INDIAN INSTITUTE OF TECHNOLOGY ROORKEE
NPTEL
NPTEL ONLINE CERTIFICATION COURSE
Mathematical Modeling:
Analysis and Applications
Lecture – 11
Introduction to Continuous Time Models
With
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Welcome to the lecture series in mathematical modeling analysis and applications. In the last lecture we have discussed discrete time modeling, and in this present lecture we will start about this continuous time models. So in the discrete time models, especially we have delt like finite difference methods that is especially called difference method we have used. And we will just discuss about what is a description advantages we have just to find in the, like discrete time models, and how we are just approaching towards the continuous time models that will discuss in this lecture.

So first we will just go for like how we can use the continuous time modeling?

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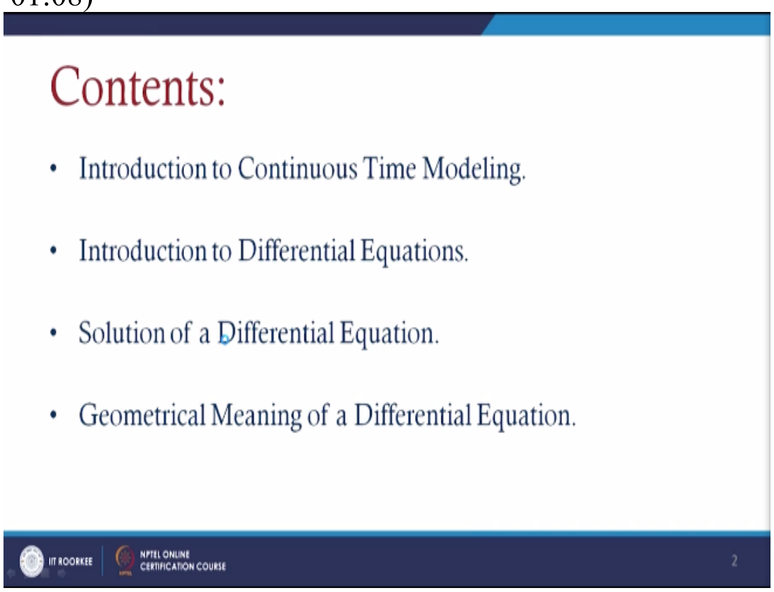

Then we will just go for the differential equations, so there itself we have just gone for the difference equations only. And then we will move towards the solution of differential equations and how we can just represent this differential equation in geometrical form that will also discuss here.

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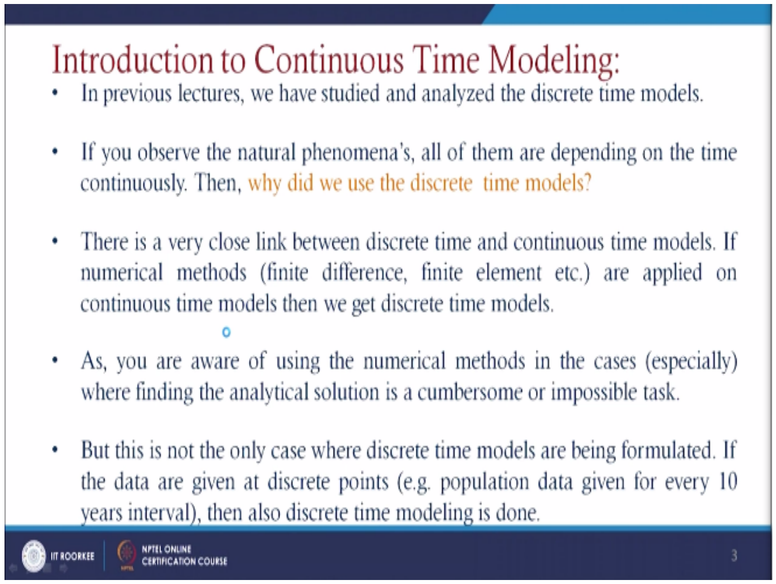

So first we will go far like, the continuous time modeling, so if you just see our previous lectures we have discussed there and analyzed this discrete time models. And if you observe the natural n's that is in natural phenomena's, all of them are depending on the time continuously. Since whenever a system starts it ends, so continuously the system proceeds in a phase manner. So if you will just consider like previous points then we can just move one by one and proceed and finally we can rise to the end points, so then why did we use the discrete time models?

