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# Mathematical Modeling: Analysis and Applications

## Lecture-01
## Introduction to Mathematical Modeling

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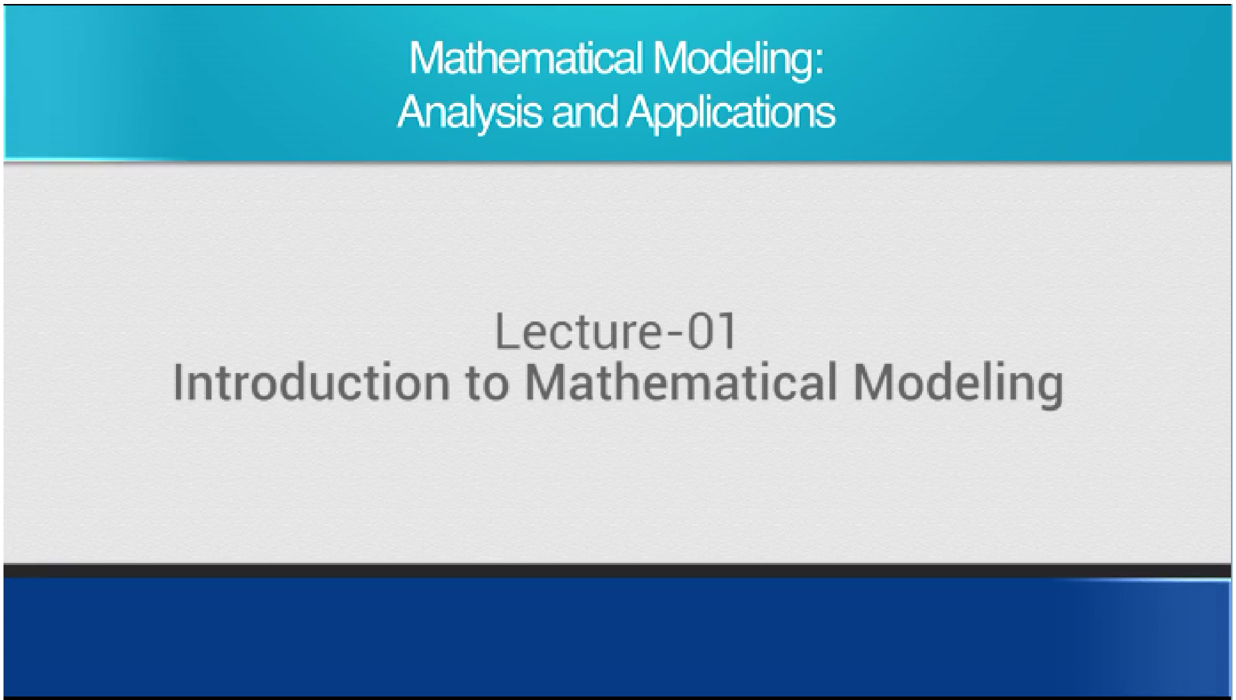
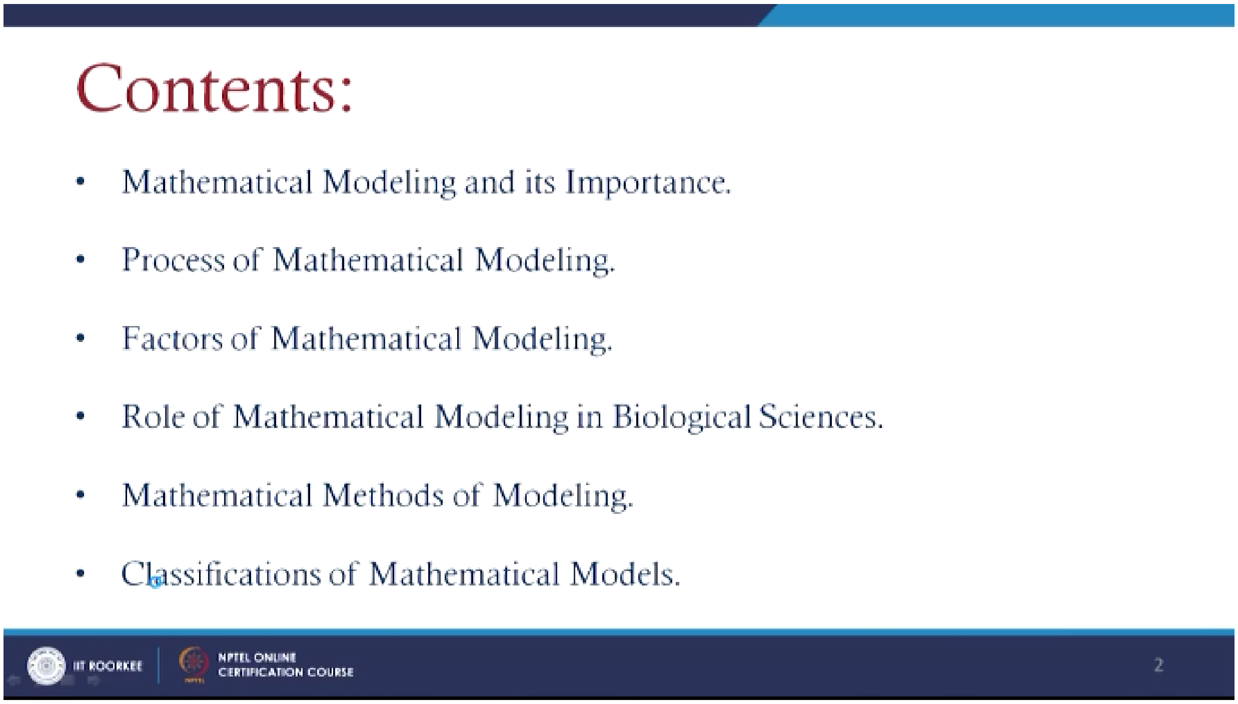




