Numerical Linear Algebra
Dr. D. N. Pandey
Department of Mathematics
Indian Institute of Technology, Roorkee

Lecture – 51
Householder matrices

Hello friends. Welcome to this lecture. In this lecture, we will discuss the concept of householder matrices and its application in a QR factorization of a matrix A. So, first let us define; what is householder matrices and what is basically role of this householder matrix.

So, here to define householder matrix basically, it is a householder matrix is given by the matrix of the form I minus 2 omega omega transpose where omega is a unit vector in R n.

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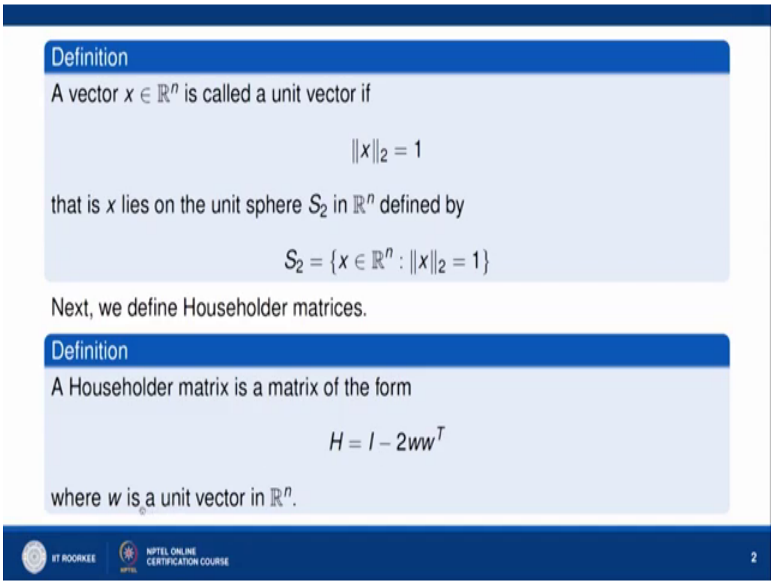

Now, how we define unit vector. So, a vector x in R n is called a unit vector if 2 norm of x is equal to 1 here, we are defining you can take any norm. In fact, norm of x is equal to 1 and or we can say that in other word, we can say that x a lies on the unit sphere S 2 where S 2 is defined as a set of all x in R n such that 2 norm of x is given as 1.

So, with the help of unit vector, you can define householder matrix is of this kind H is equal to I minus 2 omega omega transpose where omega is any unit vector in R n; So, now, once the householder matrix is defined.

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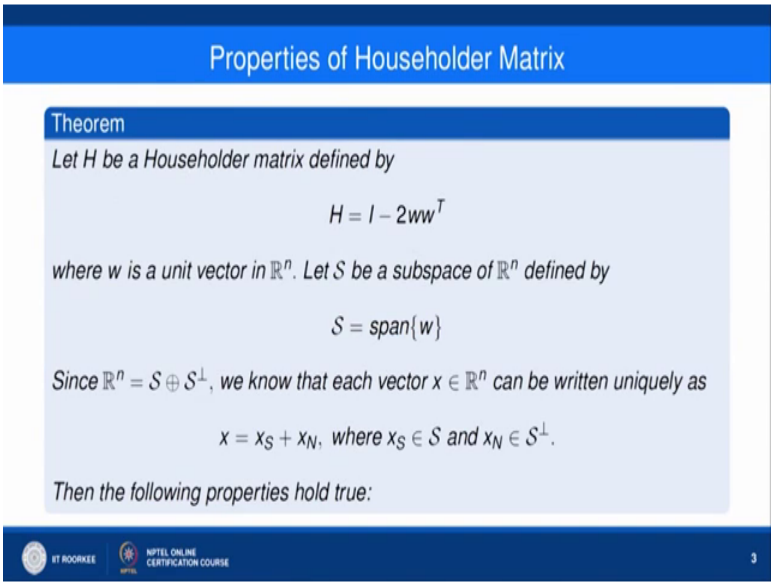

Let us consider some basic properties of householder matrix. So, so, to say that let H be a householder matrix defined by H as I minus 2 omega omega transpose where omega is a unit vector in R n and let S be a subspace of R n which is of dimension one and it is spanned by this unit vector w and we can extend this vector w to the entire basis of R n and we can generate a subspace which is known as S perp and we can write R ns S direct sum with S perp and it means that the every element in R n can be written as sum of the element of S and element of S perp and this representation is a unique representation.

