Matrix Analysis with Applications
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Lecture – 03
Rank of a Matrix

Hello friends. So, welcome to lecture series on Matrix Analysis with Applications. So, the next lecture is on Rank of a Matrix. What do you mean by rank and how it is important to solve system of linear equations we will see in this lecture and then the next lecture. So, before defining rank of a matrix let us see what do you mean by linear dependent or linear independent of elements of R n or C n.

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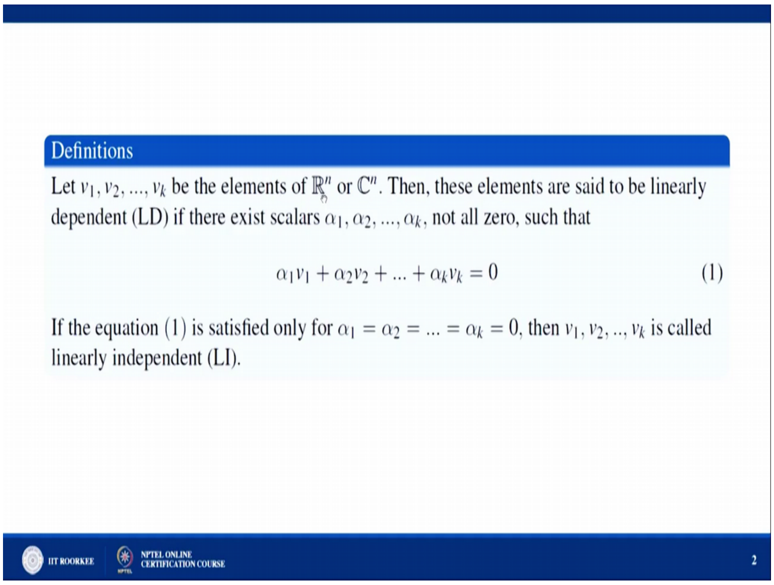

So, let v 1, v 2 up to v k let us suppose these r k elements of R n or C n. What do you mean by R n? R n means all those x 1, x 2 up to x n such that x i belongs to R for all i.

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$$\mathbb{R}^n = \{ (x_1, x_2, \ldots, x_n) \,/\, x_i \in \mathbb{R}, \forall i \}$$

$$\mathbb{C}^n = \{ (x_1, x_2, \ldots, x_n) \,/\, x_i \in \mathbb{C}, \forall i \}$$

That means, R n is simply set of n tuples such that each x i belongs to R. Now, if you think about C n and C n is simply all x 1, x 2 up to x n such that x i belongs to set of complex numbers for all i.

Now, in this definition this v 1, v 2 up to v k are the elements or vectors we also call it vectors in crn or C n, ok. That means, each v i is in is the consist of set of n tuples which is in either in R n or in C n. Then these elements or vectors are said to be linearly dependent or LD if there exist scalars alpha 1, alpha 2 up to alpha k not all 0 such that alpha 1 v 1 plus alpha 2 v 2 plus and so on up to alpha k v k equal to 0, ok.

And if the equation 1 is satisfied only for alpha 1 equal to alpha 2 equal to alpha k equal to 0 that is all alpha is are 0, then these vectors are called linearly independent. Now, let us understand what do you mean by this.

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$$v_1, v_2, \ldots v_k \in \mathbb{R}^n$$

$$v = \alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_k v_k$$

$$\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_k v_k = 0$$

$$\Rightarrow \alpha_i = 0 \text{ for all } i = 1, 2, \ldots, k$$

Then, $v_1, v_2, \ldots v_k$ are LI

Now, you have you have vectors this is v 1, v 2 up to v k. Let us suppose this belongs to R n, each v i belongs to R n, ok. Now, you take you take alpha 1 v 1 plus alpha 2 v 2 plus and so on up to alpha k v k, this is called linear combination of v 1, v 2 up to v k. These alpha is are some scalars, some constants, ok. Now, this you change alpha is this v will change, ok, if you change alpha you very alpha is this v will change, but all v are simply the linear combination of elements of v i's.

Now, if alpha 1 v 1 plus alpha 2 v 2 into plus and so on up to alpha k v k equal to 0 this implies alpha i equal to 0 for all i from 1 to k then we say that then we say that v 1, v 2 up to v k are linearly independent. That means, you take a linear combination of v 1, v 2 upto v k put it equal to 0 and there exist only one solution, which is alpha equal to 0 for all I that means, v 1, v 2 upto v k are linearly independent vectors or elements, ok.

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$$\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_k v_k = 0$$

$\Rightarrow \exists$ atleast one $\alpha_i \neq 0$, then

$v_1, \ldots v_k$ are LD.

let $\alpha_p \neq 0$, $1 \le p \le k$,

$$\alpha_p v_p = -\alpha_1 v_1 - \alpha_2 v_2 \cdots - \alpha_{p-1} v_{p-1} - \alpha_{p+1} v_{p+1} \cdots - \alpha_k v_k$$

$$\Rightarrow v_p = \underbrace{\left(-\frac{\alpha_1}{\alpha_p}\right)}_{\beta_1} v_1 + \underbrace{\left(\frac{\alpha_2}{\alpha_p}\right)}_{\beta\checkmark} v_2 \cdots \underbrace{\left(-\frac{\alpha_{p-1}}{\alpha_p}\right)}_{\beta_{p-1}} v_{p-1} - \frac{\alpha_{p+1}}{\alpha_p} v_{p+1} \cdots -\frac{\alpha_k}{\alpha_p} v_k$$

Now, if we are taking alpha 1 v 1 plus alpha 2 v 2 plus and so on up to alpha k v k and this equal to 0 implies there exist at least at least 1 alpha i which is not equal to 0, then v 1, v 2 up to v k are called linearly dependent, ok.

