**Ordinary and Partial Differential Equations and Applications**
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**Lecture - 43**
**Classification and Canonical Form of Second Order PDE-I**

Hello friends welcome to my lecture on classification and canonical forms of second order partial differential equations. There will be 2 lectures on this topic, this is first of the 2 lectures. Let us consider a partial differential equation of the type Rr + Ss + Tt + f xyz pq = 0.

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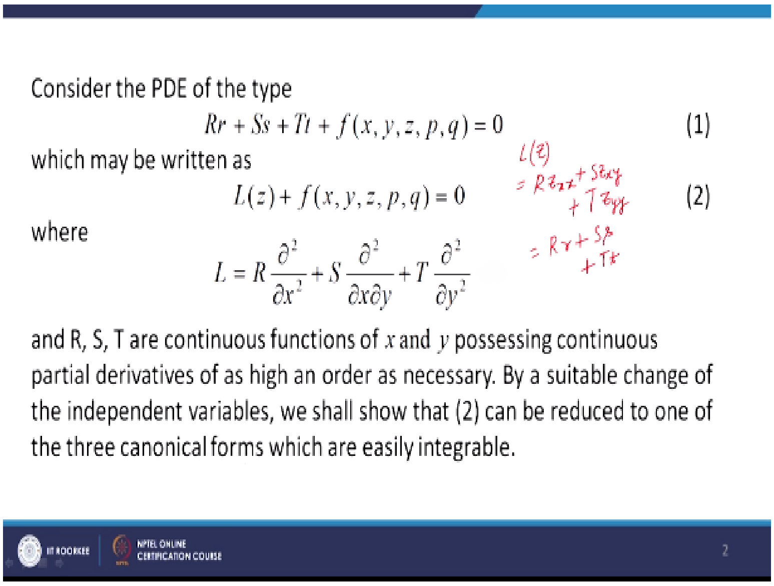


As you know p is partial derivative of z with respect to x, q is partial derivative of z with respect to y, R is second order partial derivative of z with respect to x and S is second order partial derivative of z with respect to x ny and T is second order partial derivative of z with respect to y. Now this equation can be written as Lz + f xyz pq = 0 where L is the differential operator given by R del square/del x square + S del square/del x del y + t del square/del y square.

So Lz will be equal to if we assume L to be this differential operator then Lz is Rzxx + Szxy + Tzyy and by our notice zxx is r, zxy is s, and zyy is t. So we get Rr + Ss + Tt this is Lz and so we can write this Lz + F xyz pq = 0. Now here this RST are continuous functions of x and y possessing continuous partial derivatives of as high in order as maybe necessary, by a suitable change of the independent variables x and y.

We shall show that the equation 2 Lz + f xyz pq = 0 can be reduced to one of the 3 canonical forms which are easily integrable.

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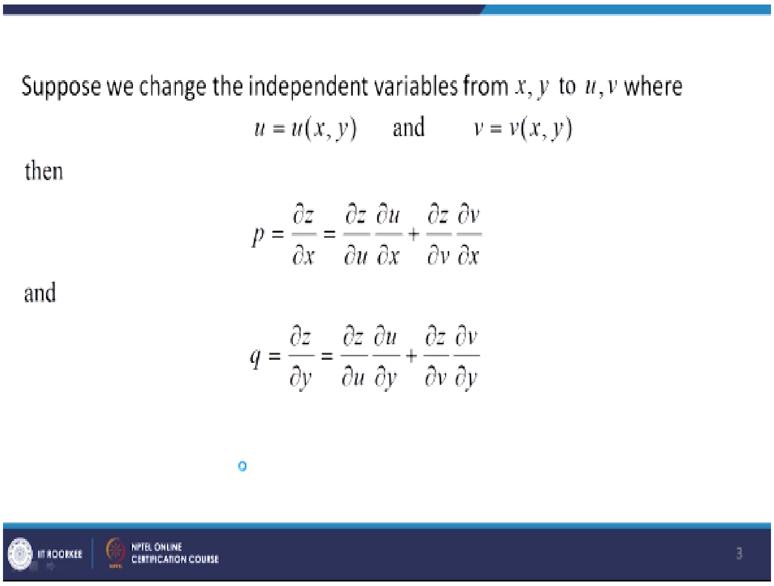


Now let us suppose that the independent variables x and y are changed to the independent variables u and v by the equations u = u xy and v = vxy then since p = partial derivative of z with respect to x by change rule of differentiation we can write partial derivative of z with respect to us, partial derivative of z with respect to xx, partial derivative of z with respect to u * ux + partial derivative of z with respect to v * vx.

