1/ Nx C_0 _N and T_0 is a time level and T is any time we have just going for the population here is credit here and to make this integration here.

So we have just make this terms as C_0 _ AN +AN we have just added or this terms in the numerated part to get in a complete solution here and then T_0 TO here and if you will just separate all this terms here then we can just get that C_0 _ AN, so first terms is just cancel here, so that's why we can just tired C_0 only, so C_0 is a constant C_0 can be taken out, so this N_0 to N, so 1 by it is there and TN.

So next time if you can see here that is a + AN/NC_0 =C_0 X C_0 _AN, so N we will cancel it out, so 1/C_0 it will be taken out and A is present there A has been taken out then N_0 to N 1\ C_0 _ AN to DN this is very easy like manipulation of things, so you can easy get the sum and then T_0 to T (K(T, T) if you integrate this terms here we just get 1by log of N by C_0 _ your 1by C_0 it is present there, so log of C_0 _ AN (C_0 _ AN by C_0 _ AN_0, since your parameter is N_0 N while we have just putting here, then this one Kx t it is coming to t is that's why upper limit by lower limit, so that is t_t_0 it is just here, so in a combined form if we just write by c_0 log off n, y/n_0 is there, so n/n_0 and this is division form of that's why this will just come to the opposite side here c_0_ An_0 /c_0_ An to t_t_0,

(Refer Slide Time: 10:15)

Example on Method of Separation of Variables:

- For simplicity let's consider $t_0 = 0$ then

$$\Rightarrow \frac{N}{C_0 - aN} = \frac{N_0}{C_0 - aN_0} e^{kC_0 t}$$

$$\Rightarrow \frac{C_0 - aN}{N} = \frac{C_0 - aN_0}{N_0} e^{-kC_0 t}$$

$$\Rightarrow \frac{C_0}{N} - a = \left(\frac{C_0}{N_0} - a\right) e^{-kC_0 t}$$

$$\Rightarrow \frac{C_0}{N} = \frac{(C_0 - aN_0)e^{-kC_0 t} + aN_0}{N_0}$$

$$\Rightarrow N = \frac{N_0 C}{(C_0 - aN_0)e^{-kC_0 t} + aN_0}$$

- To reduce the number of parameters, assume $A = C_0/a$ and $\gamma = \kappa C_0$

$$N(t) = \frac{AN_0}{N_0 + (A - N_0)e^{-\gamma t}} \qquad \text{.... 12.5}$$

- Equation 12.5 represents a particular solution to 12.4 (because the curve of N(t) passes through point $(N_0, 0)$).

IIT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE 6

And for this simplicity if you just consider here the initial time levels opposite of time level 0 has starting here t=0 and we can just tried n/C_0_ An_0= $t_{0/}$t and we can just tried c_{0-}AN, so just a reciprocal we have just using that is a we are just using this one we are just using reciprocal just we are doing here c_0 _AN/N this is nothing but c_0_ AN_0/N_0 and it will go down, so that's why it is written as e to the power|_ scale c_0 t , so this is the segment only this like division just using here, so here just we can write $c_{0/}$n _n, n cancel it out, so_ a then c_0/an_0 _a then a to the per_ scale _c_0 t and since we want to separate m, so which is dependent on time pecter here, so you can just write c_0/ m as c_0_ an_0 e to the power_ scale c_0 t +an_0/ n_0 here, so this is the complete solution of n here where n is a function of time here and two radius the number of parameters suppose if you just assume here a=c_0/a here and Y = kc_0, so always we would tried to reduce this parameters to get it may compact form.

So that's why the complete solution of n(t it is can be written as an_0/n_0+a_n_0 to the _ Y t and this equation to find here this represented particular solution top equation to find for which is just written as pulls to k, n to c_0 _n here and if you just see here r(n(t assets to the point n_0 since we are applying here and t_0 is o is the initial value we are supplied, so that's why for that particular values only this r will pass through and 0 here.

(Refer Slide Time: 12:47)

2. Homogeneous Differential Equation:

- The general form of homogeneous differential equation is:

$$\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$$

where function f(x, y) and g(x, y) are homogeneous i.e. they have same degree with respect to all the variables x and y.

