Numerical Linear Algebra
Prof. Dr. D N Pandey
Department Of Mechanical Engineering
Indian Institute Of Technology, Roorkee

Lecture - 06
Linear Dependence and Independence

Hello friends. Welcome to the lecture. In today's lecture, we will discuss the concept very very important concept of linear algebra, namely linear dependence and independence. So, first let us define what is the linearly dependence and linearly independence just look at the definition here.

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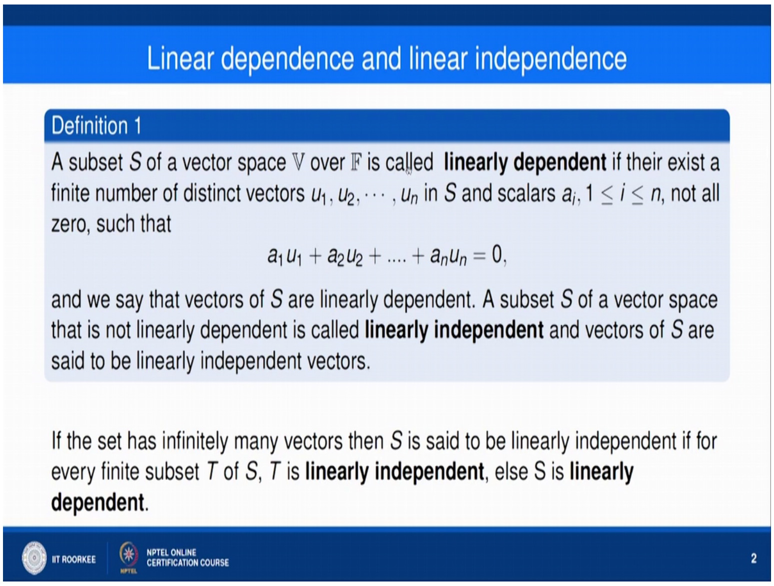

So, definition says that a subset S of a vector space v over a field F is called linearly dependent, if there exist a finite number of distinct vectors U 1 to U n which are element of S and scalars a i from i e from 1 to n not all 0 such that their linear combination is equal to 0 and these scalars are coming from this field.

So, idea is that, that if we form this system of linear equation and if this system of linear equation gives a nonzero non trivial solution in terms of ai S then we say that S is a linearly dependent set, and we call that vectors of S are linearly dependent vectors. A subset S of a vector space that is not linearly dependent is called linearly independent and vectors of S are said to be linearly independent vectors. So, this is the definition of linearly dependent set and linearly independent set. Now if we have infinite set. So, it means and then we can define linearly dependent independent ness.

So, let us define it, that if the set a has infinitely many vectors then S is said to be linearly independent if for every finite subset T of S, T is linearly independent otherwise we call S as linearly dependent set. So, let us discuss certain examples and fact about linearly dependent independent sets.

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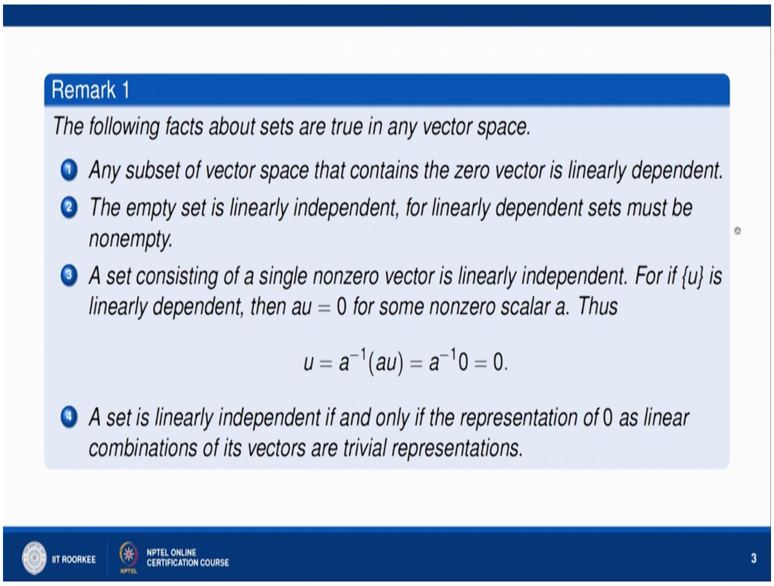

So, the following facts about sets are true in any vector space. First thing that any subset of a vector space that contains the 0 vector is linearly dependent. So, let us just look at here.

So, as for the definition says that a set S which is a subset of a vector space v which is defined over a field F.

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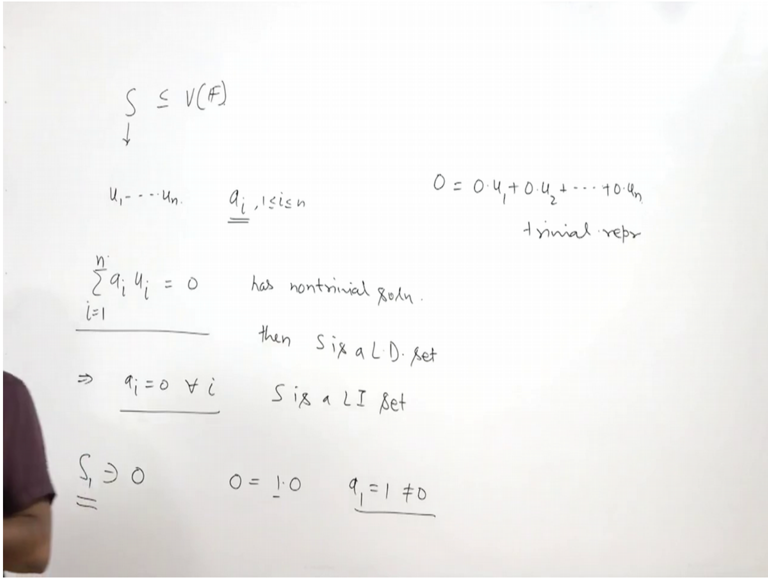

We say that S is linearly dependent, if we can find out say vectors u 1 to say u n finite number of vectors u 1 to u n and correspondingly a i is from 1 to n scalars from this field f, such that if we form this linear combination a i u i equal to 0 from i equal to say 1 to n and if this equation has nontrivial solution, then we say that S is linearly dependent in short we call this as S is a LD set. And if we do not have any nontrivial solution or means that if this linear combination is equal to 0, gives you only a trivial solution only a trivial solution means that all a is has to be 0 for all i then this case we say that S is a LI set ok.