That is also a question. So there is a very close link between discrete time and continuous time models, since sometimes if the points are like placed in a finite difference forms. That is equiv spaced points if it is placed, so it is useful or very professional to use this discrete time levels to joint it in a continuous form. So that's why we are just saying here there is a very close link between discrete time and continuous time models.

If sometimes you can just find that, if the numerical methods is applicable this means that, the solution in analytic form it is not possible, so already we have discussed that like linear equations or like quadratic equation it is easy to get this solutions in analytical form or exact solutions it can be possible. But if the points where this analytical solution will not be exist then we just go for this numerical method.

So that's why, so we have just write this statement here as, you are aware of using the numerical methods in the cases especially where finding the analytical solution is a cumbersome or impossible task, so many of this industrial problems are like engineering problems you can just find that does not exist any exact solution of this solution of this differential equations or different equations.

So mostly peoples are using numerical methods to get this solution of these differential difference equations, so if will just consider that where the discrete time models are been formulated and if the data are given at discrete points, suppose like the population levels if you just see the population counting in India. So usually like before 5 years people are predicting that this is the population level will be, this means that if suppose the population level is given at 1920, 1930, 1940, and 1950, we are just predicting that this will be the population level in 1950^{th}

year. So this is the project analysis, especially it is used using this discrete points, and then also this discrete time modeling is done in that sense, so in that sense if you just go for like further process.

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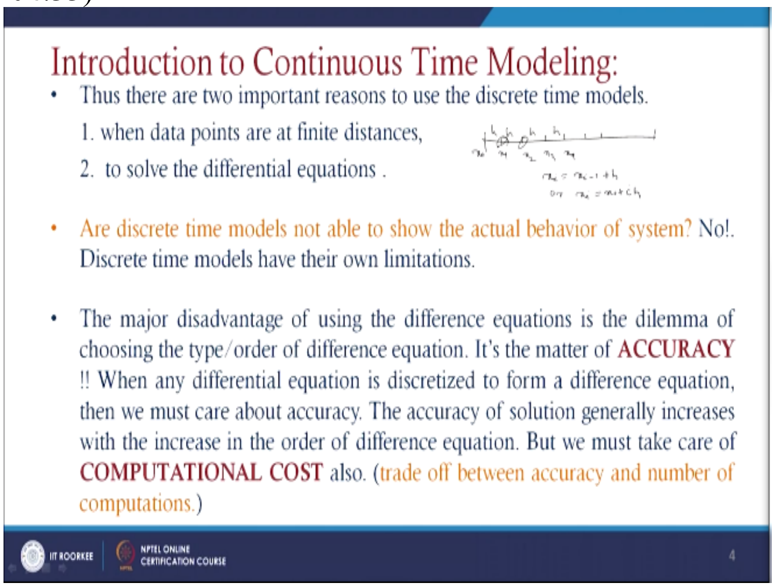

Then we will just go for this continuous modeling. So there are two important reasons to use the discrete time models, especially if you use accuracy. When data points are at finite distances, this means that we can just use this model when these points are like equiv-distanced. This means that suppose the points is starting at x0 and then x1 then x2, x3, x4 and all of placed at equal distances.

Then especially we can just denote like xis, xi minus 1 plus 8, or we can just write it as xi equals to x0 plus i8 there. So in that sense you will just see we can relate each of the following points make it or the process can be described in a like continuous steps. So second point is to solve the differential equations, so to solve the differential equation usually many difference schemes it is used to here, where all the points would be placed at equiv distanced, neither can we use this equiv distanced level or finite difference equation.

So the question is now that ARE, discrete time models not able to show the actual behavior of system? Especially the answer is exactly no, since the discrete time models having their own limitations. Since if you just consider like three different points which are just placed at equiv distanced but if the question is, ask her to find in between any points the values of that function. So if you will just use maybe there is existing error between this two points.

So whenever we are just approximating the function with this straight line here, then definitely we can just find gaps between this straight line and this function. So at that point you have to choose your limitation that the errors should be minimized here, so the major advantage of using this difference equation is the dilemma of choosing the type or the order of difference equation. It's the matter of accuracy. When any differential equation is discretized to form a difference equation, yes I have explained here.

