Matrix Analysis with Applications
Dr. S. K. Gupta
Department of Mathematics
Indian Institute of Technology, Roorkee

Lecture – 17
More on Special Matrices and Gerschgorin Theorem

Hello friends, welcome to lecture series on Matrix Analysis and Applications. So, in the last lecture, we have seen some special matrices like symmetric matrices, skew symmetric matrices, Hermitian matrices and skew Hermitian matrices. We have seen that, what are the important properties of these matrices. So, there are some more special matrices which we will see in these in this lecture and what are their properties.

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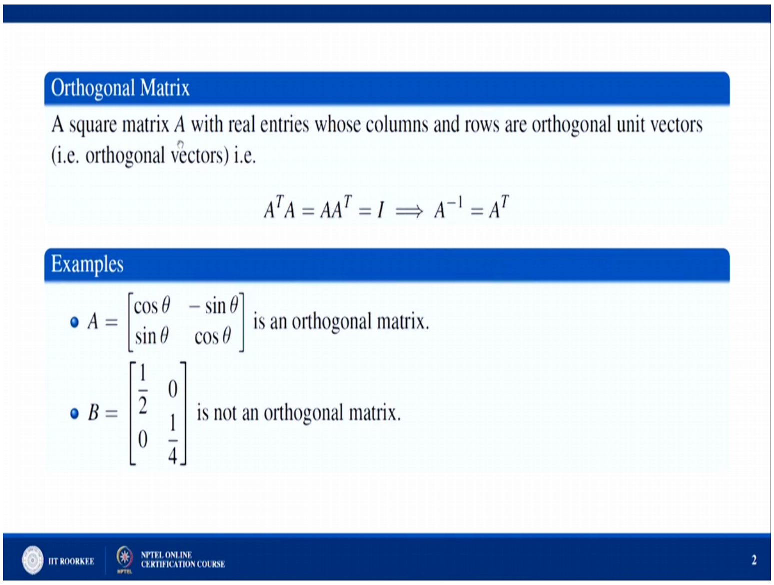

So, first is orthogonal matrix. Now, a square matrix A with real entries whose columns and rows are orthogonal unit vectors that is A transpose A equal to I. An orthogonal matrix is basically a square matrix where A inverse is nothing but A transpose. So, for such type of such type of real matrices are called orthogonal matrices. Now, first example this matrix is a first matrix we can easily verify you see.

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$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$AA^T = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I \quad \Rightarrow \quad \underline{A^{-1} = A^T} \longrightarrow \text{orthogonal matrix}$$

The matrix say simply A is equal to you see cos theta minus sin theta sin theta and it is cos theta when you take A into A transpose it is A. What is A it is cos theta minus sin theta, sin theta and it is cos theta.

And in order to take transpose of this it is cos theta minus sin theta sin theta and cos theta. So, this row into this column so cos square plus sin square which is 1; this row into this column cos into sin minus sin into cos is 0; this row this column there is sin into cos minus cos into sin is 0; and this is sin square plus cos square which is 1. So, it is identity matrix, so that means, A inverse equal to A transpose and hence it is and hence it is orthogonal matrix. However, if you see the second example and if you take B into B transpose for this matrix it is not an identity matrix. So, this matrix is not an orthogonal matrix.

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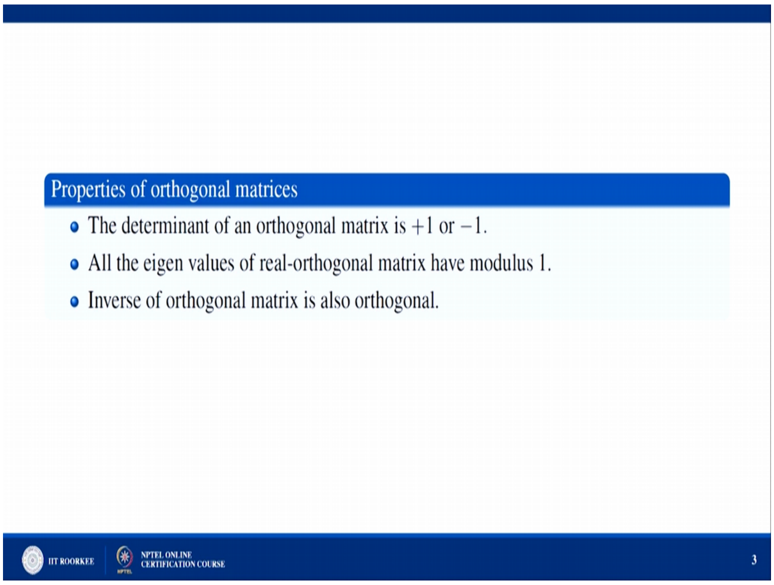

Now, what are the properties of orthogonal matrix? The first property is the determinant of orthogonal matrix is either plus 1 or minus 1. It is very easy to verify you see.

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$$AA^T = I$$
$$|AA^T| = |I|$$
$$\Rightarrow |A|\,|A^T| = 1$$
$$\Rightarrow |A|\,|A| = 1$$
$$\Rightarrow |A|^2 = 1 \Rightarrow |A| = \pm 1$$

Orthogonal matrix are simply A into A transpose is equal to identity. You take determinant both the side determinant of A into A transpose is equal to determinant of I. And this implies determinant of A into determinant of A transpose is equal to 1, because determinant of identity is 1. Determinant of A and determinant of A transpose are same, because by changing rows and columns it will not change the value of the determinant.

So, we can say that it is determinant of A into determinant of A equal to I 1. And this imply determinant of A square equal to 1 which means determinant of A is simply plus minus 1 ok.

So, from here we can say that determinant of an orthogonal matrix is either plus 1 or minus 1. So, this is the first property. Now, all eigenvalues of a real orthogonal matrix have modulus 1. So, what does it mean, how we can prove it, you see you can take any orthogonal matrix with real entries and its eigenvalues always have modulus 1. So, this is very easy to verify you see.

