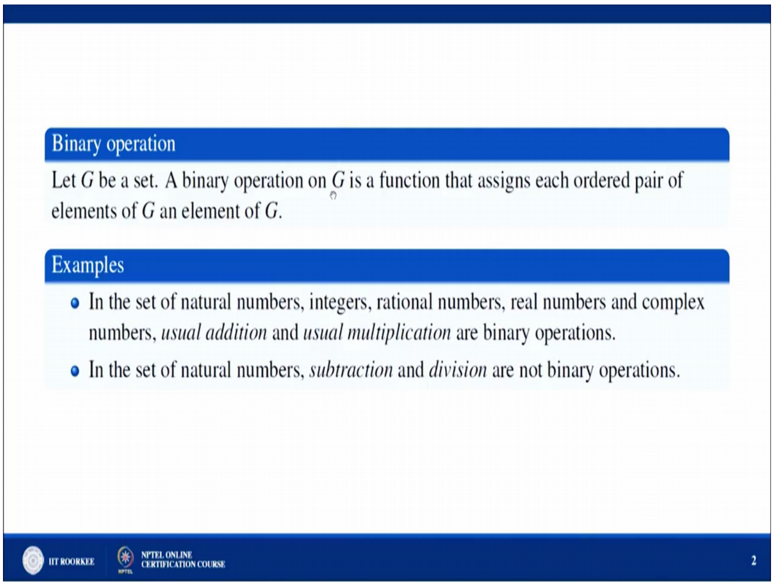
Matrix Analysis with Applications
Dr. S. K. Gupta
Department of Mathematics
Indian Institute of Technology, Roorkee

Lecture - 01
Elementary Row Operations

Hello friends. Welcome to lecture series on Matrix Analysis with Applications. So, this is the first lecture and this lecture deals with Elementary Row Operations. So, what elementary row operations are and how it is applicable to solve linear system equations, we will see in first few lectures.

(Refer Slide Time: 00:44)



So, before stating what elementary row operations are first we define binary operations.

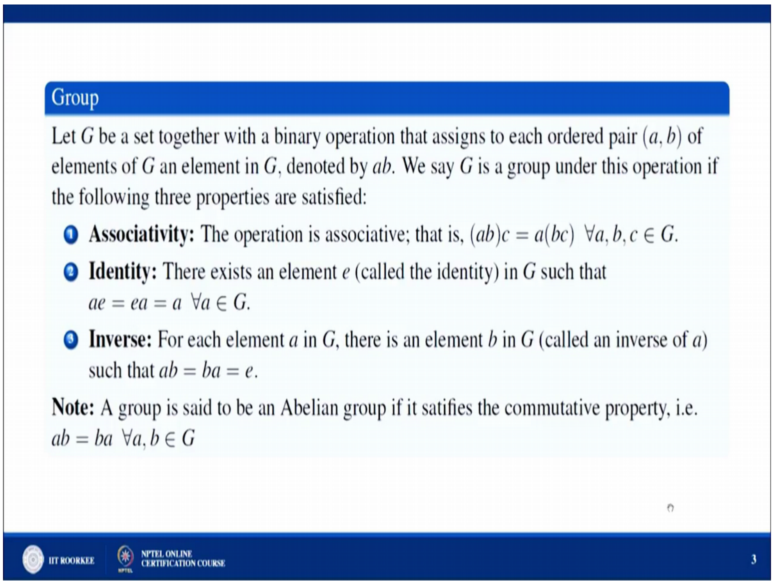
So, let G be a set a binary operation on G is a function that assigns each ordered pair of elements of G an element of G. So, what does it mean? It means if G is a set you take any two arbitrary element in that set apply the operation given to you and the resulting element if it is also belongs to a same set then we say that the operation is a binary operation. Like you take the set of natural numbers ok, and you apply usual addition.

So, we know that we if we take any two arbitrary natural numbers and we apply usual addition operation usual addition on the natural numbers then the resultant is also a natural number. That means the usual addition for the set of natural numbers is a binary operation. Similarly, if you take it set of integers here ok. And, you apply usual addition, you take any two integers add them the resultant is also a integer; that means, the usual addition over the set of integers is a binary operation. Similarly, you take the set of rational numbers or the real numbers or complex numbers under usual addition they are usual addition is the binary operation.

Now, similarly if you take the set of natural numbers and you take binary operation as usual multiplication ok. If, you take any two natural numbers you multiply them the resultant is also a natural number; that means, the usual multiplication over the set of natural numbers is a binary operation ok. Now, similarly set of integer's rational numbers, real numbers and complex numbers, the usual multiplication is the binary operation. Now, if you take a set of natural numbers, subtraction and divisions are not are not binary operations.

Suppose you take two natural number say 1 and 2, 1 minus 2 is equal to minus 1, which is not an integer. That means, subtraction over set of natural numbers is not a binary operation. Similarly, if you take division you say you take two natural numbers say 2 and 3 you divide them 2 upon 3. So, 2 upon 3 is not a natural number so; that means division is not a binary operation.

(Refer Slide Time: 03:52)



So; that means, binary operation is the operation, which when applying any two elements of a set G the resultant element must be in the same set. Then, that operation is called binary operation.

Now, come to group when we say that a set is the group under some binary operation. So, let G be a set together with a binary operation that assigns to each ordered pair a comma b of the elements of G an element in G, denoted by a into b or ab. We, say G is the group under this operation if the following three properties are satisfied ok. First of all the operation, which we are defining on the set G is the binary operation.

Binary operation means is satisfy closure property; that means, you take any two arbitrary element on the set G the resultant element is also in G ok. And what are the other three properties which are set G should satisfy to form a group number 1: associativity. Associativity means you take any 3 arbitrary element abc in G. If, you take brackets in the first two elements or you take the bracket in the last two elements, the values are same that is associativity ok.

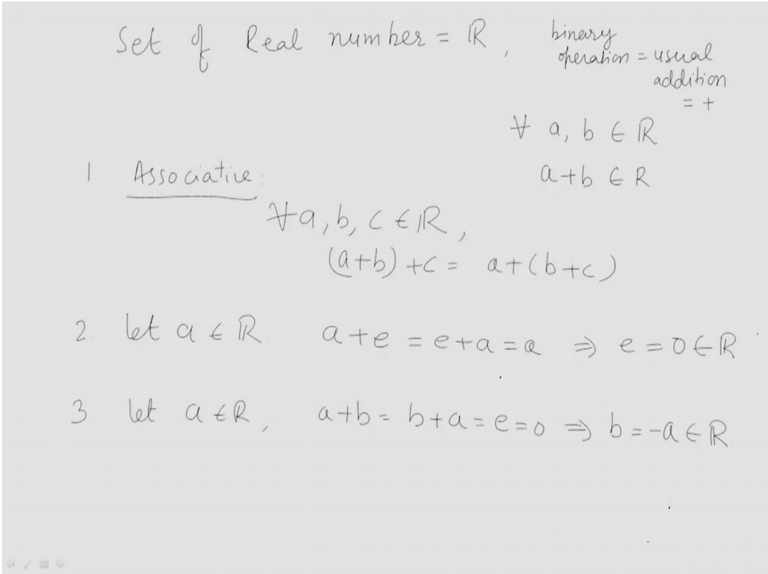
Then the identity; identity means if there existed element e which is also called the identity in G such that a e equal to ea equal to a for every a in G. Then the third properties inverse means for each a in G, there is an element b in G called the inverse of a such that a b equal to ba equal to e, ok.

So, if d three property hold on a binary operation applied on G, then we say that the G under that binary operation is the group. So, what are the four properties which we have discussed?