So if you will just approximate any function with the straight lines at the difference points, we can just find that if the points are suppose in equiv distanced then we can just find their exist gap between these two points here, and especially if you just choose this discretized points also very close to each other then this error are difference between this function and this straight line it will be minimized but the error will exist there, this may cause like if it is like large scale problem it may cause a huge problem there, since small errors it will be added and it will just create a big problem there.

Especially practical example if you'll just consider like if a small error is introduced in like a row system suppose 100 scientist are working. So then this small error first introduced by one people there and then this error will be propagated to the 100 people there, so this error will be like increased in exponential way. Since in each of this process you can just find the error will introduced and this error will get increased.

So that's why we are just saying that discrete time model having own limitations. And if we will just go for an accuracy level especially we are just finding that this difference between this straight line and this I call or the function it is just exist in, always we will just go for how much accuracy it is providing this solution for us. So for that you will just consider like a differential equation is discretized to form a difference equation, and then exactly we have to care about the accuracy. And the accuracy of solution generally increases with the increase in the order of difference equation.

But we must take care of the computational cost since if we will just introduce this error to minimize this one we have to go for like several iterations or we have to use like several of routines to minimize this error there, so it will have like huge human cost it is required or it will take long time to get the accuracy of this glass of solutions. So if you will just go for this like to overcome this limitation of the discrete time models we need an evolution in formulation of a model.

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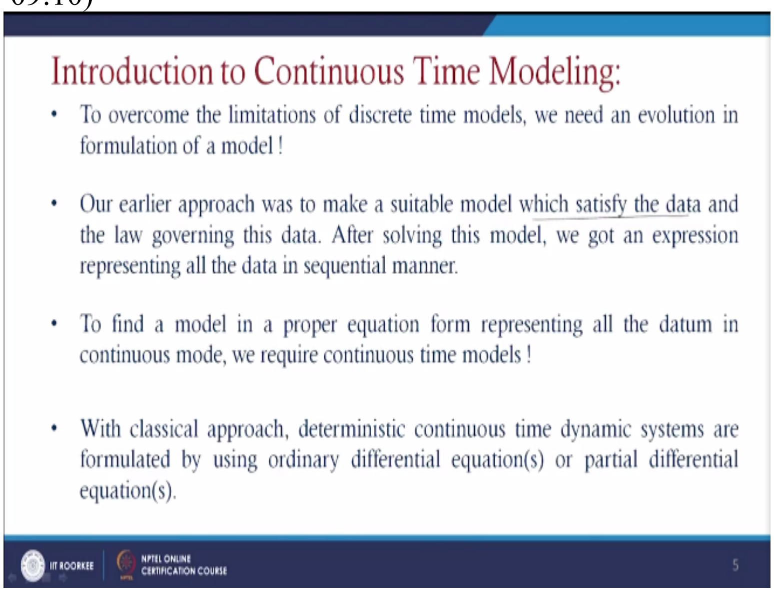

So our earlier approaches you'll just see, was to make a suitable model which satisfy the data and the law governing this data. This means that whatever this data points it will be a just received their, it should satisfy the system there itself. Or the natural law whatever it has been satisfied by the system, it should be satisfied there. So after solving this model we can often an expression representing all the data in sequential manner.

So first we will just formulate a mathematical model then we just generate the data and this data should satisfy the natural law whatever it is satisfied by the system and then we can just put this data in a sequence manner to represent it in a complete face, so to find the model in proper equation form representing all the data sets in continuous mode.

So if it is not in continuous mode then we can distinguish how the system is behaving like a time process, so that's why we require this continuous time models. With the classical approach, especially deterministic continuous time dynamics systems are formulated by using ordinary differential equations or partial differential equations.

So especially you just see any natural system there involves their differential equation or the partial differential equation on since one of this variable it tends according to the other variable there. Suppose if you are playing or you are purchasing something, so according to that you can just say that, one variable gets tends according to the other variable and this changes is occurring at a different levels so these different levels we are just setting as points there, like if the system is gets changed like equal intervolves there.