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$$AA^T = I = A^TA \quad \text{— (1)}$$

$$AX = \lambda X, \; X \neq 0 \quad \text{— (2)}$$

$$\bar{A}\bar{X} = \bar{\lambda}\bar{X}$$

$$\Rightarrow A\bar{X} = \bar{\lambda}\bar{X}$$

$$\Rightarrow \bar{X}^T A^T = \bar{\lambda}\bar{X}^T$$

$$\Rightarrow \bar{X}^T A^T A = \bar{\lambda}\bar{X}^T A$$

$$\Rightarrow \bar{X}^T I = \bar{\lambda}\bar{X}^T A$$

$$\Rightarrow \bar{X}^T X = \bar{\lambda}\bar{X}^T AX$$

$$\Rightarrow \bar{X}^T X = \bar{\lambda}\bar{X}^T (\lambda X)$$

$$\Rightarrow (1 - |\lambda|^2)(\underbrace{\bar{X}^T X}) = 0$$

$$\Rightarrow |\lambda| = 1$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$\bar{X}^T X = (\bar{x}_1 \; \bar{x}_2 \cdots \bar{x}_n)\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

$$= |x_1|^2 + |x_2|^2 + \cdots + |x_n|^2$$

$$\neq 0 \quad , \; X \neq 0$$

You see we have a orthogonal matrix definition of orthogonal matrix is A into A transpose equal to identity or it is also equal to A transpose A. Now, you take A X equal to lambda X X is not equal to 0. So, this means lambda is an eigenvalue of A and corresponding eigenvector is X. Now, you take conjugate both the side because lambda maybe complex entry also complex value also. Now, this implies because A is real so A bar is itself so it is A into X bar is equals to lambda bar X bar.

Now, you take transpose both the sides. So, if you take transpose both the side, it will be X bar transpose A transpose is equal to lambda bar X bar transpose because lambda bar is A scalar quantity its transpose will be itself. Now, this will be this implies X bar transpose. Now, you multiply both sides by A matrix A post multiplication. So, you get this expression. A transpose A from this expression from expression 1 is identity. So, this implies X bar transpose identity is equals to lambda bar X bar transpose A.

Now, again you post multiply both the sides by vector X. So, this implies X bar transpose X is equal to lambda bar X bar transpose A X. And A X is nothing but lambda X from 2. So, this implies X bar transpose X is equal to lambda bar X bar transpose lambda X, because A X is equal to lambda X from 2. So, this implies this implies 1 minus mod lambda whole square into X bar transpose X is equal to 0, because lambda into lambda bar is mod lambda square.

Now, X bar transpose X is never 0, because if X is suppose this vector x 1 x 2 up to x n then X bar transpose X will be x 1 bar x 2 bar and so on up to x n bar into x 1 x 2 up to x n. And when you multiply this row with this column, this is mod x 1 square plus mod x 2 and so on up to mod x n square. And it will be 0 only when all x'is are 0, because modulus are the positive quantities.

So, this will be never be 0, because x is not 0, because x is not 0, hence this is not 0, because this is 0 only when x is 0 ok. So, this quantity is not 0. So, this implies mod of lambda is equal to 1. So, we can say that all the real all the eigenvalues of an orthogonal matrix have modulus 1. Now, the next property is inverse of an orthogonal matrix is also orthogonal.

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$A \rightarrow$ orthogonal matrix $\Rightarrow \quad AA^T = A^TA = I$

$B = A^{-1}$ To show: B is also an orthogonal matrix

$$BB^T = I$$

$$\begin{aligned} BB^T &= (A^{-1})(A^{-1})^T \\ &= (A^{-1})(A^T)^{-1} \\ &= (A^TA)^{-1} \\ &= I^{-1} \\ &= I \end{aligned}$$

$B^TB = I \quad , \quad BB^T = B^TB = I$

So, suppose you have a A which is an orthogonal matrix. So, since it is an orthogonal matrix. So, what does it imply, it imply A into A transpose is equal to A transpose A is equal to identity. Now, you take B as inverse of A. And you have to show that B is also orthogonal matrix. So, we have to prove that B is also an orthogonal matrix. So, in order to show this, what we have to show, we have to show that B into B transpose is equal to identity or B B transpose B equal to identity.

Now, what is B, B is A inverse. You take this side left hand inside into A inverse whole transpose, this is equal to A inverse into A transpose inverse. This is equal to A transpose into A whole inverse, because A into B whole inverse is B inverse A inverse. And this is equal to I inverse because A transpose A is I. So, this is I. So, similarly this is B transpose B we can show is equal to I, and hence we can say that B B transpose is equal to B transpose B is equal to I. So, this implies B is also an orthogonal matrix. So, these are the property of orthogonal matrices.

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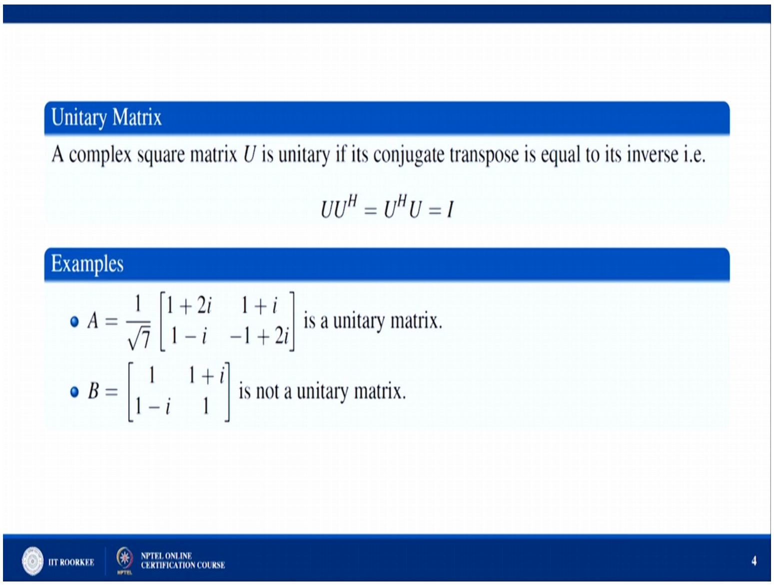

Next is unitary matrix. Now, a complex unitary matrix is a complex square matrix in which conjugate transpose is equal to its inverse. If you take the conjugate transpose and is equal to its inverse, then we say that that matrix is a unitary matrix. Now, for example, you consider this matrix A. So, what is matrix A here.