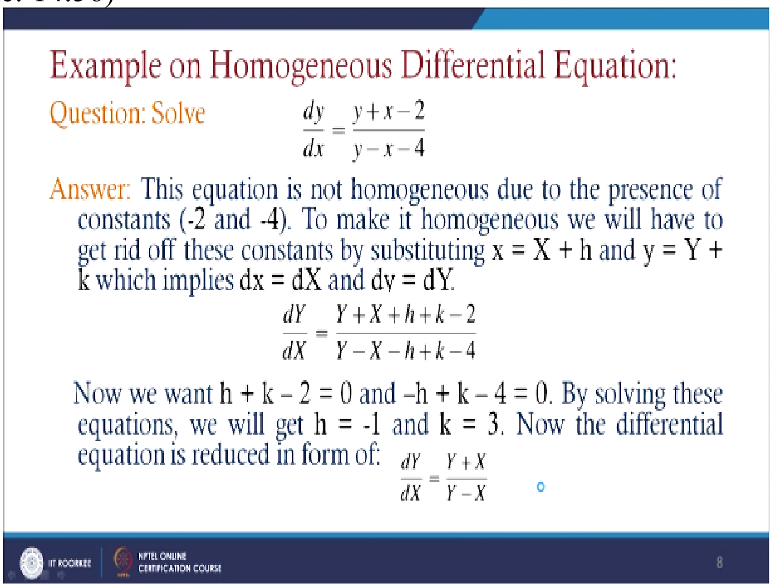
- The basic idea of solving this type of differential equation is to convert it into a simple form by substituting y/x = v or y = vx, then $\frac{dy}{dx} = v + x\frac{dv}{dx}$
- After this, use the method of separation of variables and proceed for solution.
- Sometimes, differential equations are not directly presented in homogeneous form. They need to be reduced into that one.

IIT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE 7

So next we just go for this homogenous differential equation the general form of this homogenous differential equations is the d y /dx=f(x, y) where this function of f(x , y) and g(x, y) are homogenous, homogenous means we say that all terms has the degree same that is they have some degree with respective variables x, y, so if x, y is multiplied and just consider this degree has to they are if it is x" just considered the degree of that homogenous, the basic idea of solving this type of differential equations it into a simple form of substituting the y , y x=v, since called it on homogenously here, so whatever they solved in terms of y , y x if you will put and cancel it out and the idea is we want to established this relationship between this two variables y and x in sufficient that all the terms can be cancel it out.

Since it is same degrees are existing for all the terms and if you just substitute this stuff y, y = and we can write this y = b, x here we just entered the variable v which can be separate it out, since it involves all the product terms that in terms of y and x/ here, then we can have like if we just differentiate at y as a v, x here, so we can just write d, y / d, x since we have to variable here, so that's why we tired d x that is 1 here then +X , x into d, y / d, x and after this we just used the method of separation of variables to get the solutions, sometimes they differential equations homogenous form then we have to reduce this equations to homogenous form if certain constants are involved at homogenous equations.

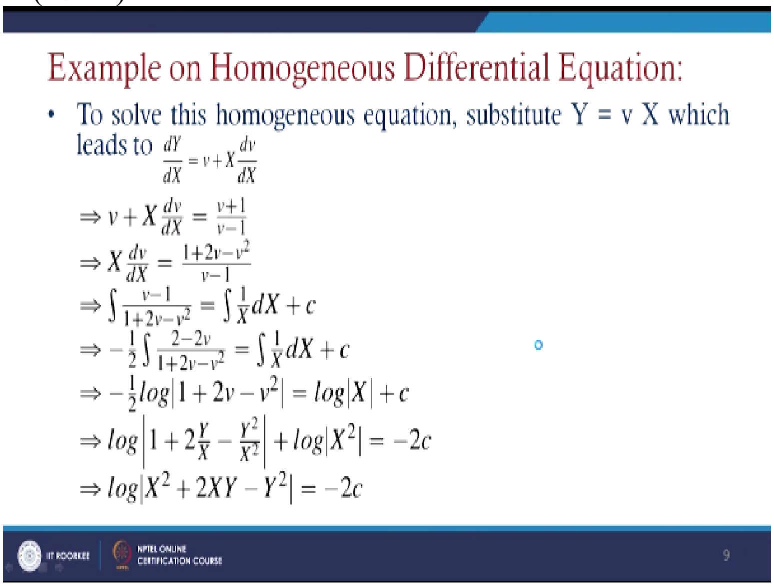
(Refer Slide Time: 14:56)



So like examples of homogenous differential equation here we can just see that y+x_2 it is not a homogenous part y_x_4 it is not a homogenous part here, since this constant 2 and _4 is artificial there if you just see here degree of differential 1 therefore if you see the degree of differential here 0 here, so that's why this is not a homogenous form here and to make it homogenous we have to like a get rate of this constant if you just put here suppose small x is capital x +h and small y = Y + k here which implies we can directly since h is a constant here we can just write d, x is a d, x here and d, y has a d, y here, h and k both are constant , and then we can just replace small y and small x by this capital Y and capital X here, so capital d, y/ d, x can be represented y +x + h+ k _ 2.