Then we can just use this discrete time modeling and if you all just join all these points in like sequence form then we will have a continuous model there. All this time modeling, we can just use that s time process then we can just establish this relationship with each of the points in a continuous face that s time process or s1 of this variable dependent on other. And that's why depending on that we can just say that it can just formulate the relationship between the dependent variable with independent variable in a complete sense. And if it involves then we can just say it is a differential equation or a partial differential equation.

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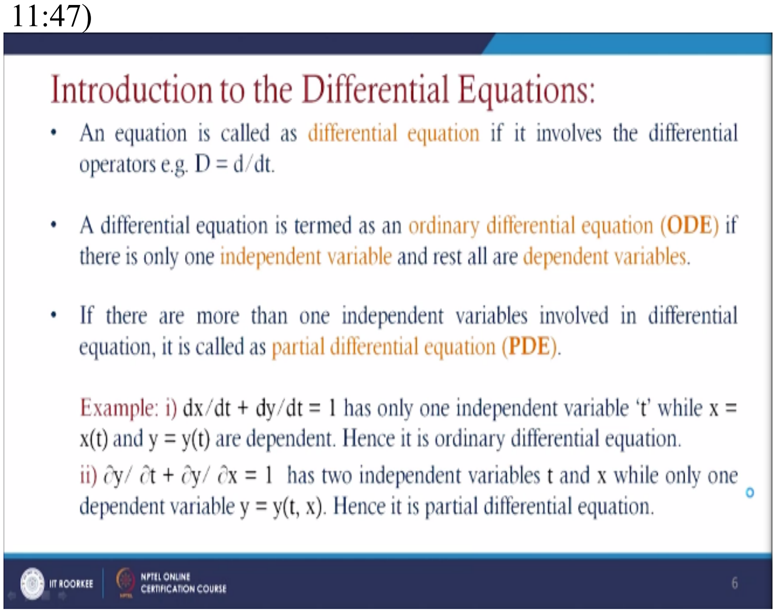

So if you just go for this introduction to the differential equations, an equation is called a differential equation if it involves the differential operators. So especially this differential operator is denoted as capital D, which is represented either d by DT, dx or Dy, so any of this variables you can just involve here to get the equation as in differential form. But this capital D is always generated as a differential operator and d by d of some factors, which can be expressed as capitalty.

And secondly a differential equation is termed as an ordinary differential equation or especially called as ODE, if there is only one independent variable involve in that equation, and rest all are dependent variables. So just if you'll just see here, like suppose a equation can written as dy by dx, since you are just writing d is a operator here which is operated on y which can written as d by dy dx here, and which can be written as, suppose y or we can just write dy by dx equals to x suppose.

And then we can say it as an ordinal differential equation, since we will have one dependent variable here, and one independent variable here. And especially this is called ordinal differential equation, if there is only one independent variable it will just say here so x is the independent variable and y is the dependent variable here. And here also the same thing y is a dependent variable x is an independent variable.

So in continuous pairs if you will just write this one then we can just write this one as dx by dt plus dy by dt this equals to suppose 1 here, if you all just see here, this dependent variable is like x here y here but the independent variable if you'll just see here, only t is here the independent variable. And if there are more than one independent variable involved in a differential equation, then we can just say it is a partial differential equation, more than one independent variable means you can just say that, suppose partial difference equation is written as the deeloi by del t plus del y by del x is equals to one here.

If you'll just see here t and x both are independent variables and the dependent variables are like y here, so that's why it is called a partial differential equation. And especially if you'll just see here for this ordinal differential equation the xy, dt plus dy dt is equals one, has only one independent variable t, while x is represented as a x of t and y is represented as y of t are dependent variables.

Since t is the independent, so x must be dependent on t to get the exchanged with respective t there. Similarly if you'll just see here, t is independent variable and y will get change with respective t there, that's why y is called the dependent variable, and t is an independent variable.

So similarly here also in the partial differential equation, if you'll just see here, del y by del t plus del y by del x is equals to 1 here, as two independent variables t and x, only one dependent variable y which will gets hence with respective to t also with respective to x here, since t and x are like independent variable, they will not get changed.

But this y factor that will get changed in respective to t also y also gets changed with respective x there, so that's why we writing here only one dependent variable y which is dependent on t and x both. Hence it is a partial differential equation.