Let us suppose, let us suppose that alpha i which is not equal to 0 is alpha p, ok, is alpha p. So, let alpha p is not equal to 0 where p is between 1 to k, p is any element between 1 to k, ok. Then you leave alpha p, v p in this side and all the remaining terms over here on the right hand side. So, this will be minus alpha 1 v 1 minus alpha 2 v 2 and so on, minus alpha p minus 1 v p minus 1 minus alpha p plus 1 v p plus 1 and so on minus alpha k v k.

And since alpha p is not equal to 0, so you can always divided by alpha p, so this implies v p will be equal to minus alpha 1 alpha p v 1 minus alpha 2 upon alpha p v p and so on minus alpha p minus 1 upon alpha p v p minus 1, minus alpha p plus 1 upon alpha p v p plus 1 minus alpha k upon alpha p into v k. So, yeah, ok.

So, so that means what? This is alpha 2. So that means what? That means, that means if vectors are linearly dependent, then there exist at least 1 vector which can be expressed as linear combination of remaining vectors. So, you can always put it some beta 1, you can always take it some beta 2, you can always some take it some say beta p minus 1 and so on. So, what I want to say that if vectors are linearly dependent, then there always exist at least 1 vector or 1 element in that set which can be expressed as linear combination of remaining vectors. And if vectors are linearly independent then none of the vector can be expressed as linear combination of remaining vectors, ok.

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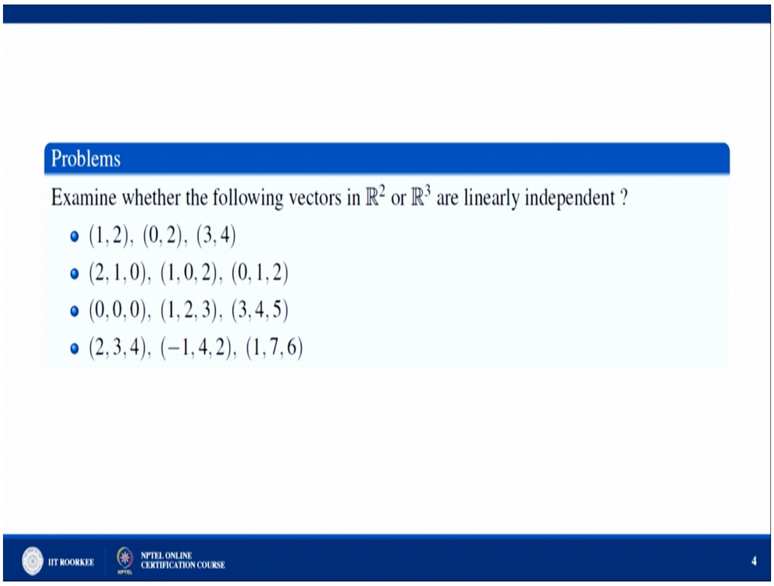

Now, let us discuss this thing by few examples. So, what do you mean by linearly independent or dependent vectors let us discuss all this things.

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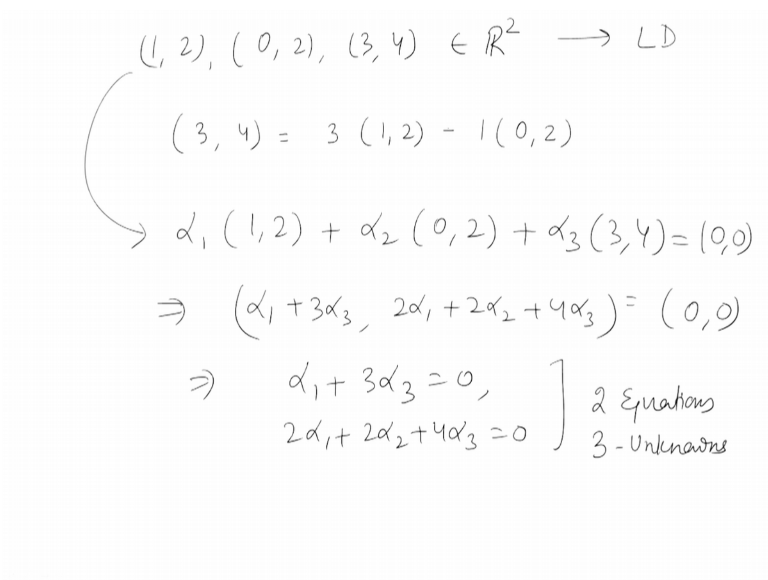

So, the first problem is this is 1 2, then 0 2, and then 3 4. These vectors are in R 2, these vectors are in R 2. We have to see whether this set of vectors are linearly independent or dependent.