Number one the operation must be an a binary operation, number 2 associativity number 3 identity; identity means you take any arbitrary element a in G there exist e such that a e equals to ea equals to a for every a in G. Inverse means for any a in G there exists a element b in G such that this result hold. Moreover this group will be called an Abelian group, if it satisfies commutative property also. That, means, ab equals to b a b equal to b a for all ab in G ok. So, let us discuss this by an example.

Suppose you take a set of real numbers set of real numbers ok.

(Refer Slide Time: 06:38)



Let us (Refer Time: 06:45) denoted by capital R ok. And under which binary operation it must be mentioned. So, binary operation which we are defining here is suppose usual addition. So, we know that usual addition on the set of real numbers on the set of real numbers is a binary operation, because if you take any two elements a comma b belongs to R, then a plus b if this is usual addition we are denoting by plus then a plus b is also in R for all a b in R.

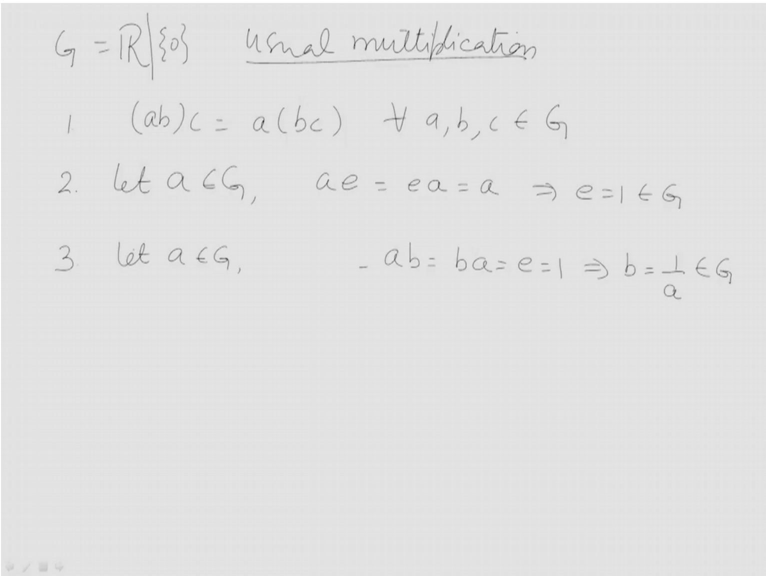
So; that means, this plus which is applying on the two elements a b in R is the binary operation ok. Now, if we have to see that are whether this set of real numbers under this binary operation forms a group or not. So, it so, this is the binary operation now we have to see the other three properties are satisfying or not number one property is associate associativity a associative property.

So, associative property is obviously satisfied because addition always satisfied associativity, you take you for a for all abc in R in R a plus b plus c is equals to a plus b plus c. You take bracket in the first two elements or you take bracket in the last two elements resultant is same.

Now, next is to see whether identity at exist or not you take any element a in R ok. Now a plus e is equal to e plus a must be a for any a in R and this implies e equal to 0 and 0 is in R. So, this belongs to R; that means, for any a in R identity element is 0, which exist and is the and belongs to the set of natural real numbers. The third property is inverse you take any a in R then a plus b plus is equal to b plus a should be equal to e e is 0.

So, this implies b is equal to minus a, which also belongs to R, suppose you want to find out the inverse of 2 2 is the real number. So, inverse of 2 is minus 2 which is also in R. So, we have shown that all the properties are satisfied; that means this set R over the binary operation usual addition forms a group. Now, if we see the same set of real numbers same set of real numbers over multiplication if you say same set of real numbers, over usual multiplication usual multiplication.

(Refer Slide Time: 10:06)

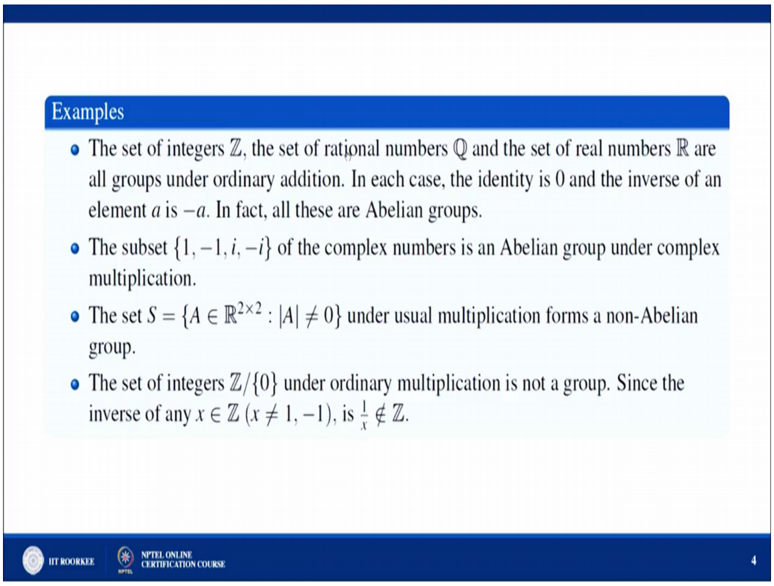


So, we have to exclude 0 because inverse of 0 is not defined here. So, we are here to exclude 0 ok.

Now, if we are taking this set the set of real number excluding 0 under unusual multiplication it forms a group. Usual multiplication is a binary operation because if you take any two arbitrary element from the set from this set say G and multiply them then the resultant is also a real number. So, it is a binary operation. Now, the first property assosiativity holds because a into b into c is equal to a into b into c for all a b in a or a b c in G the second property is identity. You take any element in G say a belongs to G, then a into e should be equals to e into a should be equals to a for all a and G and this implies e equal to 1 which is in G. So, there this means there exist an identity element in G.

And third is inverse if you take if you take any a in G. So, it is a into b should be equals to b into a should be equals to e which is 1. So, this implies b is equals to 1 by a which also belongs to G. Suppose you want to find out multiplicative inverse of 2. So, it is 1 by 2, which is in set of real numbers are excluding 0 so; that means, this set G under binary operation usual multiplication constitute a group.

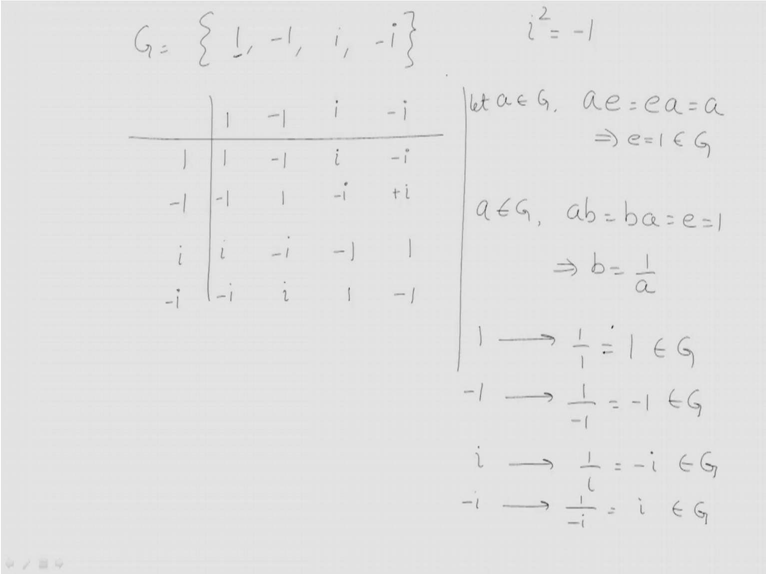
(Refer Slide Time: 12:01)



So, here are some examples you see the set of integers Z, the set of rational number is Q the set of real numbers R are all groups under ordinary addition a usual addition.