Similarly, partial derivative of z with respect to y can be written as partial derivative of z with respect to u * uy + partial derivative of z with respect to v * vy. So we have p and q by using the change rule.

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Hence

$$r = \frac{\partial^2 z}{\partial x^2} = \frac{\partial^2 z}{\partial u^2}\left(\frac{\partial u}{\partial x}\right)^2 + 2\frac{\partial^2 z}{\partial u \partial v}\frac{\partial u}{\partial x}\frac{\partial v}{\partial x} + \frac{\partial^2 z}{\partial v^2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{\partial z}{\partial u}\frac{\partial^2 u}{\partial x^2} + \frac{\partial z}{\partial v}\frac{\partial^2 v}{\partial x^2}$$

$$s = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial u^2}\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + \frac{\partial^2 z}{\partial u \partial v}\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right) + \frac{\partial^2 z}{\partial v^2}\frac{\partial v}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial z}{\partial u}\frac{\partial^2 u}{\partial y \partial x} + \frac{\partial z}{\partial v}\frac{\partial^2 v}{\partial y \partial x}$$

$$t = \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial u^2}\left(\frac{\partial u}{\partial y}\right)^2 + 2\frac{\partial^2 z}{\partial u \partial v}\frac{\partial u}{\partial y}\frac{\partial v}{\partial y} + \frac{\partial^2 z}{\partial v^2}\left(\frac{\partial v}{\partial y}\right)^2 + \frac{\partial z}{\partial u}\frac{\partial^2 u}{\partial y^2} + \frac{\partial z}{\partial v}\frac{\partial^2 v}{\partial y^2}$$



Now when we go to the second order partial derivatives that is we find R, S and T, okay, R, S and T. R will be = second order partial derivative of z with respect x, S will be delta square z/delta x delta y and T will be delta square z/delta y square. So when we find the second order partial derivatives of z with respect to x and y we arrive at these expressions, r = this expression, s = this expression and t = this expression.

I will show you how we arrive at the equation for r. Similarly, you can derive the equations for s and t. The values of how we get the value of r, let us in the next slide I will show you how we get this.

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we have $p = z_x = z_u u_x + z_v v_x$ or $\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x}$

$$r = \frac{\partial p}{\partial x} = z_{xx} = \frac{\partial}{\partial x}(z_u u_x + z_v v_x)$$
$$= \frac{\partial}{\partial x}(z_u u_x) + \frac{\partial}{\partial x}(z_v v_x)$$
$$= \frac{\partial}{\partial x}(z_u)\, u_x + z_u u_{xx} + \frac{\partial}{\partial x}(z_v)\, v_x + z_v v_{xx}$$
$$= (z_{uu} u_x + z_{uv} v_x) u_x + z_u u_{xx} + (z_{uv} u_x + z_{vv} v_x) v_x + z_v v_{xx}$$
$$= z_{uu}(u_x)^2 + z_{uv} u_x v_x + z_u u_{xx} + z_{uv} u_x v_x + z_{vv}(v_x)^2 + z_v v_{xx}$$
$$= z_{uu}(u_x)^2 + 2 z_{uv} u_x v_x + z_u u_{xx} + z_{vv}(v_x)^2 + z_v v_{xx}$$

$$\Rightarrow \frac{\partial}{\partial x}(z_u) = \frac{\partial}{\partial u}(z_u) \cdot u_x + \frac{\partial}{\partial v}(z_u)\, v_x = z_{uu} u_x + z_{uv} v_x$$

Similarly

$$\frac{\partial}{\partial x}(z_v) = \frac{\partial}{\partial u}(z_v)\frac{\partial u}{\partial x} + \frac{\partial}{\partial v}(z_v)\frac{\partial v}{\partial x} = z_{uv} u_x + z_{vv} v_x$$



So we have p = zx which is = zu * ux + zv * vx. Now z xx r = partial derivative of p with respect to x which is zxx this = partial derivative of the righthand side with respect to x. Now let us apply this differential operator on the 2 terms inside. So we have, now zu and ux are functions of x and y, so I can write using the formula for the derivative having product of 2 functions.

We have partial derivative of u with respect to x * ux + zu * partial derivative of ux with respect to x will be uxx and similarly here partial derivative of zv with respect to x * vx + zv * partial derivative of vx with respect x so we will have vxx. Now in order to, our problem is to find the partial derivative of zu with respect to x. z is the function of xn/xn/r function of u and v. So z is the function of u and v okay.

