Matrix Analysis with Applications
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Lecture - 09
Linear Transformations

Hello friends. Welcome to lecture series on Matrix Analysis with Applications. So, in the last few lectures we have seen that what vector spaces subspaces are, and what are their basic properties. We have also seen, we have also studied a basis and dimension of vector spaces and subspaces. Now this lecture basically deals with linear transformation, what linear transformation is and how we can find out a linear transformation from V to W.

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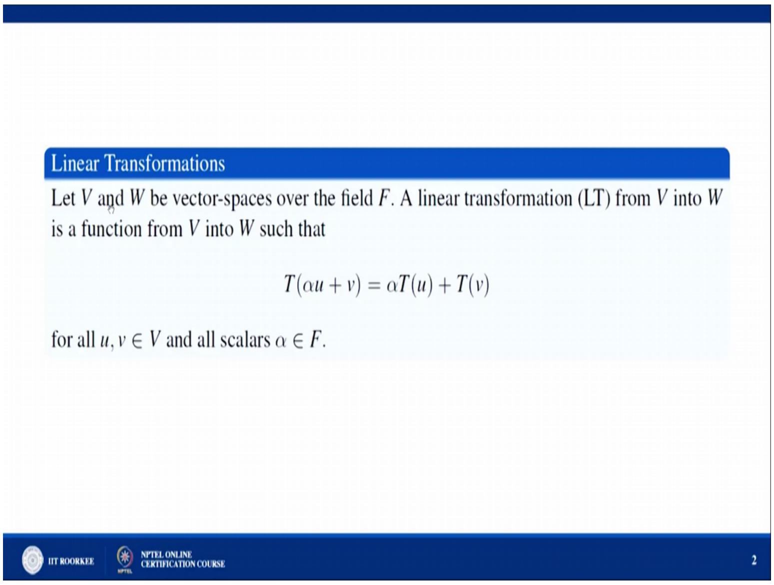

So, let us see now what linear transformation is you see let V and W be 2 vector spaces over the field F. A linear transformation which is also say LT. From V into W is a function, from V into W such that T of alpha u plus v is equal to alpha T u plus T v for every u v in v and for all scalars alpha in v.

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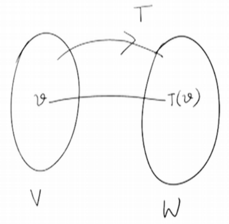
$T : V \to W$



$\forall\, v_1, v_2 \in V,\ \forall \alpha \in F$

$T(\alpha v_1 + v_2) = \alpha T(v_1) + T(v_2)$

OR

1. $T(v_1 + v_2) = T(v_1) + T(v_2)$
2. $T(\alpha v_1) = \alpha T(v_1)$

$\forall\, v_1, v_2 \in V,\ \alpha \in F$

So, basically what linear transformation is basically, you see that T is a linear transformation from a vector space v to vector space W over the field F. If for every v 1 v 2 in V, and for every alpha in field T of alpha v 1 plus v 2 is equals to alpha times T of v 1 plus v 2. Now this alpha v 1 is nothing but a scalar multiplication of a alpha with a element of a vector space.

So, I am not putting dot here its understood that it is alpha dot v 1 ok. Similarly, this is this T v 1 is a element of W you see we have a we have vector space V we have vector space W and we define a linear transformation T from V to W if we have any element say V here the image of this element in W is simply T V ok. So, this T V is nothing but a element of a vector space W this vector space W and this T v 2 is also element of vector space W.

So, this alpha into T of v 1 means a scalar multiplication of a alpha with element of W ok. So, I am not putting dot here its understood this is nothing but alpha dot T of V fine. So, basically if these property hold for every v 1 v 2 in V and for every scalar alpha in field that we say that T is a linear transformation. Now this property can also be stated as we can also say that T of v 1 plus v 2 is equals to T of v 1 plus T of v 2 first property, and second property is T of alpha v 1 is equals to alpha of T of v 1 for all v 1 v 2 in V and alpha belongs to field ok. So, we club these 2 property here in this step ok.

So, we can also state this single property as these 2 properties. So, if a if a function T from V to W satisfy these 2 property then we say that T is a linear map or a linear transformation. Now, let us discuss few examples of linear transformation the first example is we have considered T.