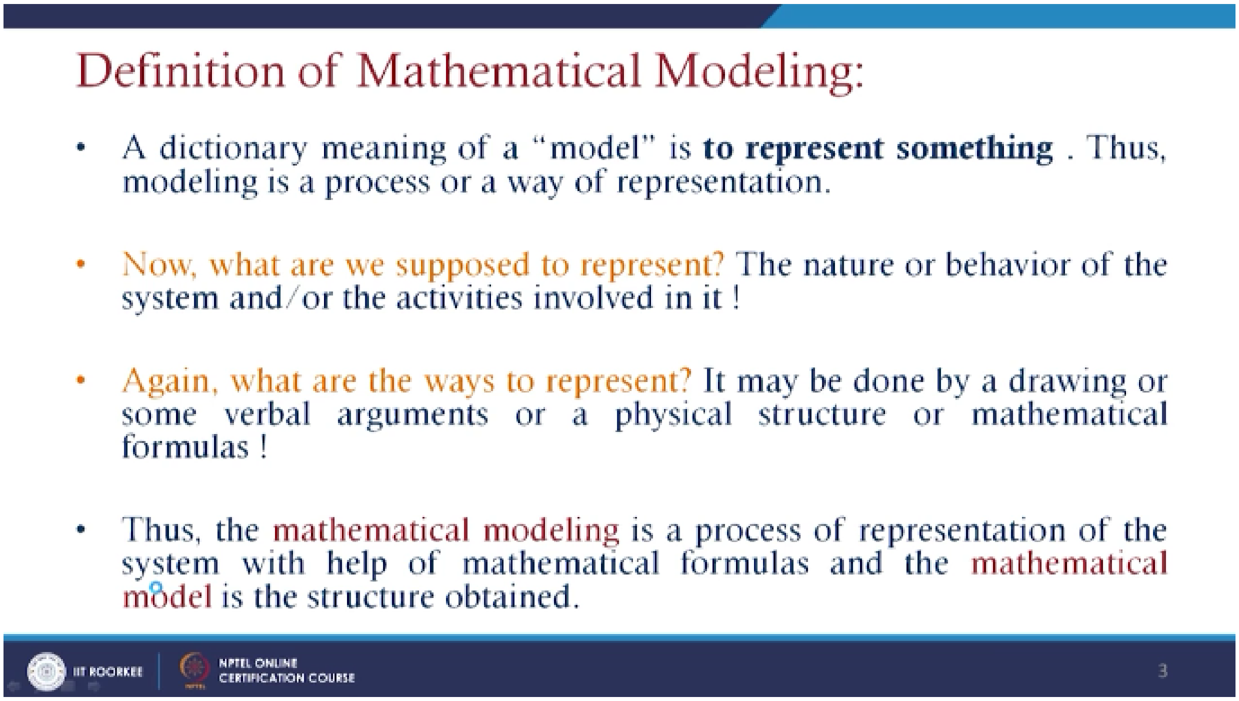
Welcome to the lecture series on Mathematical Modeling Analysis and Applications. In this lecture series we'll cover of the mathematical models that build with a like biological models or different class of real class problems, and especially how we can just formulate a mathematical model based on a real system that we will just discuss.




And if we will just go for this mathematical modeling, in the first step we can just find that how we can just formulate a mathematical model based on its importance. And in the second process we can just go for what is especially called a mathematical model, and how we can just develop a mathematical model based on this real system or a physical system or physical phenomena.

And third part we can just discuss that how the factors are affecting in a mathematical model, and how this mathematical modeling behaving in a biological system or in ecological system that we will discuss.

And then we will just go for this development of mathematical model, and based on its classifications, especially this classification may include like when we are just going for the mathematical modeling and biology, mathematical modeling and sociology, mathematical modeling and economics, so different class of mathematical models how we can just develop.

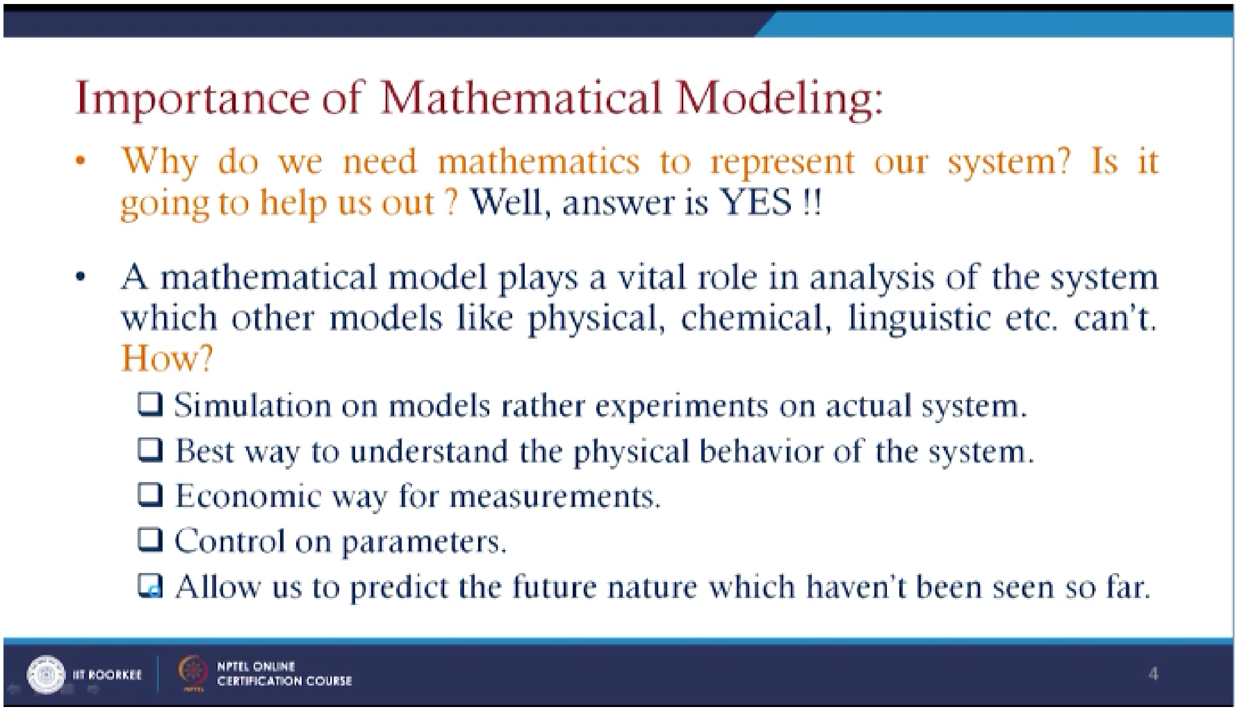



So if we will just go for this basic definition of mathematical modeling, so the abstract model that we can just find by considering a different behavior of the system, considering a mathematical relationship that is especially called a mathematical model, so this model means it is just to represent something, thus modeling it is a process or a way of representation of any system, so system means the collection of entities especially called as a system.

Now since we are just representing here, the nature or behavior of the system or the activities involving in it then if we supposed to represent all this natural systems or like natures or behaviors in a proper form then we can just get a mathematical model there.