In each case the identity element is 0 and the inverse of an element a is minus a. In fact, all these are Abelian groups, because they satisfy commutative property also. If, you take the set of integers you take a into b or b into a resultant is same, the values are same; that means, the set of integers. In fact, forms an Abelian group similarly set of rational numbers set of real numbers also forms an Abelian groups. Now you take this set 1 minus 1 iota minus iota you take this set.

(Refer Slide Time: 12:57)



Now, set is 1 minus 1 iota minus iota, we know that iota square is minus 1. And, what is the binary operation under which binary operation we are seeing that it will it will forms a group or not under usual multiplication. We know that the usual multiplication is a binary operation for this G have why it is binary operation this you can easily see you take 1 minus 1 iota minus iota ok.

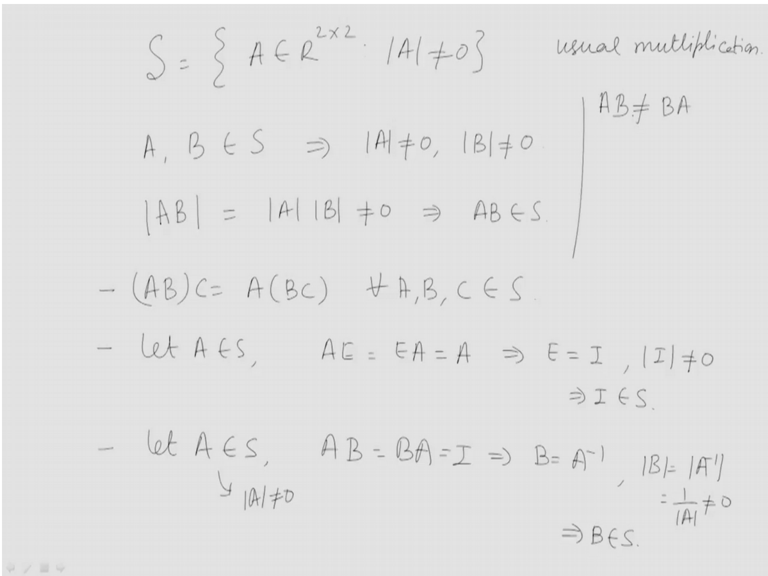
You take 1 minus 1 iota minus iota. Now, you multiply 1 with 1 is 1 1 with minus 1 is minus 1 1 with iota is iota this is minus iota minus 1 with 1 is minus iota, then plus 1 minus iota iota square is minus 1 it is plus iota ok. Iota with 1 is iota it is minus iota iota square is minus 1 minus iota square is 1 it is minus iota it is plus iota it is minus iota square is 1 and it is minus 1. Now, you have you we have seen all the possible multiplication of the elements of G with itself and we have seen that all the elements in this set are in G itself. That means, this usual multiplication on this that G is in binary operation that is clear, because if you multiply any element of G with itself all the elements all the elements are in G itself.

That means user multiplication on this G is binary operation. So, first property is hold now we see we have to see the associative property. So, associativity is always hold in multiplication in usual multiplication is always satisfied, then we have to see identity element has the since of identity element. If you take any a any a in G any a in G then a a into e should be equals to e into a should be e for all a in G this implies e is equals to 1, which is in G you have see you see here this is 1 which is in G so; that means, identity element also exist which is in G. Now, the existence of inverse, if you see the existence of inverse so, you take any a in G the for inverse a b should be equals to ba should be equals to e which is 1.

Then this implies b is equals to 1 by a if you take the inverse of 1 inverse of 1 is 1 by 1 which is 1, which is in G, if you take inverse of minus 1 minus 1 inverse is 1 upon minus 1 which is minus 1 is also in G. Inverse of iota which is 1 upon iota which is minus iota it is also in G and inverse of minus iota is 1 upon minus iota which is iota it is also in G. So, inverse of all the elements exist a identity element exist a sensitivity property holds. So, we say that this set G under this binary operation I mean by usual multiplication constitute a group.

Now, you said this you take this set the set as which is set of all 2 cross 2 matrices, which whose determinant are not equal to 0; that means, invertible matrices of order 2 cross 2. Now, now if you take so, so here binary operation which we are choosing is the usual multiplication it forms a non Abelian group. Now let us see how.

(Refer Slide Time: 16:53)



So, we are taking all those 2 cross 2 matrices, whose determinant are not equal to 0. And, what is the operation we are applying operation is usual multiplication.

Now, you take any A comma B belongs to S, this means determinant of A is not equal to 0 and determinant of B is not equal to 0. If, you take the multiplication of these 2 matrices A into B and take the determinant the determinant of A B is equals to determinant of A into determinant of B, which is also not equal to 0 this implies A B belongs to S; that means, this usual multiplication forms a binary operation.

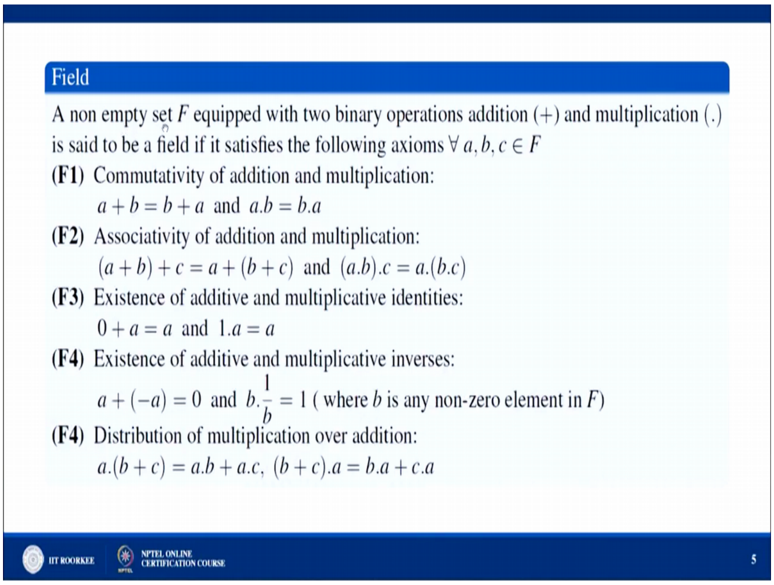
On this set S ok. Now, we have to see associativity the matrix is satisfy associative property we already know that A B into C is same as A B C for all A B C in S, this is this is always satisfied in case of matrices. Now, existence of identity element you take you take any A in S A into some E should be equals to E into A should be A. So, this implies e is equals to I and determinant of I is since is not equal to 0. So, this implies I also belongs to S. So, this guarantees the existence of identity elementiness. Now, we have to see now we have to see existence of inverse element, you take any A in S then A B should be equals to B A should be equals to I and this implies B is equals to A inverse.

Now, inverse exists because determinant is not equal to 0 and this and since determinant is not equal to 0, then determinant of A inverse is also not equal to 0 determinant of B will be what? Determinant of A inverse and which is equals to 1 upon determinant of A which is also not equal to 0 because determinant because from here determinant of a is not equal to 0 and this implies B belongs to S.

So, we have shown the existence of inverse element also in S. So, hence we can say that this as constitute a group under usual multiplication. Now, it is an it is a non Abelian group, why non Abelian because if you multiply A into B or you multiply B into A, they need not be equal A into A B, it need not be equal to B into A for all A B in S ok.

So, it is the group, but it is not an Abelian group. Now, the set of integers Z excluding 0 under ordinary multiplication is not a group, because if you can clearly see, if you identity element is there identity element under multiplication is 1, but if you take element say 2, it is inverse is 1 by 2, which is not a which is not in which is not in this set of integers excluding 0. Hence it will not constitute a group. So, this is all about group. Now, we come to field and then we go to matrices ok. Now what is the field let us see let us quickly see a non-empty set F equipped with two binary operations, addition and multiplication is said to be a field if it satisfies the following axioms for all a b c in F.