Since we have to replace directly the variables here and similarly this is in nothing but the y +k _x _h then _4 and now we want to make this h + k _2= 0 and_ h _ k =0 and if we just take this constant equals to 0 here then we can have a homogenous equations, so if you just solve this equations like h + k _ 2 =0 and _ h + k = 4 =0 then we will get h = _ 1 and k = 3 here and now the differential equations reduced in the form of capital d y / d x = y + x / y _ x, since this factors is 0 here.

(Refer Slide Time: (17:21)

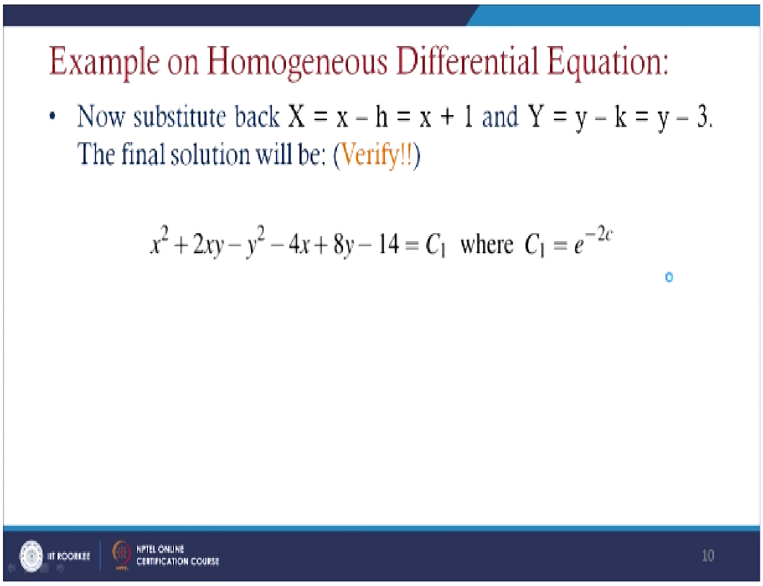


To solve this homogenous equation now we will just substitutes y = v x then we will have here d y / d x as v + x we will have liked v / d x, so then if you just substitute that equation and we can just find details like we will have this equations that has d y / d x this equals to y = x and y _ x, so directly if you just substitute here then v + x d v / d x which can be written as y as capital y you will just see here which can be written as v x here + x / v x _x here. So that's why we can just write this one as v + 1/v _ 1 here, since x can be taken comment here now we will have this variables like v and x, so once we will have this equation v + x d v / d x = v + 1 _ v _ 1 and just separate this one.

Since we can just write this one as x (d v / d x then as v + 1 / v _ 1 _ v here and now we can just add this one year v + 1 _ b x b _ 1 / b _ 1 and obviously once you are just getting this complete lead terms here then we can do x / x this is d v / this whole term here and once you are just putting in this form that is just coming like x d v / d x but is nothing but 1 + 2 v _ b^2 in terms it will just give you this is v _ 1 then we will just write v _ 1 + 2 v _ 1 b^2 and b v = 1 / x b x + c and if you will just take this integration here then we can just find this one as, since now we will have like 1 + 2 v _ b^2.