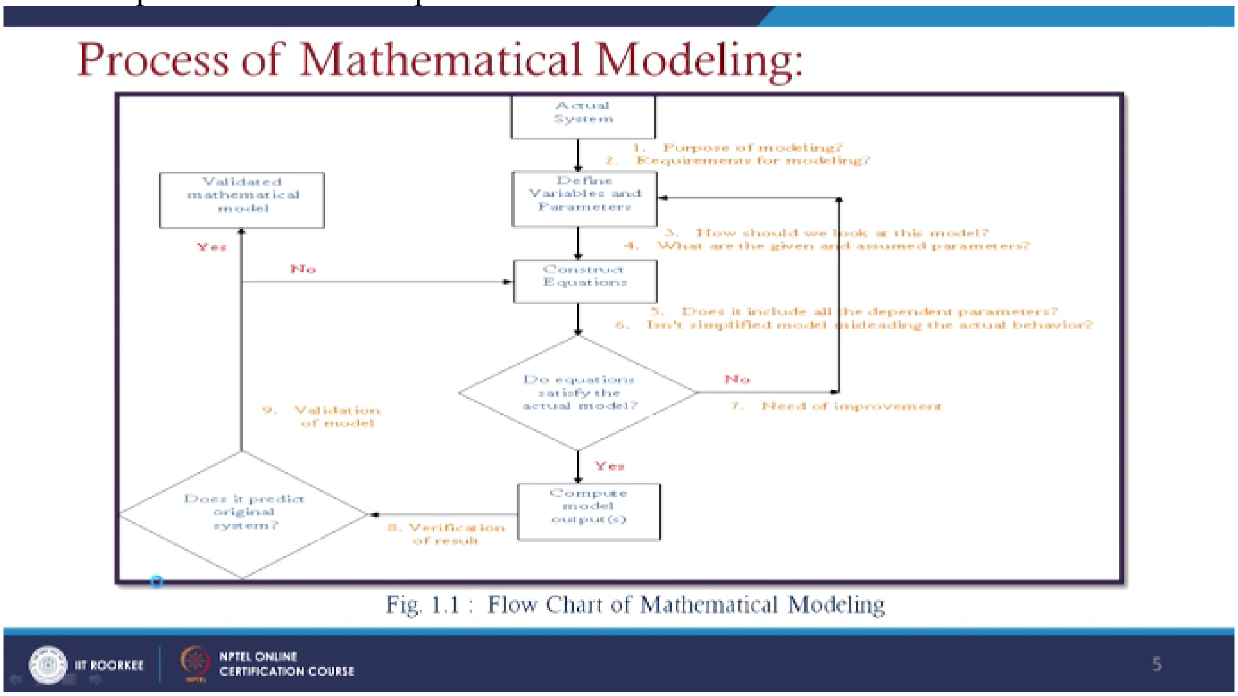
Again what are the ways to represent all this? Whether we are just representing in a graphical sense or in a picturized form or in a mathematical model, so this mathematical model how it is differing from all other systems, especially if you'll just see, you're just constructing something or you are just like representing a visual obstruct suppose, so that is especially it is not a represented in mathematical form, but that especially we can just say that are also models.

So if it may be done by drawings some verbal arguments or physical structure or in mathematical formulas we can just represent the systems. Thus the mathematical model is a process of representation of the system with help of mathematical formulas and the mathematical model is the structure to be obtained. So especially if you will just establish this relationship with this different mathematical formulations, either it is in the form of algebra or in the form of analysis, or it is in the biological sense or ecological system sense then we can just say we have a mathematical model there.




So if we will just go for this importance of mathematical modeling, why we do need mathematics to represent our system, is it going to help us out? Well, the answer is yes, since the mathematical model plays a vital role in analysis of the system which other models like physical, chemical, linguistic etcetera, it cannot do, since if you'll have a like a system then the system can be divided into two types, one you can just say it is abstract model, another one it is like a real experimental model, so in the abstract model we can just simulate or we can just try to find this solution in two forms, either we can have a like physical model, another one we can just say it is a mathematical model, and if we will have a mathematical model then we can just find the solution of the system in two forms, so that is either analytical solution or a simulated result. If we will have a simulated result then we can just compare this simulated result with the actual system whether the system is preserving the actual behavior or nature of the system or not.

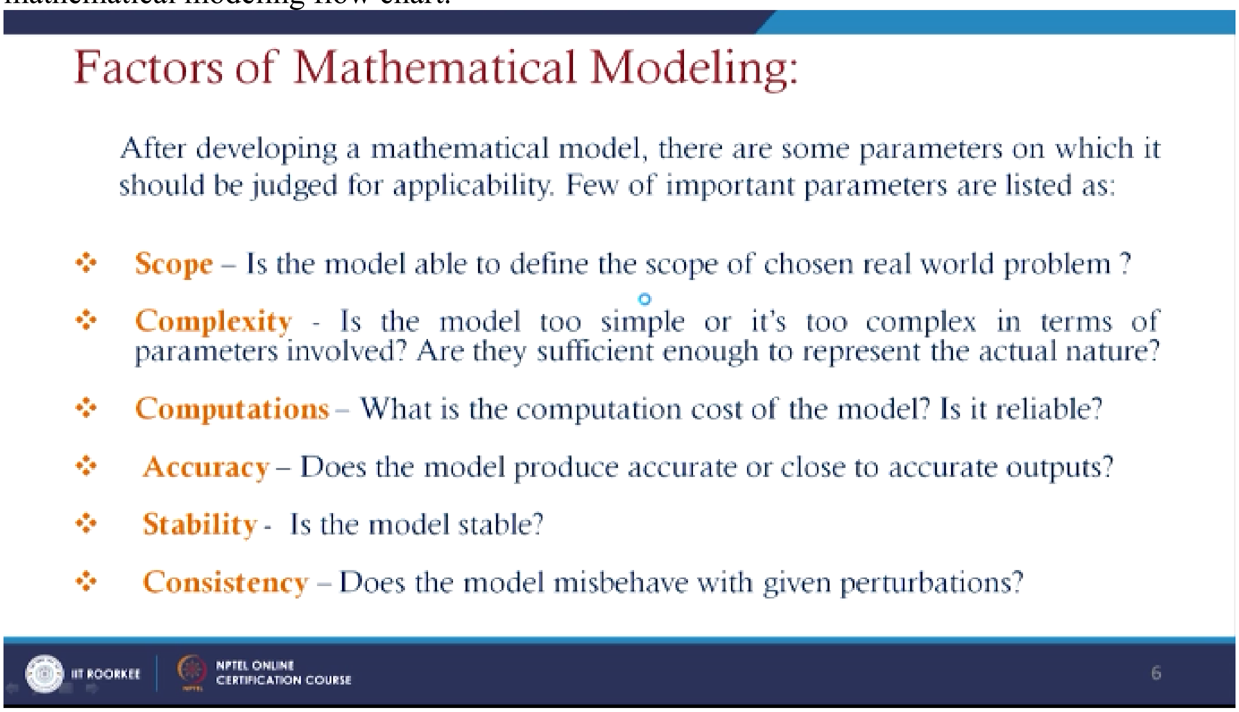
So if a mathematical model plays a vital role in the analysis of the system like suppose physical, chemical or linguistic systems which cannot do, then how we can do that one? So especially if we will just do the simulation on models rather experiments on actual system since already I have told you that system is divided into two types, one it is called like physical, or the experimental system, another one is like a abstract model or a model which can be just constructed. So if this models can be constructed so then we can just try to understand the actual or the physical behavior of the system in a concrete sense, so sometimes it is necessary to find the economic way for the measurements or control on parameters or allow us to predict that future nature which have not seen so far. Suppose if you'll just go for like a population level, so the total population size it depends on like the birth rate and death rate at that time level, so if you just construct like the mathematical model based on this population birth rate or death rate which is proportional to the total population size, so we have to consider the past data to verify the model, and based on that past data we can just to say that how the future model will be, so this can be predictor or it can be made through mathematical modeling only, so this is our last point that allow us to predict future nature which have not been seen so far.