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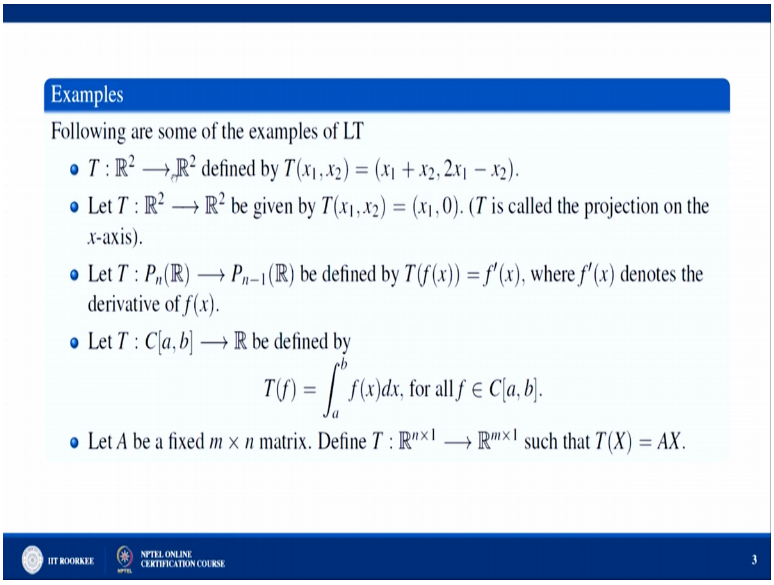

From vector space R 2 to a vector space R 2 which is defined by T of x 1 x 2 as x 1 plus x 2 to x 1 minus x 2.

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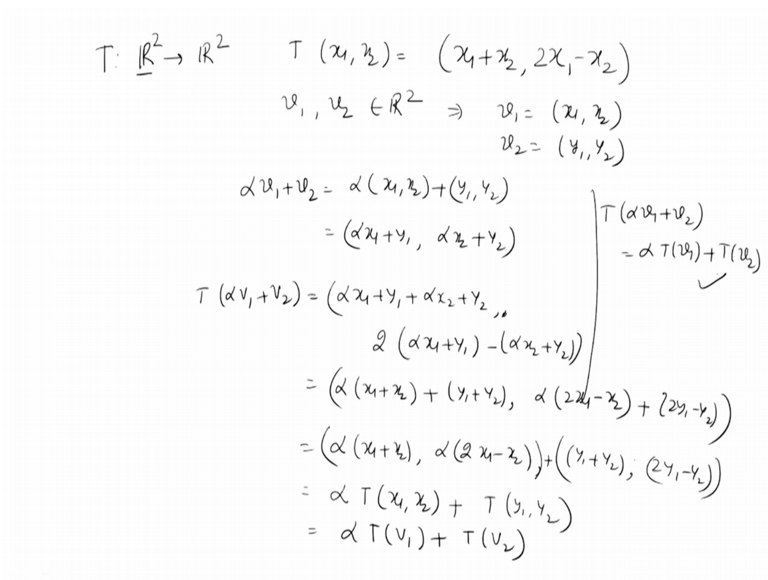

So, let us see, we defined T from R 2 to R 2 as T of x 1 x 2 is equal to x 1 plus x 2, and it is it is 2 x 1 minus x 2. Now suppose you want to show that this is a linear map or linear transformation from R 2 to R 2. So, how can we proceed for this, so let us consider v 1 v 2 as 2 elements in R 2 2 element of this vector space R 2? So, this implies v 1 is something say x 1 x 2 and x 2 is something y 1 y 2 and now we have to show that if it is a linear map. So, we have to show that T of alpha v 1 plus v 2 is equals to alpha times T v 1 plus T v 2 this we have to prove.

If it is a linear map, so how can you proceed for this let us find the T alpha v 1 plus v 2 first it is alpha times x 1 x 2 plus y 1 y 2. So, it is alpha x 1 plus y 1 and it is alpha x 2 plus y 2 ok. So, T of alpha v 1 plus v 2 will be equals to T of this element; that means, it is defined by this definition. So, it is simply T alpha x 1 plus y 1 plus alpha x 2 plus y 2 which is sum of these two element in the first component and 2 times of a component minus second component as a second element. So, it is it is 2 times 2 times x 1 which is alpha x 1 plus y 1 minus x 2 is alpha x 2 plus y 2.

Now, this can be written as alpha x alpha times x 1 plus x 2 ok, plus y 1 plus y 2 comma it is alpha times 2 x 1 minus x 2 plus 2 y 1 minus y 2 ok. Now this can be written as alpha times x 1 plus x 2 comma alpha 2 x 1 minus x 2 comma y 1 plus y 2 plus comma it is, it is 2 2 y 1 minus y 2 ok. Addition of these two if you add this with this you get the first component if you add this with this we get the second component. Now this is nothing but you can easily verify this is alpha times T of x 1 x 2, T of x 1 x 2 will be nothing but x 1 plus x 2 as a first component and 2 x 1 minus x 2 in the second component, and alpha time this will give the first component plus and this is T of y 1 y 2.