Now, you can you can simply see here if you can see the if you see by observation you can see. You can always see that 3 comma 4 can be expressed as 3 times 1 comma 2 plus or either it is 0 comma 2 you see, it is 3 times that is 3. Now, you want 4, so it is 6, 6 minus 2, it must be 1; it is 6 minus 2 is 4, ok. So, we are express this vector 3 comma 4 as a linear combination of these 2 vectors. So, what does it mean? It means that this vectors are linearly dependent. So, this is linearly dependent, LD, this set is LD, ok. Because we are express 1 vector as a linear combination of the remaining vectors.

Other way out is, other way out is you take a linear combination of these vectors, and put it equal to 0. So, what does it imply? It imply it is alpha 1 plus 0 plus 3 alpha 3 comma it is 2 alpha 1 plus 2 alpha 2 plus 4 alpha 3 equal to 0 0. So, this implies this is equal to 0 0 means each component is 0, that means, alpha 1 plus 3 alpha 3 is equal to 0 and 2 alpha 1 plus 2 alpha 2 plus 4 alpha 3 equal to 0. Now, this is you see we are we are having 2 equations and there are 3 unknowns and equation is homogenous right hand side is 0, ok. So, you can arbitrary choose any value of alpha 3 may not be 0, ok.

Then you can find out alpha 1 and alpha 2 so that means, there exist there exist some alpha which is not equal to 0 and that means, that this set of vectors are linearly dependent, ok.

(Refer Slide Time: 10:29)

$$(2,1,0),\ (1,0,2),\ (0,1,2) \in \mathbb{R}^3$$

$$\alpha_1(2,1,0) + \alpha_2(1,0,2) + \alpha_3(0,1,2) = (0,0,0)$$

$$\Rightarrow \begin{cases} 2\alpha_1 + \alpha_2 = 0 \Rightarrow \alpha_2 = -2\alpha_1 \rightarrow \alpha_2 = 0 \\ \alpha_1 + \alpha_3 = 0 \\ 2\alpha_2 + 2\alpha_3 = 0 \Rightarrow -2\alpha_1 + \alpha_3 = 0 \Rightarrow \alpha_3 = 2\alpha_1 \rightarrow \alpha_3 = 0 \\ \alpha_1 + 2\alpha_1 = 0 \Rightarrow \alpha_1 = 0 \end{cases}$$

Now, for second problem if you see a second problem 2 1 0, it is 1 0 2 and 0 1 2, these are in R 3, and you have to see where this set of vectors are linearly independent or dependent. So, you take the linear combination of these vectors, put it equal to 0, ok. So, this implies 2 alpha 1 plus alpha 2 equal to 0, then alpha 1 plus alpha 3 equal to 0, and then 2 alpha 1 alpha 2 plus 2 alpha 3 equal to 0, ok.

So, what is it imply? From here we are getting alpha 2 as minus 2 alpha 1, when we substitute it here it here we are getting from here it is minus 2 alpha 1 plus alpha 3 equal to 0 that means, alpha 3 is equals to 2 alpha 1, ok. And when we substitute it over here in this equation alpha 1 is alpha 1 and alpha 3 is 2 alpha 1 it is equal to 0.

So, this implies alpha 1 equal to 0, ok. And when you substitute alpha 1 equal to 0 here from here we are getting alpha 2 equal to 0 and from here we are getting alpha 3 equal to 0. So that means, we are getting only one solution which is the 0 solution that means, all alpha is the 0 - alpha 1 is 0, alpha 2 is 0, alpha 3 is 0 it is the only solution of this equation that means, this vector are linearly independent.

(Refer Slide Time: 12:14)

$$\underbrace{(0,0,0)},\ (1,2,3),\ (3,4,5) \rightarrow LD$$

$$(0,0,0) = 0(1,2,3) + 0(3,4,5)$$

$$(2,3,4),\ (-1,4,2),\ \underbrace{(1,7,6)} \in \mathbb{R}^3$$

$$(1,7,6) = 1(2,3,4) + 1(-1,4,2)$$

$$\rightarrow \underline{LD}$$

Now, if you take this set 0 0 0, 1 2 3, and 3 4 5 the next example, next problem. Now, this is clearly linearly dependent. Why it is linearly dependent? Because this vector 0 0 0, this 0 0 0 can always we expressed as 0 times 1 2 3 plus 0 times 3 4 5. That means, that means, 1 of a vector of this set can be expressed as linear combination of remaining 2 vectors this means set is LD, set is linearly dependent. Now, if you see the next problem it is 2 3 4 then minus 1 4 2 then 1 7 6, ok.

Now, you have to see whether this set is LI or LD. So, you take linear combination of these 3 vectors put it equal to 0 and if you find only one solution that is all alpha is are 0 that means, this vectors are linearly independent otherwise linearly dependent. One thing you can easily observe that this vector is equal to the sum of these 2 vectors see 2 minus 1 is 1, 3 plus 4 is 7, 4 plus 2 is 6 that means, 1 7 6 is equal to 1 times 2 3 4 plus 1 time minus 1 4 2.

So that means, one vector can be expressed as linear combination of a remaining vectors, this means this clearly means set is LD set is linearly dependent, ok. So, in this way we can check whether set is linearly dependent or independent. Now, we will see some properties of dependent or linearly independent sets.