So if you'll just go for this development of a mathematical model, so really we will have a actual system then we will just define certain variables like a, either you can just develop some mathematical relationships or you can just choose any type of parameters, and then we can just establish this logical relationships with this different variables by considering certain postulates or approximations. Then we will have certain class of equations, if this equations will satisfy or justify the accurate model then it will go for this like a computational model, so if you'll just compute this model data we will have outputs here, so then that outputs can be verified with the exact system or the actual system how it just behaving there, so sometimes we can just collect this data, we are just plotting that one or we are just putting in a different forms, so to show that is the computed data or like collected data or this calculated data is representing the real system in that sense or not, suppose water is flowing if we are just developing certain models, then we are just solving that one, finally we are just getting like certain dot points or like certain number of streamline points, so if we'll just plot the streamline by considering suppose different points, then exactly if it is just showing how this water is flowing there, then we can just say that like our model is valid in that sense.

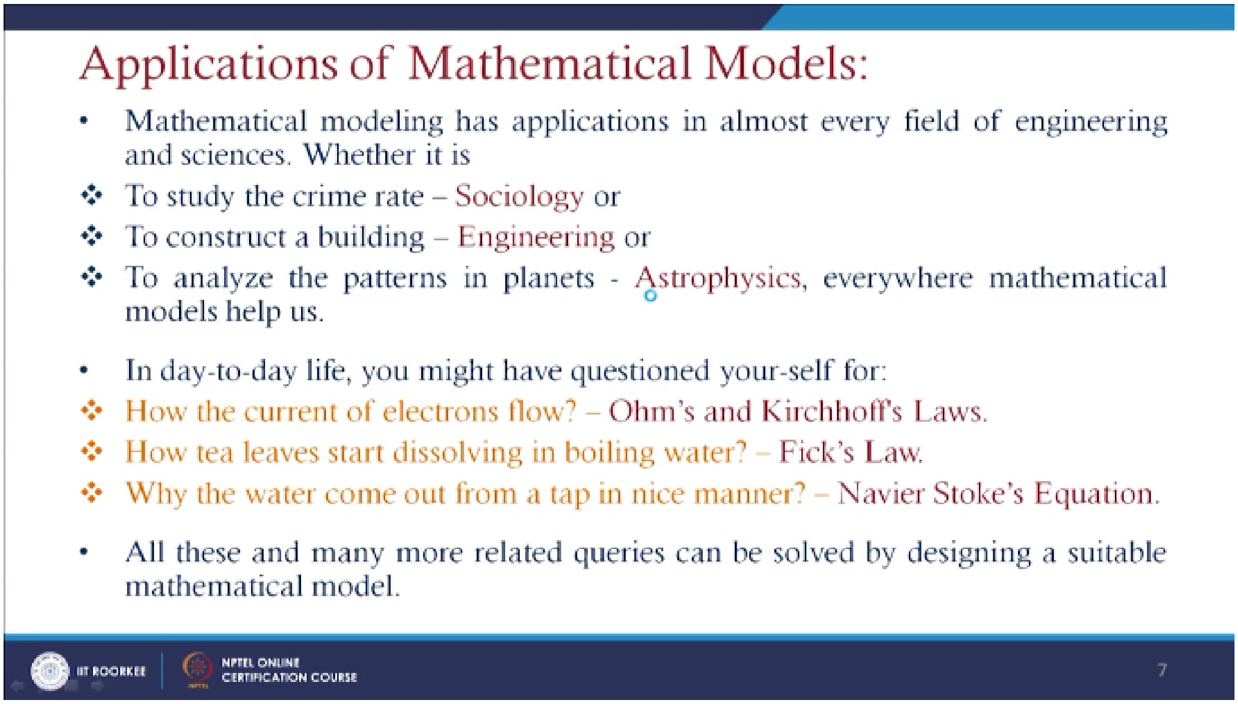
So if the verification is not proper than we will just go for like construct of the equations again, so certain parameters we can just interchange or certain variables we can just like adjust, then we will have different model or we will have a like a new corrector model there, so in that model sense we can just look at this model, what are given and assumed parameters we are just using there, so if that parameters will justify the actual system in a complete form then we can just say that all this defined parameters are correct to us, so sometimes if this simplified model misleading the actual system then we need to improve this system in again by choosing different variables or by choosing this different assumptions in a new level, once it will just satisfy this complete model or the original system then we can just say that it is a validated mathematical model, especially if you just see here so we are just keeping the actual system, then this actual system takes like a this defined variables or the parameters and once this parameters it is just assumed then this parameters are formulated in the form of a complete mathematical equations here.

And once we will have the set of equations so then we can just go for the actual model there, if it is the equations or exactly satisfying this computed model or the defined data there, then we are just going for the verifications of the result there. And does it predict the original system? If it is not satisfying then we are just going again the construct of equations that by assuming or just redefining these variables in different forms, so this flow chart is completely explaining the mathematical modeling flow chart.