So zu is again, when we differentiate z with respect to u, zu is again a function of u and v, so it is partial derivative with respect to x can be found from the partial derivative of this one from this equation. This equation is valid for any function z of u and v so it will also be valid for zu. So I can write from this equation replacing z by zu.

And what I get is zuu * ux + zuv * vx similarly we can find the partial derivative of zv with respect to u with respect to x okay, similarly, now this zv is also a function of u and b so I can replace here z by zv, so delta/delta x zv = delta/delta u of z v * delta u/delta x + delta/delta v of zv * delta v/delta x and this is what, this is zuv * ux and we have zvv * vx, so let us put these values here.

So first we put this value, so this is equal to zuu * ux + zuv * vx * ux + zu * uxx + now we put the value of this partial derivative of zv with respect to x. So zuv * ux + zvv * vx * vx + zv * vxx and then you can write it as zuu * ux whole square * ux and zuv * ux* vx + zu * uxx + zuv * ux vx + zvv * vx whole square and then zv * vxx and this term and this term are same so I can write zuu * ux square + 2 times zuv * ux * vx + zu * uxx + zvv * vx whole square + zv * vxx.

Now let us see if this is the value of r, so r is zuu * ux whole square, we have zuu * ux whole square then you have 2 zuv, ux vx which we have here and then we have zvv * vx whole square. So we have zvv * vx whole square and then we have zu * uxx, and then we have zv * vxx. So we get zv * vxx. So this is how we get r. You can similarly find the other 2 second order partial derivatives S and T, the procedure is the same.

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Putting the above values of $p, q, r, s$ and $t$ in (1) and simplifying, we get

$$A\frac{\partial^2 z}{\partial u^2}+2B\frac{\partial^2 z}{\partial u\partial v}+C\frac{\partial^2 z}{\partial v^2}+f\left(u,v,z,\frac{\partial z}{\partial u},\frac{\partial z}{\partial v}\right)=0 \qquad (3)$$

where

$$A=R\left(\frac{\partial u}{\partial x}\right)^2+S\frac{\partial u}{\partial x}\frac{\partial u}{\partial y}+T\left(\frac{\partial u}{\partial y}\right)^2 \qquad (4)$$

$$B=R\frac{\partial u}{\partial x}\frac{\partial v}{\partial x}+\frac{1}{2}S\left(\frac{\partial u}{\partial x}\frac{\partial v}{\partial y}+\frac{\partial u}{\partial y}\frac{\partial v}{\partial x}\right)+T\frac{\partial u}{\partial y}\frac{\partial v}{\partial y}$$

$$C=R\left(\frac{\partial v}{\partial x}\right)^2+S\frac{\partial v}{\partial x}\frac{\partial v}{\partial y}+T\left(\frac{\partial v}{\partial y}\right)^2$$

Now let us put these values of pq RST, we have found the values of pq RS and T. Let us put these values in the equation 1, in this equation, let us put the values of RST pq okay and then simplifying we get, you can simplify that equation you will arrive at A times z uu + 2v times zuv + C times zvv + a function of uvz, zu, zv = 0.

So we have changed from the independent variables xn by to the independent variables u and b and the equation 1 now gives us this equation 3. Here the values of ABC are A is R times ux square + S times ux uy + T times uy square and B is R times ux vx + 1/2 of S times ux vy + uy vx + T times uy vy. C is = R times vx square + S times vx vy + T times vy whole square and f uvz zu, zv is the transformed form of this f; f xyz pq = 0. So this is the transformed form of that.

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and $f\left(u,v,z,\frac{\partial z}{\partial u},\frac{\partial z}{\partial v}\right)$ is the transformed form of $f(x,y,z,p,q)$.

Now we shall determine $u$ and $v$ so that the equation (3) reduces to the simplest possible form. The procedure becomes simple when the discriminant $S^2-4RT$ of the quadratic form (4) is everywhere either positive, negative or zero. We shall discuss these three cases separately.

Now we shall determine the functions u and v okay, we have changed from the independent variables xn y to the new variables u and v. We will determine u and v so that the equation 3, reduces to the simplest possible form. Why we do this are doing this to make the equation integrable. So the procedure become simple when the discriminant S square-4RT of the quadratic form 4.

This is the quadratic form Rux square + S ux uy + T uy square, okay, so this quadratic form is everywhere either positive or negative or 0. Let us discuss these 3 cases separately. First we discuss the case when S square – 4 RT of the quadratic form 4 is positive.