Now, based on this definition we want to prove the first thing that any set say S 1 which contains say 0 element. So, it means that if we have a set S 1 which contains 0 then this set S 1 is linearly dependent why? Because if you look at we can write a 0 as 1 dot 0. So, it means that we are forming a linear combination like 0 and 1 is a scalar coming from this field, then I can write a 0 as 1 dot 0. So, it means that here your a 1 is equal to 1 which is not equal to 0.

So, here we have a nontrivial solution for this equation. So, we say that S 1 is linearly dependent. So, or you can say that this has another meaning that if we look at this here that here 0 can be represented as linear combination of u i. So, 0 has a several representation say. So, I can write a 0 as 0 dot u 1 0 u 2 and so on. So, this is known as trivial representation. So, this is a trivial representation. So, if 0 has only a trivial representation, we call that S is a LI set, and if 0 has a nontrivial representation like this it means that sum of a is are nonzero then we call that as is a LD set is it ok.

Now, let us move to second remark, it says that the empty set is linearly independent because when we try to define a linearly dependent set, we always requires a nonempty set to define a linearly dependent as so, but empty set does not contain any element. So, we can say that trivially that empty set is linearly independent. Now let us move to next remark it says that a set consisting of a single nonzero vector is linearly independent.

So, it means that if we have a singleton set for example, take this singleton u and here u is non 0 vector then if we take a u equal to 0 for some nonzero scalar a it means that we are assuming that that this singleton u is linearly dependent then since it is an a is nonzero, then we can multiply by a inverse and we can have u as a inverse applying on a of u now here a of u is nothing, but 0. So, a inverse applying on 0 which gives you 0. So, this implies that u has to be 0. So, it means that if a singleton set is linearly dependent, then that element has to be a 0 element. So, it means that if a set having only 1 vector which is nonzero, we can say that singleton set is linearly independent set.

Now, let us consider the fourth point; a set is linearly independent if and only if the representation of 0 as linear combination of its vectors are trivial representation, that we have just pointed out that a 0 can be written as summation a i u i equal summation ai ui, where all the ai's are nothing, but 0. Then if we have this kind of representation which is known as trivial representation, we say that set is linearly independent otherwise set is linearly dependent.

Now, let us come to first example let us consider this the set S which is consisting.

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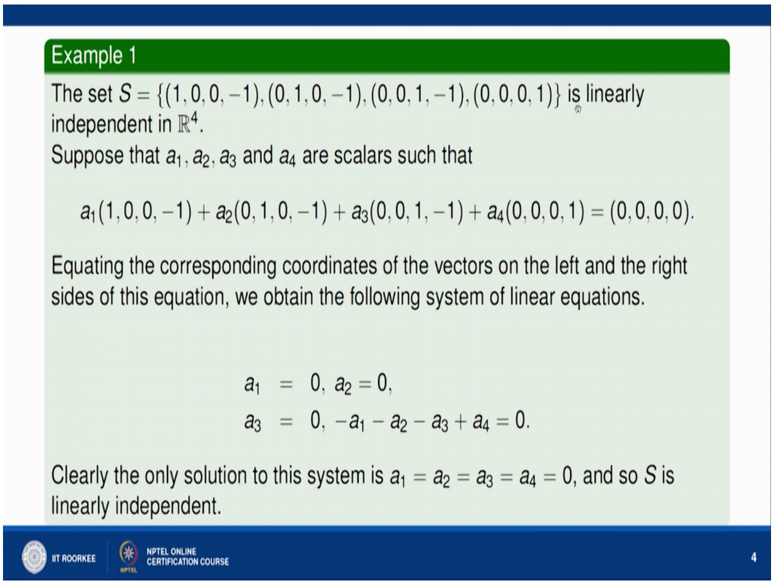

Say 4 element 4 vector, 1 0 0 minus 1 0 1 0 minus 1 0 0 1 minus 1 0 0 0 1 is linearly independent R4. Let us see whether this set is linearly independent or not. So, here we want to check whether this set S which is subset of R 4.

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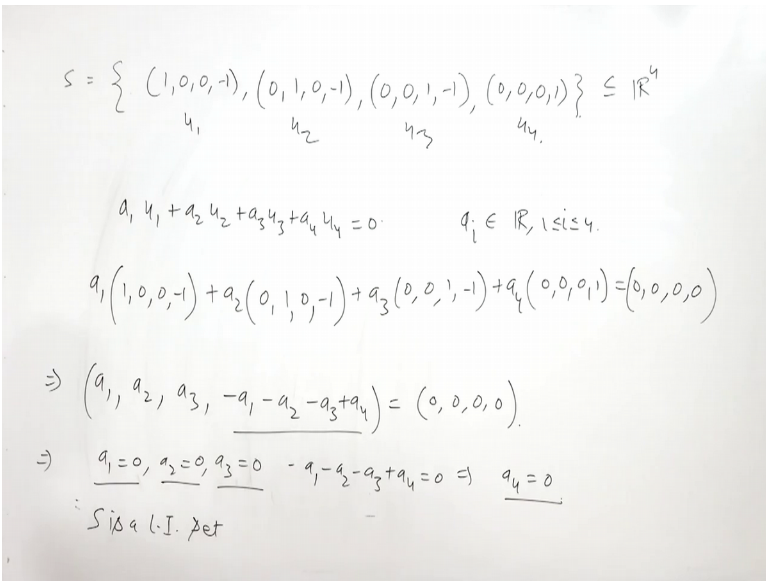

And here R 4 is a vector space and we want to check whether this set S is linearly independent or dependent.

So, let us take the element here and form a linear combination. So, here if you call this as u 1, u 2, u 3 and u 4 and from this combination a 1, u 1 plus a 2, u 2 plus a 3, u 3 plus a 4, u 4 is equal to 0 and then try to find out what is the solution in terms of a is. So, let us and here all these a is are nothing, but from R i equal to 1 to 4.

So, here let us say a 1 and u 1 is 1 0 0 minus 1 plus a 2 0, 1, 0 minus 1 plus a 3 0, 0, 1 minus 1 plus a 4 0, 0, 0, 1 equal to 0.

So, here this 0 is nothing, but 0 vector or you can write it like this. So, if we equate. So, you can write this as say a 1, and then here we have a 2, and here we have sorry here we have then we have a 3 yeah then we have minus a 1, minus a 2, minus a 3, plus a 4 is equal to 0 0 0 0 and if we equate both the things because this is a vector, this is vector and then it will be equal if it is equal in component wise.