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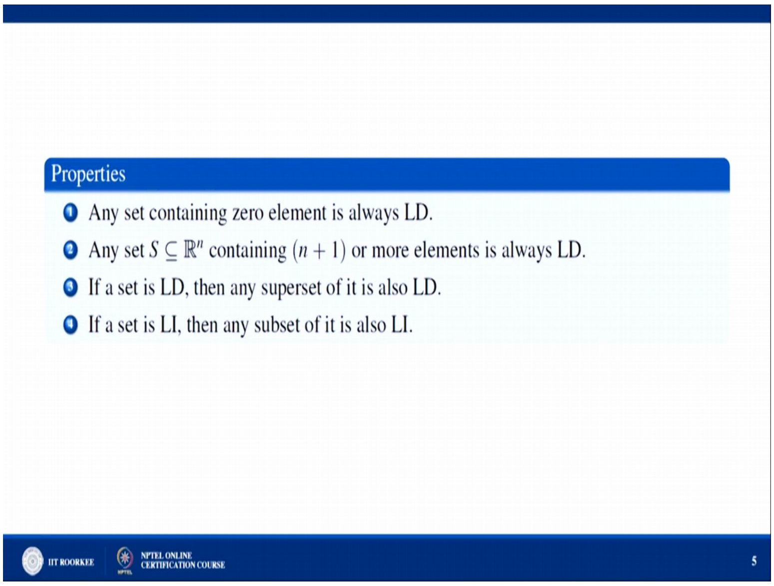

The first property is any set containing 0 element is always LD. This is very easy to show you see. If you are having any set containing 0 element then that 0 element can always be expressed as linear combination of the remaining vectors were scalars are also 0, ok. So, since this 0 vector can be expressed as linear combination of remaining vectors. So, the set is clearly LD.

As we have also seen one of the example you see we have already seen in this example the third example that how why this set is LD, because this 0 0 0 can be expressed as 0 time this plus 0 time this. That means, this vector 0 vector can be expressed as linear combination of remaining 2 vectors, ok.

The next property is any set s with a subset of R n containing n plus 1 or more elements is always LD, this is easy to prove you see. You see we have some set say v 1, v 2 up to v n up to v n plus k suppose, and all v i's are in R n, ok.

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$$\{v_1, v_2, \ldots v_n, \ldots v_{n+k}\} \qquad v_i \in \mathbb{R}^n \ \forall i, \quad k \geq 1$$

$$v_1 = (v_1^1, v_1^2, \ldots v_1^n)$$

$$v_i = (v_i^1, v_i^2, \ldots v_i^n), \ \forall i$$

$$\alpha_1 v_1 + \alpha_2 v_2 + \cdots + \alpha_{n+k} v_{n+k} = 0$$

n- equations

$(n+k)$ unknowns, $k \geq 1$

LD

So, this is the set containing n plus 1 or more elements k is k is greater than equal to 1. So, this set is containing n plus 1 or more elements and each vi's you see if you are talking about v 1. So, v 1 is in R n that means, it is v 1 1 it is v 1 2 and so on up to v 1 n and similarly if you talk about v i. So, v i is simply v i 1 v i 2 and so on up to v i n for all i, ok. Now, if you take the linear combination of this vector. So, v alpha 1 v 1 plus alpha 2 v 2 and so on up to alpha n plus 1 v alpha n plus p v n plus p equal to 0.

Now, when you substitute v 1 from here to here, similarly v 2 which is a n tuples n tuple set of n tuples, similarly v n plus p then when you multiply with alpha 1 multiply v 2 by alpha 2 and so on you will get n equations with n plus p unknowns, ok, p n plus k sorry it is n plus k here it is k, here also it is k. So, we are getting n equations with n plus k unknowns, k is greater than equal to 1.

So, more unknowns and equations are less that means, many solutions. And many solution means there exist non-zero solution and non-zero solution means set is LD set is linearly dependent. So, so this set is always LD, ok. The third property is if the set is LD then any super set of it is also LD, ok.

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$$\{v_1, v_2, \ldots v_p\} \rightarrow LD$$

$$\exists\ v_k,\ 1 \le k \le p,\ \text{such that}$$

$$v_k = \alpha_1 v_1 + \cdots + \alpha_{k-1} v_{k-1} + \alpha_{k+1} v_{k+1} + \cdots + \alpha_p v_p.$$

$$\{v_1, v_2, \ldots v_p, \ldots v_{p+m}\}$$

$$\underline{v_k} = \alpha_1 v_1 + \cdots + \alpha_{k-1} v_{k-1} + \alpha_{k+1} v_{k+1} + \alpha_p v_p + 0 v_{p+1} + \cdots + 0 v_{p+m}$$

$$\rightarrow \underline{\underline{LD}}$$

This is again easy to show you see if we have a set say v 1, v 2 up to say v p say this set is LD. So, if this set is LD that means, there exist that means, there exist some v k where k is between p 2 1 such that such that this v k can be expressed as linear combination of remaining vectors, alpha p v p.

Now, take a superset of this set, say superset is v 1, v 2 up to v p and so on up to say v p plus m take a superset of this set containing elements more than this containing this set and some more elements. Now, if alpha k can be expressed as linear combination of the remaining vectors, then this alpha k can also be written as alpha 1 v 1 and so on up to alpha k minus 1 v k minus 1 plus alpha k plus 1 v k plus 1 plus alpha p v p plus 0 times remaining vectors, that means, that means linear combination of remaining vectors.

So, this set will also become linearly dependent, because we have expressed one element v k which in the which is in this set and can be expressed as linear combination of remaining vectors, ok. So, this set is also LD. So, we have shown that if you have any set which is LD then any super set of this set is also LD, ok.