(Refer Slide Time: 20:44)



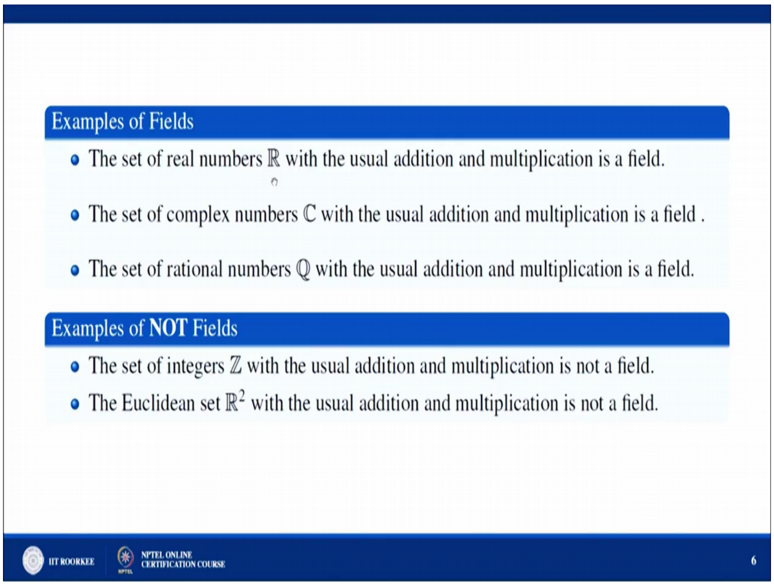
Now, here in fields instead of one binary operation we are having two binary operations. The first binary operation we are denoting by addition, it may be any operation, but we are denoting it by addition and the second binary operation we are denoting by multiplication. Now, the first property is commutativity holds of for addition and multiplication; that means, a plus b should be equals to b plus a for all a b in F and a into b should be equals to b in b into a.

In case in case of both the binary operations number 1 number 2 a associativity of addition and multiplication must hold. Existence of identity existence of additive and multiplicative identities should exist that is 0. We are denoting 0 as the additive identity and 1 as the multiplicative identity. So, 0 plus a should be a and 1 dot a should be a for all a in F.

Then, existence of additive and multiplicative inverses so, here minus a is simply additive inverse of a and 1 by b is simply multiplicative inverse of b where b is a non-zero element in F ok. And the distribution of multiplication over addition, from right also and form left also from left and from right this must hold. So, what I want to say basically that in that in case of field we are having two binary operations, one we are calling as addition, other we are calling as multiplication. So, F respect to addition must be an Abelian group and all non-zero elements in F, over multiplication must constitute and Abelian group and that and the next property is distribution of multiplication over addition ok.

So, if these three property hold; that means, with respect to addition f must be an Abelian group, with respect to multiplication the set excluding 0 in F must be an Abelian group and distribution of multiplication over addition this property must hold. So, if these broadly if these three property holds, then we say that F is the field.

(Refer Slide Time: 23:22)



Let us see few examples based on this now you see the set of real numbers.

Now, if you see set of real numbers and which the binary operation. The first binary operation is usual addition and the next binary operation usual multiplication. Now, usual addition forms an Abelian group for this set of real numbers, we have already seen ok. And, if you exclude 0 from the set of real numbers it will also forms an Abelian group respect to multiplication and dot and multiplication distribute over addition also from left and from right also.

So, we say that the set of real numbers with the usual addition a multiplication forms of field is a field. Now, you see the set of complex numbers usual addition and usual multiplication again the two binary operations are usual addition usual multiplication. Now, you see the set of complex number set of complex numbers and the usual addition constitute a Abelian group this we can easily see.

All the 4 properties I mean it is a binary operation additions the binary operation number 1, associativity identity is 0, and the inverse is inverse is minus a of any a and c. So, and also a into b is equals to b into a for all elements a b and c. So, it constitute a Abelian group over usual multiplication. And, you if you exclude 0 from this C then this will also constitute Abelian group over usual multiplication ok.

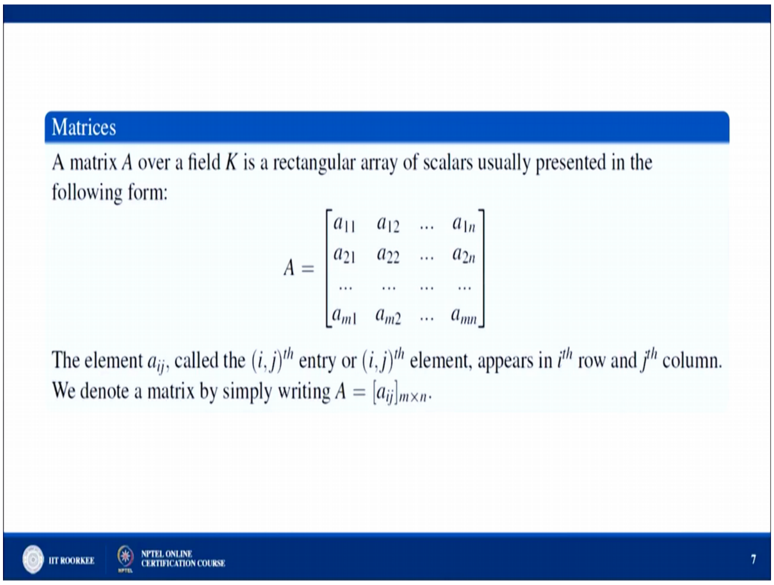
And, multiplication satisfy distributive over addition also so, it will constitute a field. Similarly, the third example set of rational numbers with the usual addition usual multiplication is also a field. Now, see some examples, which are not which are not fields suppose we are considering set of integers. Now, set of integers under usual addition set of integers under usual addition constitute Abelian group it is true, but under usual multiplication it does not form a group even because if you take a element say 2 in Z. So, it is inverse is multiplicative inverse is 1 by 2 which is not in Z. So, this set of integers does not constitute group under multiplication.

So, it will not constitute a field. Similarly, if you there Euclidean is space say R 2 R 2 is all x y such that x y are in R. Now, under usual addition it again constitute a group I mean Abelian group. In fact, because associative property hold identity is 0 comma 0 and inverse of any a comma b in R 2 is minus a minus b, which is also in R 2, but if you take the if you see respect to multiplication excluding 00.

There are other element say 1 comma 0, which is which are in R 2, but it is inverse does not exist. Hence, this R 2 under usual multiplication does not constitute a group. So, it is not a field, now a come to matrices. So, we defined group and field because we want to define matrices over a field k matrices are always defined over a field ok. So, that is that is why we first clear out that what do you mean by a field? Ok.

Now, we define matrices, what do you mean by matrices?

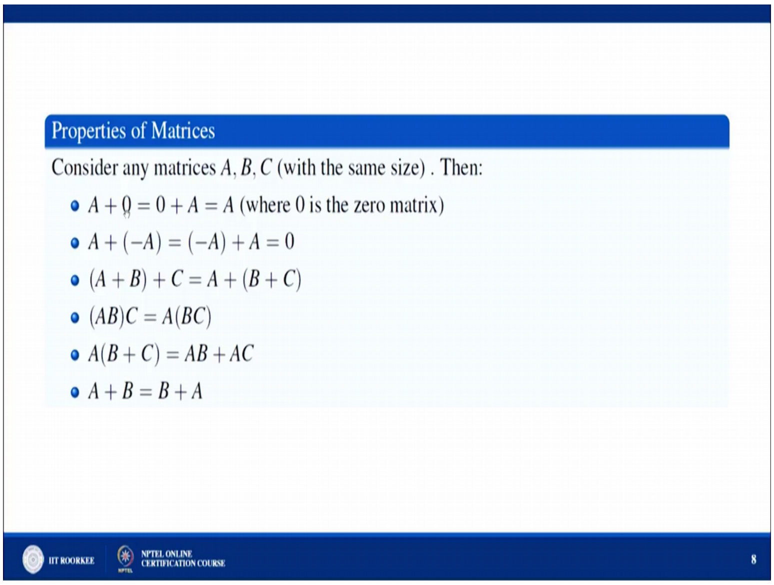
(Refer Slide Time: 27:04)



So, A matrix A over a field K, K may be any field is a rectangular array of a scalars usually rep presented in the following form. So, any matrix A can be represented as a rectangular form a 1 1 a 1 2 a 1 3 up to a 1 n and similarly if m n a m 1 m 2 and a m n. So, if you take any a I j aij means i j-th entry of this matrix a i j-th entry means the element in the i-th row and in the G th column.