So, that is nothing but alpha times T v 1 plus T of v 2. So, we have shown that this property holds for every v 1 v 2 and alpha belongs to field; that means, this map is a linear transformation ok. Now similarly we can easily show that T from R 2 to R 2 which is given by this expression which is also called projection on the a axis is also a linear map it follow the same lines as we did earlier in the first example. So, now, the third one is we consider T from the set of all polynomials degree less than equal to n over the field R, to all polynomials of degree less than equals to n minus 1 over R time by T of f x is equal to f dash x. Now this T is also called differential operator ok, where f dash T know the derivative of f x now this is also all linear map.

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$$T(f(x)) = f'(x)$$

$$f, g \in P_n(\mathbb{R}), \quad \alpha \in \mathbb{R}$$

$$T(\alpha f + g) = (\alpha f + g)' = \alpha f' + g'$$
$$= \alpha T(f) + T(g)$$

How it is a linear map you can simply see here, we have defined T of f x as f dash x where f dash x is nothing but derivative of f x now you take any f and g in P n x P n R ok, and alpha belongs to field here is R you take alpha f plus g T of this. So, T of this will be nothing but alpha f plus g whole derivative by this definition and this is nothing but alpha of f dash plus g dash and this is alpha times T of f plus T of g.

So, we have shown that the property of linear transformation hold for every f and g in vector space P n over the field R; that means, this will be a linear transformation. Now, similarly if we define a integral of a to b of a function f x where f x is a continuous function from a to b then this is also a linear map ok.

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$$T(f) = \int_a^b f(x)\,dx$$

$$f, g \in C[a,b], \quad \alpha \in F$$

$$T(\alpha f + g) = \int_a^b (\alpha f + g)(x)\,dx$$

$$= \int_a^b (\alpha f(x) + g(x))\,dx$$

$$= \alpha \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

$$= \alpha T(f) + T(g). \checkmark$$

So, this is very easy to show again because if you take a if you take T of f x as all a to b f x d x, I mean T of f I think T of f. It is T of f if you take T of f defined as this, now you take any f and g in set of continuous function in the interval closed interval a to b and any alpha in field.

And you take alpha f plus g the T of this that will be equals to integral a to b alpha f plus g x in to d x, which is equal to integral a to b alpha f x plus g x whole d x and which is equal to alpha times integral a to b f x d x plus integral a to b g x d x and this is equal to alpha times T of f plus T of g. So, we have shown that this property hold for every f and g in the set of continuous function the closed interval a to b and alpha belongs to field this means this map is a linear map.

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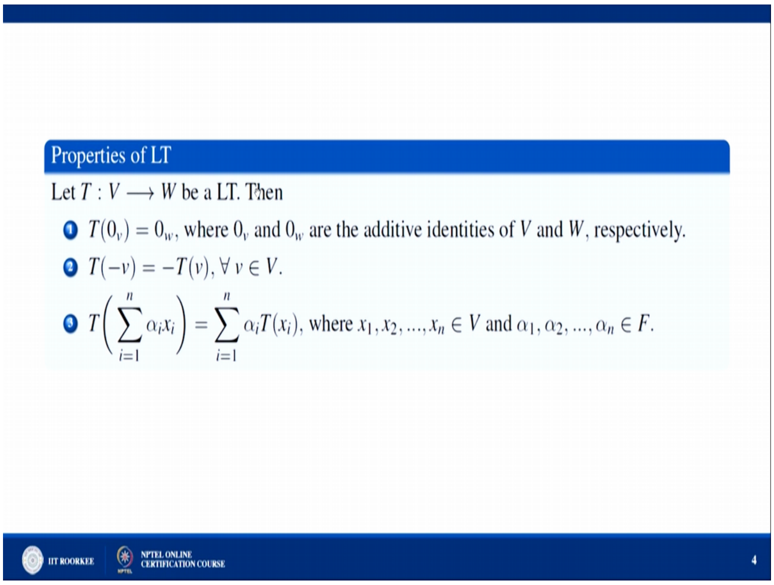

Now, similarly if we see the last example you see we have considered a be a matrix we have fix matrix of order m cross n. And we define T as R n cross 1 means it is a it is a column vector basically, to a column vector of m dimensional space such that T x equal to a x. Now again this, a is fixed if you take any x and y in R n cross 1 and take c x plus y. So, it is easy to show that it is nothing but alpha times T x plus T y, so it will be a linear map.

Now, let us see some basic properties of linear transformation, the first property is if we consider a linear map from V to W, V and W are the vector space over the field F then the first property is T of 0 of v 0 of v means v means additive identity of V, always map to additive identity of W ok.