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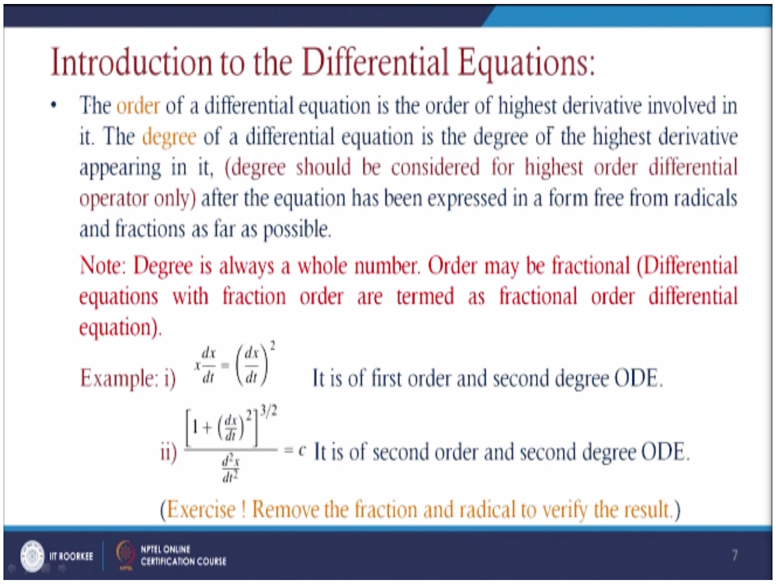

So next we will just go far like order of differential equation this is very important aspect to solve the differential equations. The order of a differential equation is the order of highest derivative involved in it. And the degree of a differential equation is the degree of the highest derivative appearing in it. so degree should be considered for the highest order differential operators only, since sometimes people get confused that what is a, if suppose some of the terms that have involved like highest order and some have involved like lowest order but highest degrees.

So they are confused to that what is the order and degree of that differential equation. So you have to keep it in your mind that degrees would be considered for the highest order differential of order only. So the operator which has the highest order the degree can be considered for the completive function equation and after the equation is expressed in a form free form from radicals and fractions form free from radicals and fractions as far as possible.

So if some fractional numbers are here so it is difficult to say what is the order or degree of those differential equations. And the degrees is always a whole number, it can add way fractional number. But order maybe fractional since nowadays peoples are using like homo-toppy analysis method or raw matter fundamental method, or like in differential transformation method. So this involves the differential equations with fractional order are termed as fractional order differential equations.

Especially for this we have just consider certain examples here, that is x dx by DT this equals to dx by DT whole square. If you just see here it is a first order and second degree ODE here, since if you just see the order of this differential equation termed as one here, and order of this equation is of two here. But if you just see here it is called to be first order second degree, degree means if you just see here the degree is 2 here, and order here is one here, one here.

So that’s why we are just saying that this will take the highest degree the termed involved as of highest order here. So that’s why this is called first order and second degree ODE here. And if you just go for like calculation of the order and degree of a this differential term here, or the differential equation here, when it is of second order and second degree ODE here, so it can be like as we have discussed here degrees always a whole number and first we have to make this as radical free so that's why if you just made this square of, for like numerator and denominator, we can just find this one as like square here and this will be like whole cube here so which can be radical free and we can say that since it is second order and second degree is always present there so that's why it is called as second order and second degree of ODE.

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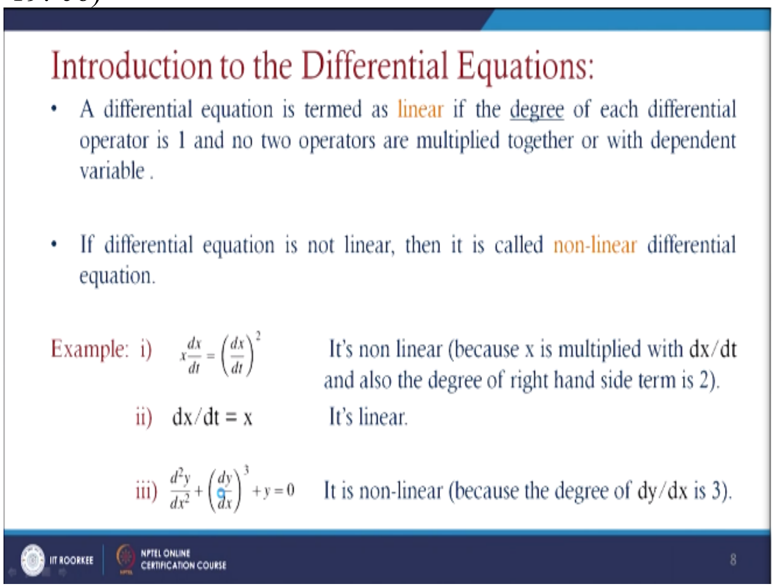