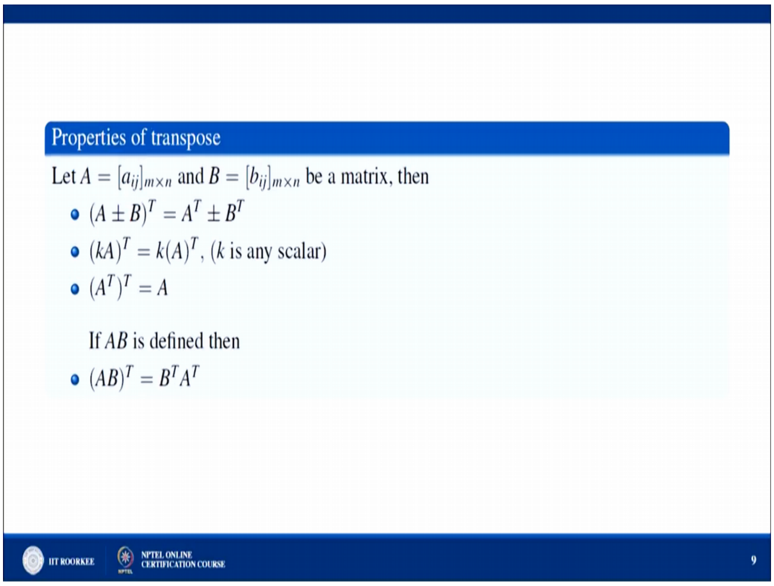
Suppose, we are talking about a 2 2 a 2 2 is the element in the second row and in the second column. If, you are talking about a 2 n a 2 n means element in the second row and n th column ok. So, we denote a matrix by simply writing a equals to a i j and this denote the order of the matrix the order of the matrix is m into n, because number of rows here are m and number of column here are n. So, the order of the matrix is m cross n rows into columns. So, there are general representation of a matrix A, which is a i j means any element i j-th i is varying from 1 2 m and j is varying from 1 to n.

(Refer Slide Time: 28:33)



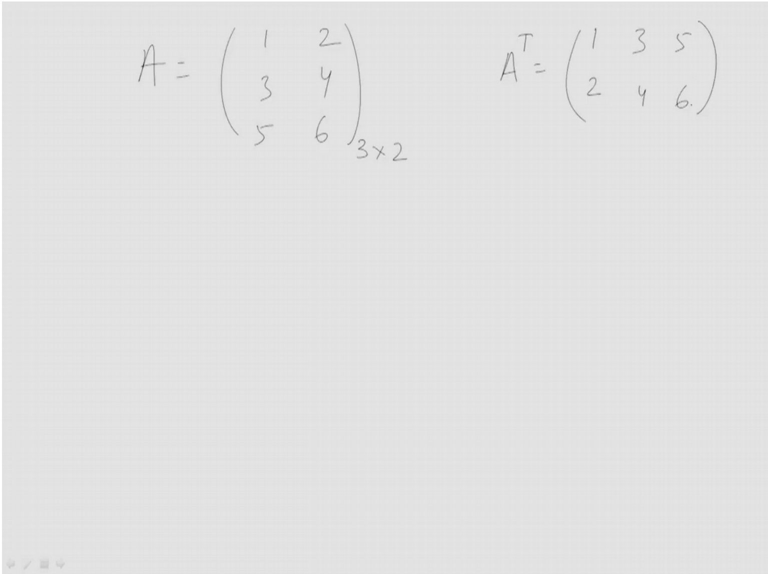
These are some simple properties of a matrices we already know these things that a plus 0 equals to 0 matrix ok, because 0 plus A equal to A A minus A equal to 0 A plus B associativity property hold. Respect to addition multiplication then it distribute dot distribute over addition, because it satisfy this property A plus B equals to B plus A these three are the very basic properties this is already holds in matrices.

(Refer Slide Time: 29:03)



Now, properties of transpose what do you mean by transpose? Transpose means you change interchange rows and columns.

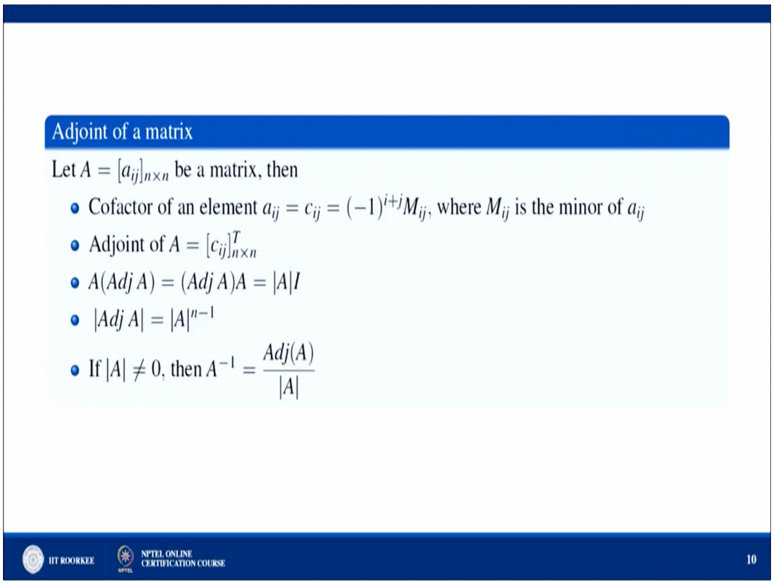
(Refer Slide Time: 29:13)



Suppose A is any matrix, which is suppose 1 2 3 4 5 6. It is it is having 3 rows and 2 columns and you want to find out a transpose.

So, a transpose means you simply write you simply convert rows into columns that is 1 2 3 4 5 6. So, this is the way this is the way of writing a transpose means you interchange rows by columns. The first row in first column, second row in second column, third row in third column, ok. So, these property hold for transpose also A plus minus B whole transpose is A transpose plus minus B transpose, k into a whole transpose is k times A transpose where k is any scalar, transpose of transpose is itself. And if A into B is defined then A into B whole transpose is equal to B transpose into A transpose.