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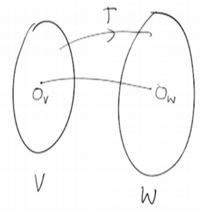
$$T(\alpha v + p) = \alpha T(v) + T(p)$$

$$\forall\, v, p \in V$$

$$\forall\, \alpha \in F$$



$p = 0_v$

$$T(\alpha v) = \alpha T(v) + T(0_v)$$

$$\alpha = 1,\ v = 0$$

$$T(0_v) = T(0_v) + T(0_v)$$

$$T(0_v) = 0_w$$

$$\text{let } T(0_v) = w \in W$$

$$w = w + w$$

$$w + (-w) = w + w + (-w)$$

$$0_w = w + 0_w \Rightarrow w = 0_w \Rightarrow \boxed{T(0_v) = 0_w.}$$

Now it is very easy to show we are considering here T from vector space V to W. What I want to show if we have a additive identity here which we which we denote as g 0 of v, 0 v means additive identity of vector space v always map to additive identity of vector space w by this linear transformation T. So, it is very easy to show you see that for a linear map alpha v plus say p is equals to alpha times T v plus T p. And this is true for all v and p belongs to v and for all alpha belongs to field, since it is a linear transformation. Now you first in this in order to we have to show that T of 0 v is equals to 0 w this we have to show.

So, you first put p equal to 0 since it is true for every alpha belongs to field and p and v belongs to v. So, it will be true for p equal to 0 also 0 means 0 of course, v ok. So, this means it is 0 of alpha v is equals to alpha times T v plus T of 0 v, because additive identity plus an element of vector space is itself that is why we are having here alpha v. Now in order to show that now you take say alpha equal to say you take v equal to 0 ok, you take alpha equal to 1 and v equal to 0 since it is true for every alpha and every v. So, it will be true for true for alpha equal to 1 and v equal to 0 also. So now, we will obtain 1 dot v is always v. So, it is v I mean v is 0 here 0 of v which is equals to 1 dot T of v will be T of v and v is 0, so it is T of 0 v plus T of 0 v.

So, let T of 0 v is basically let us suppose it is w. So, we are obtaining w equal to w plus w now w is a element of capital W it is a vector space, now if it is a vector space. So, its additive inverse will exist, so you can always add with additive inverse of w both the sides element which is inversely identity element.

So, it is 0 of w is equal to w plus 0 of w, so this implies w equal to 0 of w and this implies w is nothing but w is nothing but you see T of 0 of v. So, this T of 0 of v is equals to T of 0 of w, so this is a first most property that if it is a linear map. So, T of 0 will always may map to 0 of w additive identity of w, the second property is T of minus v minus v is additive inverse of v always map to or always equal to negative of T v; that means, additive identity of T of v. So, again it is easy to show you see we have to show that T of minus v is equal to minus of T v for every v in v ok.

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$$T(-v) = -T(v), \quad \forall v \in V.$$

$$T(\alpha v + p) = \alpha T(v) + T(p), \quad \forall v, p \in V, \ \forall \alpha \in F$$

Put $p = 0_V$

$$T(\alpha v) = \alpha T(v) + T(0_V) = \alpha T(v) + 0_W = \alpha T(v)$$

Put $\alpha = -1$

$$T(-v) = -T(v)$$

Now, we know that if it is a linear transformation then T of alpha v plus p will be equals to T alpha v plus T p for all v p belongs to vector space V and for all alpha belongs to field, this we already know now you put p equal to 0 first since it is true for every p. So, it will be true for p equal to 0 also 0 means 0 of v ok. So, it will be T of alpha v the left hand side will be equals to alpha times T of v plus T of 0 v; now T of 0 of v is equal to 0 of w. So, it is alpha times T v plus 0 of w which is equals to alpha times T v. Now you substitute put alpha equal to minus 1, if you put alpha equal to minus 1 we have already shown you the vector spaces that minus 1 dot v is nothing but minus v.

So, it is T of minus v and it is minus 1 dot element of vector space W will be minus times that element that is minus of T v. So, we have shown this property also the last property is T of alpha 1 x 1 plus alpha 2 x 2 and so on up to alpha n x n is equals to alpha times sum of alpha alpha i s T of x i s, where x i s are the element in vector space and alpha is element is field again it is easy to show the result is very trivial.