So how a mathematical model is affected by different factors, that we will just discuss in this slide, so after developing a mathematical model there are certain parameters on which it should be judged for applicability, few of important parameters if you just see, so the scope of this model is the model is able to define the scope of the chosen real world problem, so this means that once you're just developing a mathematical model whether that fits properly with the exact system or the actual system or not, so then the complexity of the problem, is the model is too simple or it is too complex, since if it is a complex mathematical model or a complex equations are involved it is difficult to find the solution also sometimes, since lots of parameters it will get involved and it is not sufficient enough to represent the actual nature, so then if we are just formulating this model whether it is simple or complex then we will just go for the computations.

What is the computation cost of this model? Since sometimes maybe this model will take like one day computation or sometimes it may require a huge amount of devices to get the result there of, so that we have to keep it in mind that when we'll just go for a simplified model or a complex model, so once this result is just get it out then whether this model producing accurate or closed to a solution which should be produced by the actual system or not, that we have to see also.

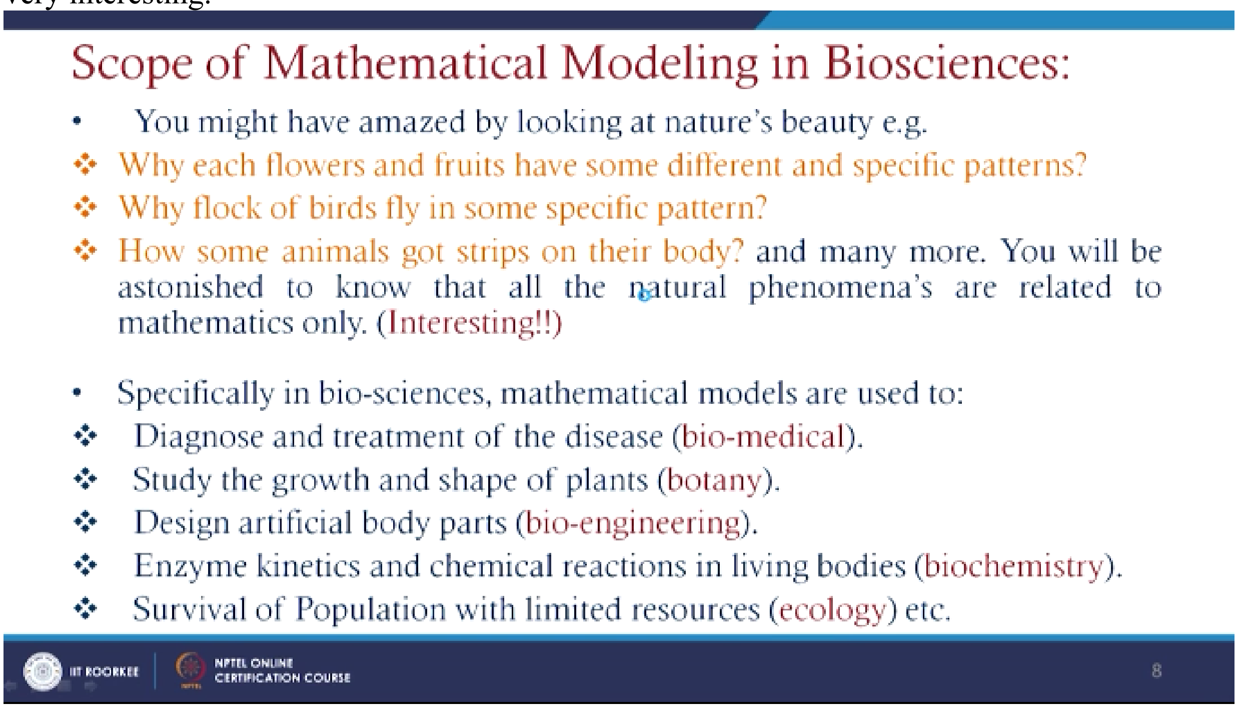



Sometimes the systems are just destabilized to you itself, this means that if we are just going towards the actual model then it will not go towards the exact solution there, it will just deform from the actual system, then we will have like instability condition, so we have to go for the stability of this model, that how the stable condition or how this parameters, if we are just defining in the system so it should provide a stable solution to us. Then we will just go for the consistency, if the system is not consistent then we will not have a fusible solution, so if we don't have a fusible solution then we cannot say that our system is fully equipped with this mathematical model, so then we'll just go for the applications of mathematical models, sometimes if the real system if we are just doing the experiments, many times it is not possible to do or to conduct the experiments in actual sense also, if suppose in a like paper mill, suppose if the temperature is go beyond like, go above like 1000 degree kelvin, so it is not easy to handle if there are small accidents are going on, so especially what people are doing is they are just formulating a mathematical model first, then they are just going for the experiments.

So in that sense where we can just use this mathematical models that we will just take care, so if you just see mathematical modeling has applications in almost every field of engineering and sciences, whether it is in a like, to control the crime rate in sociology or to construct a building, so especially if you just see fracture mechanics it is just, if you just see it is not a set of equations only, like stress components are there, strain components are there, different tangential factors it is just involved, that is nothing but the mathematical models only.

So if you just go for the analyzed patterns of planets suppose, in astrophysics so you can just fine that they require also a mathematical model to visualize all this like computations or like the moment of this planets in the space, in day to day life you might have question for yourself, how the current of electrons flow? So especially if you just see this always satisfy this ohms and Kirchhoff's laws, so that is nothing but the representation of some of this mathematical relationships with certain number of variables.

So how tea leaves start dissolving in boiling water? Fick's law of diffusion, why the water come out of the tap in a nice manner, Navier Stoke's equation, so if you just see Navier Stoke's equation it involves only the partial difference allegation with certain nonlinear terms are present. So all these are many more related queries can be solved by designing a suitable mathematical model, so if you just go for this scope of mathematical modeling in a ecological sense or in population balance modeling or in biological sciences, you might be amazed by looking at nature's beauty also sometimes, why each flowers and fruits have some different and specific patterns? You can just find that it follows a mathematical relationship there, why flock of birds fly in some specific pattern? How some animals got strips on their body? So always you can just find their establishers or relationship with the mathematical terms there, and so many things you can just find that everywhere the beauty of mathematics is present, you will be astonished to know that all the natural phenomena's are related to mathematics only, so this is very interesting.