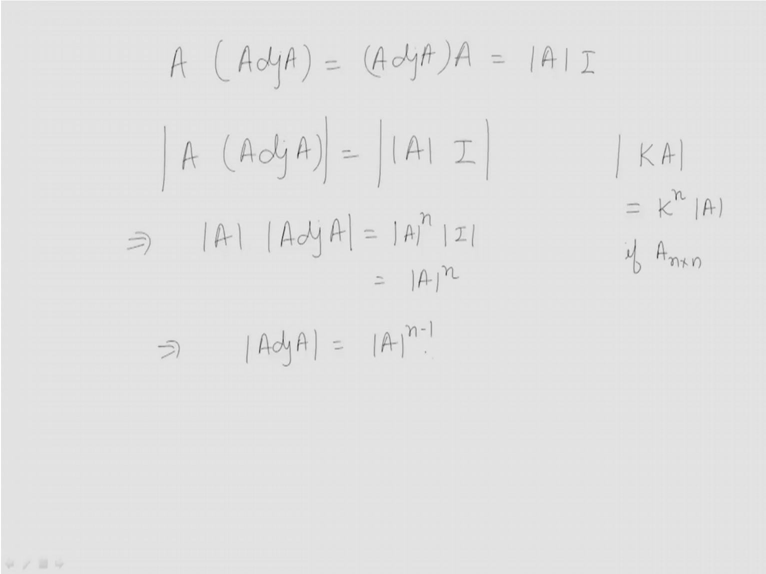
(Refer Slide Time: 30:15)



Now, adjoint of a matrix: let A b a n cross n matrix or square matrix of order n cross n. Then, how to find adjoint first you first find cofactor of a element a ij, which is given by a minus 1 raised to power minus minus 1 raised to power i plus j M ij where M ij is the minor of a ij.

This we already know adjoint of a is simply you make the matrix of cofactors and then take the transpose, that will be the adjoint of A matrix A. And, we also denoted by this expression adjoint of A, then the third properties A into adjoint of A is equal to adjoint of A into A is equal to determinant of A times identity matrix of the same order of course, n cross n. Then determinant of adjoint of a is determinant of a raised to power n minus 1, this is very easy to prove you can simply see here.

(Refer Slide Time: 31:23)



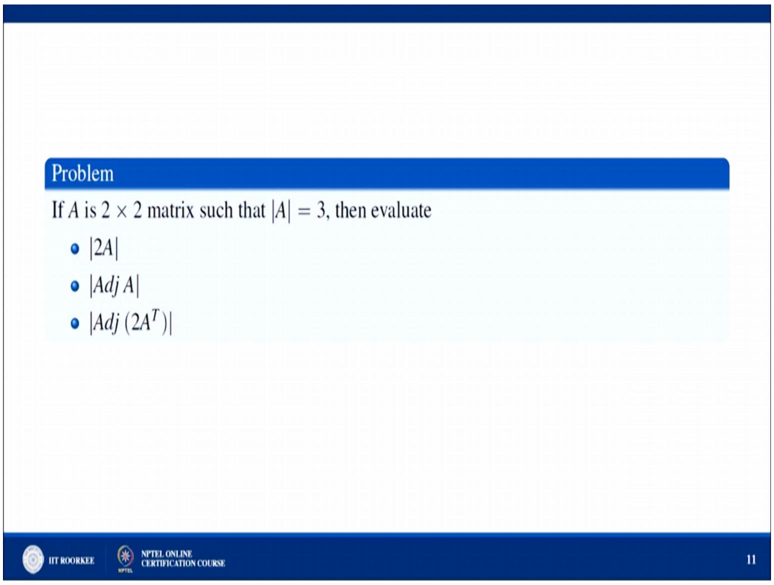
As you have seen that A into adjoint of A is equals to adjoint of A into A equal to determinant of A times identity. So, you take this is this is A into adjoint of A is equal to determinant of a times identity. Now, you take determinant both the sides. Now, this is A into B the determinant of A into B is equal to determinant of A into determinant of B.

And, also determinant of k into A where k is any scalar and A is A matrix of order n cross n is simply equal to k raised to power n determinant of A. If A is A matrix of order n cross n ok. Because, this k is multiplied with all the elements of A and when you take the determinant, you can take the you can take the common from each row first row second row up to n th row. So, k raised to power n will be common and then it will be determinant of A.

So, here determinant of a works as k, because it is a scalar, scalar quantity and I is the matrix of order n cross n. So, it is determinant of a raised to power n and determinant of I. Determinant of I is 1 so, it a determinant of I. So, this implies determinant of adjoint of A is simply determinant of a raised to power n minus 1.

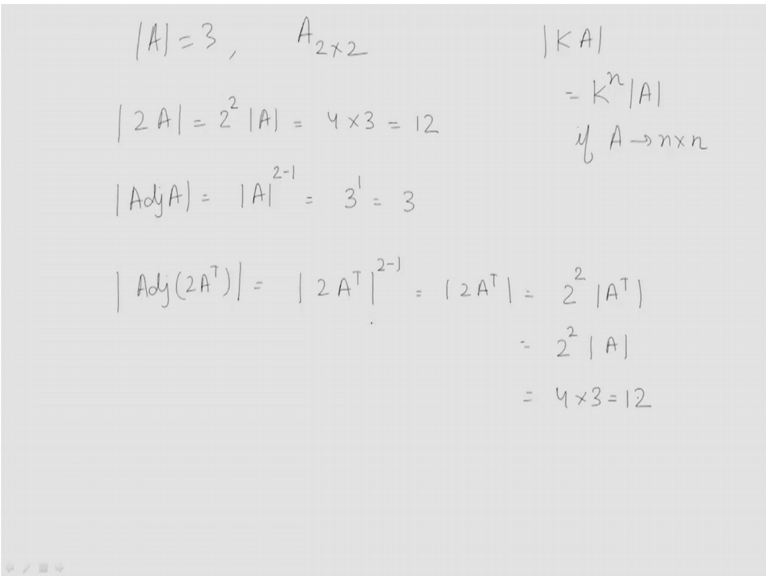
Also we know this thing that if matrix is invertible then inverse exist and inverse is given by adjoint of a upon determinant of A.

(Refer Slide Time: 33:15)



Now this is the very simply problem let us see, just to illustrate few properties of matrices.

(Refer Slide Time: 33:20)

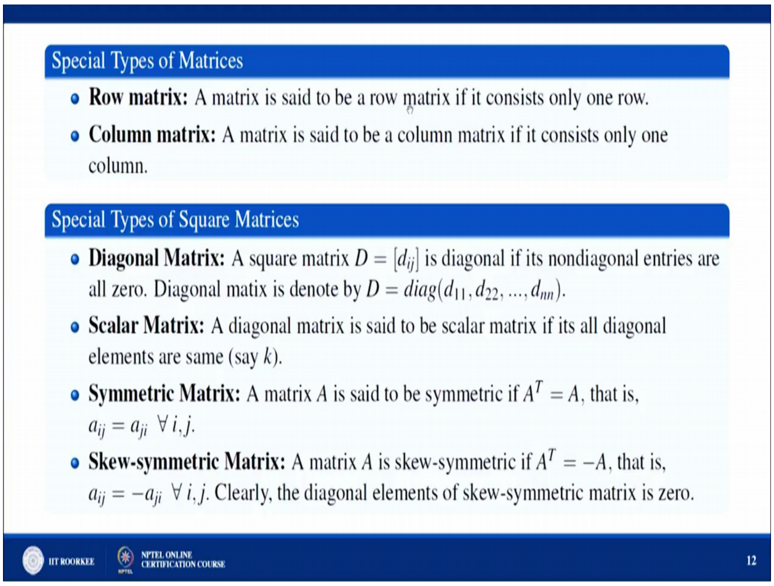


Now, here determinant of A is 3 and A is a matrix of 2 cross 2 order.

If, you want to find out determinant of 2 A so, it will be simply because we know that determinant of k into A is equal to k raised to power n times determinant of A. If A is a matrix of order n cross n, here matrix of order 2 cross 2 and k is 2. So, it is 2 raised to power 2 determinant of A, which is 4 into 3 which is 12. If, we want to find out at determinant of adjoint of A it is simply determinant of a raised to power n minus 1 here n is 2 and determinant of A is 3.

So, 3 raised to power 1 which is 3 if you want to find out determinant of adjoint of 2 A transpose. So, it is simply determinant of 2 A transpose whole raised to power 2 minus 1, which is determinant of 2 A transpose, which is equals to 2 raised to power n 2 raised to power n, which is 2 into determinant of A transpose, determinant of A transpose and A are same. So, it is determinant of A which is equals to 4 into 3 which is 12 ok. So, in this way we can simply solve this problem.