It means that any vector in x in R n can be written as x of S plus x of N whereas, x of S is a member in S and x of N is a member of S perp then the following properties holds true.

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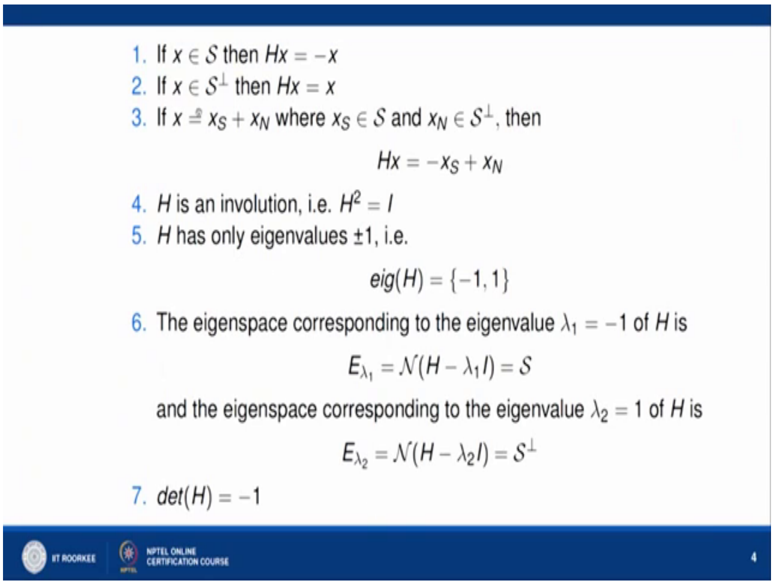

So, first property is that if x belongs to S, then if we operate H on x, then it is coming out to be minus x similarly if we take x in S perp, then image of x under H is going to be X itself and if x is equal to x S plus x N where x S is a member in S and x N is a member of S perp then H of x is given by minus x of S plus x of N and H is an involution matrix or you can say that H square is equal to I and H has only 2 only Eigenvalues plus minus 1 it means that Eigen space of Eigenvalues of H is minus 1 and 1 and the Eigen space corresponding to the Eigenvalue lambda 1 equal to minus 1 of H is the S.

So, it means that I gain space corresponding to minus 1 is S and Eigen space corresponding to lambda 2 equal to 1 is S perp.

Or in other word, you can say that the geometric multiplicity of lambda 1 equal to minus 1 is 1 and geometric multiplicity of lambda 2 equal to 1 is N minus 1 and determinant of H is equal to minus 1 that we can calculate.

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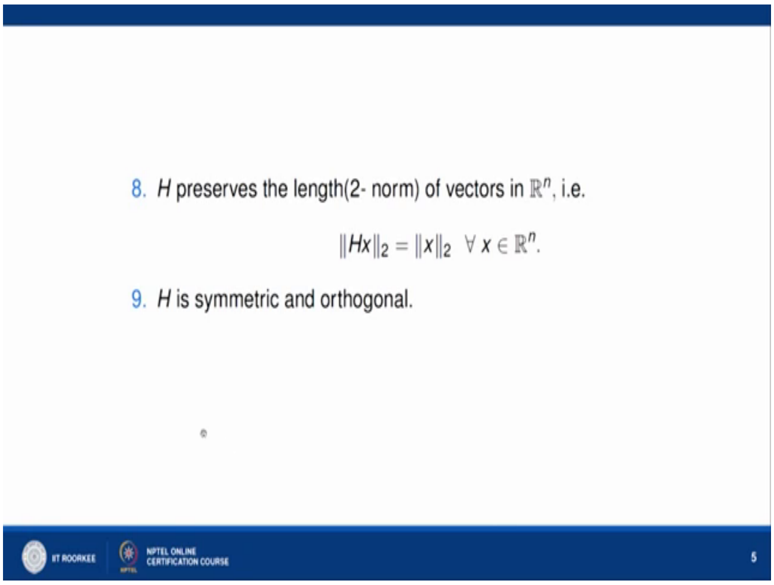