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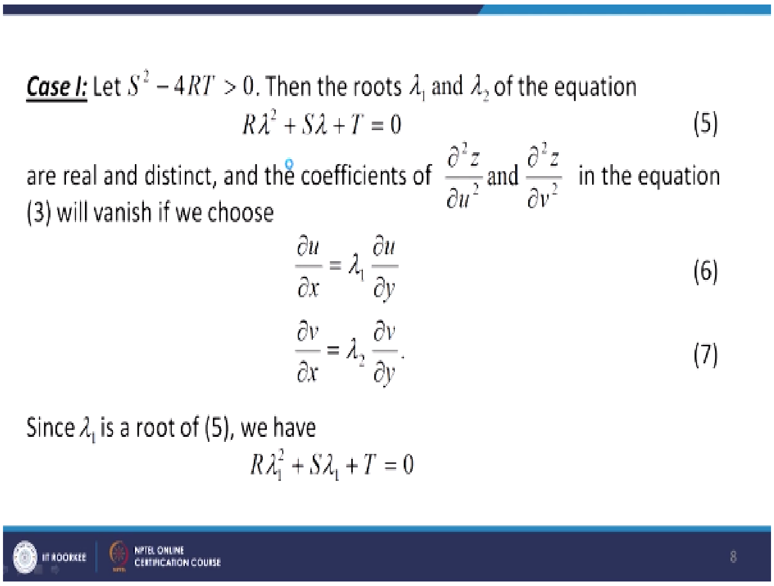


So let us consider the case when S square – 4 RT is > 0 then the equation, R lambda square + S lambda + T, so we have this. Then the roots of the lambda 1 lambda 2 of the equation R lambda square + S lambda + T = 0, are real and distinct because this is the second order equation in lambda if you find the roots of this then if the discriminant S square – 4 RT is > 0 the roots of this equation will be real and distinct.

And the coefficients of del square z/del u square and del square z/del v square in the equation 3 will vanish. Let us see here. The coefficient of this second order derivative in u and the second order derivate in v that is A and C will vanish if we choose our u and v in such a way that delta u/delta x = lambda 1, delta u/delta y, delta v/delta x = lambda 2 * delta v /delta y where lambda 1 and lambda 2 are functions of x and y.

Because RST depend on x and y. Now since lambda 1 is the root of this equation R lambda square + S lambda + T = 0 it will satisfy this equations, so we will have R lambda 1 square + S lambda 1 + T = 0. Now what we do is let us use equation #6, this, ux = lambda 1 uy so if you use this what we get here A = you see we are using ux = lambda 1 times uy okay.

So what is R, this will give you A = R times ux square, ux square means lambda 1 square, uy square + S times ux, ux is lambda 1 * uy * uy + T times uy square. So what we can do, we can write or A = R lambda 1 square + S lambda 1 + T times uy square, okay, since lambda 1 satisfies the equation R lambda i square + S lambda + T, R lambda 1 square + S lambda 1+T = 0, so we get A = 0, okay.

Similarly, if you use this equation vx = lambda 2 vy then C will be = vx = we are using vx = lambda 2 vy, so we will have C = R times lambda 2 square vy square + S times vx is lambda 2 vy * vy + t times vy square. So we can write it as R lambda 2 square + S lambda 2 + t * vy square and lambda 2 is the root of r lambda square + S lambda + T = 0. So since lambda 2 is the root of this we have R lambda 2 square + S lambda 2 + T = 0.

So this will be = 0. So by using the relations ux = lambda 1 uy vx = lambda 2 vy we notice that in the equation this A and C become 0. So we get 2 v * z uv + this = 0 and now what we do is so we get this = 0.

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Hence, using (6), we get

$$A = R\left(\frac{\partial u}{\partial x}\right)^2 + S\frac{\partial u}{\partial x}\frac{\partial u}{\partial y} + T\left(\frac{\partial u}{\partial y}\right)^2$$

$$= \left(R\lambda_1^2 + S\lambda_1 + T\right)\left(\frac{\partial u}{\partial y}\right)^2 = 0$$

Again, since $\lambda_2$ is a root of (5), we have

$$R\lambda_2^2 + S\lambda_2 + T = 0$$

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Again since lambda 2 is the root of 5, R lambda square + S lambda + T = 0 we get R lambda 2 square + S lambda 2 + T = 0. So we get C = 0 and now what we do is let us write the equation #6 in this manner.