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$$T(\alpha_1 x_1 + \underbrace{\alpha_2 x_2 + \cdots + \alpha_n x_n}_{X})$$

$$= \alpha_1 T(x_1) + T(X)$$

$$= \alpha_1 T(x_1) + T(\alpha_2 x_2 + \underbrace{\cdots + \alpha_n x_n}_{Y})$$

$$= \alpha_1 T(x_1) + \alpha_2 T(x_2) + T(Y)$$

You see T times alpha 1 x 1 plus alpha 2 x 2 and so on alpha n x n if you take this now you take this element as suppose capital X. So, we know the property of vector space, so by the property of vector space this will be equals to alpha 1 times T x 1 plus T capital X

Now, it is alpha 1 x 1 plus alpha 1 T x 1 plus T of alpha 2 x 2 and so on alpha n x n, again you take this as say capital Y again you apply the property of vector space I mean linear transformation. So, this will be alpha 2 T of x 2 plus T of Y. So, similarly if you extend this up to n times, so we will get the same result bit we are having here.

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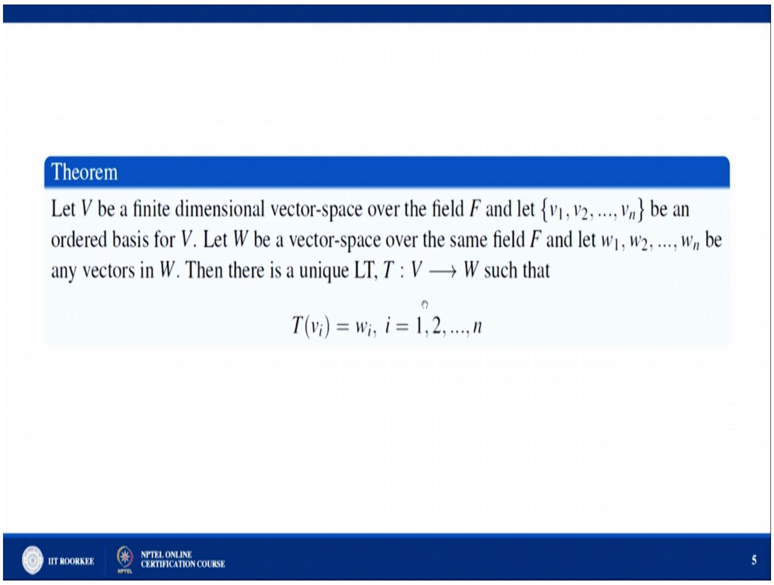

Now, the next theorem is let V be a finite dimensional vector space over the field F and let v 1 v 2 up to v n, v n ordered basis for V ok. Let W be a vector space over the same field F and let w 1 w 2 up to w n be any vectors in capital W. Then there is a unique linear transformation T from V to W such that T of v i equal to w i for i equal to 1 to n ok, that now what I want to say basically in this theorem that you are having a linear transformation T from V to W ok.

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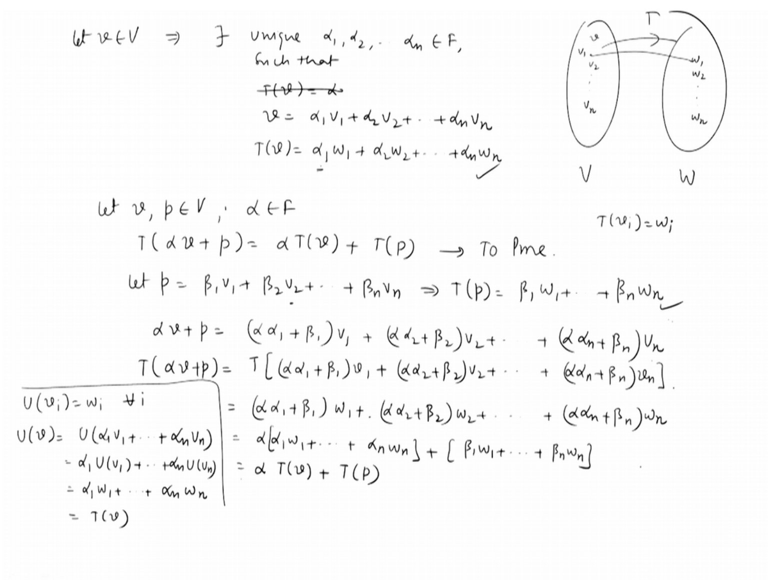

Here you are having from vector say v 1 v 2 v n which is in ordered basis of capital V ok. So, they will always they will exists a unique linear transformation T this T such that such that T of v i will map to w i w 1 w 2 up to w i ok, this is the main result that they will exist a unique linear transformation T from V to W such that this happens.

Now, proof is very easy you see here you take any let v belongs to v ok, if any v belongs to this space and this we already know that v 1 v 2 up to v n is a basis is a ordered basis for v ok. If it is a basis this means any element v in this vector space can be written as linear combination of element of the basis because, it is it is a basis; that means this span of this will generate the entire vector space V.