And H preserve the 2 norm vectors in R n, it means that 2 norm of H of x is same as 2 norm of x for every x in R n and H means householder matrix is symmetric and orthogonal matrix.

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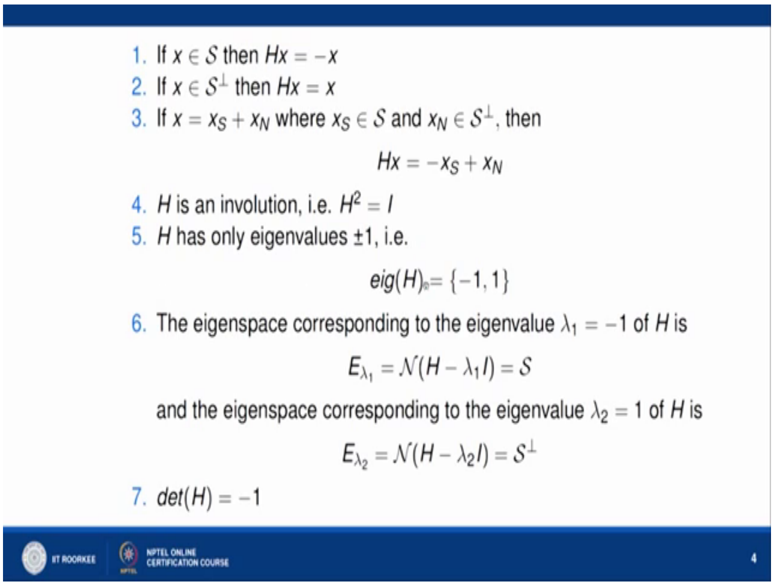

So, the these are some set of basic properties of householder matrix and we try to give a very simple or very quick proof of these properties.

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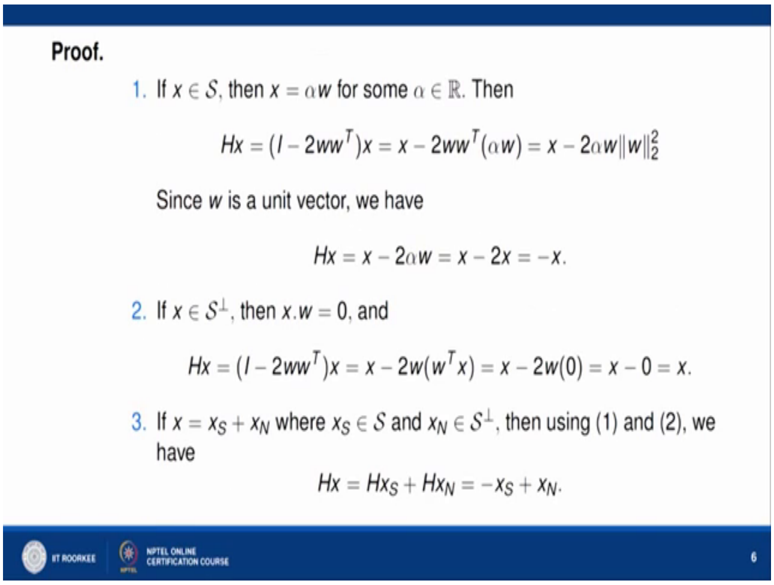

So, let us start with one. So, here if x belongs to S then x is alpha omega where omega is the vector unit vector which is spanned the space S. So, x is equal to alpha omega and if we operate H on this alpha omega, then it is nothing, but I minus 2 omega omega transpose x that is x minus 2 omega omega transpose. Now we are writing x as alpha omega, then alpha you can take it out. So, x minus 2 alpha omega and omega transpose omega is nothing, but 2 norm of omega whole square.