So, we can say that a 1 equal to 0, a 2 equal to 0, a 3 equal to 0, and when you put last 1 is minus a 1, minus a 2, minus a 3, plus a 4 equal to 0 when you simplify you will get a 4 equal to 0. So, here if we form if we take this linear combination which is equal to 0, then we can say that we have only a trivial solution that is all a is are nothing, but 0. So, we can say that this S is a LI set nearly independent set and the vectors u is are linearly independent vectors here.

Now, let us move to second example. So, in the second example we have another set of R4.

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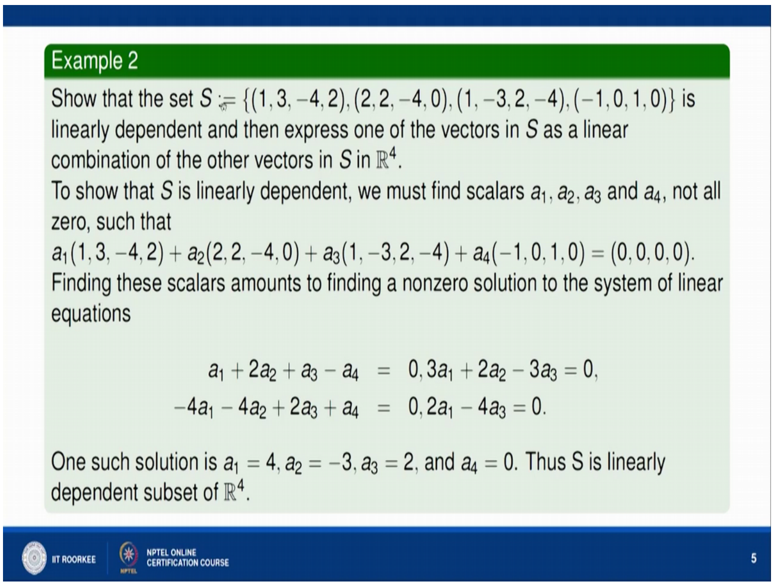

So, we need to check whether this is a LI set or LD set. So, here example is that we have to show that this set is linearly dependent, and then we try to express one of the vectors in S as a linear combination of others vectors in S in R4.

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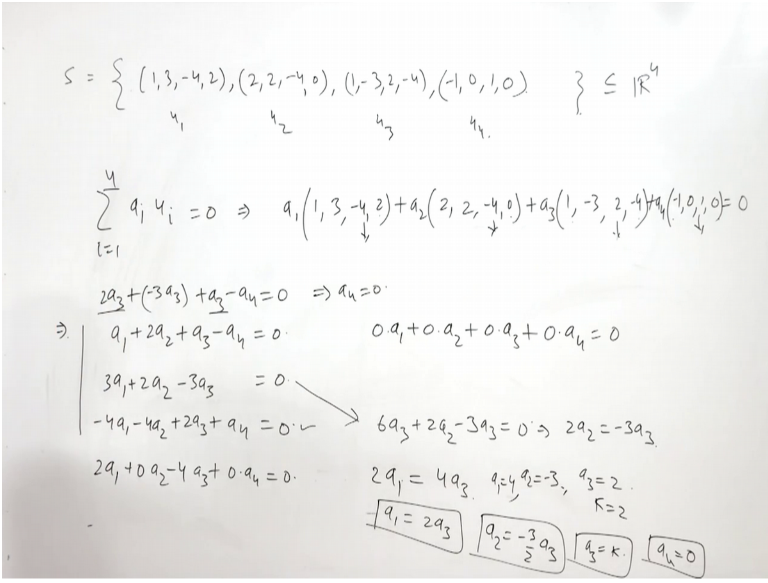

So, as the example 2 says that we have to show that the set S which consists of these 4 vectors say U 1 which is 1 3 minus 4 2 u 2 S 2 2 minus 4 0 and u 3 S 1 minus 3 2 minus 4 and u 4 as minus 1 0 1 0.

So, this set we want to show that it is linearly dependent and not only we want to show that it is linearly dependent, but we can show that in this case that one of the vectors can be written as linear combination of other vectors. So, first let us start to show that this is a linearly dependent set. So, let us form a linear combination of these vectors u 1 to u 4. So, let us say that a 1 a i u y summation i is from 1 to 4 is equal to 0. So, this is nothing, but a 1 let us say 1, 3 minus 4, 2 plus a 2 2 2 minus 4 0, plus a 3 and it is 1 minus 3 2 minus 4 plus a 4 and here we have minus 1 0 1 0 equal to 0 vector.

So, if we write it here then we can say that we can write down a system of linear equation in terms of a i s. So, we can say that a 1, plus 2 a 2, plus a 3, minus a 4 is equal to 0. And here we have 3 a 1, plus 2 a 2, minus 3 a 3, equal to 0; then minus 4 a 1, minus 4 a 2, plus 2 a 3, plus a 4 is equal to 0 and this we are looking at the third component. So, third equation is nothing, but we are looking at only the third component.

So, here last equation is what; 2 a 1 plus 0 times a 2 minus 4 times a 3 plus 0 times 0 times a 4 equal to 0. So, here this system of this linear equation is equal to 0 gives rise to the system of linear equation in terms of a i and if and we want to find out the solutions here. We have these 4 equation a 1 plus 2 a 2 plus a 3 minus a 4 equal to 0, 3 a 1 plus 2 a 2 minus 3 a 3 equal to 0, minus 4 a 1 minus 4 a 2 plus 2 a 3 plus a 4 equal to 0 and 2 a 1 minus 4 a 3 equal to 0. And as we pointed out that if we add these 2 equation then we are nothing getting nothing, but the minus of this equation.

So, it means that this system of linear equation has more than one solution or you can say that infinite (Refer Time: 14:09) many solution and one such possible solution is given by this that a 1 equal to 4, a 2 equal to minus 3, a 3 equal to 2, and a 4 equal to 0 and if you look at here we have say nonzero solutions for these a is. So, we can say that S is linearly dependent. So, this can be obtained if you look at here then this can be obtained by here. So, you can say that a 1 is nothing, but 2 of a 3.

So, you can get a 1 as 2 of a 3, then just look at here the second equation then second equation is what you simply write down the value of a 1 then it is what? 6 of 3. So, from this equation you can write 6 of a 3 plus 2 of a 2 minus 3 of a 3 is equal to 0. So, here we can write it that 2 of a 2 is equal to minus 3 of a 3. So, here we can write it minus 3 of a 3 how we are getting? 6 minus 3 three and then you if you go take it at other side we have 2 of a 2 as minus 3 of a 3. So, we can write down that a 2 is nothing, but minus 3 by 2 a 3 and a 3 is basically free you can take this as k.