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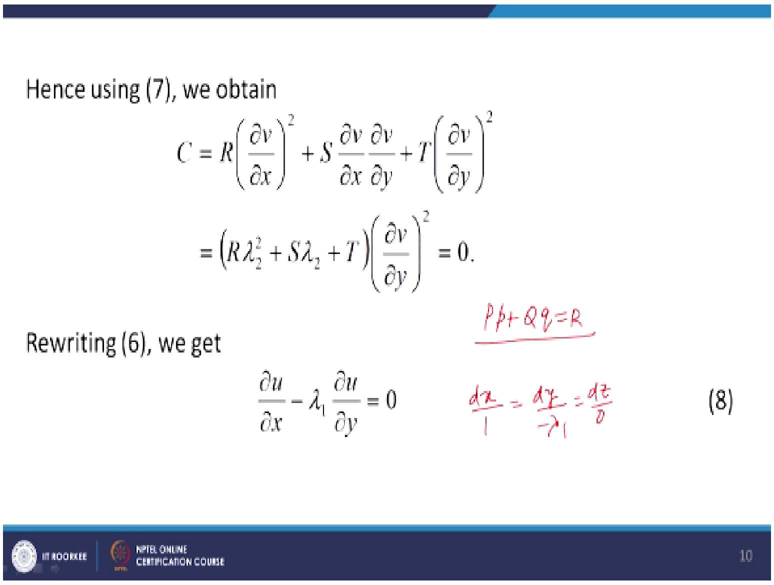


This equation ux = lambda 1 uy we will write it in this form ux – lambda 1 uy = 0 now it is a Lagrange's equation this is of the form Pp + Qq =R it is of this form okay. So what we have here, here the dependent variable is u, independent variables are x and y. So this p and q here are this ux and uy, P is 1, Q is –lambda 1 and R = 0 so we can write the characteristic equation for this Lagrange's equation.

We will have dx/1=dy/-lambda 1 = dz upon 0 okay, so this is how we arrive at the Lagrange's auxiliary equations.

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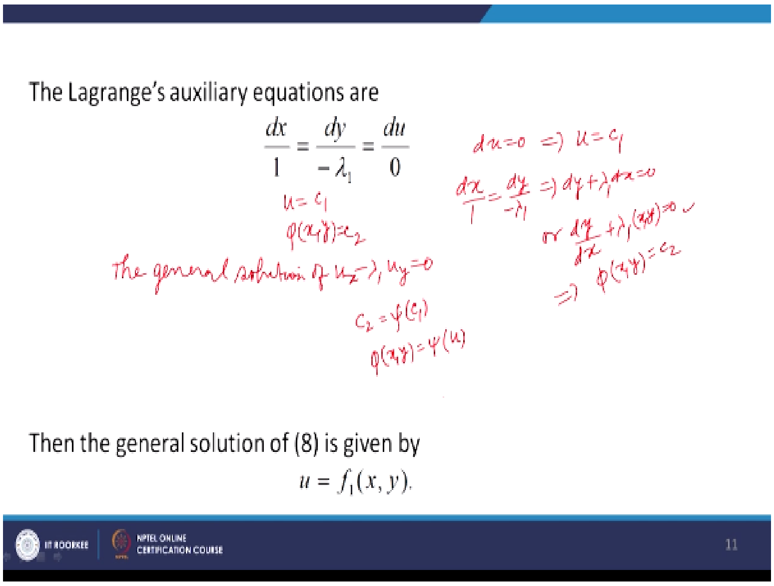


Now let us take from this characteristic equations or auxiliary equations we notice that du = 0 so we get u = some constant let us say C1 and dx/1 = dyy – lambda 1 this gives you dy + lambda 1, dx = 0, okay, or we can say dy/dx + lambda 1 xy = 0 okay, lambda 1 is the function of x and y, so this will give us a solution of the form say phi xy = C2 okay, so this will give you a solution of the form phi xy = C2 and now u = C1 is the one solution of this.

Okay so there are 2 solutions are there u = C1 phi xy = c2 okay and therefore the general solution of this equation, ux – lambda 1 uy = 0, the general solution of uy = 0 will be a function of you can say C2 will be a function of C1 or C1 will be a function of C2. So C2 will be a function of C1, okay, so we can say it is the function of let us say psi of C1, okay. So we can say that C2 is phi xy, okay.