Now, since omega is a unit vector. So, it is coming out to be 1. So, it is nothing, but x minus 2 alpha alpha omega and 2 alpha omega is 2 x y because alpha omega is x. So, this is nothing, but x minus 2 x which you simplify then it is coming out to be minus x. So, if x belongs to S then H of x is equal to minus of x.

Now move. So, the one is quite obvious you simply apply and you can see that now if x belongs to S perp, it means that x dot omega is equal to 0 right and then we calculate H of x which is nothing, but I minus 2 omega omega transpose x when you simplify that this is x minus 2 omega omega transpose x. Now, omega transpose is x is nothing, but x dot omega and it is coming out to be 0. So, this is x minus 0, it means this x; so, H x is equal to x when x is in S perp. So, this is also straightforward.

Now, let us prove that if x is an any vector in R n, then we can uniquely represent x as x of S plus x of N where x axis in S and S n is an S perp, then using 1 and 2 these we say that H of x which is nothing, but H of x S plus H of x n. Now H of x S is nothing, but minus of x S that is clear from one and H of x N is x N that is clear from this 2. So, H of x is equal to minus x of S plus x of n.

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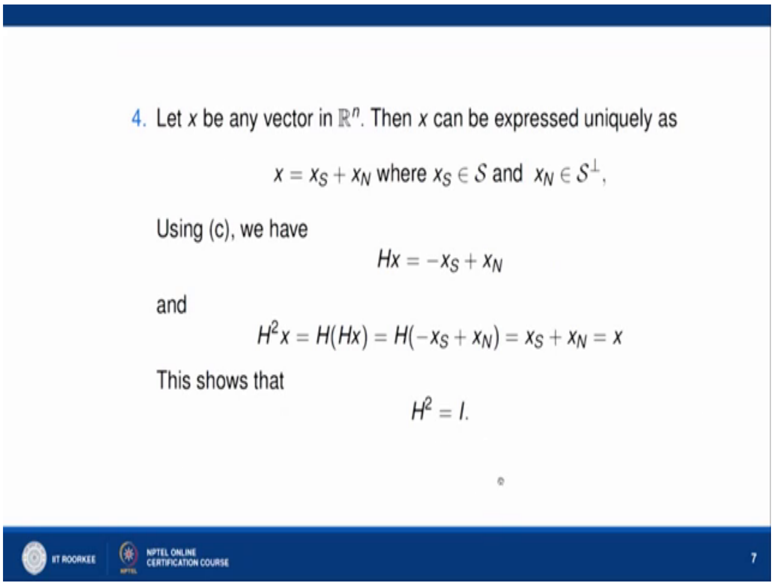

So, that gives you the proof of third property now look at the fourth property that a H square is an H is in evolution or we can say that H square is equal to I for that we already know that H of x is equal to minus x of S plus x of N where x is any vector in R n and x S and x N is the part of x in S and S perp and this representation of x is unique.

So, when you operate H on this H of x then H will be operated on minus x of S plus x of N now again using one and 2 H of minus x of S is minus of minus x. So, that is x of S plus x f N. So, this is nothing, but x. So, H square x is equal to x and this is true for every vector in r of N. So, we can show that this implies that H square equal to I. So, we can repeat this process for each E I where E I is the unit vector you can say and that basis element standard basis element of R n. So, H square is equal to I.