Now, how to find out this 4 a 4? So, if you put all the values of a is here then a 1 is basically 2 of a 3, and 2 of a 2 is nothing, but minus 3 of a 3 plus a of 3 minus a 4 equal to 0. And if you simplify this 2 of a 3 plus 1 of a 3 and it is 3 of a 3 and this gives you that a 4 equal to 0. So, here we have a 4 or equal to 0. So, here we can give any value to this k and we have infinite many solutions to this. So, once a solution is that you take k as to. So, let us the, this as 2. So, a 3 is equal to 2, then your a 2 is nothing, but minus 3 and a 1 is nothing, but that is 4 here. So, here a 4 minus 3 and 2 and 0 is one such solution that we have pointed out in our slide.

Now, in this case you can write down one of the vector in terms of others. So, let us take this solution that a 1 is equal to 4 let me write it here as a 1 equal to 4.

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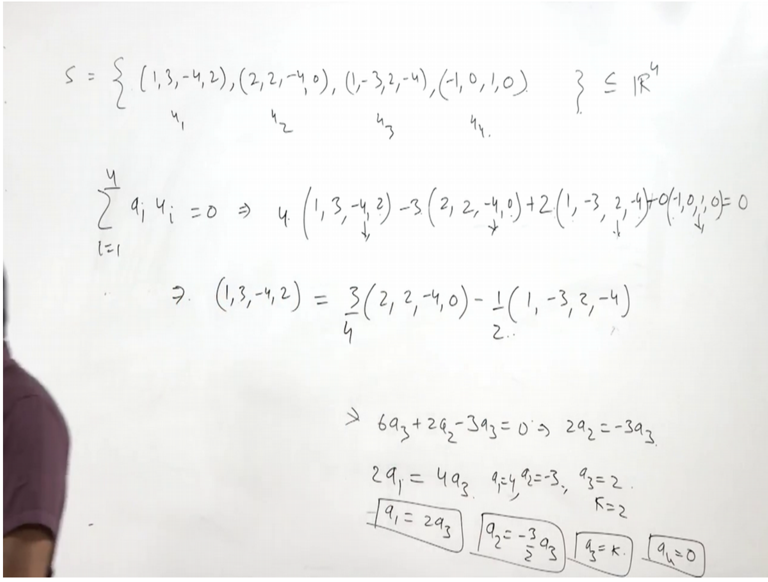

And a 2 is what? A 2 we have pointed out that is it is equal to minus of 3. So, here it is minus of a 3, a 3 is simply k. So, a 3 is nothing, but 2 here. So, an a 4 is coming out to be 0.So, here we have 0 equal to this. So, it means that we can write down this as 1 3 minus 4 2 4 you just take it the other side and this can be written as 3 times 2 2 minus 4 0, minus 2 times 1 minus 3 2 minus 4 and if you divide it by 4 then you will get this relation. So, minus 2 by 4 can be simplified by as minus 1 upon 2. So, here what we have shown here we have shown that S is a linearly dependent set and one of the vector say a 1 3 minus 4 comma 2, this can be written as linear combination of element of say it is u 2 and u 3. So, it means that U 1 can be written as linear combination of u 2 and u 3.

Similarly, if you rearrange this, this time I have taken this to other side if you take this element go other side then you can write down u 2 in terms of U 1 and u 3 similarly you can write down u 3 in terms of U 1 and u 2 u three. So, in case of linearly dependent set you can always write one of the element as in a combination of other element of this set statement of this theorem.

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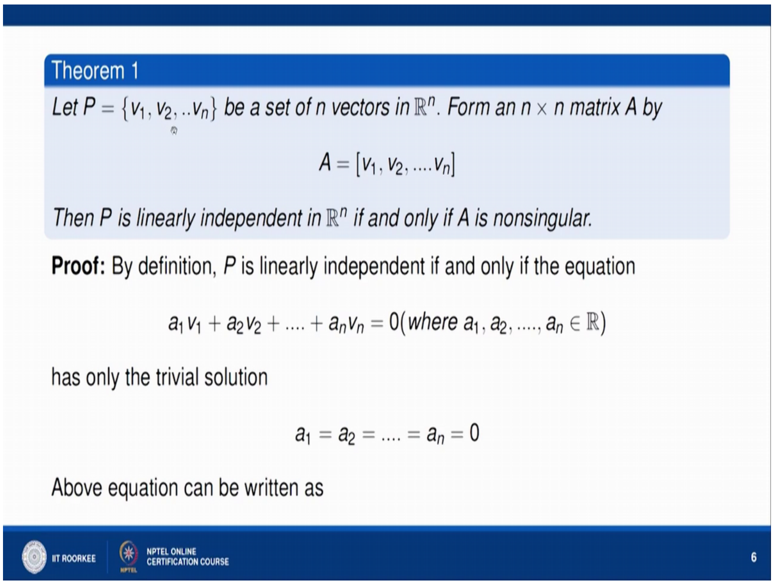

One is that let P contains these an element n vectors we want to v n be a set of n vectors in R n then we can check the linearly independent as of this set by forming a n cross n matrix a by saying that a equal to v 1 to v n, here v 1 is an element of Rn. So, v 1 is first first column, v 2 is second column, and v n as n th column then we can say that this set p is linearly independent in R n if and only if this matrix a is nothing, but a non singular matrix.

So, how we can prove it. So, let us move here by definition P is linearly independent if and only if the equation, this linear combination summation a i vi equal to 0. So, here we have a 1 v 1 plus a 2 v 2 plus a and v n equal to 0, when all these coefficients are coming from real numbers. Then we say that p is linearly independent if this equation has only a trivial solution.

Now, this equation can be written in terms of system of linear equation a x equal to 0.