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$$A = \frac{1}{\sqrt{7}} \begin{pmatrix} 1+2i & 1+i \\ 1-i & -1+2i \end{pmatrix} \qquad \bar{A}^T = A^H = \frac{1}{\sqrt{7}} \begin{pmatrix} 1-2i & 1+i \\ 1-i & -1-2i \end{pmatrix}$$

$$A A^H = \frac{1}{7} \begin{pmatrix} 1+2i & 1+i \\ 1-i & -1+2i \end{pmatrix} \begin{pmatrix} 1-2i & 1+i \\ 1-i & -1-2i \end{pmatrix}$$

$$= \frac{1}{7} \begin{pmatrix} 1+4+1+1 & 0 \\ 0 & 7 \end{pmatrix} = I$$

A is an unitary matrix

Here A is 1 upon under root 7 times 1 upon root 7 times it is 1 plus 2 iota 1 plus iota 1 minus iota and minus 1 plus 2 iota. Now, if here you take A conjugate transpose this which is A H we denoted by A H if you find this it is one upon under root 7 it is 1 minus 2 iota 1 minus iota 1 plus iota minus 1 minus 2 iota.

Now, if you take A into A H which is A conjugate transpose, it will be equal to 1 upon 7 times 1 upon 2 1 plus 2 iota 1 plus iota 1 minus iota minus 1 plus 2 iota, it is A ; and A conjugate transpose is 1 minus 2 iota 1 minus iota 1 plus iota and minus 1 minus 2 iota. Now, when you multiply these two matrices what we obtain you see, you see this row this column. This is nothing but 1 plus 4 plus 1 minus 1.

When you multiply this row with this column it is yeah ok, we have you see you see when you find A conjugate transpose here, so you have to take the conjugate also you have to take the transpose also. So, when you take the transport here, so what you will obtain it is it is 1 plus iota and it is 1 minus iota. You have to take you have to take the conjugate and then that then the transpose also. So, here also it is 1 plus iota and here it is 1 minus iota.

Now, when you multiply this row with this column, it is 1 plus 4 and it is 1 plus 1. When you multiply this row with this column, you can verify easily that it comes out to be 0. You can verify. And this row this column, when you multiply this row with this column, it is 0 and it is again something 7. So, what we obtain is simply identity; this you can easily verify by simple mathematical calculations. So, we can verify that A into A conjugate transpose is identity, hence we can say that A is an unitary matrix.

Now, if you if you see if you take this matrix B, and if you multiply this matrix B by the by its conjugate, conjugate transpose, then it is not coming out to be an identity matrix. So, we can say that it is not a unitary matrix.

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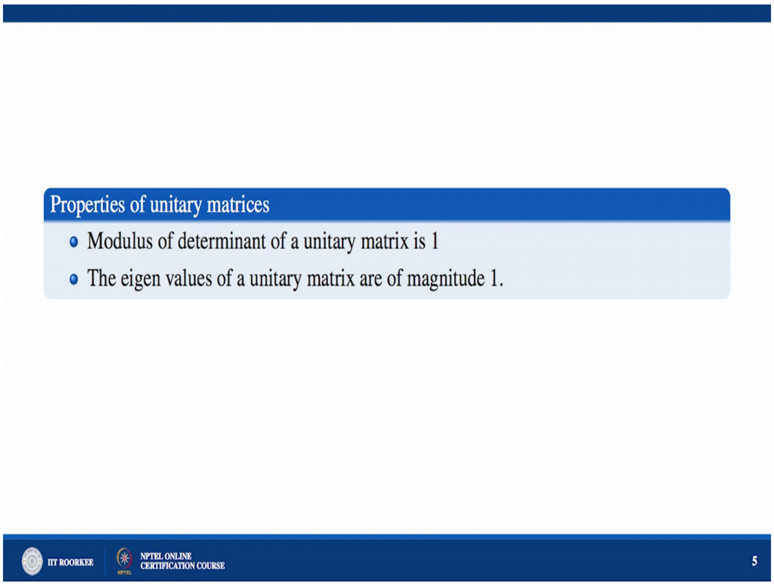

Now, what are the properties of unitary matrices? The first property is modulus of determinant of a unitary matrix is 1. The second property is the eigen value unitary matrix are of magnitude 1. So, this property we can easily show you see.

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$$AX = \lambda X, \quad X \neq 0$$

$$\bar{A}\bar{X} = \bar{\lambda}\bar{X}$$

$$\bar{X}^T\bar{A}^T = \bar{\lambda}\bar{X}^T$$

$$\bar{X}^T\bar{A}^T A = \bar{\lambda}\bar{X}^T A$$

$$\bar{X}^T I = \bar{\lambda}\bar{X}^T A$$

$$\bar{X}^T X = \bar{\lambda}\bar{X}^T A X$$

$$\bar{X}^T X = \bar{\lambda}\bar{X}^T \lambda X$$

$$(1 - |\lambda|^2)(\bar{X}^T X) = 0$$

$$\because \bar{X}^T X \neq 0 \Rightarrow |\lambda| = 1$$

This follows a same line as we did for orthogonal matrices. You see here A X equal to lambda X; X is not equal to 0. You take bar both the sides. Now, you take transpose. When you multiply both sides by now multiply both sides by matrix A post multiplication. A bar transpose A is identity. Again post multiplication by vector X, and it is X bar transpose X is equals to lambda X bar transpose lambda X lambda bar. And it is 1 minus mod lambda square again same lines for same lines as we did for orthogonal matrices. Now, since X bar transpose X is not equal to 0. So, this implies mod lambda is equal to one. So, like orthogonal matrices unitary matrices are also have eigenvalues unitary matrix are also have modulus 1.