So then we will just go for like differential equations, is termed as linear if the degree of each differential or degree of each differential operator is 1 and no two operators are multiplied together or with dependent variable. So especially we can just write this basic definition of linearity is that the degrees should always 1 there. And no product of definite variable and the derivatives should be present there, and sometimes the differential equation is called non linear if it is non linear differential equation means the degree should be one and it should be free from the product of dependent variable and derivatives then we can just say that it is a linear equation.

So the two like explain this factors if you just write the differential equation as x dx by DT this equals to dx by DT whole square. So if you see here x is multiplied with dx by DT here and also the degree of this dx by DT it is 2 there, so that's why we can just say that it is non linear equation. But if you just see here dx by DT is equals to x here, so its degree is 1 here, and x power x is not multiplied with any of researchers so that's why it is called linear equation.

And if you say this third equation here d square y by dx square plus Dy by dx whole cube plus y is equal to zero. It is non linear because the degree of this Dy by dx term is 3 here. So that's why it is a non linear equation.

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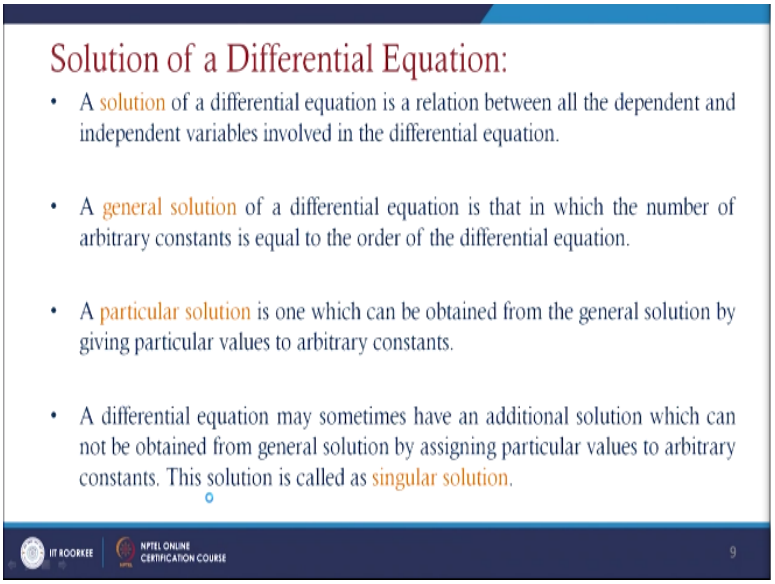

So further if you just proceed for this solution of differential equation here, a solution of a differential equation is nothing but a relation between all the dependent and independent variables involved in the differential equation. This means that obviously whatever the solution it will just come, if we will just put in differential equation then it will just provide solution there. Or it should b e satisfied this differential equation there.

So especially whenever we are just obtaining this solution for a differential equation we are just saying there is a general solution, there is a particular solution is existing for the differential equations, why it is existing? That since whenever we just approaching towards the solution of differential equation it always involve certain constants. And if the constant is involving there, then a generalize solution we can just obtain from this differential equation.

So that's why the basic definition of this general solution is that, a general solution of a differential equation is that in which the number of arbitrary constants is equal to the order of the differential equations. That means whatever this order of this differential equation the same number of constants will appear in the solution itself.

So particularly we can say that, a particular solution is existing for a differential equation or a particular solution can be often from the general solution by giving particular values to the arbitrary constants, some boundary condition is prescribed or if we are using certain methods to find a particular solution from the general solution then we can just get it by choosing this arbitrary constant values are we can just determine this arbitrary constant values to get the particular exhibition.