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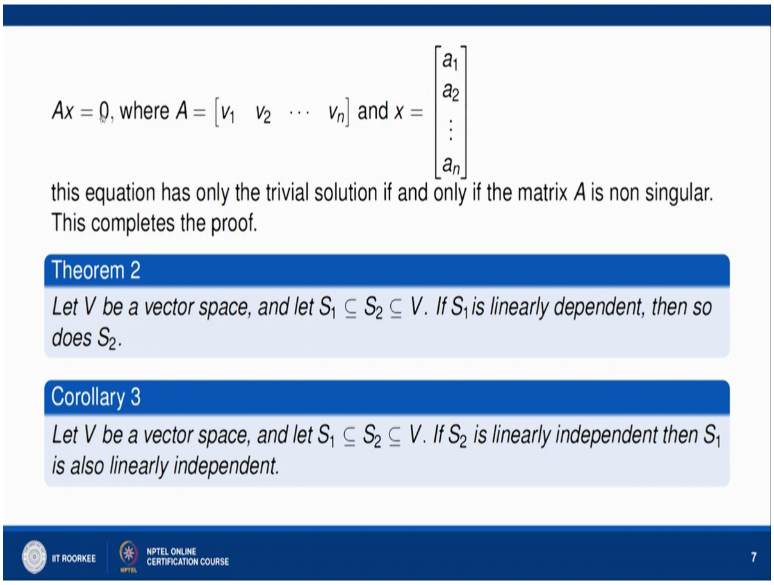

So here a x equal to 0. So, I am writing this equation as.

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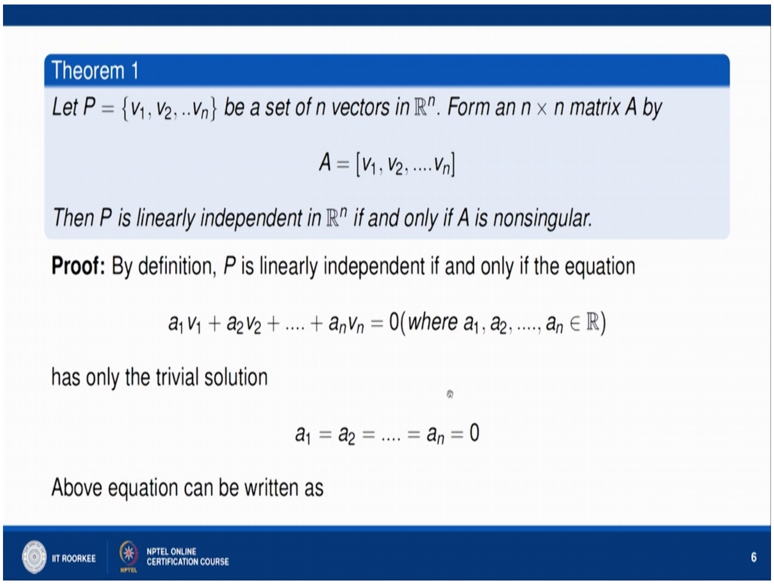

Matrix equation a x equal to 0 where a is nothing, but v 1 to v n where v 1 is first column, v 2 a second column Vn as n th column and x is unknown which is nothing, but a 1 to a n. So, we can say that this equation has a trivial solution provided a x equal to 0 has a trivial solution now a x equal to 0 has trivial solution only, if and only if the matrix is nonsingular. So, which complete the proof. So, it means that in R n if we have n a set of n vectors, then by forming this matrix we can check easily check that this set is linearly independent or not. So, only the end thing we need to check is the similarity or non similarity of the matrix a.

Now, let us move to next theorem; theorem 2 which says that let v be a vector space and let S 1 is a subset of S 2 is a subset of V, then if S 1 is linearly dependent then. So, does S two. So, theorem says that we have a.

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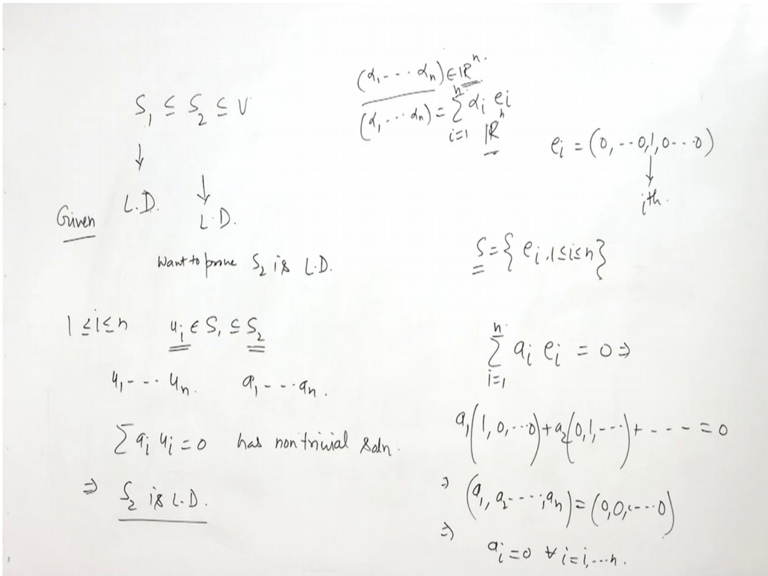

So here a x equal to 0.S 1 a subset of S 2 is a subset of vector space v and we say that if S 1 is a L D set. So, if S 1 is L D set then its superset S 2 is also L D, this is what we want to prove. So, this is given. So, given that S 1 is a L D set want to prove that want to prove that S 2 is also LD. So, how we can prove here. So, let us since S 1 is given as linearly dependent set. So, it means that by definition of linearly dependent ness we can always find out a finite number of vectors in S 1 say you want to say u n and constant coming from the field say let us say that a 1 to a n such that this linear combination a i u i equal to 0 has nontrivial solution.

So, that is by definition of linearly dependent set nontrivial solutions. Now if you look at since all these u is are coming from S 1 for all i, i is from 1 to n now S 1 is subset of S 2. So, you can say that u i is all u i is all element of S 2 here. So, taking u i as element of S 2 and a is coming from the scalar field we can say that this linear combination summation a i u i equal to 0 has nontrivial solutions. So, this implies that S 2 is linearly dependent. Because by definition S 2 is linearly dependent means we if we can find out a finite number of vectors finite number of vectors from S 2 and constant a 1 to some constant a 1 a is from scalar field such that this summation this linear equation summation a i u i equal to 0 has nontrivial solution. So, this implies that S 2 is L D which we want to prove.

So, that is the small informal proof of this theorem 2 corollary of this theorem which says that let v be a vector space and let S 1 is a subset of S 2 is a subset of v if S 2 is linearly independent then S 1 is also linearly independent. So, this I can say that here S 2 is what S 2 is a set and if S 2 is linearly independent then subset of linearly independent set is also linearly independent. So, if you look at the theorem, theorem says that superset of a linearly dependent set is linearly dependent and this corollary says that subset of a linearly independent set is linearly independent. So, this can be easily proved with the help of this theorem.