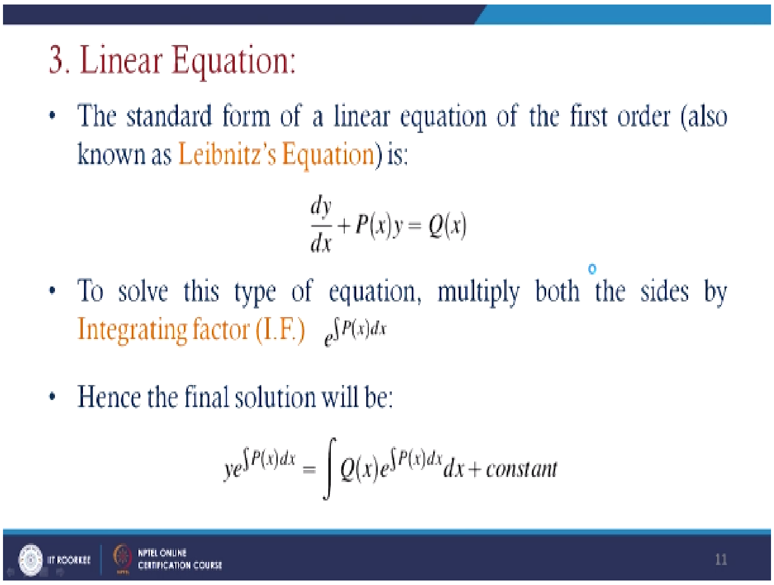
Here we have to multiplied here 2 divided by 2 then you can have like _ of log of 1 + 2 v _ b^2 this equals to log x + c and the complete solutions is just represented as log off of x^2 + 2 x y _ y^2 is _ 2 c here, so once you are just obtained in this solutions of capital x and capital y here then you can just replace that one, since we will just have a like small x is capital x + h and small y = capital y + k

(Refer Slide Time: (20:30)



We can just substitute their terms, so if you will substitute capital x to small x _ h and h is coming as _ 1 there, so that's why x + 1 is coming capital = y _ k, so that's why k is coming as 3 y _ 3 and the final solution you can obtained if you will just put on this variables in this equation here log off x^2 + 2 x _ y + _ 2 c you can have this solution like x^2 + 2 x _ y^2 _ 4 x + 8 y _ 14= c_1 where c_1 has given as 8 y / 2 c this is just little panamolisem if you will just do you can just get it.

(Refer Slide Time: (21:14)



So we will just go for linear equations, so this stunner form of linear equations of the first order also sometimes it is called as Leibnitz equations this is written in the form of d y / d x + p(x) y = Q (x) and most of this engineering problems you can just find the equation which is of for first order then that is represented in this form, since one of this variable y is multiplied with p x term and rest of the terms is free from y terms there to solve this type of equation especially this multiplied by some of this integrity factors some time it may be p x d x sometimes may be any factor of for x sometimes it may be any factor of y.

So many forms of this integrating factors it has been multiplied with this differential operators to make it in an integral sense factor they are, so whenever we will just multiplied this factors, so this differential terms then it can be integrated easily, so that's why it is called integrating factors.

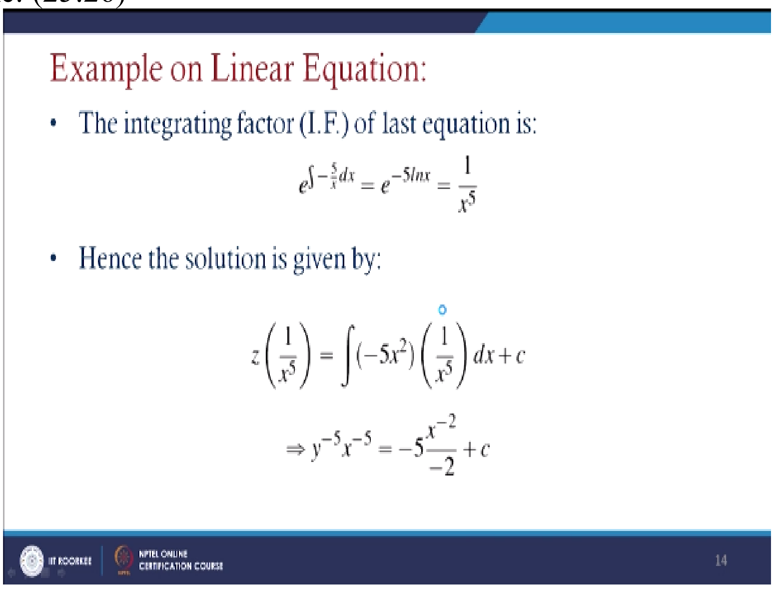
So if we are using like del m by del y _ del in by del x the equation will be represented by in the form of like m x y, dx n x / y this equals to 0, so many class of integrating factors it can be multiplied to get this solutions, so either it may be like 1/ m x _n y it my b 1/ mx+ n y or it may be del m by del y _ del n / del x or it may be like del m / del y +del n / del x, so whatever it has been multiplied cutting this multiplied factors it will cut change any way we will have this solutions.

So mostly this people are using integrate factor that is in the form of 8 to the par p x (d x) for this class of problems and the final solution it will be multiplied then easily it can be integrable. So that's why we are just writing this arrow y e { p x since it is multiplied with the solution here then right hand side we can just write it as a integration of q (x) e{ p(x) dx + constant and some equations can be converted to the standard liner equation form this equation is called Bernoulli's equation.