So phi xy = psi of u, or we can say u is the function of xny, okay, so these are general solution of this equation. This is the function of x and y so psi u = a function of x and y so we can say that u is the function of x and y and therefore u = function of xy the general solution of this equation ux – lambda 1 uy = 0.

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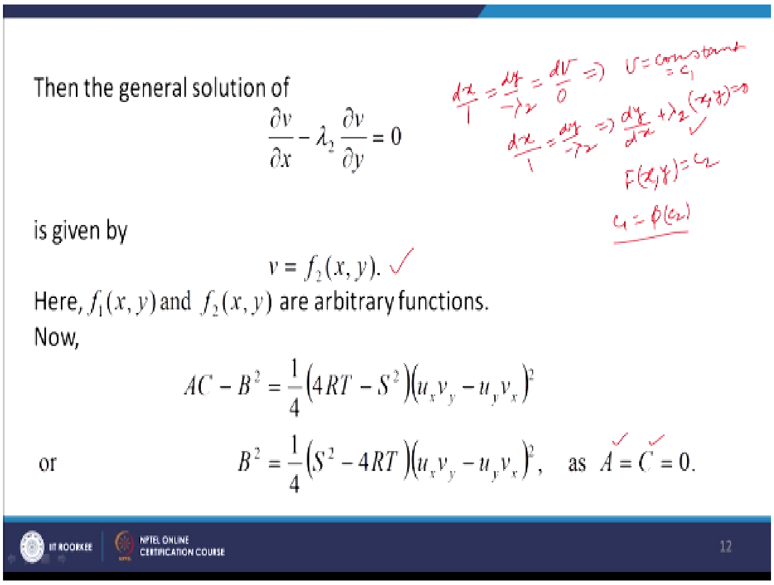


And similarly when we find the general solution of vx-lambda 2 vy = 0, this is Lagrange's equation, the auxiliary equations are dx/1 = dy/-lambda 2 = dv/0. So this will give you v = constant. We will have dx/1 = dy/-lambda 2 we will imply that dy/dx + lambda 2 xy = 0 so this will give you a general solution of this equation will then be of the form say some function of xy = a constant.

Let me say this is constant say C1, this is constant of C2, then we can say that since C1 and C2 are functions of each other, okay, v is the function of x and y, so we can take v to be = f2 xy. C1 is the function of C2. So we can say that v is function of x and y and f1 and f2 are arbitrary functions here. Now let us see this AC-B square. We have the expressions of A, okay, B and C.

So from the expressions of AB and C when you find this expression AC-B square you can easily prove that AC-B square is 1/4 times 4 RT-S square * ux vy – uy vx whole square. And since A and C are 0s we will have –B2 = 1/4 4 RT-S square this term will become 0 so – B square will be equal to this and we can multiply by -1 so B2 = 1/4 S square -4 RT * ux vy – uy vx whole square.

Now this quantity is nonnegative okay so and S square -4RT, S square is > 4 RT so this is also, this is positive okay, so positive * this non negative quantity makes it this one.

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Since, $S^2 - 4RT > 0$, we have $B^2 > 0$.
Hence we may divide both sides of (3) by B. Then noting that $A = C = 0$,
(3) transforms to the form

$$\frac{\partial^2 z}{\partial u \partial v} = \phi\left(u, v, z, \frac{\partial z}{\partial u}, \frac{\partial z}{\partial v}\right) \quad (9)$$

which is the canonical form of (1) in this case.



Since S square – 4 RT, 4 RT is > 0 we have v square > 0 okay, B square okay ux * vy-uy vx u and v are functions of x and y they are arbitrary function. S square is > 4 RT okay, so we can assume that this is strictly positive. So B square is strictly positive okay, and that means B is not = 0 so when B is not = 0 we can divide the quotient 3/B.

This equation A and C are 0s 2B * delta square z/delta u delta v + a function of uvz zu zv = 0 will then be divided by 2v we can divide by 2v and arrive at this equation, z uv = some function phi of uvz zu zv which is the canonical form of the equation 1 in this case and it can be then easily integrated. Now let us look at problem on this.