The next properties if a set is LI then any subset of it is also LI, if it is not if any subset of it is not LI then it is LD and if it is LD then the super set of this set will also be LD however it is LI. So, this contradict statement 3, ok. So, therefore, if a set is LI then any subset of it is also LI.

Now, come to the rank of rank of a matrix. How we defined rank of a matrix? Say we are having a matrix of order m cross n.

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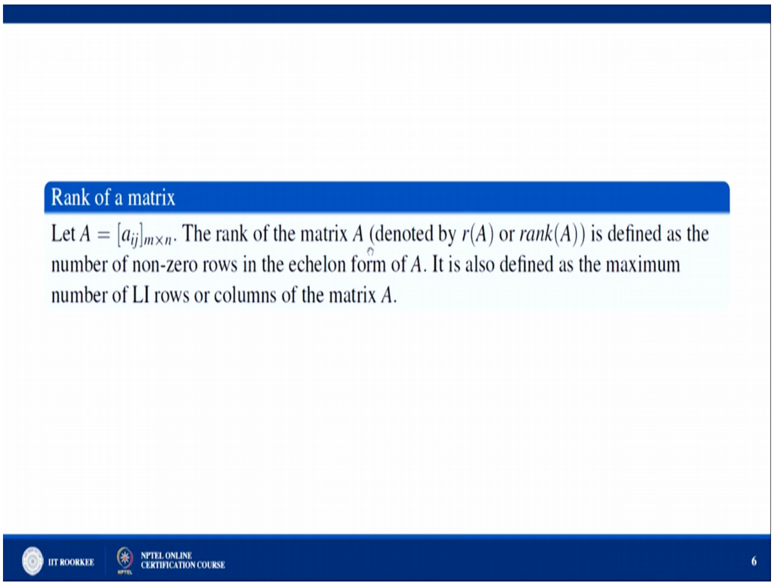

Then the rank of matrix A denoted by r A or rank of A is defined as the number of non-zero rows in the echelon form of a matrix. We have already discuss the echelon form of a matrix. Now, number of non-zero rows in the echelon form is called its rank or it can also we defined as maximum number of linearly independent rows or column of the matrix A, ok. If you take if you take you see if you take, if you have a matrix A a 11, a 12 and so on up to a 1n; a 21, a 22 and so on a 2n it is of order m cross n.

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$$A = \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_m \end{array} \\ \begin{array}{cccc} a_{11} & a_{12} & \cdot & a_{1n} \\ a_{21} & a_{22} & \cdot & a_{2n} \\ \cdot & & & \\ a_{m1} & a_{m2} & \cdot & a_{mn} \end{array} \begin{array}{l} \to R_1 \\ \\ \\ \to R_m \end{array} \\ m+n \end{array}$$

Now, you take the first column say C 1 has a 1 vector the second column a second vector and the third this m th column as a m th vector, ok, sorry it is n, ok. If this number of linearly independent vectors number of independent columns is the rank. I mean we are taking the 1 column as a 1 vector the second column as second vector the n th column as n th vector.

Similarly if you take the row this is first row and the first vector R 1 and n th row m th row as the m th vector; So, the number of linearly independent rows or columns considering 1 column as a 1 vector or considering 1 row as a 1 vector, ok. So, number of rows or number of linearly independent rows or columns is the rank of the matrix.

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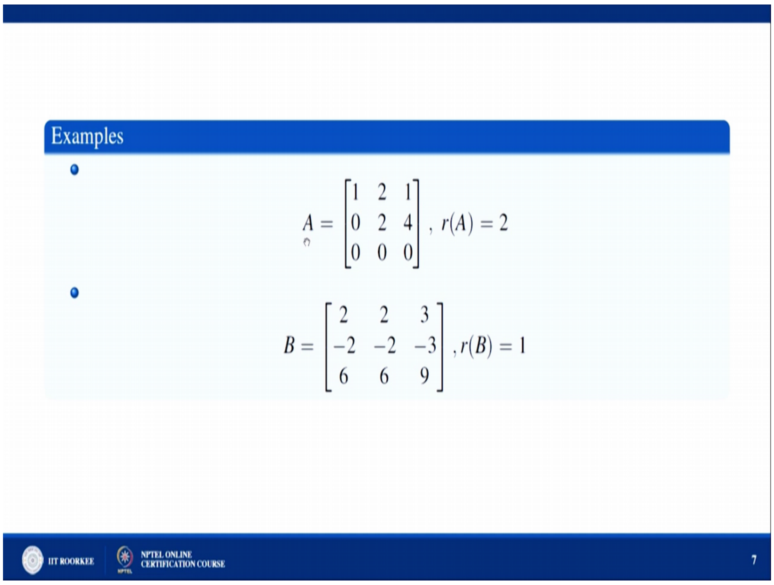

Now, let us discuss few examples you see. Now, this is the matrix a. Now, this is the echelon form of some matrix is clear because you see the first leading non-zero element is 1 in the first row and in the below this all elements are 0.

The, in the second row the first leading non-zero element is 2 and below this all elements are 0 and all rows containing only use element are at the bottom of the matrix, ok. So, now, how many number of non-zero rows it is having it is having only 2 rows first row and the second row. So, that we can say the rank of this matrix is 2, ok. We can also say like this you see this vector 1 2 1 can never be expressed as alpha times 0 2 4, ok. So, these two vectors are linearly independent. So, maximum number of linearly independent rows with this matrix is having is only 2, ok. So, rank of this matrix is 2.