Since arbitrarily if you just see constant values can be changed according to the solution process, so that's why we are just saying it as a particular solution there. This means that, for the particular values for this constant this solution will satisfy the differential equation. So further if you just go for like differential equation may sometimes have an additional solution which cannot be often from general solution by assigning a particular value to arbitrary constant this solution is called singular solution.

Sometimes you can just find the regular solutions or singular solutions, if some dividend or multiplied their where this analyticity preserves that point in that level, then if the dividend or

multiplied with the differential operator then we can just find this singular solution there, so that's why we are just saying that sometimes this solution only exist but an additional solution can be added with the general solution there, but that can be often using the particular values to the arbitrary constants. And this is especially called singular solution.
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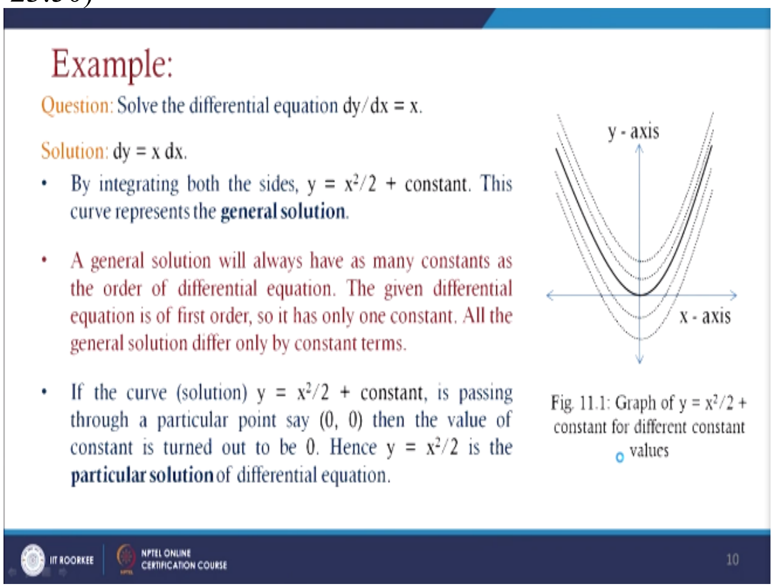

And if it just go for like a geometrical aspects of this solution of differential equation, so we have just considered one simple example here, that is solve the differential equation by dy by dx is equals to x here. So it is just integrate this like Dy term, and it is just multiply here dx term in the right hand side. Then we can just find the solution as y as x square by two plus constant there and if you just plot this solution then we can just often general solutions.

If you see for a different constant value we will have like different cost here. And that's why a general solution will always have as many constants as the order of differential equation. And the given differential equation is of the first order, so it has only one constant and all the general solution differ only by constant terms.

So whatever this constant value, you can just choose arbitrarily the solution will like shift their positions according to that one. Since it is a linear equation only, so that's why we are just adding one constant and this constant will shift the positions of this like a horse one point to the other point, and the trained of the graph remains same.

And if you just go like the curve of the solution like y equal to x square by 2 plus constant is passing through a particular point say suppose 0, 0 here, then the value of a constant is turned out to be 0 there, so whatever this constant values just give you, or it has been provided so accordingly this point will just getting a shift meant with the axis. Hence y equals to x square by 2 is the particular solution of the differential equation.

Since the particularly we are just assigning one values equal to zero there, so that's why we are obtaining this graph there so that is why it is called a particular solution and for constant of a different values, that means that the different constant values if you just provide we will have a different like shiftments of the graph. So if you just go like for any particular differential equation of first order and first degree,

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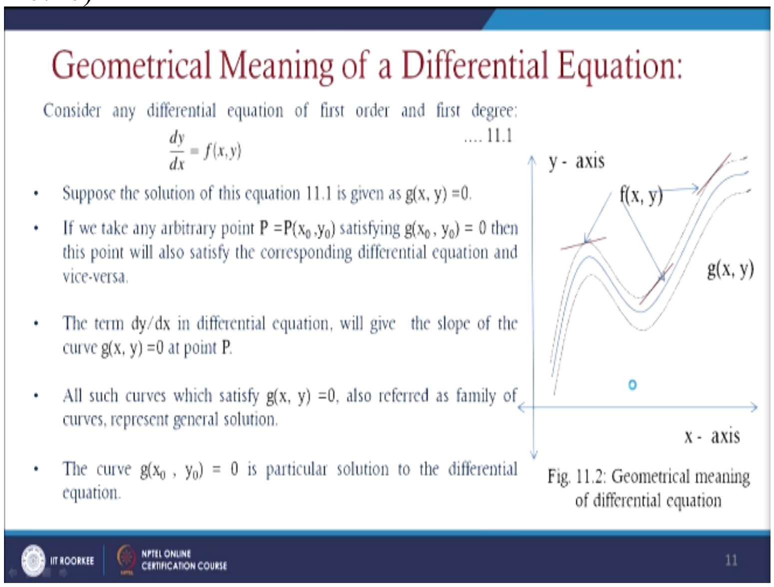