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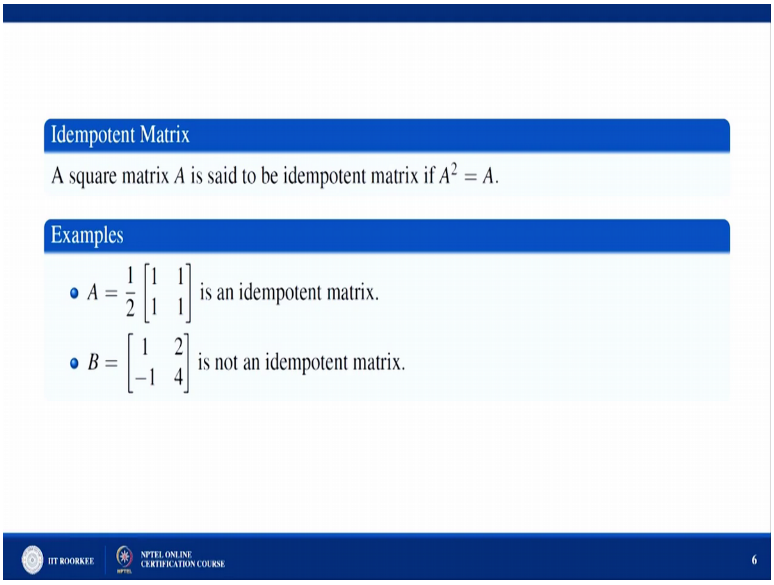

Now, square matrix now idempotent matrix, A matrix A is said to be idempotent if A square equal to A. You can easily verify if you square this matrix. So, you get matrix itself I mean A itself; however, square of this matrix is not an idempotent. So, these are very easy to verify.

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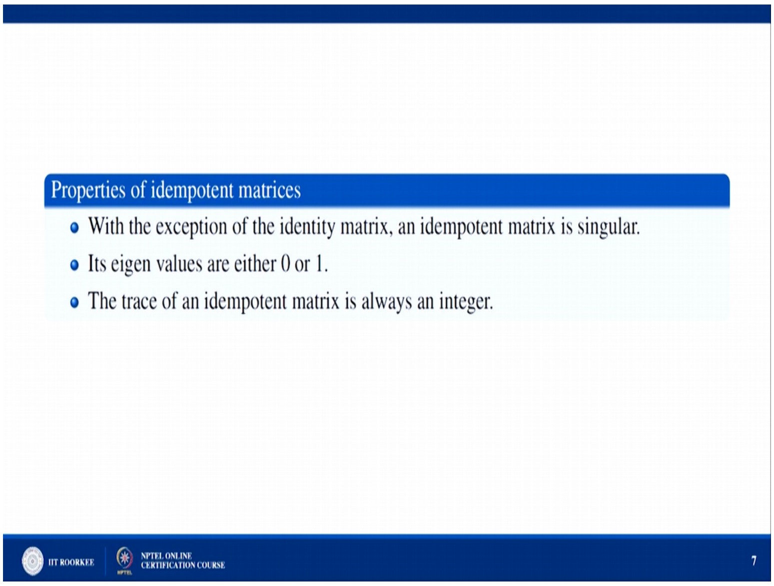

Now, what are the properties of idempotent matrices. The first property is with the exception of identity matrix and idempotent matrix is singular. Identity matrix is idempotent you see I square is equal to I. But if A is not equal to I you see, idempotent matrix is what A square equal to A.

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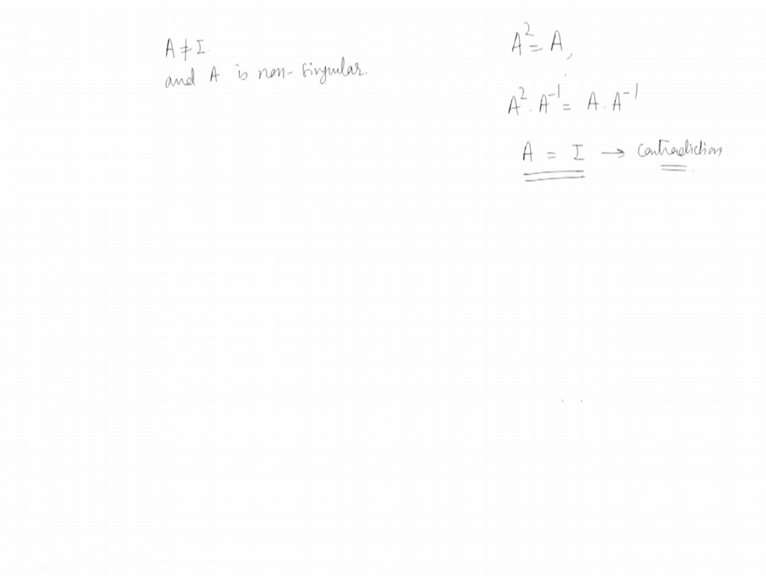

Suppose A is not equal to I, and A is non singular ok; non singular means inverse exist. And if you apply A inverse both the sides here, it is A square into A inverse which is equals to A into A inverse. It is betweens A is equal to identity have which is not which is not here, because A is not equal to identity contradiction.

So, that means, that means, if A square equal to A that means, idempotent matrices and if A not equal to I it must be singular, because if it is non singular then we are getting contradiction. So, that means, an idempotent matrix as (Refer Time: 15:46) identity matrix is always singular that is inverse does not does not exists. Next is this eigenvalues are either 0 or 1. So, again let us try to prove it because A square equal to A.