(Refer Slide Time: 34:48)



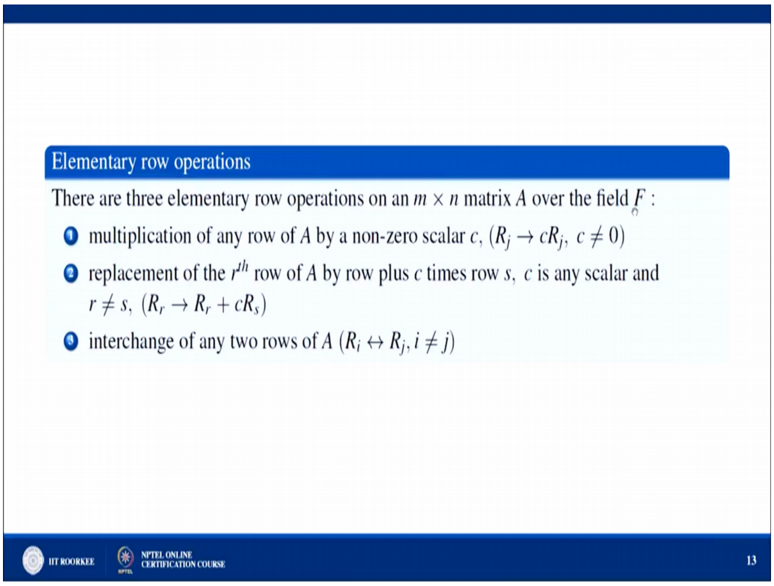
So these are some special type of matrices, row matrix is a matrix is said to be a row matrix if it consists only 1 row, column matrix A matrix said to be a column matrix if it consist of only 1 column.

Diagonal matrix are a square matrix is said to be diagonal if it is non diagonal entries are all 0, that a diagonal matrix. A scalar matrix a diagonal matrix said to be scalar if it is all diagonal elements are same say k. Symmetric matrix means a matrix a is said to be symmetric if a transpose a equal to a that is a i j equals to aji for all i j.

We will discuss more about symmetric and askew symmetric matrix in detail later on. Askew symmetric matrix A matrix A said to be skew symmetric if a transpose is equal to minus A, that is aij equal to minus a ji for all I and j and also a diagonal matrix is few symmetric matrix are 0 now what are elementary row operations.

So, first we are talking about matrices over the field F or over the field k. So, these scalars whatever we are talking about comes on the field, if the field a set of real numbers then the scalars will be a set I mean real. And, if we are talking about the set of complex number the scalars come from the set of complex numbers. Now, what are elementary row operations let us see there are 3 elementary row operations on an m cross n matrix a over the field F.

(Refer Slide Time: 36:31)



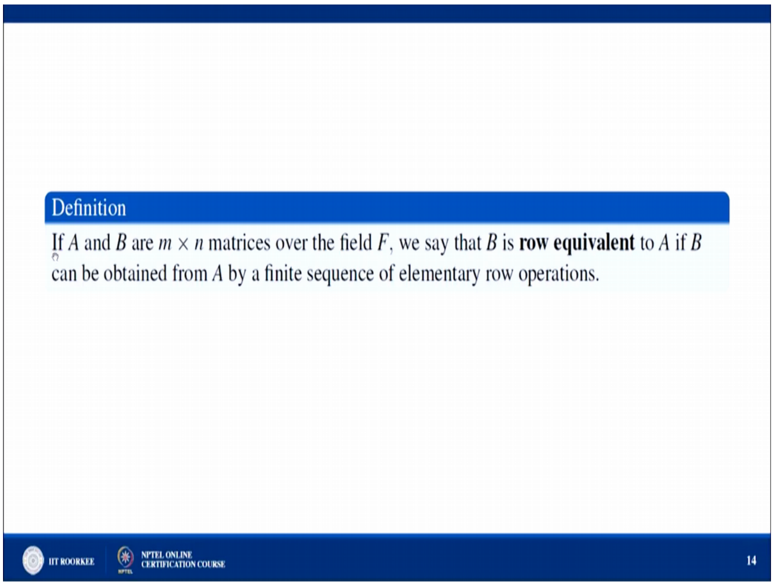
What are they number one? Multiplication of any row of A by a non-zero scalar c, you see if you take any row R j of a matrix A. And you multiply that row by a non zero scalar c then this is the first elementary row operation which we can apply on a matrix a the second elementary row operation is replacement of the R th row of a by row plus c times S row.

Where c is any scalar and R is not equal to s; that means, you take any R th row and you replace this R th row by the R th row plus c time some other s th row. Then this is the second elementary row operation on any matrix A and you can always interchange any 2 rows you can interchange i-th row of a j-th or j-th with by i-th i is not equal to j.

So, these are the 3 basic elementary row operations number 1 1 is you can multiply and row by any non-zero scalar c, then you can always for any row you can always take row plus c time some other row rs and you can always interchange any 2 rows of a.

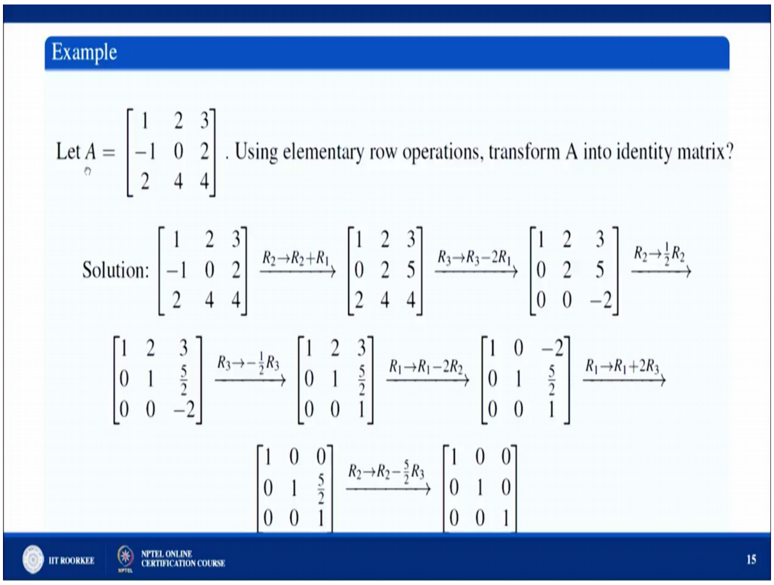
So, these are the 3 elementary row operations now let us discuss this is by an example now first thing is let us let us this also first definition.

(Refer Slide Time: 38:09)



If, A and B are m cross n matrices over the field F, we say that B is row equivalent to A, if B can be obtained from A by the finite sequence of elementary row operations. You see you have a matrix a and you apply a some elementary row operations on that matrix and you get a new matrix B, then we say that the matrix B is row equivalent to A.

(Refer Slide Time: 38:39)



Now, you you take now you we discuss this example. You take a let us suppose you take A equal to 1 2 3 minus 1 0 2 2 4 4. Now using elementary row operations transform a into identity matrix. Suppose we have to apply elementary row operation on this matrix A and we have to convert this matrix into an identity matrix.

So, how can we convert this? So, let us see area of solution let us discuss solution first. So, this is matrix A this is the matrix A. So, in the identity we have to take this element as 1 the first element as 1 and in that column all that all the elements must be 0.

Similarly, similarly for the second element and for the third element I mean in diagonal. Now to make 0 here which element a row operation we will apply to make 0 here we will take this row R 2 and we add with R 1 because minus 1 plus 1 will become 0 here.