Here q of x term it is the word with a e power term here and if it is a just put in this form then we can just like divide y to the { n in the left hand side here and then we can just reduce another variable to the { 1 _ n (z) and we will have N _ d x + p x z= Q (x), since if you just see here directly we can just reduce this equations as 1 to the { n d y / d x + p x y 1_ n = q (x) and especially if you put y { 1 _ 1 1_ 1 is z then this is p (x) z this is q (x) this will the reduced factor of inside them and directly.

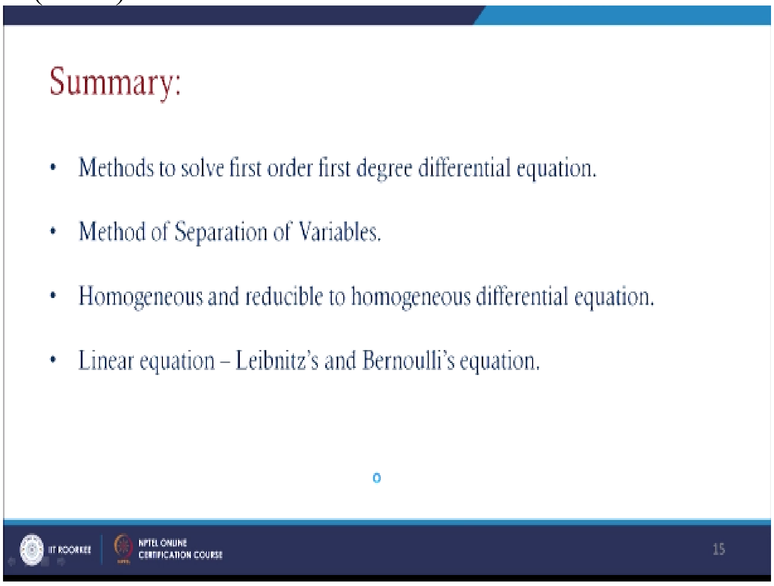
If you are said considering one example here d y / d x y= x^3 y6 this equation is either in the Leibnitz form and the Bernoulli's form here we have to reduce into the Bernoulli's form / divide the hole equation by x and then convert into Leibnitz form = y^{-5} here which means that we can just write this equation as d z/ d x = _ 5 $^{-6}$ d y / d x then this simplified linear equation or d z/ d x _ 5 z/x = _ $5x^2$, since all this variables it has been capture their directly you can just solve substitute.

(Refer Slide Time: (25:26)



And the integrating factor of last equation since it is just radius into leibnitz form here then if you just see here this is written as p (x y = q x), so that's why you can just write this one as $e^{-5/}$ x d x= e^{-5lnx} it is dividing factors and which gives you $1/x^5$ is integrating factor and the solution it can be written as Z x $(1/x^5)$ = {(_5x^2 $(1/x^5$) d x +c and the solution it will jut converted as y^{-5} x^{-5} = _5^{x-2} / _2 + c.

(Refer Slide Time: (26:08))



In this lecture we have discussed everything about the methods to solve the first order of degree differential equation method of separation of variables homogenous and reducible to homogeneous differential equation linear equation thank you for listening to this lecture.

FOR FURTHER DETAILS CONTACT
COORDINATOR, EDUCATIONAL TECHNOLOGY CELL
INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
ROORKEE- 247667
EMAIL: ETCELL@IITR.EMET.IN,ETCELL.IITRKE@GMAIL.COM
WEBSITE: WWW. NPTEL.IITM. AC. IN
ACKNOWLEDGEMENT
PROF.AJITH KUMR CHTURVEDI
DIRECTOR, IIT ROORKEE
NPTEL COORDINATOR
IIT ROORKEE
PROF.B K GANDHI
SUBJECT EXPERT
DR. AMEEYA KUMAR NAYAK
DEPARTMENT OF MATHEMATICS
IIT ROORKEE
PRODUCED BY
SARATH.K V
GRAPHICS
BINOY.V. P
PRODUCTION TEAM
MOHANRAJ.S
PANKAJ SAINI
WEB TEAM
DR.NIBEDITA BISOYI
NEETESH KUMAR
JITENDER KUMAR
VIVEK KUMAR

DHARMAVEER SINGH
GAURAV KUMAR
AN EDUCATIONAL TECHNOLOGY CELL
IIT ROORKEE PRODUCTION