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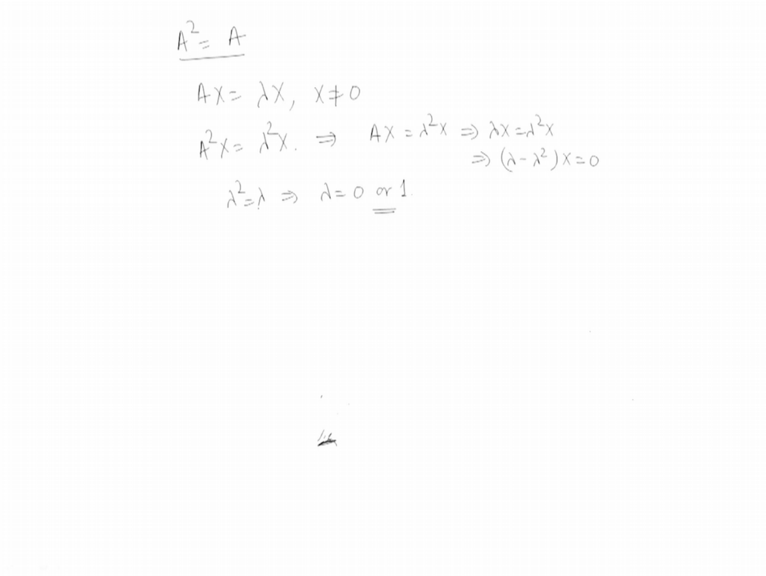

Now, A X equal to lambda X, X not equal to 0, so that means, A square X is equals to lambda square X as we have already know this thing and. So, so that means, so we what we have concluded from here we have concluded that lambda square is equal to lambda because A square equal to A. So, we have concluded lambda square equal to lambda.

So, lambda square equal to lambda. So, the it implies lambda is either 0 or 1, because A square is equal to X because here you can see that A X A square equal to A is equals to lambda square X. And this implies A X is lambda X and this implies lambda minus lambda squared times X is not equal to 0. And since X is since X is not equal to 0, so lambda square equal to lambda hence lambda is either 0 or 1. Now, since eigenvalues are either 0 or 1 so sum of Eigenvalues, which is equal to trace of a matrix is always an integer in fact a positive integer. So, trace of an idempotent matrix is always a positive integer.

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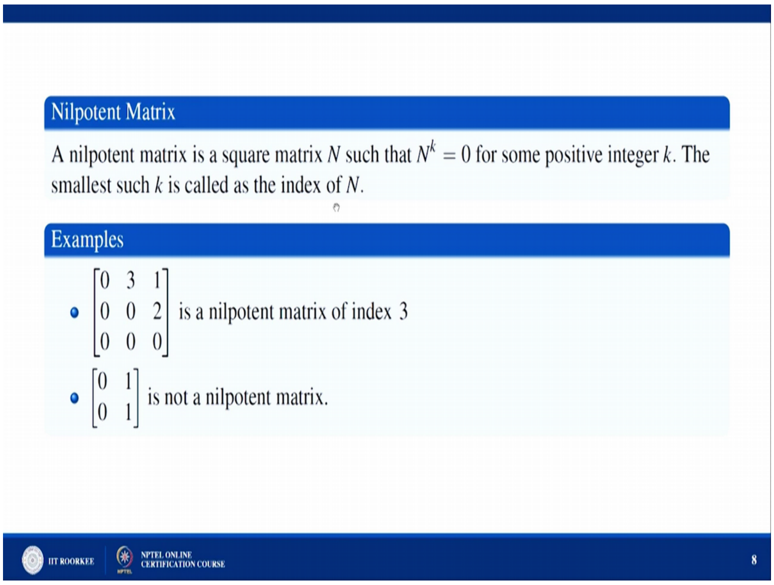

Next is nilpotent matrices. Now, a nilpotent matrix is a square matrix N such that N raised to power k is 0 for some positive integer k ok. And the smallest such k is called an index of matrix N. So, suppose you consider this matrix.

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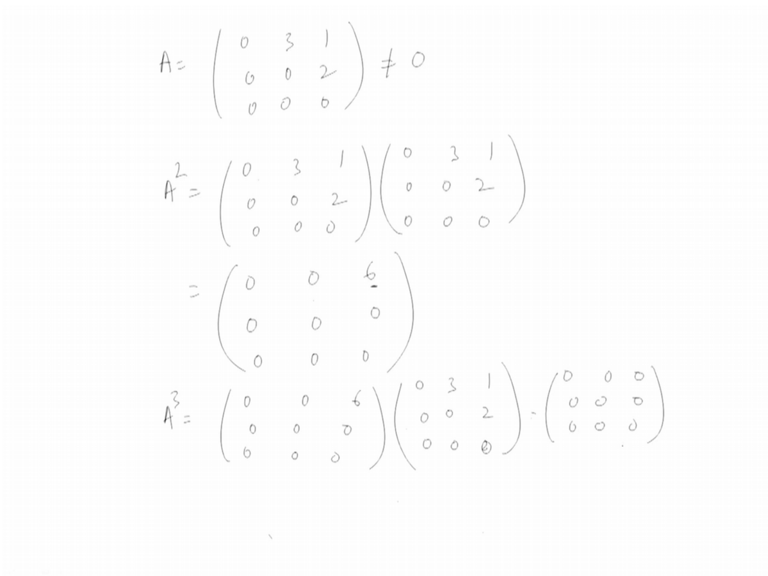

So, this matrix is basically this is 0 3 1 0 0 3 1 0 0 2 0 0 0. Now, when you take a square of this matrix this is not equal to 0, I mean 0 matrix. When you take a square of this matrix it is 0 3 1 0 0 2 into 0 0 0 into 0 3 1 0 0 2 0 0 0; then you can easily verify this row this column is 0; this row this column is this row this row this column is again 0 this row this column is 6. This row this column is 0; this row with this column is 0; this row with this column is 0; this row with this column all other entries are 0.