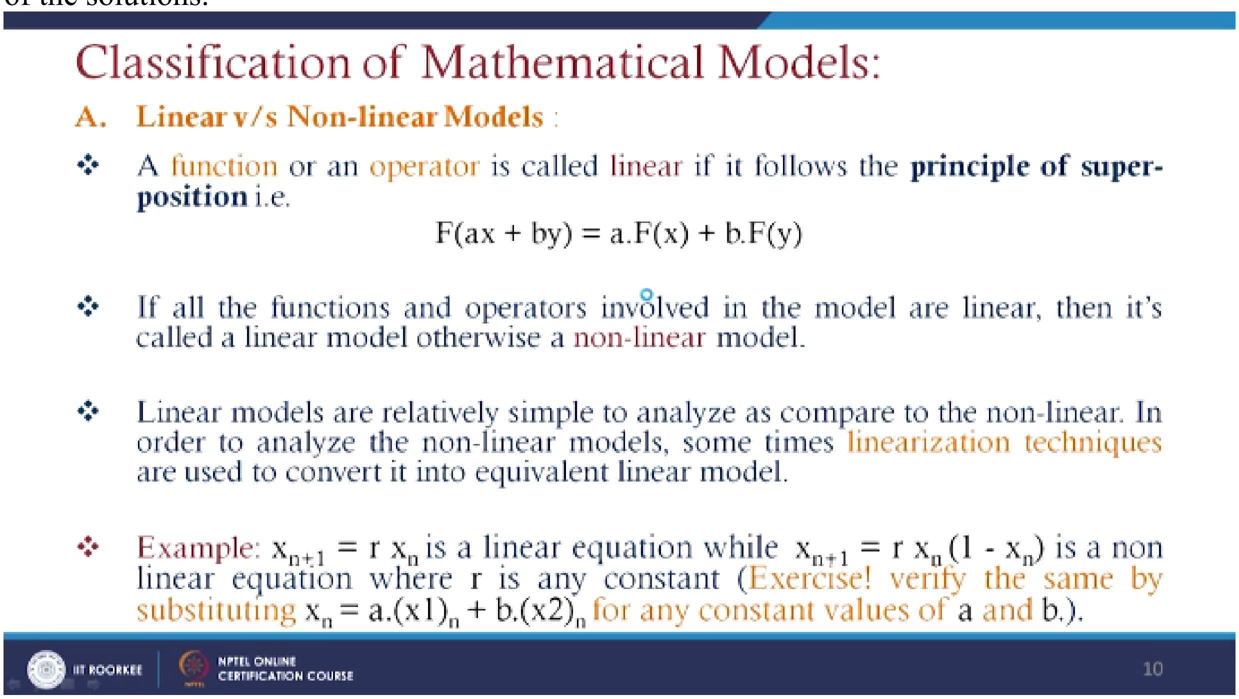

Specifically in a biosciences you can just find that mathematical models are used to diagnose and treatment of the diseases, study the growth and shape of plants, design artificial body parts, enzyme kinetics and chemical reactions in living bodies, then survival of population with limited resources, that is ecology.

So in mathematical modeling we'll just use different mathematical methods, especially any real world problem can be modeled by three basic methods, one it is called forward modeling that is we will have like inputs and we will just go for outputs, especially if you, you can just see for example like a triangle, so if P and V are known to you then S it can be computed by using P and V, and inverse modeling, so this means that you'll have this output and you will or you want to know the input there, especially if you just discuss here astrophysics then you can find that when this solar rays are coming and when this rays are imparting on the ground, from the ground data itself we are just going in a back form or in inverse modeling form to find that what composition of gases or radiation effect it is just acquiring in solar space.

Then in the third we will just go for mixed modeling, so in a biomedical imaging sense you can just find that some data they are using forward modeling sense or some of the data it is used for inverse modeling to get the complete solution, so especially that is used for many physical scenarios that is especially called mixed modeling.

Now any system if you just see we will have either it is in forward modeling sense or in the inverse modeling sense or the mixed modeling sense, so if you just go for this mathematical technique is used in the modeling are like define in a 5 forms here, that is first one is classical approach, so classical approaches are used for like differential, integral, integro-differential or difference equations. Second one is just stochastic approach, in the stochastic approaches we are using statistical methods, probabilistic models, or stochastic differential equations.

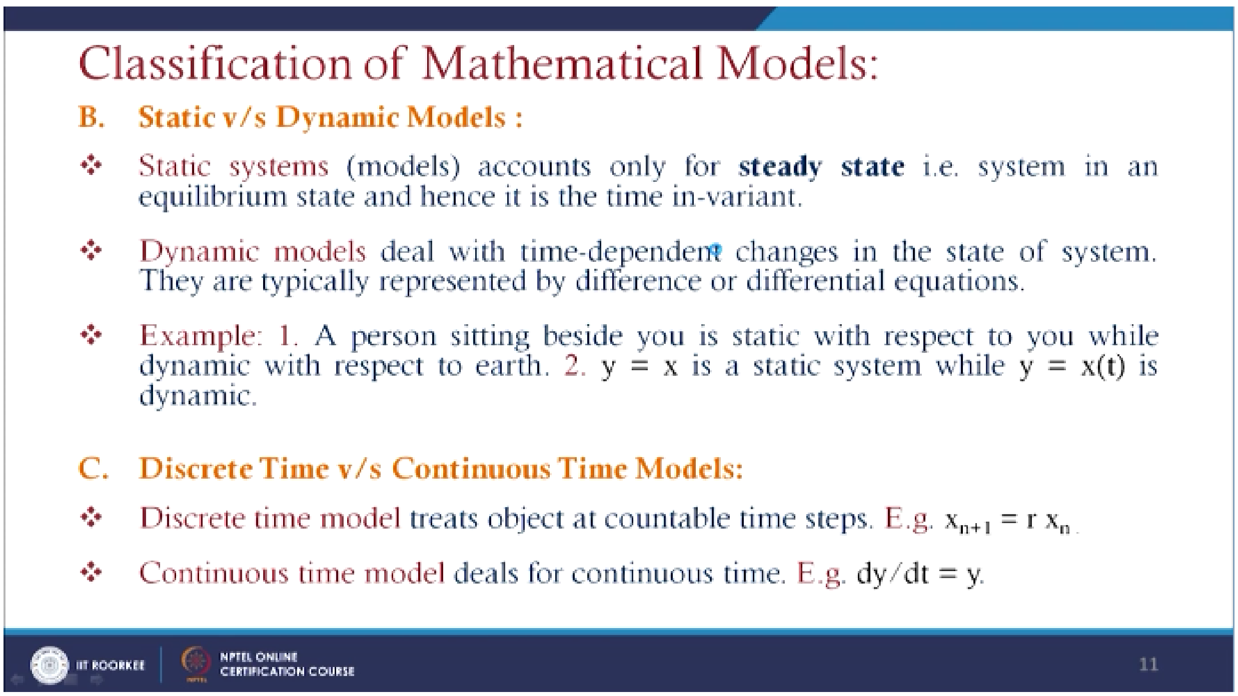
And if you just go for this third class of technique is that is computer simulation, in the computer simulation mostly we are just using analytical methods to find the exact combination of the solutions.