Now, see here if you have this matrix the first row is 2 2 3, the second row is minus 2 minus 2 minus 3, third row is 6 6 9. Now, if you multiply the first row with minus 1 you will get the second row that means, the second row is simply minus 1 time the first row, ok. So that means, these 2 rows are linearly dependent similarly if you multiply this first row by 3 you will get the third row that means, this row and this row is also dependent.

So, how many maximum number of linearly independent rows with this matrix is having? Only 1, that is 2 2 3 because other 2 rows are linearly dependent on the first row. So, maximum number of linearly independent rows with this matrix is having only 1. So, the rank of this matrix is 1. You can also you can also find the echelon form of this matrix and then you can see the number of non-zero rows with this matrix is having.

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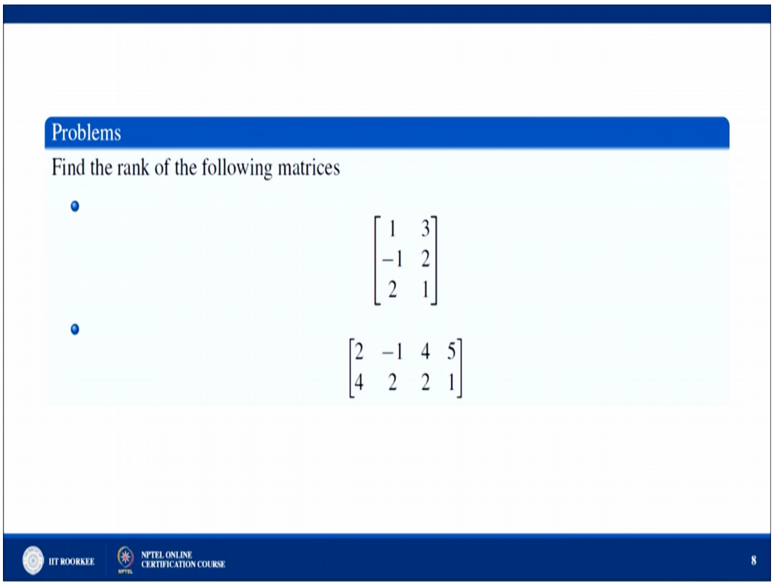

Now, let us find the echelon form of these matrices. I mean echelon form and then the rank.

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$$A = \begin{pmatrix} 1 & 3 \\ -1 & 2 \\ 2 & 1 \end{pmatrix}_{3\times 2} \sim \begin{pmatrix} 1 & 3 \\ 0 & 5 \\ 0 & -5 \end{pmatrix}_{3\times 2} \quad \begin{matrix} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$R_3 \to R_3 + R_2$$

$$\sim \begin{pmatrix} 1 & 3 \\ 0 & 5 \\ 0 & 0 \end{pmatrix}_{3\times 2} \quad r(A) = 2$$

$$B = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \quad r(B) = 1$$

$$A \to m \times n \qquad r(A) \leq \min\{m, n\}$$

Now, what is the matrix we were having? It is 1 3, minus 1 2, and 2 1. Let us find the echelon form of this matrix the echelon form will be you see 1 3 you will make 0 here with the help of this. So, how you will make this in the in R 2, you will apply R 2 plus R

1. So, that will be 0 this is 5 you will again make 0 here with the help of this that is R 3 you will take R 3 minus 2 times R 1. So, that is 0 this minus 2 time this and this minus 2 times this is minus 5, ok.

Now, in this in this is the first non-zero element you will make 0 in the bottom. So, in the third row again you will take R 3 plus R 2. So, what will be the final form 1 3, 0 5, 0 0. So, what is the echelon form of this? This is the echelon form of this matrix. And what is the rank? Rank will be simply rank of this matrix ih this matrix is a the rank of this matrix is simply 2 because number of non-zero row are 2, ok. In fact, if it is this order is 3 cross 2 you see, that order of this matrix is 3 cross 2, this is 3 cross 2.

So, rank can never be more than 2, because whatever element you are having here you will always make 0 with the help of this to convert this matrix in it is a echelon form. So, maximum linearly independent rows or maximum non-zero rows with this matrix is having is only 2, ok. It maybe, it may be less than 2 also, but the maximum is only 2 you see if you are having this type of matrix 1 2, 2 4, and then 36 what the echelon form of this matrix 1 2, 0 0 and then again 0 0. So, the rank of this matrix is simply if this is B, the rank of this matrix is simply 1, ok.

So, what I want to say that if it if a matrix has been having an order m cross n. The rank of this matrix can never exceed n and similarly can never exceed m. So, it will be always less than or equals to minimum of m or n, ok.

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$$A = \begin{pmatrix} 2 & -1 & 4 & 5 \\ 4 & 2 & 2 & 1 \end{pmatrix}_{2\times 4} \qquad r(A) \leq 2$$

$$\sim \begin{pmatrix} 2 & -1 & 4 & 5 \\ 0 & 4 & -6 & -9 \end{pmatrix} \qquad R_2 \to R_2 - 2R_1$$

$$\underline{r(A) = 2}$$

Now, you have second example suppose it is 2 minus 1 4 5, 4 2 2 1. So, it is having 2 rows and 4 columns. Of course, by simply saying the order of the matrix I can say that rank of this matrix if it is A, then rank of this matrix is always less than equal to 2. This I can say surely, because rank of a matrix is always less than equal to minimum of m or n because rank can never exceed m and can never exceed n.