So, let us check this entry once again. So, this row with this column both the last last column; this row with this column is 0, it is 6, it is 0 though it is 6. Now, when you find A cube A cube of this matrix that is A into A A square into A. So, what we obtain this row this column is 0, this row this column is 0, this row this column is 0 and all other entries are 0. So, it is a simply a 0 matrix. So, what we have concluded we have concluded that this A matrix is a nilpotent matrix of order 3.

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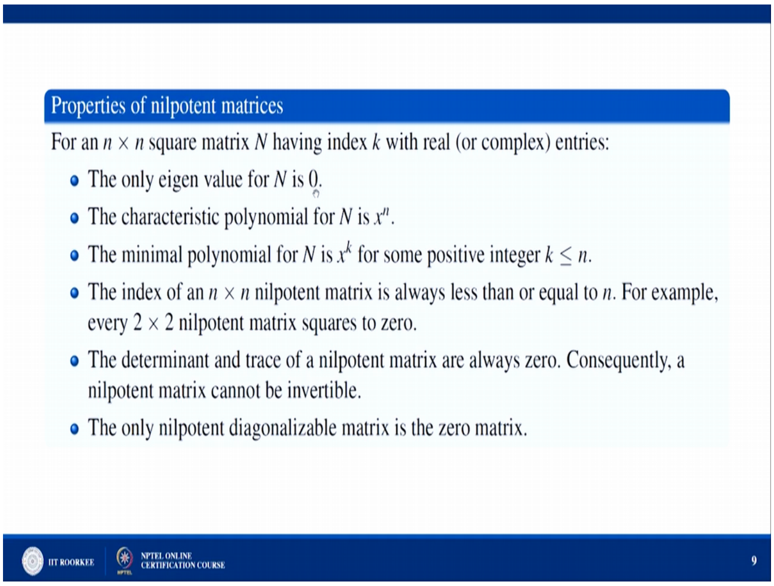

Now, what are the property of nilpotent matrices the first property is. The only eigen value of N is 0 I mean if N is a nilpotent matrix it has only eigen value of this is 0 so that is easy to show you see.

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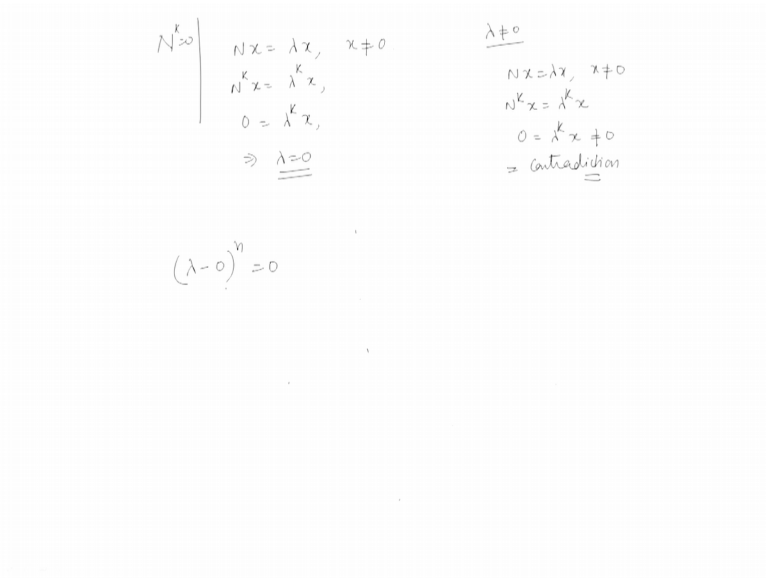

Suppose N is a nilpotent matrix then that means, N x is equal to lambda x x is not equal to 0. Then N raised to power k x is also equal to lambda raised to power k x. And N raised to power k is 0 because N is a nilpotent matrices. So, 0 is equals to lambda raised to power k x. So, this implies lambda equal to 0 because x is not equal to 0. So, the only eigen value of nilpotent matrix is equal to 0. So, this can also be proved suppose N is a nilpotent matrix and lambda is not equal to 0 let us suppose.

So, if lambda is not equal to 0, then we obtain then N x equal to lambda x for x not equal to 0; N raise to power k x is equal to lambda raise to power k X now the 0 is equal to lambda raised to power k x. And this implies this is a contradiction, because lambda is not equal to 0, x is not equal to 0 which is not equal to 0. So, this is a contradiction basically. So, this means lambda cannot be lambda is lambda cannot be I mean cannot be the only eigenvalue of lambda is 0 only. It cannot be different from 0.

The next is the characteristic polynomial for N is x raise to power n. Now, since all the eigenvalues of lambda all the eigenvalues of n is are 0. So, the characteristic polynomial of N will be lambda minus I mean 0 raised to power n equal to 0 because 0 is the only eigenvalue of N so that is why it is x raise to power n.

Now, the minimal polynomial of N is of course, x raised to power k here k is less than equal to n, because minimal polynomial is the lowest degree polynomial to satisfy n. Now, next is the index of an n cross n nilpotent matrix is always less than equal to n index is always less than equal to n that is very easy to show because Caylay Hamilton theorem is always satisfied. If the minimal if k this k in the minimal polynomial is less than n so that will be the order I mean index of the nilpotent matrix otherwise by the Caylay Caylay Hamilton theorem n is the index. So, this point can be easily verified using the using these two points.

For example, every 2 cross 2 square matrices matrices squares to 0 always 2 cross 2 non zero matrices in fact. Now, the determinant and trace of nilpotent matrix is always 0 because all eigenvalues are 0, so trace would be 0 sum of eigenvalues and the determinant will be 0 which is equal to product of the eigenvalues of course. So, consequently a nilpotent matrix cannot be invertible. The only nilpotent diagonalizable matrix is a 0 matrix.