Fourth one is operation research, so usually the optimization technique is are control theory is used to find the solution process, and other methods like fuzzy logic, topology, differential geometry etcetera, it is used to find certain class of problems. So then we will just go for this classification of mathematical model, so especially if you just see this classification depends on whether we'll have a linear system or we'll have a nonlinear system, especially if you'll have linear system we can just say that it is a linear model then if we will have like nonlinear system then we can just say that it is a nonlinear model there, so if you just go for this like comparison of linear and nonlinear model we can just find that, any function or any operator it is said to be linear, if it follows the principle super position that is, especially if you just write any function which can be defined as a F(ax + by) which can be written as A into F(x) + B into F(y) where A and B are constants, and X, Y are the variables, then we will have a linear operator.

If all the functions and operators involved in the model are linear, then it is called a linear model otherwise it is a nonlinear model, so linear models are relatively simple to analyze as compare to the nonlinear, in order to analyze the nonlinear models sometimes linearization techniques are used to convert into a equivalent linear model, so the popular linearization model is Newton's linearization technique.

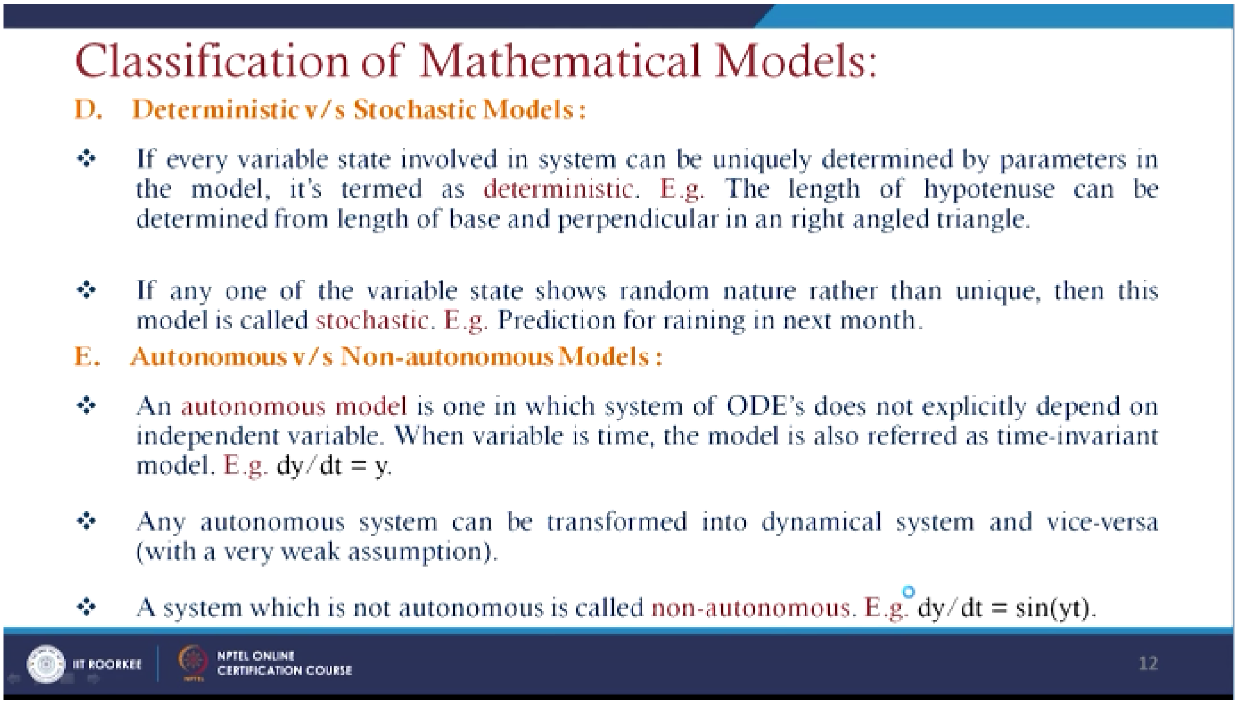
For examples suppose you'll have a equation like X n+1 = R Xn is a linear equation, while X n+1 = R Xn (1 – Xn) is a nonlinear equation where R is any constant, so you can have this exercise, verify the same by substituting Xn = A into (x1) N + B into (x2) N for any constant values of A and B.




So second classification of mathematical model deals with a static versus dynamic models, so static systems means it accounts only for steady state that is if a system is an equilibrium state and it is invariant with respect to time, we can just say the system is in static.

Dynamic model means with respect to time if the state of the system getting changed, they're typically represented by difference or differential equation is called a dynamic model. For example, if a person is sitting beside you is static with respect to you while dynamic with respect to earth suppose, since earth is rotating with respect to time.