So, if you take any element V in this vector space that can be written as linear combination of elements of v i s ok. So, this implies there will exist unique alpha 1 alpha 2 up to alpha n in field such that T of we will be alpha I mean sorry, T V will be equals to alpha 1 v 1 plus alpha 2 v 2 and so on up to alpha n v n ok. Now for this v for this v, we can define T of v as alpha 1 T of v 1 as w 1 T of v 2 as w 2 and so on. Now this T is well defined and it is cleared T of v i is w i, now we have to show that it is this map is linear and number 2 this is unique.

So, for linear how we can show linear you can take say v and p are 2 elements in w, I mean in v you take any alpha belongs to field and we have to show that alpha T of alpha T plus p is equal to alpha T v plus T w i mean T v. So, how can we show this, so we have to prove this thing T of alpha v plus p is equals to alpha times T v plus T v. So, this to prove now let p since p is also some element in vector space v.

So, this p can be written as some linear combination of element of v i s. So, this will be beta 1 v 1 plus beta 2 v 2 and so on up to beta n v n, and this implies T of p will be beta 1 T of v 1 is w 1. So, it is w 1 plus and so on up to beta n w n. Now you take alpha v plus p because we have to show this result for l for T as a linear map.

Now, alpha p plus v will be what it is alpha alpha 1 plus alpha alpha 1 plus beta 1 times v 1 this is p this is v plus alpha alpha 2 plus beta 2 times v 2 and so on. alpha alpha n plus beta n times v n and what is T of alpha v plus p this will be T of this this will be T of T of alpha alpha 1 plus beta 1 ok, v 1 plus alpha alpha 2 plus b 2 times v 2 alpha alpha n plus beta n times b n.

Now this is equals to it is alpha it is you see it is alpha alpha 1 plus beta 1 times T of v 1 which is w 1 plus alpha alpha 2 plus beta 2 times T of v 2 which is w 2 and so on alpha alpha n plus beta n times T of v 1 which is w n. So, it is alpha alpha 1 alpha times alpha 1 w 1 and so on up to alpha n w n and plus b beta 1 w 1 and so on up to beta n w n which is equals to alpha times T of v plus T of p from here and from here.

So, we have shown that T T of alpha p plus alpha v plus p is equals to this this means this is a linear map then next thing to show that it is this linear map is unique. So, so in order to show that this linear map is unique you consider a linear map u such that T of v i s w i for all i. Then if you write u of v from here you see if you take any v in again in v, then that v can be uniquely expressed as alpha 1 v 1 plus alpha 2 v 2 and so on up to alpha n v n. So, what will be u of v then u of v will be by this expression that will be u of alpha 1 v 1 and.

So, on up to alpha n v n and this will be alpha 1 u of v 1 and so on up to alpha n u of v n and this will be alpha 1 w 1 and. So, on up to alpha n w n ok; that means, it is equals to T of v again from here so; that means, since the expressions are same; that means, transformation is unique So, we have shown that this is a linear map and a transformation is unique hence we have proved the theorem ok.

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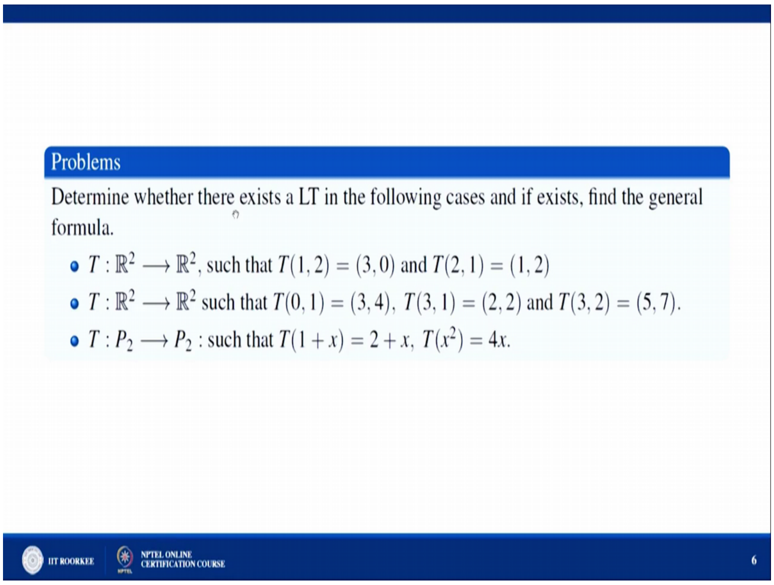

Now, the next is determine whether there exists a linear transformation in the following cases and if exist find the general formula. So, let us start with a first problem if we are having say T from R 2 to R 2 which is defined as this expression then can then can be find, a linear transformation such that these property hold how can we see. So, let us see let us start with a first problem.