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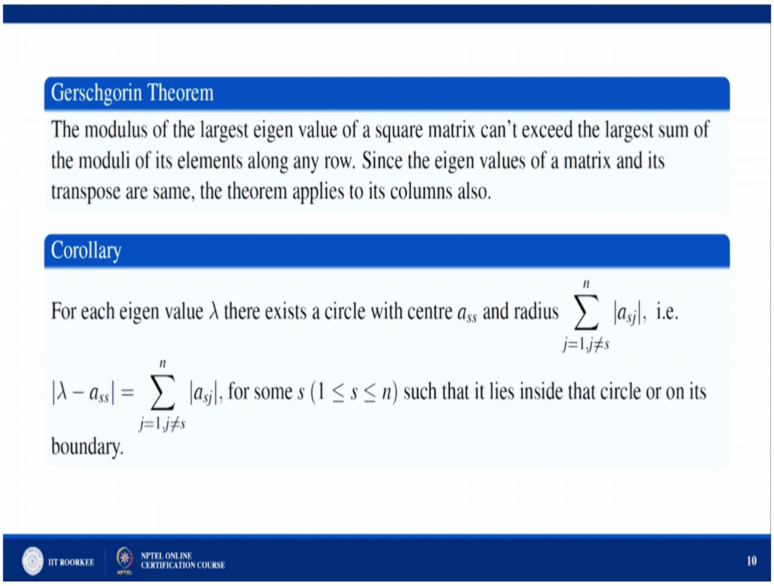

Now, next is Gerschgorin theorem. Now, what is Gerschgorin theorem what it is state let us see. The modulus of the largest eigen value of a square matrix if you want to see what is the largest eigen value of the modulus of largest eigen value of a matrix then this theorem by this theorem, we can find at least the range. The modulus of the largest eigen value of a square matrix cannot exceed the largest sum of the moduli of its element along any row. Since, the eigen value of a matrix and its transpose are same, so the theorem is also applicable to column also.

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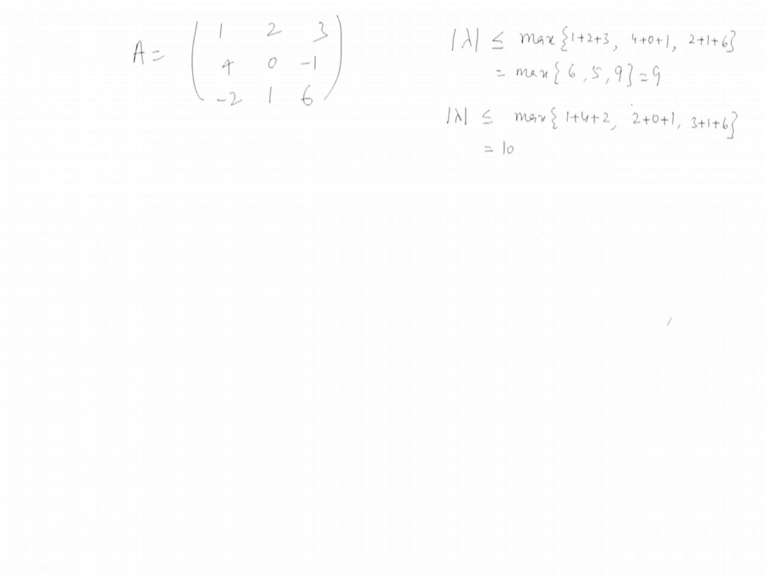

So, what this theorem means basically this theorem means suppose you have a matrix like this say 1 2 3 4 0 minus 1 minus 2 1 6. So, this theorem states that that modulus of lambda, lambda is A eigen value largest eigen value of this. It will be less than equals to maximum of maximum of mod of 1 is 1; mod of 2 is 2; mod of 3 is 3 that is 1 plus 2 plus 3 4 plus 0 plus 1 because modulus of minus 1 is 1. And it is 2 plus 1 plus 6. So, this is equal to maximum of it is 6 5, and it is 9 so which is 9.

Now, when you see the column you see mod lambda is less than equals to maximum of column you when you see it is 1 plus 4 plus 2 1 plus 4 plus mod of this is 2, it is 2 plus 0 plus 1 and it is 3 plus 1 plus 6 which is 10. So, we can say that mod of maximum lambda of this eigen this matrix is less than equal to is less than equal to I mean 9 the intersection of these two. So, how how can we prove this theorem, how can we show this theorem, so proof is simple you can see.

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$$AX = \lambda X, \quad X \neq 0 \qquad X = (x_{1r}, x_{2r}, \ldots, x_{nr}) \neq 0$$

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n1} & \cdots & \cdots & a_{nn} \end{pmatrix}$$

$$\Rightarrow a_{i1}x_{1r} + a_{i2}x_{2r} + \cdots + a_{in}x_{nr} = \lambda_r x_{ir}, \quad i = 1, 2, \ldots, n$$

$$|\lambda_r x_{ir}| \leq |a_{i1}||x_{1r}| + |a_{i2}||x_{2r}| + \cdots + |a_{in}||x_{nr}|$$

let $|x_{sr}| = \max\{|x_{1r}|, |x_{2r}|, \ldots, |x_{nr}|\}$

$$|\lambda_r| \frac{|x_{ir}|}{|x_{sr}|} \leq |a_{i1}| + |a_{i2}| + \cdots + |a_{in}|, \quad \left|\frac{x_{ir}}{x_{sr}}\right| \leq 1, \ \forall i$$

$$i = s, \quad |\lambda_s| \leq |a_{s1}| + |a_{s2}| + \cdots + |a_{sn}| = \sum_{j=1}^{n} |a_{sj}| \leq \max_i \left(\sum_{j=1}^{n} |a_{ij}|\right)$$

You see if you have a you take A x equal to lambda x x is not equal to 0. Now, you suppose x is suppose x 1 r x 2 r and so on up to x say n r. We are it which is not equal to 0. So, you see A is this matrix A is a 1 1 a 1 2 and so on up to a 1 n a 2 1 a 2 2 and so on up to a 2 n and so on a n 1 and so on up to a n n.