Now, the echelon form of this matrix is 2 minus 1 4 5, you have to make 0 here with the help of this to find its echelon form. So, this row in R 2 you will make a elementary row transformation R 2 minus 2 times R 1. So, it is 0 this minus 2 times this is 4, this minus 2 times this is minus 6, this minus 2 times this is minus 9. So, how many number of non-zero is it is having? It is having 2 rows. So, we can say that rank of A is 2, ok.

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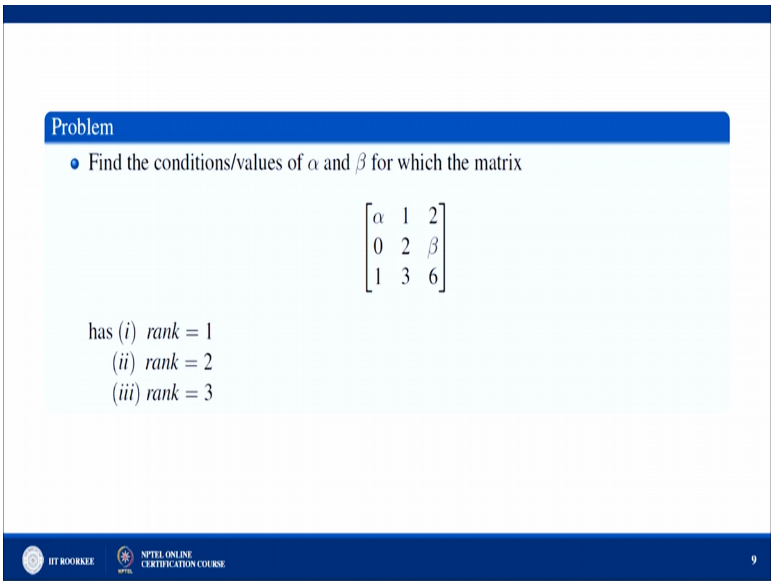

Now, the next problem is suppose you want to find out the conditions on unfair beta for with this matrix is the rank 1 2 or 3. So, this is also a simple problem let us try to find it. So, what are the matrix we are having?

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$$A = \begin{bmatrix} \alpha & 1 & 2 \\ 0 & 2 & \beta \\ 1 & 3 & 6 \end{bmatrix} \sim \begin{bmatrix} 1 & 3 & 6 \\ 0 & 2 & \beta \\ \alpha & 1 & 2 \end{bmatrix} \quad R_3 \leftrightarrow R_1$$

$$\sim \begin{bmatrix} 1 & 3 & 6 \\ 0 & 2 & \beta \\ 0 & 1-3\alpha & 2-6\alpha \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 3 & 6 \\ 0 & 2 & \beta \\ 0 & 0 & \frac{(1-3\alpha)(4-\beta)}{2} \end{bmatrix} \quad \boxed{R_3 \to R_3 - \frac{(1-3\alpha)}{2} R_2}$$

$$(2-6\alpha) - \left(\frac{1-3\alpha}{2}\right)\beta = (1-3\alpha)\left(\frac{4-\beta}{2}\right)$$

Matrix is alpha 1 2, it is 0 2 beta, it is 1 36. So, first find its echelon form and then only we can say the rank of this matrix A. So, so for echelon form it is alpha and it is 0. So, first you will because it is a parameter alpha which may be 0 also we do not know. So, first interchange the third row with the first row, ok. So, it is 1 3 6, it is 0 2 beta, it is alpha 1 2. We have make R 3 with R 1 interchange R 3 with R 1.

Now, it is 1 3 6 it is 0 2 beta you make 0 here with the help of this. So, this minus alpha time this is 0, this minus alpha time this is 1 minus 3 alpha, this minus alpha time this is 2 minus 6 alpha. Now, you will for the echelon form you will make 0 here with the help of this, again you will make a elementary row transformation 1 3 6, 0 2 beta, it is 0, it is 0. What operation you have applied in R 3? It is R 3 minus 1 by 2 times it is 1 minus 3 alpha times R 2, to make 0 here. And the same operation you will apply here so what you will obtain you see it is 2 minus 6 alpha minus 1 minus 3 alpha by 2 times beta.

So, what you will get? It is 1 minus 3 alpha into 4 minus beta upon 2. So, this element will be 1 minus 3 alpha into 4 minus beta upon 2. Now, now for which values of alpha and beta rank of this matrix is 1 you see, this element is non-zero. So, rank will always be 2 or more than 2 rank is either 2 or 3 rank in never be 1.