The generalized like representation of this equation is like Dy by dx equal to f of xy. Suppose the solution of this equation is given an as g of x y equals to zero. Since obviously you can just like a dy terms can be separated or a it can be taken right hand side to the left hand side and x terms can be separated to the right hand side then we can just obtain a solution by using different methods of that will discuss in the next lecture.

And the solution especially since this differential equation involves x and y terms both so that's why this solution will also includes both the variables there x and y. and if we take in arbitrary points, suppose p of x0, y0 satisfying g of x0,y0 this equals to zero. Then the point will also satisfy corresponding differential equation and vice-versa. And especially if you see here the term Dy by dx in differential equation will give the slope of the curve g of xy equal to zero at point p. and all such curves if you just see here, the turns ends are existing at this points if you just see here, and all such curves which satisfy g of xy equals to zero.

Since g of xy it is just satisfied or it is represented in this curve here as referred as familiar of cost represents general solution. Accordingly we have the steps on that shiftment occur according to the constant values and each of this like points we can have slope which is nothing but the representation of dy by dx term there and the curve g of x0, y0 equals to zero is particular solution to the differential equation that we have explained in the previous slide.

So this is the geometrical representation of this differential equation. Sometimes people are also representing this transect lines according to this differential operators. And furthermore also you can just find that manifolds or some other factors it can be associated with this differential forms.

So from this lecture we have,

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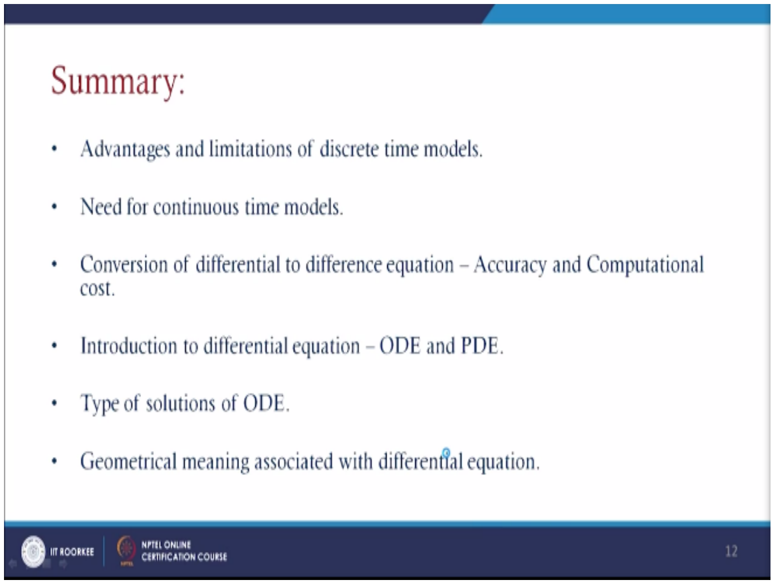

to find that the advantages and disadvantages of the discrete time models and then we have discussed about continuous time models, how we are just representing this continuous time models in the different forms, then we have discussed about conversion of differential to difference equation if its equation points are present, then equation stressed points are there when, we are just approximating function with a like a lion or some other effectors.

Then exactly if the state line is satisfying the exact solutions, then this curve is differing at many points which will not provide here accurate solutions. So that's why accuracy and computation cost we have to see when we are just applying this difference method for the differential equations. Then we have introduction the classification of a differential equation linearity and non linearity for this ODE, and type of solutions of ODE, and whatever the representation for the differential equation in a geometrical form, Thank you for listening this lecture.

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