So, what we will obtain you see you have to multiply each row of a with X. So, this implies it is a a i 1 x 1 r plus a i 2 x 2 r and so on up to a i n x n r is equal to lambda r x i r and this i is from 1 to n. You when you put i equal to 1 it is a 1 1 x 1 r a 1 2 x 2 r and so on up to a a 1 n x n r which is lambda r x 1 r, and similarly for i equal to 2 to n. Now, you take modulus both the sides and mod of this because mod of a plus b is less than equals to mod of a plus mod of b. So, we can say that is less than equals to mod of a i 1 into x 1 r plus mod of a i 2 into x 2 r and so on mod of a i n into x n r.

Now, let x suppose s r it is s r is a maximum of mod x 1 r mod x 2 r and so on mod x n r. Suppose, out of maximum of modulus of these xi's x s r is a maximum one ok. And of course, it is not equal to 0 because x is not equal to 0 so maximum cannot be 0. Now, you can divide now you divide this equation by this by mod of this x s r. When you divide both the sides, so inequality will not change; and it is less than equals to mod a i 1 plus mod a i 2 plus and so on plus mod a i n. Because mod of x i r mod of x i r upon mod of x s r is less than equal to 1 for all i because this is a maximum one. So, we can all we can get this inequality.

Now, this is valid for every i. So, this is valid for i equal to s also. So, for i equal to s what we obtain mod of lambda s is less than equals to mod of a s 1 plus mod of a s 2 plus and so on mod of a s n so which is equals to summation of j varying from 1 to n mod of a s j. Now, if it is less if it is less than equal to sum of this, so it will be less than equal to maximum of maximum over i basically sum of j from 1 to n mod of a i j.

So, so what does it mean, it mean that sum of sum modulus of some other rows. So, we can say that the eigen value of matrix A is less than equals to maximum of sum of the rows. Now, we have one result based on this that is for each eigenvalue lambda there exists a circle with centre a s s and radius this radius this such radius this such that it lies inside the circle or on its boundary.

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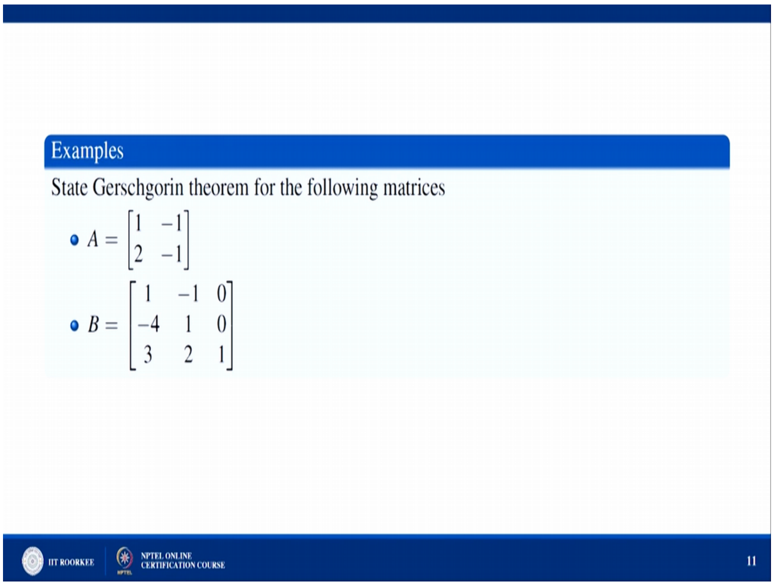

Let us verify this result by a example suppose, we have this matrix A.

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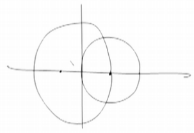
$$A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix} \qquad \left.\begin{aligned} \lambda_1 + \lambda_2 &= 0 \\ \lambda_1 \lambda_2 &= -1+2 = 1 \end{aligned}\right\}$$

$$-\lambda_1^2 = 1 \Rightarrow \lambda = \pm i$$

$$\lambda_1 = i, \quad \lambda_2 = -i$$

$$|\lambda - 1| \leq 1,$$

$$|\lambda + 1| \leq 2$$



What is the matrix A here. The matrix A is 1 minus 1 2 minus 2 2. So, what are eigenvalues of this matrix if you quickly see, so eigenvalues of this matrix are simply sum of eigenvalues is 0 and the product of eigenvalues is simply minus 1 and plus 1 plus 2 ok, so it is 1. So, eigenvalues what are eigenvalues then that we can easily find out you solving these two equations. So, these are nothing but you see you see minus lambda 1 square is 1. So, lambda equal to lambda equal to plus minus iota. So, one lambda is iota; another lambda is minus iota. This you can easily verify; sum is of 0 and the product is 1.

Now, now if you see lambda minus a s s here first the first row is 1 is less than equals to sum of these two other than diagonal elements that is one. Now, lambda plus 1 centre here is minus 1 less than equals to excluding this entry because we are we are we are excluding j equal to s here. So, that is why so it is equal less than equal to 2.

Now, when you draw these two here centre is 1 and radius is 1; centre is 1 radius is one means this circle. And here centre is minus 1 and radius is 2 radius is 2 mean this circle something like this. Now, when you see the first lambda which is iota its modulus is 1; its modulus is 1 so it is inside the circle that means, inside the second circle. And its eigenvalue its modulus is also 1 and it is also inside a second circle.

So, what this is what this theorem states that there exists in A for each eigenvalue lambda there exist a centre a s s and the radius such that it lies inside a circle or on its boundary. So, the same theorem can also be verified on this example. So, hence we have seen that what are the other what are the properties of some special matrices like a unitary matrices, orthogonal, idempotent and so on and also how we can find out the range of eigen values if matrix given to us using Gerschgorin theorem.

Thank you.