So, let us suppose that this S 1 is not linearly independent. So, it means that S 1 is linearly dependent. So, by previous theorem if S 1 is linearly independent dependent then its superset is also linearly dependent, but it is given that S 2 is linearly independent. So, this implies that S 1 can S 1 cannot be linearly dependent set. So, S 1 has to be linearly independent is it ok.

So, this corollary can be easily proved with the help of this theorem. So, we can summarize these theorem and this corollary is that, if we have a linearly dependent set then all its super sets must be linearly dependent and this corollary can be seen that if we have a linearly independent set then all its subsets must be linearly independent is it ok.

So, now let us move to very important theorem 4.

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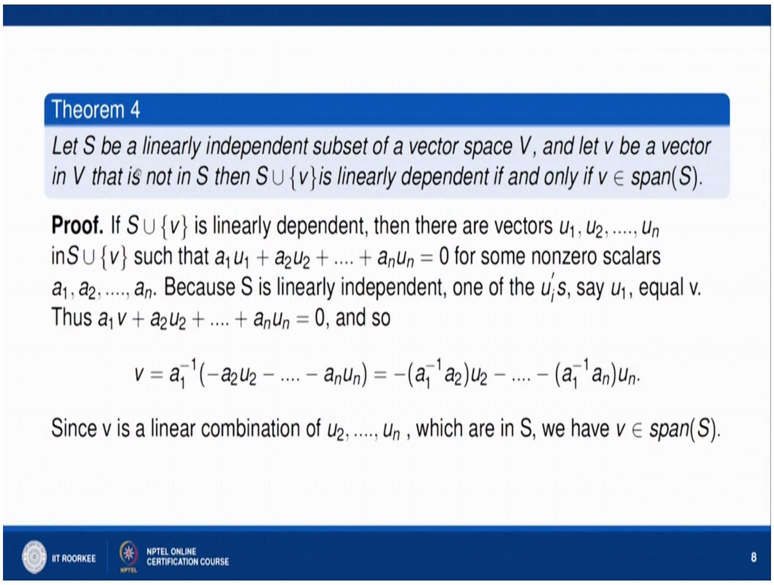

Which says that let S be a linearly independent subset of a vector space V and let v be a vector space v be a vector in vector space v that is not in S, then S union; this singleton element v is linearly dependent if and only if v belongs to span of S. Now what is the importance of this theorem? If you look at the previous corollary, corollary says that the subset of linearly independent set; set is linearly independent, but we do not know how that what is what about the superset of linearly independent set. So, this theorem gives a small hint here, that if we have a linearly independent set like this say S is a linearly independent set, and if we join if we add a new vector say v which is not in S, then this will be linearly independent if and only if v is not in a span of S. So, this theorem will help us to construct new linearly independent set, as a superset of a given linearly independent set.

So, let us say that since S union v is linearly dependent, then by definition we can always find out some vectors say u 1 to u n and S union v such that a 1 u 1 plus a 2 u 2 plus a n, u n equal to 0 for some nonzero scalars a 1 to a n. So, now. So, this is by definition right now we already know that S is linearly independent. So, it means not that these u i must contain at least 1 element this v, because if u i and none of u i is v, then this simply says that S is linearly dependent, but S is linearly independent. So, it means that 1 of the u is say U 1 equal to this v. So, it means that we have a 1 v plus a 2 u 2 plus a n u n equal to 0.

Now, here so, this follows from the linearly independent ness of this S. Now here my claim is that this a 1 is also non zero why? Because if this a 1 is 0 then this is nothing, but a 2 u 2 plus a n u n equal to 0, and which has a nontrivial solutions. So, this implies that S is linearly dependent, but that is not that will contradict the fact that S is linearly independent. So, it means that a 1 has to be nonzero.

So, if a 1 is non zero then we can multiply by the inverse of a 1, and we can write down this v as a 1 inverse of minus of a 2 u 2 minus of a 3 u 3 and minus of a n u n. And if you simplify this can be written as minus of a 1 inverse a 2 u 2 minus and minus a a 1 inverse a n u n. So, this can be say that this can be written as that v is written as linear combination of u 2 u 3 up to u n. So, it means that v is a linear combination of u 2 to u n and all these elements are coming from where these elements are coming from this S.

So, we can say that v is belonging to span of S that is what we wanted to prove. So, it means that we have started with that S union v is linearly dependent, and what we have proved here that we belongs to span of S. Now we try to prove the other part the other part is that if we take v belongs to span of S then we try to show that S union v is linearly dependent. So, let us move here.

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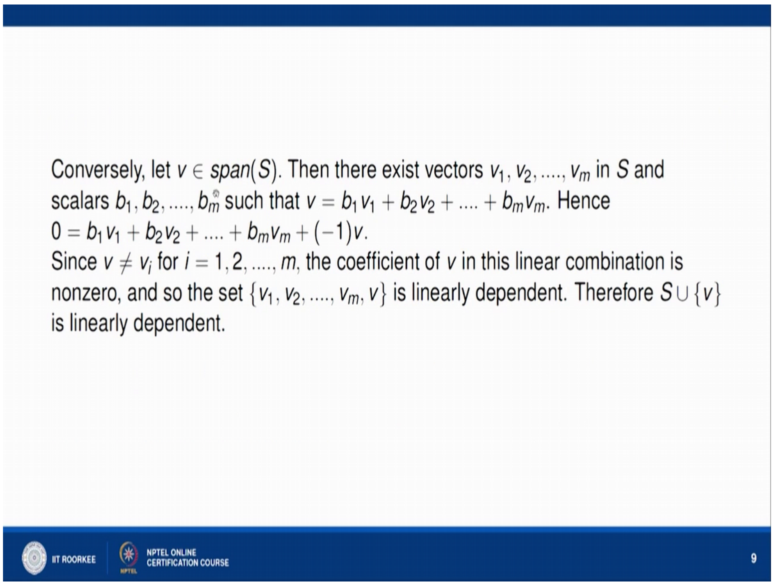

So, let v belongs to a span of S. So, if v belongs to span of S means v can be written as linear combination of elements of S. So, let us take elements v 1 to v m in S and scalars corresponding scalar b 1 to b m says that v can be written as b 1 v 1 plus b 2 v 2 plus b m v m. Now when you write it here this can be taken in the right hand side and we can write that 0 can be written as b 1 v 1 plus b 2 v 2 plus b m v m plus minus 1 to minus 1 into v.

Now, it may happen it looks like the this 0 has a nontrivial representation, but this 0 has nontrivial representation if we none of this v i is equal to v, because if some of say v i is equal to v then this will be canceled out by the corresponding v is, and we can say that 0 has only trivial representation, but this cannot happen because v is not in S.