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$$T: \mathbb{R}^2 \to \mathbb{R}^2, \quad T(1,2) = (3,0), \quad T(2,1) = (1,2)$$

$(x,y) \in \mathbb{R}^2$

$(1,2) =$

$(x,y) = \alpha(1,2) + \beta(2,1)$

$\{(1,2),(2,1)\} =$

$\Rightarrow x = \alpha + 2\beta, \quad y = 2\alpha + \beta$

$[(1,2),(2,1)] = \mathbb{R}^2$

$$\alpha + 2\beta = x$$
$$2\alpha + \beta = y \quad \times 2$$

$$-3\alpha = x - 2y \Rightarrow \alpha = \frac{2y-x}{3}, \quad \beta = y - 2\alpha$$

$$\beta = y - \frac{2}{3}(2y-x) = \frac{-y+2x}{3}$$

$$(x,y) = \left(\frac{2y-x}{3}\right)(1,2) + \left(\frac{2x-y}{3}\right)(2,1)$$

$$T(x,y) = \left(\frac{2y-x}{3}\right)T(1,2) + \left(\frac{2x-y}{3}\right)T(2,1)$$

$$= \left(\frac{2y-x}{3}\right)(3,0) + \left(\frac{2x-y}{3}\right)(1,2) = (\quad , \quad)$$

So, here it is T of R 2 to R 2 and here T of T of 1 2 is 3 0, and T of 2 1 is 1 2 first of all first of all we have seen that these two elements are in R 2.

First of all let us see first of all observe that is are they linearly independent. So, answer is yes we can’t write 1 2 as a liner combination of 2 1 ok. So, yes they are linearly dependent now what will be what will be the dimension of R 2 dimension of R 2 will be 2 and if there exist any 2 linearly independent vector in R 2. So, that will be the basis of R 2, I mean that will be that the span of those two elements will generate the entire vector space ok.

Now, these elements 1 2 and 2 1 in R 2, so the span of these 2 elements a span of these 2 elements will be definitely R 2, because they are linearly independent thus what is a what is the dimension of R n, dimension of R n is n ok. And if you are having any n dimension any set containing n linearly independent vectors for that will be the dimension that will be the basis of R n there are infinite basis of R n, they have infinite basis of R 2 1 of the basis is this you take any 2 linearly independent vectors in R 2 the span of this will be definitely generate R 2 this is 1 of the basis ok.

Now is there exist any linear transformation such that this equal to this and this is equal to this. So, and if yes how can you find that, so you see you take any x y in R 2, any x y here now since this span of this generates entire R 2 so; that means, there will exist some scalars alpha and beta such that such that this x y can be written as linear combination of these 2 vectors ok.

So, this implies x is equals to alpha plus 2 beta and y is equals to 2 alpha plus beta. So, it is alpha plus 2 beta is equal to x and 2 alpha plus beta is equals to y. Now you multiply this by 2 and subtract these 2 equations what we will obtain this is minus 3 alpha. Multiply by 2 and subtract with this of first equation and this equals to x minus 2 y and that implies that implies alpha is equals to 2 2 y minus x by 3. Now what will be beta beta will be nothing but y minus 2 alpha. So, beta will be y minus alpha is 2 by 3 times 2 y minus x.

So, it is 3 y minus 4 y that is minus y plus 2 x upon 3, now this x y is alpha times the first element the first vector in beta time the second vector. What is alpha? Alpha is 2 y minus x upon 3 times 1 2 and plus 2 x minus y upon 3 times 2 comma 1 now what is T of x comma y since T is linear. So, it is 2 y minus x upon 3 times T of this because it is a it is some scalar plus 2 x minus y upon 3 times T of 2 1. So, this is equals to 2 y minus x upon 3 times what is T of 1 2 it is given here 3 0 and plus 2 x minus y upon 3 times T of 2 1 is given as 1 2, now you can simplify this and we can easily find out what is T of x y ok. So, in this way we can find out a linear transformation T from R 2 to R.

Now, if you see the second example ok, here what here 3 elements are given to you that is T of 0 1 is 3 4 T of 3 1 is 2 2 and T of 3 2 is 5 7 of course, 0 1 3 1 and 3 2 are not linearly independent, because the dimension of R 2 is only 2 and here we are having 3 vectors. So, you take any 2 any 2 linearly independent I mean any 2 arrive vectors says 0 1 and 3 1. Find out a linear transformation as we did in the example first here and if that linear transformation satisfy the third expression also then there exist such linear transformation otherwise, otherwise we say that that linear transformation does not exist. So, if you have a conditions here in the first example we are having two conditions only and the vectors are linearly independent here we are having we are having 3 vectors ok.