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***Example:*** Let us consider the equation $Rr + Ss + Tt + f(x,y,z,p,q) = 0$

$$r - x^2 t = 0.$$

$R = 1, S = 0, T = -x^2$ Hence $S^2 - 4RT = 0 - 4 \cdot 1(-x^2) = 4x^2 > 0$

$R\lambda^2 + S\lambda + T = 0$

or $\lambda^2 - x^2 = 0 \Rightarrow \lambda = \pm x \Rightarrow \lambda_1(x,y) = x$ & $\lambda_2(x,y) = -x$

Now we choose u & v such that $\frac{\partial u}{\partial x} = \lambda_1 \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x} = \lambda_2 \frac{\partial v}{\partial y}$

$\Rightarrow \frac{\partial u}{\partial x} = x \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x} = -x \frac{\partial v}{\partial y}$

$\frac{\partial u}{\partial x} - x \frac{\partial u}{\partial y} = 0$ Lagrange's equation

$\frac{dx}{1} = \frac{dy}{-x} = \frac{du}{0} \Rightarrow du = 0$ & $dy + x\,dx = 0$

$du = 0 \Rightarrow u = c_1$

$dy + x\,dx = 0 \Rightarrow y + \frac{x^2}{2} = c_2$

$c_1 = \phi(c_2) \Rightarrow u = \phi\left(y + \frac{x^2}{2}\right)$

Let us choose $u = y + \frac{x^2}{2}$ ✓

$\frac{\partial v}{\partial x} = -x \frac{\partial v}{\partial y} \Rightarrow \frac{dx}{1} = \frac{dy}{x} = \frac{dv}{0} \Rightarrow dv = 0$ and $dy - x\,dx = 0$, $dv = 0 \Rightarrow v = c_3$, $dy - x\,dx = 0 \Rightarrow y - \frac{x^2}{2} = c_4$

$v = \psi\left(y - \frac{x^2}{2}\right)$

Let us choose $v = y - \frac{x^2}{2}$ ✓



Say r-x square t = 0 so what we do here is that if you compare it with these trended form Rr + Ss+ Tt +f xyz pq = 0 then we notice that R = 1, S = 0 and T = -X square, hence S square – 4

Rt is S0 4*1 so 0-4*1*-x square so we get 4 x square which is positive. So S square – 4 RT is > 0. Therefore, the equation let us go to this equation R lambda square + S lambda + T = 0.

This equation will have 2 real and distinct roots so let us write the roots for this equation R lambda square + S lambda + T = 0. So let us write R lambda square + S lambda + T = 0. Let us find the roots of this. R = 1, so we have lambda square and then S = 0, T = -x square so lambda square –x square = 0 which gives lambda = +/- x. So we get the 2 roots. Lambda 1 xy = x and lambda 2 xy = -x, this is real and distinct roots.

Now let us consider the 2 equations, we write these 2 equations, this one, ux = lambda 1 uy vx = lambda 2 vy, we choose these equations, so ux now we choose u and b such that ux = lambda 1 times uy and vx = lambda 2 vy. So what do we get? This is = lambda 1 is x so x times uy, okay and similarly vx = - okay now let us solve this equation, for the value of u. so this is –x, this is Lagrange's equation.

So in the case of Lagrange's equation here we have instead of z we have u okay, so we have dx/1 = dy/-x = du upon 0, which implies that du = 0 and dy + xdx = 0, okay, du = 0 gives us u = some constant say C1 and dy + xdx = 0 gives us y + x2/2 = C2. Now C2 is the function of C1, or C1 is the function of C2 so we can write C1= function of C2 that is C1 is u.

Okay so u = phi of y + x square/2 where phi is an arbitrary function. Let us choose u to be = y + x square/2 and similarly the other equation, this gives you the Lagrange's auxiliary equations as dx/1 dy/x = dv/0, so this gives you dv = 0 and dy-x dx = 0 and dv = 0 gives you v = some constant let us say C3 and dy-xdx = 0 gives you y-x square/2 = some constant say C4, okay.

So C3 is the function of C4, so you can say that v = some function say psi of y-x2/2. Let us choose v to be y-x square/2, okay. So we find the value of u and the value of v. With this choice of u and v A and C will be 0 and we will get the equation of this form okay. So let us see how we arrive at this canonical form so let us go to this equation.

R-x square t = 0 let us change R and T and X here which are, R is a partial derivative of z with respect to x, second order derivate, T is a second order derivative of z with respect to y so let us change them to the corresponding derivatives with respect to the independent variables u and v.