So, v is not in S means v cannot be equal to any of these v is. So, v is not equal to v i for i is 1 to m. So, it means that the coefficient of v in this linear combination has to be nonzero. So, it means that this minus 1 cannot be cancelled by any of these v is. So, it means that this 0 has a non triple representation in terms of v 1 to v m and v. So, it means that S union v is linearly dependent. So, what we have proved here that, if we start with say linearly independent set say S, then we can add a new vector into this set only when this set v is not in a span of S. So, it means that if v belongs to span of S, then this union will make this set as a linearly dependent set.

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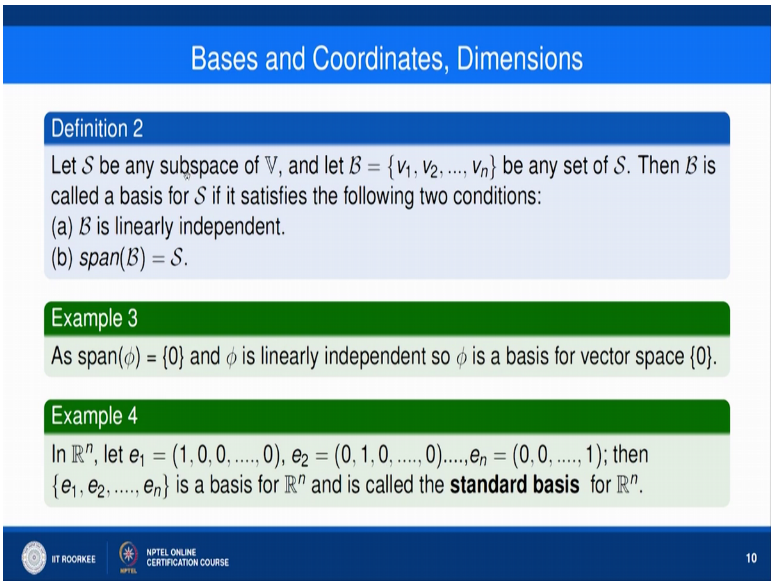

So, let S be an any subspace of V and let B is a set v 1 to v n be any set of S, and we say that b is called a basis for this subspace S if it satisfies the following 2 condition. The first condition is that this set B has to be linearly independent set, second that the span of this set B is whole of S and if we have these 2 property holds then this set B is known as basis for this subspace S. So, let us take a very important example 3, which says that since span of phi is equal to singleton set 0, and we already know that phi is linearly independent then we can say that this singleton empty set phi is a basis for vector space 0.

Now, moving on next example and next example we are considering this set R n and it is quite popular, R n and here we try to find out a basis for this R n. So, we say that e 1 which is the vector here 1 0 0 0 0, e 2 as only the second place is 1 rest are all 0 e (Refer Time: 31:50) is n th place is 1 rest are all 0, then this set e 1 to e n is a basis for R n. So, we need to prove that this set e is are linearly independent and every factor of R n can be written as linear combination of these e is; these are this is this is quite a trivial example, but quite important example.

So, here we have R n and here we have e i, e i is basically what 0 0 0 1 all zeroes and here. So, this is only i th place. So, i th place is 1 rest all 0.So, we want to show that these the set S which is nothing, but e i, i is from 1 to n is a basis here. So, S is a basis for r n and this has a name very special name which is known as standard basis of r n. So, to show that it is linearly independent set. So, we form this a i e i equal to 0, i is from 1 to n and if you write down. So, here we have what a 1 1 0 0 0 plus a 2 0 comma 1 0 0 and so on. If you look at if you simplify equal to 0 if you simplify this it is nothing, but a 1 a 2 and a n is equal to 0 comma 0 comma 0.So, this implies nothing, but a is are all 0 for each i 1 to say n.

So, lines is quite easy here, now to show that this span the whole of R n you take any element in R n. So, let us take any element in R n say let us say alpha 1 to say alpha n it is an element of Rn here, now here we may write it as transpose of this, but since we are using this as a row vector. So, I am assuming alpha 1 to alpha n also as a row vector here otherwise we have to apply everywhere the transfers here.

So, we can say that this factor which is an element of R n here this can be written as alpha 1 to alpha n can be written as summation alpha i e i i is from 1 to n this can you can easily check. So, it means that this every vector in R n can be written as linear combination in terms of e is, and all these e is are linearly independent. So, we can say that they set e 1 to e n forms a basis of R n and it is known as standard basis of R n.

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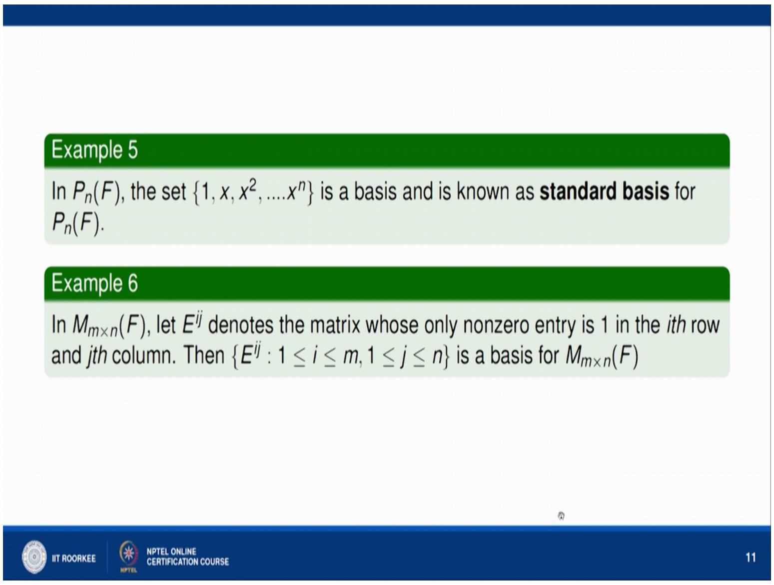

So, moving on next example, here we consider say subset of a polynomials. So, P n F represent the polynomials of order say n of order n, and we try to claim this set is 1 x x square x n is a basis and it is also known as stand basis for P n F means set of all polynomials of degree n. So, to show that it is a basis we need to prove 2 things first is that these are LI and second thing is that every polynomial of degree at most n can be written as linear combination of this.

So, that this part that it is that this span be a polynomial any polynomial of degree n is quite easy.