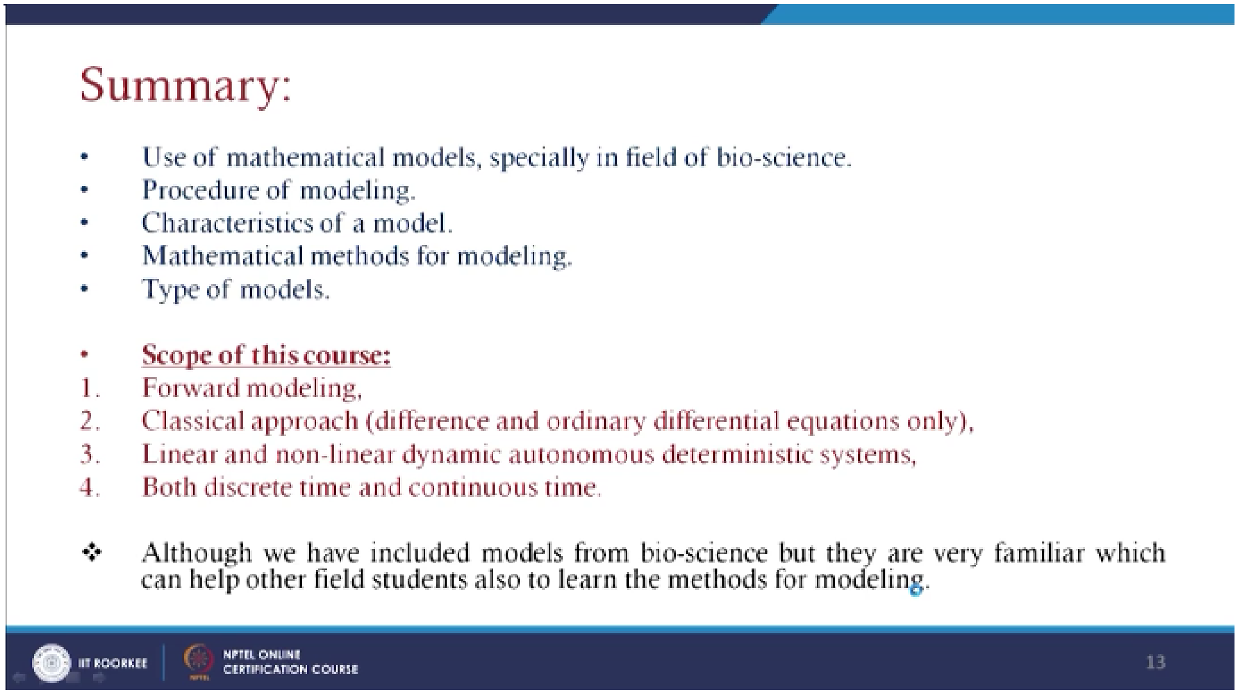
Second example if suppose you are just considering Y = X is a static system, while Y = X(t) where X is a function of time is a dynamic system. So third classification of mathematical model deals with discrete time versus continuous time models, discrete time model means it, we can just treats the object at countable time steps, that is especially X n+1 = suppose R of Xn here, and continuous time model deals for continuous time that is dy/dt this equal to y. If you just see this simple differential equation you can just find that Y get sense with respect to T here, so that's why continuously it got sense so that's why we are just representing in this sense here.




Then we will just go for like deterministic and stochastic models, if every variable of the state involve in a system can be uniquely determined by parameters in the model it is termed as deterministic, suppose the length of hypotenuse can be determined from length of base and perpendicular in an right angle triangle, already I have explained this one for this forward modeling. And second, stochastic models means if anyone of the state variable shows random nature rather than unique, then this model is called stochastic model, that is especially prediction for raining in next month, so if you just like expect the data what it will happen then we can just say that it is in a stochastic sense.

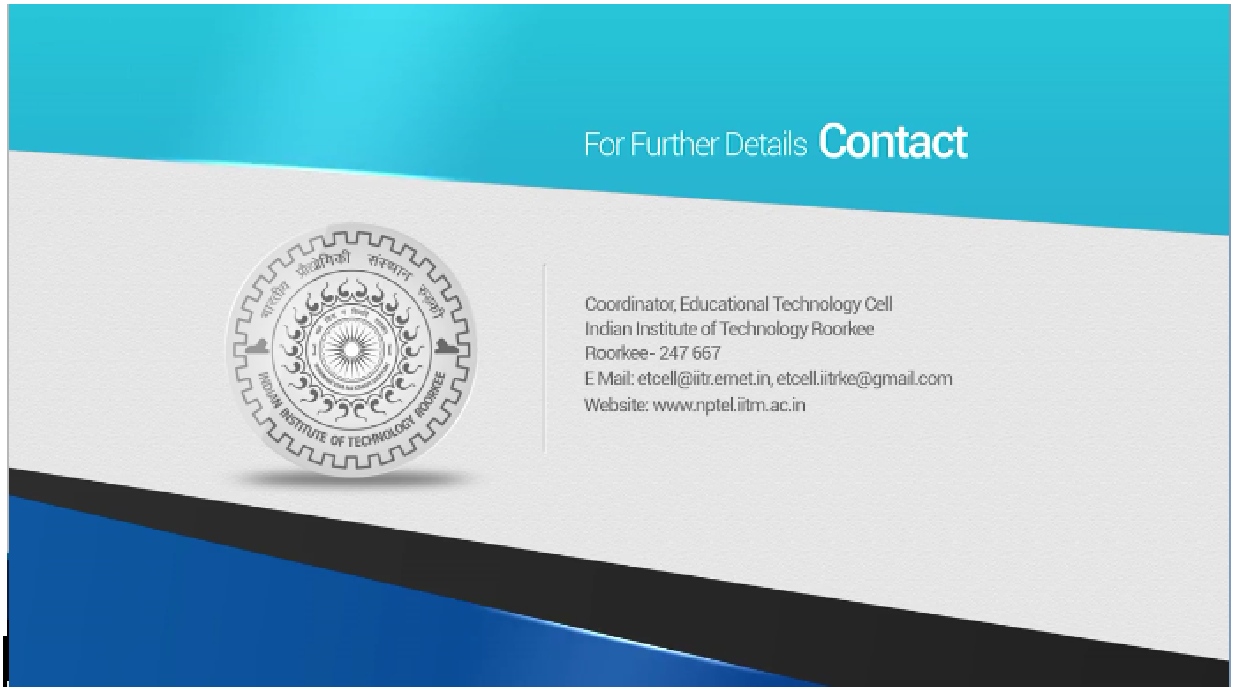
Then we will have like autonomous versus non-autonomous models, an autonomous model is one in which system of ordinal differential equations does not explicitly depend on independent variables, when variable is time the model is also referred as time invariant model that is dy/dt = y suppose. And any autonomous system can be transformed into dynamical system and vise-versa with a very weak assumption especially, and a system which is not autonomous is called an non-autonomous system, so especially if you just consider this differential equation that has a dy/dt this equals to sin y(t) suppose, so with respect to time so both the sides it got sensed so we can just say the system is non-autonomous there.

So finally we are just concluding based on this basic definitions and classifications of mathematical model here, if we'll just use this mathematical models in different fields of like engineering or bio-sciences, we can have like different models, and first we can just develop this model then we can just define some of this approximations with respect to this variables, and sometimes we can just define this logical operations to combine all this mathematical variables, then we will have proper model.




So how we can just formulate a mathematical model, that is procedure of modeling, then characteristics of model if you'll have a mathematical model how we can just find different characteristics or how we can just like predict different characteristics from this model, then mathematical models for modeling, so different models can be used and then the type of models, so different class of models we have just to discuss till now.

So scope of this lecture especially forward modeling classical approach, linear or nonlinear dynamic autonomous or deterministic systems, both discrete time and continuous time can be used for the mathematical modeling. Although we have included models from bio-science but there are very familiar which can help other field students also to learn the methods for modeling. Thank you for listen this lecture.

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