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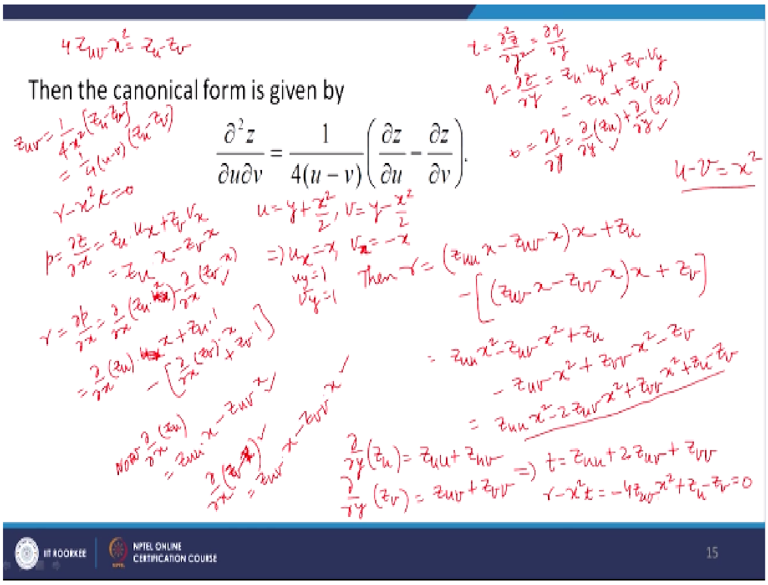


So we consider r-x square t = 0 okay, first we find in order to find r let us find first p, p = partial derivative of z with respect to x so this is zu * ux zv * vx. Now what is our choice. Our choice of u is y+z square/2. So u is y+z square/2 and v is y-x square/2, so this gives you ux = x and this gives you vx = -x. So this is zu * x and zv * -x, okay, so let us now find r, r is partial derivate of p with respect to x.

So partial derivative with respect to x of zu * ux – partial derivative of zv * x with respect to x, okay, so what do we have partial derivative with respect to x of zu * ux, ux is x here, this is x okay, so this is x and then + zu * derivative of x with respect to x, so 1 and then minus derivative of zv with respect to x * x + derivative of x with respect to x so that is 1.

So what we get now in place of z here let us put zu because this relation is valid for any function z of u and v. so it is also valid for zu because zu is the function of u and v, so delta/delta x of zu will be = now delta/delta x of zu = let us put in place of zu so zuu * x – zu when you put in place of z here this is partial derivative of z with respect to v so this becomes we get zuv * x.

So this is how we get the partial derivative of zu with respect to x. Similarly, partial derivative of zv with respect to x we can get delta/delta x of zv with respect to x. So this term we can find, okay, let me write only this. Partial derivative of zv with respect to x. So in place of z we put zv here and we get zuv * x – zvv * x, okay now let us put these value, this value and this value here, okay.

So then r = partial derivative of zu with respect to x so zuu * x – zuv * x * x + zu – now let us put the value of partial derivative of zv with respect to x here. So zuv * x – zvv * x * x + zv and what we get, zuu * x square – zuv * x square + zu and what we get here this x gets multiplied to this and we get – zuv * x square and we get + zvv * x square and we get –zv, okay, so what we get we get z uu x square – 2 zuv x square + zvv x square + zu – zv, okay.

So we have found the value of R, now let us find the value of T. So for Tt is this, okay, so first we find q okay, q = this, this is delta q/delta y, so first we find this so this is zu * uy + zv * vy, now uy = how much? uy = 1 okay, from this relation vy is also 1. So we get here zu + zv okay, now t = delta/delta y of zu + delta/delta y of zv, okay, so we have the value of delta z/delta y for any function z of u and v.

So replacing z by zu there we have, okay, so zuu + z when you replace by zu so we get zuv and when we do this for zv function, zv is also function of u and v, so replacing z by zv we get zuv + zvv so this imply the t = we add these 2 values this and this okay, so we get z uu + 2 z uv + zvv, okay. We have got the values of R and T and what is x here if you see x square subtracting u and subtracting v from u okay.

We get u-v you can see u is y + x square/2, v is y-x square/2 so u – v = x square okay, so let us put the value here. R = this, this is R okay, this is R and when I multiply t by x square okay and subtract what we get r-x square * t how much is that, from this we subtract x square times t, so x square * z uu will cancel and then x square * zvv that will also cancel okay and here x square when you multiply and subtract – 2 x square z uv okay will add to this okay.

So we will get -4 zuv * x square + zu-zb = 0 okay, or we can say I take this term to the other side so 4 zuv * x square = zu-zv, okay, or I can say zuv = 1/4 x square * zu-zv, okay, I have already said x square u-v so we get 1/4 u-v * zu-zv okay, so this is how we get zuv = 1/4 * u-v zu-zv so this is the required canonical form. With this I would like to conclude my lecture, thank you very much for your attention.