So, we make first elementary row operation in R 2 row and we replace R 2 by R 2 plus R 1 ok. Now, this minus 1 plus 1 is 0 0 plus 2 is 2 2 plus 3 is 5.

So, this is the first elementary row operation which we have applied in this matrix. Now, the next is we are to make 0 here because we want to make identity here. Now to make 0 here we have to take the third row and we have to subtract 2 times the first row, I mean we have to replace the third row by R 3 minus 2 times R 1, then it will become 0 2 minus 2 times 1 become 0. So, this minus 2 time this 0, this minus 2 minus this become 0, this minus 2 time this become minus 2.

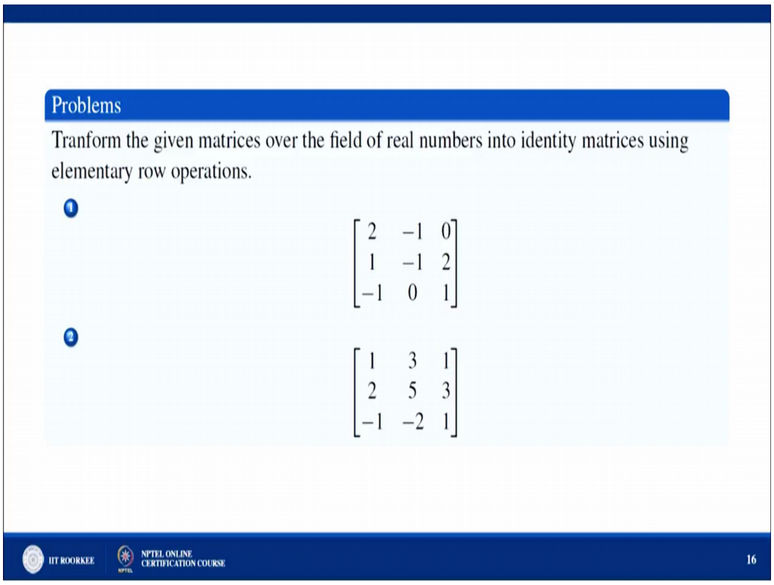
Now, now we have to make 1 here to make identity so, replace this row by 1 by 2 this row I mean replace R 2 by 1 by 2 R 2, because you want to make 1 here ok. So, you replace R 2 by 1 by 2 R 2 we get another matrix which is this 1 2 3 0 1 5 by 2, because we divided by 2 here and 0 0 minus 2, now we want to make 1 here.

So, you divide this by minus 2, if you divide this by minus 2 or replace this row by minus 1 by 2 times R 3, then it is 0 it is 1 2 3 0 1 5 by 2 0 0 1. Now, you have to make 0 here 0 here and 0 here to complete the identity matrix. To make 0 here with the help of this row you simply take this row row 1 and you subtract it with twice of row 2; that means, in the row 1 you apply the elementary row operation R 1 minus 2 times R 2. So, this is minus 2 times this is one, this minus 2, times this is 0, this minus 2 time, this is minus 2 and all the all other elements remain the same.

Now, you are to make 0 here. So, to make 0 here we take the help of this 1. So, this plus 2 times this I mean R 1 you replace R 1 by R 1 plus 2 times R 3. So, this plus 2 time this is 1 this plus 2 time this is 0 this plus 2 time this is 0. And other elements remain the same now you want to make 0 here to complete identity matrix. So, this minus 5 by 2 times this row I mean R 2 you replace R 2 by R 2 minus 5 by 2 times R 3. So, this will be the identity matrix.

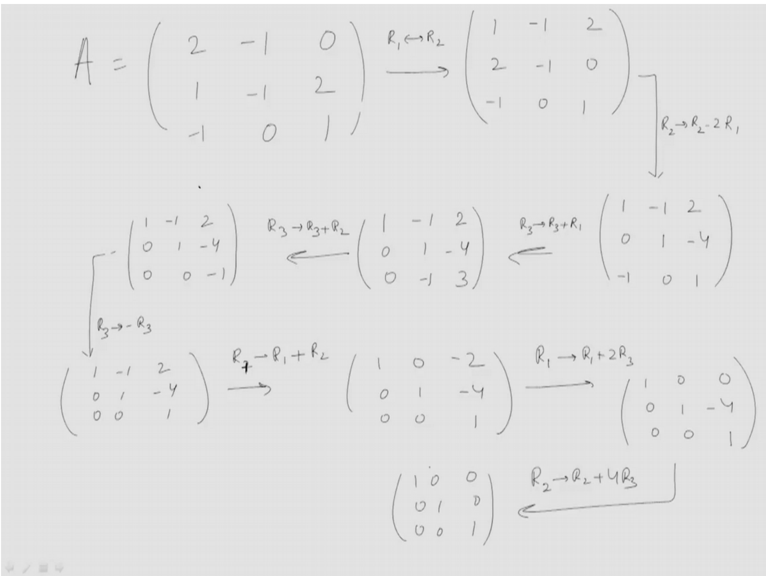
So, we have applied series of series of elementary row operations to get to convert matrix A into an identity matrix ok. Now, if you talk about this matrix say this matrix. So, this matrix is obtained from the matrix A by two elementary row operations. So, we can say that this matrix is a row equivalent matrix to A or in fact, we can say any matrix any matrix up to here are the row equivalent forms of the matrix A, because they are they are obtained by applying some elementary row operation on the matrix.

(Refer Slide Time: 43:26)



Now, let us discuss this example suppose discuss first example suppose we discuss to convert this into an identity matrix by applying elementary row operation.

(Refer Slide Time: 43:38)



So, what is the matrix you see matrix is 2 minus 1 0 it is 1 minus 1 2 it is minus 1 0 1 it is a matrix. Now, by applying elementary row operation you want to transform this matrix into an identity matrix. So, how you will proceed?

In the first column you see if there is any one, I mean any element any element 1 then you interchange those rows first ok. So, first what you do you take you replace R 1 and R

2 you interchange R 1 and R 2. So, this is 1 minus 1 2 it is 2 minus 1 0 it is minus 1 0 1 2 make our calculation easy. The other way out is you divide it by a 2 and then apply the elementary row operations now here now you want to make 0 here with the help of this.

So, to make 0 here with the help of this you replace R 2 by R 2 minus 2 times R 1 then only it will become 0. So, it is 1 minus 1 2 it is 0 this minus 2 time this is 1 this minus 2 time this is minus 4 it is minus 1 0 1. Now you want to make 0 here with the help of this. So, replace R 3 by R 3 plus R 1. So, this will be 1 minus 1 2 it will be 0 1 minus 4 it will be 0 minus 1 3.

Now, this is already 1 you want to make 0 here because you want to complete identity matrix. So, again you have apply elementary row operation you replace R 3 by R 3 plus R 2. So, this will be 1 minus 1 2 it will be 0 1 minus 4 it will be 0 minus 1 now it is minus 1 you have to make 1 here.

So, you multiply this by minus 1 you replace R 3 by minus of R 3. So, this is 1 minus 1 2 0 1 minus 4 0 0 1. Now, you want to make 0 here with the help of this. So, you replace R 2 R 1 by you replace R 1 by R 1 plus R 2. So, it is 1 0 minus 2 it is 0 1 minus 4 it is 0 zero 1 ok. Now, you want to make 0 here with the help of this. So, this plus 2 times this. So, in R 1 you take R 1 plus 2 times R 3. So, it is 1 0 minus 2 0 1 minus 4 0 it is 00. So, this is this will be simply 0 and it is 0 1.

Now, you want to make 0 here with the help of this or this plus 4 time this will give you the identity matrix; that means, you replace R 2 by R 2 plus 4 times R 3. So, it will be an identity matrix now. So, these are the elementary row operations which we apply to convert this matrix A into an identity matrix, similarly we can push it for the second problem.

Thank you very much.