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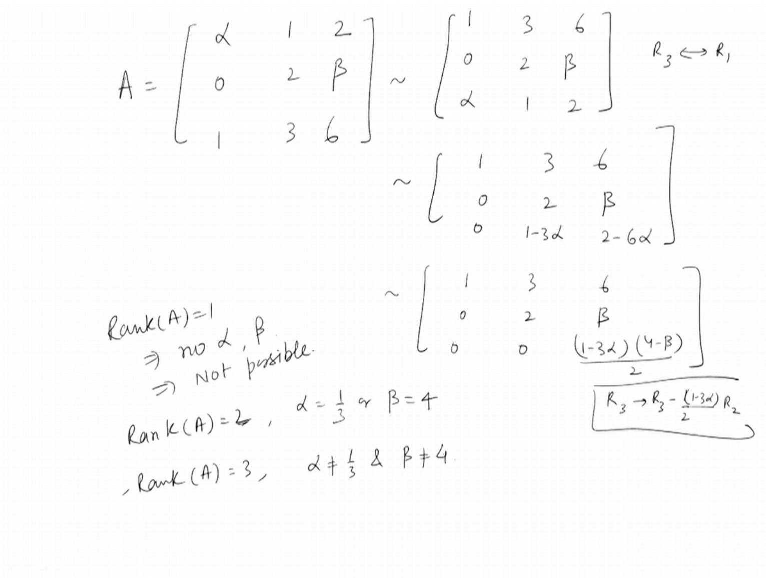

So, the first on the answer is for rank 1. No, alpha beta, because this case is not possible, I mean not possible this case is not possible. For no values of alpha and beta rank is 1, ok.

Now, for rank, rank 2. Now, rank of this matrix is 2 if this quantity is 0, and this quantity because if this is 0 then we are having 1 0 row, ok. Now, this entire row cannot be 0 because this is 2, ok. So, rank will be 2 if either alpha is 1 by 3 or beta is 4 and rank of A will be 3 if this is not equal to 0. So that means, alpha should not equal to 1 by 3 and beta should not equal to 4. So, whenever you want to find out echelon form of any m cross and matrix you first write is echelon form and then try to find its number of non-zero rows. From number of non-zero rows you can find its echelon, I mean from number of non-zero rows you can find its rank.

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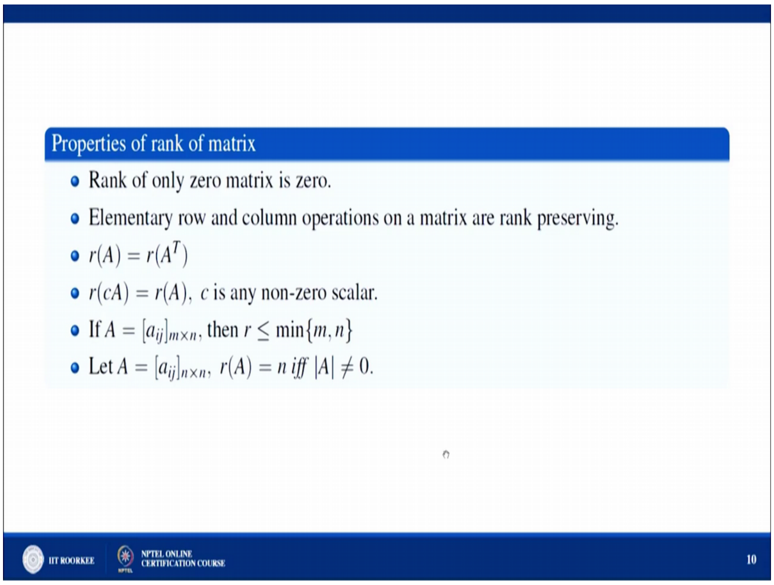

Now, we have some properties of rank of a matrix. The first properties rank of only 0 matrix is 0, only the null matrix has a rank 0, ok. Of course, rank of a matrix can never be infraction because it is number of non-zero rows, ok. Elementary row and column operations of a matrix are rank preserving that means, it does not change the rank of a matrix if you apply a row operations or you apply a column operations, it will not alter the rank of the matrix rank. Rank remain the same because it is number of maximum number of linearly independent rows or columns, and hence rank of a same as rank of a transpose.

Rank of c times A, where c is any non-zero scalar is always equal to rank of A you see. If you multiply a matrix by a non-zero scalar it will not change the number of linearly maximum number of linearly independent rows or columns it is having. So, rank will remain the same. This I already explained a rank of A is always less than equals to minimum of m or n.

Now, if you have a square matrix of order n cross n and rank is n. Now, if rank is n this means matrix does not have any all 0 row because it order of the matrix is n cross n and rank is n that means, it does not have any in the echelon form of the matrix it does not have any row containing all 0 elements. And that means, the determinant of the matrix is not equal to 0, because we have already discuss this thing that suppose A is a matrix and

B its echelon form, B is its echelon form then determinant of A is some alpha time determinant of B, where alpha is not equal to 0, ok.

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$$A \longrightarrow B \text{ (Echelon)}$$

$$|A| = \alpha\,|B|, \quad \alpha \neq 0$$

So, if this if echelon form does not have any 0 I mean all 0 row it is a square matrix, then determinant is not equal to 0. So, determinant of A will not be equal to 0, and if determinant is not equal to 0 that means, the echelon form of the matrix does not have any row which contain all 0 that means, rank of that matrix is n.

So, from here we can also conclude that if rank of a matrix is less than n which is n minus 1, n minus 2 or any other thing, then determinant of a is always 0. Because if rank of a matrix is less than n minus 1 that means, it contains at least 1 row containing all 0 element. So, determinant of the echelon form of the matrix is 0 and hence the determinate of the original matrix is also 0, ok. So, these are some of the property of rank of a matrix.

Thank you.