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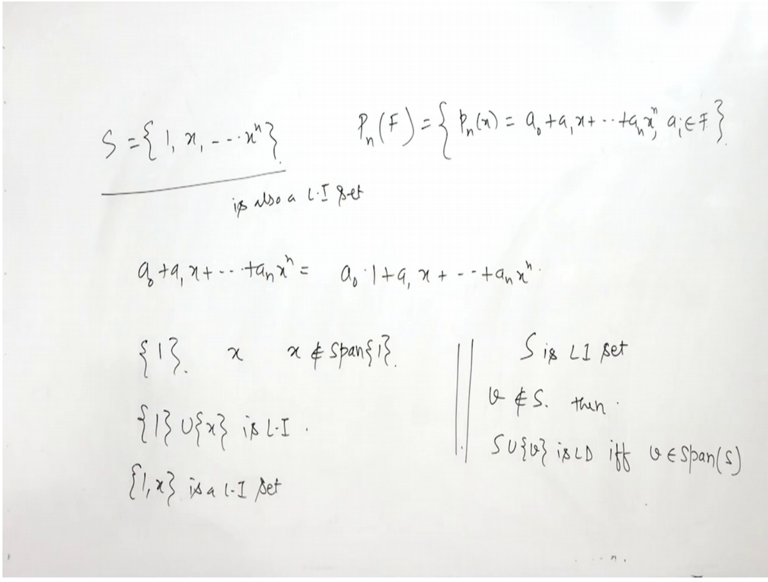

So, here we want to show that this 1 x x n, this forms a basis for P n F what is P n F? It is a set of all polynomials which can be written as say a set of all polynomials say of p n x which can be written as say a naught plus a 1 x plus so on a n x n all these a is are coming from this field f. So, so first of all take any element of P n F. So, it will be of this kind say a naught plus a 1 x plus so on a n, x n and you can easily see that this can be written as linear combination of element of this set call this as say S.

So, this can be written as a naught into 1 plus a 1 into x and a n and into x power n. So, proving that the span of S is P n F is quite easy, but to prove that this set S is basically a linearly independent set is quite nontrivial. So, let us start with this one. This one is a non 0 vector. So, this is a this singleton 1 is basically linearly independent set. Now let us consider the next element that is x, now x can be cannot be written as a linear combination of this one. So, it means that here x is not in a span of this 1.

So, it means that this 1 union x is LI, this follows from the previous theorem which says that if we start with a LI, set that if S is a LI set and take any element v which is not in S then S union v is LD, if and only if v belongs to a span of S. Now here if we look at this S is nothing, but singleton [FL] singleton 1 and this x is not in a span of 1 then 1 union x has to be li. So, it means that this 1 x is a LI set and we can keep on doing and we can say that this set S is also a LI set. So, here by repeated application of this result, we can show that this set S is a LI set.

So, we here we have shown that this is LI set and this is span the whole of P n F. So, we can say that it is a basis for set of all polynomials of degree at most end, and we call this set S as a standard basis for P n F. And next example m cross n it is a set of all m cross n matrix whose entries are coming from this field f then here we try to construct the basis for this. So, here basis is given in terms of E i j what is E i j? E i j denote the matrix whose only nonzero entries is 1 in the i th row and j th column, then this if we define our matrix like this the E i j then they set e i j form a basis for the set this vector space M of matrix of order m cross n.

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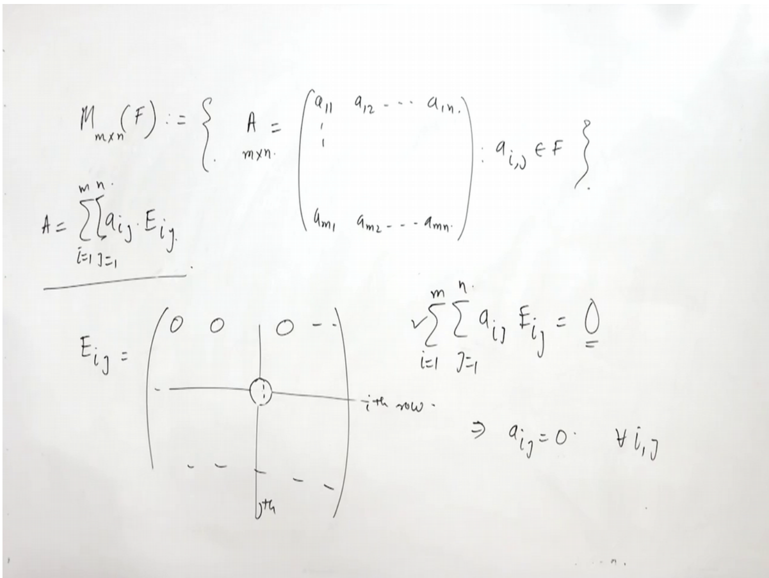

So, let us consider here. So, here if we take any element here element is that we have a matrix of m cross n order, and whose entries are coming from this field f we can write this as say a 1 1 a 1 2 up to a 1 n and so on a m 1, a m 2 and so on a m n and here a i j is coming from this field F. So, this is the vector space m of matrix of order m cross n and we try to find out say basis of this and claim that if we take E i j.

So, what is E i j? E i j is the matrix whose the i th row and j th column. So, here we have i th row and j th column and here if we have only one entry here rest all 0.So, only one a nonzero entry one here is at this place rest are all 0, and we say that this e i j forms a basis for this vector space M. So, here to prove that this is a linearly independent you simply form this summation a i j E i j equal to 0.

Now, here 0 is a 0 matrix and we can show that here i this is double summation here i is from 1 to say m and j is from 1 to n. And if we form this then we can prove that by equality of matrix you can show that a i j is nothing, but 0. So, is that. So, this is not a very difficult thing to show. So, you just show that by writing all e i j here and forming this kind of linear combination you can show that all a i js are 0 for all i and j, then to form to show that this form a span spanning set, you can simply say that you can write down this A as nothing, but a i j, E of i j, i is from 1 to say m and j is from 1 to n. So, you can write down your matrix A in as a linear combination of these a i j like this.

So, we can say that by this you can say that any element a of this vector space can be written as linear combination of these E i j, and this by this exercise you can show that all these E i j is nothing, but linearly independent vectors. So, we can say that the basis for this vector space is nothing, but E i j, i is from 1 to m and j is from 1 to n.

So, here I am I will stop here, in next class in next lecture we will discuss more about the properties of basis and with the help of basis we try to define what is the dimension of a vector space and with the help of this notion we also try to define the coordinates of a given vectors. So, here I am stopping and thank you for listening us.

Thank you.