So, basically if you have to see that if you have to see that such linear transformation exist then you can write 3 2 or 3 1 anyone vector as a linear combination or remaining 2 and try to see that weather whether the expressions are also same or not. Images are also same or not if they are not; that means, that linear transformation does not exist. Now for the third example you see it is defined from P 2 to P 2 P 2 is a polynomial degree less than equal to 2.

So, what is the dimension of P 2 dimension of P 2 will be 3. So, for a unique to in order to find the unique L T unique linear transformation from P 2 to P 2 we must have at least three independent conditions, but here the conditions are only to. So, this means there exist infinitely many linear transformation there exist infinitely many linear transformation from P 2 to P 2 here. And if you are interested to find out 1 such a linear transformation then you take a vector independent of this and this take any image of this and that then you can find out say for example.

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$T: P_2 \to P_2 \qquad T(1+x) = x+2$

$T(x^2) = 4x$

let $T(x) = x^2$

$$a + bx + cx^2 = \alpha(1+x) + \beta(x^2) + \gamma(x)$$

$$\Rightarrow a = \alpha, \quad \alpha + \gamma = b, \quad \beta = c$$

$$\Rightarrow \gamma = b - a$$

$$a + bx + cx^2 = a(1+x) + cx^2 + (b-a)x$$

$$T(a + bx + cx^2) = a\,T(1+x) + c\,T(x^2) + (b-a)\,T(x)$$

$$= a(x+2) + c(4x) + (b-a)x^2$$

$$= (b-a)x^2 + (4c+a)x + 2a$$

Here what is given to us given to you T is from P 2 to P 2 ok, now T of 1 plus x as x plus 2 and T of x square is 4 x only two conditions are given to you.

So, there will be infinitely many linear transformations from P 2 to P 2 satisfying these 2 equations, suppose we are interested to find out 1 such linear transformation. So, how can we proceed you let you take T of say T of say x as say x square this is this will make these vectors these linearly independent. These 3 vectors are linearly independent you can easily verify that these 3 vectors are linearly independent. Now you take any polynomial of P 2 say a plus b x plus c x square that can be written as alpha times 1 plus x plus beta times x square plus gamma times x square, because these 3 vectors are linearly independent and how many vector these are 3 and the and the dimension of P 2 is 3.

So, this will form a basis of P 2, so any vector in P 2 can be written as linear combination of element of the basis. Now this implies this if you take here the constant here is alpha. So, a is equal to alpha the coefficient of x is alpha plus gamma and that is equal to b the coefficient x square is beta which is c, now this implies gamma is equals to b minus a because alpha is a. So, we can say that a plus b x plus c x square will be alpha times alpha is a 1 plus x plus beta is c x square and gamma is b minus a times x.

Now, you take T of a plus b x plus c x square that will be equals to a times T of 1 plus x c times T of x square plus b minus a times T of x. And that will be equal to a time T over 1 plus x is x plus 2 plus c T of x square is 4 x plus b minus a T of x is x square. So, that will be equal to basically b minus a times x square plus 4 c plus a times x plus 2 a. So, this is a our required linear transformation 1 such linear transformation.

So, there are infinite ways to consider the third expression. So, there are infinite linear transformation of such type, but one such linear transformation is this. So, in this way we have seen that what linear transformation is and what are the basic properties of linear transformation now there may be some examples say.

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$T: \mathbb{R}^2 \to \mathbb{R}^2 \qquad T(x,y) = (1+x, y)$

$T(1,0) = (2,0)$

$T(0,1) = (1,1)$

$T(1,1) = (2,1) \neq (2,0)+(1,1)$

$T: \mathbb{R}^2 \to \mathbb{R}^2, \qquad T(x,y) = (x^2, y)$

Say you consider T from R 2 to R 2 as T of say x y as 1 plus x comma y, now it is not a linear transformation we can easily verify this you see if you take T of 1 0 it is simply it is simply 2 comma 0. If you take T of 0 1 it is again 1 and 1 if you take T of sum of these 2 that is 1 and 1 sum of these 2 is 1 and 1 that must be equal to sum of these 2 by the property of linear transformation, but by the definition it is coming 2 comma 1 which is not equal to sum of these 2. So, that means it is not a linear transformation. Now similarly if you defined say T from R 2 to R 2 as T of x y is equal to say x square comma y. It is also not a linear transformation. It is very easy to show you simply give a counter example for this, ok.

So, in this lecture we have seen that what linear transformations are and what are the basic properties of linear transformation. In the next lecture we will see some more properties of linear transformation.

Thank you.