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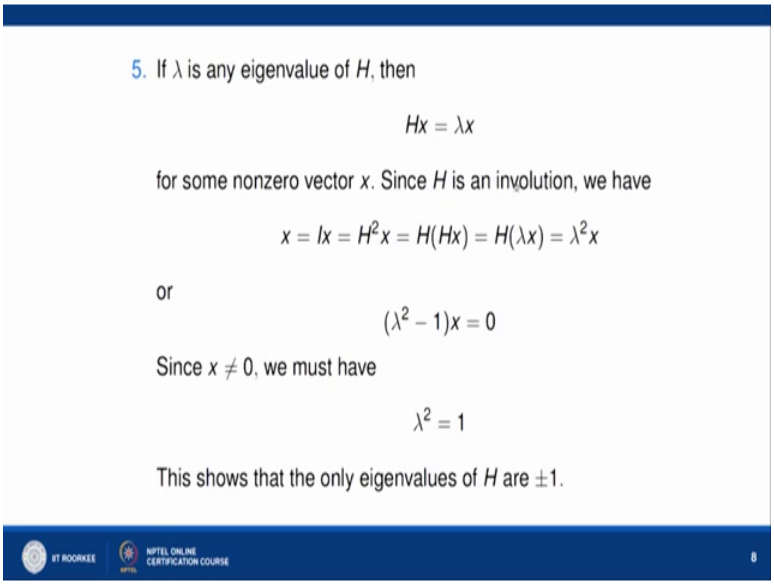

Now in fifth, we try to find out the Eigenvalues of matrix H for that lambda is an Eigenvalue of H if H of H of x equal to lambda x and we want to find out that what are the Eigenvalues of lambda. So, for that we start with the this that x can be written as I of x and we already know that I is H square you can say that H is squares I.

So, here I can say that x is equal to H square x now H square x I can write it H of H of x. Now H of x we already knew know; that it is lambda of x. So, H of lambda x is equal to lambda of H of x. Now, again H of x is equal to lambda x which is given as lambda square x.

So, it means that x equal to lambda square x or you can simplify, then it is lambda square minus 1 x equal to 0. Now, x is an Eigenvector corresponding to lambda, it means that x is a nonzero vector. So, this thing this lambda square minus 1 into x is equal to 0 possible only when lambda square is equal to 1 or we can say that lambda is equal to plus minus 1. So, it means that Eigenvalues of H is either 1 or minus 1. So, this says that Eigenvalues of um this householder matrix is either 1 or minus 1.

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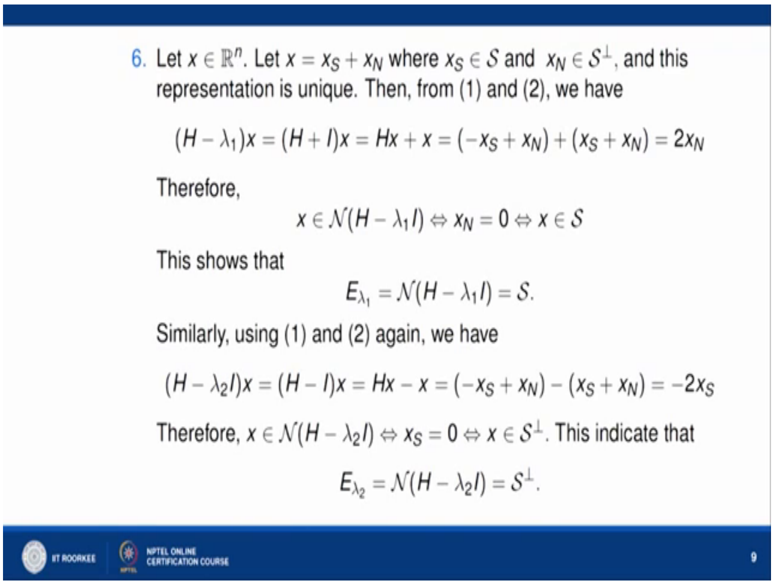

Now, coming out about the Eigen space here, we want to find out say Eigen space corresponding to lambda 1 equal to minus 1 and lambda 2 equal to minus 1 for that please observe that x if you take any x in R n, then it has a unique representation given as x of S plus x of N where x of S is element S here and x N is an element in S perp then if you operate H minus lambda I I into x, we want to find out say the eigenvector corresponding to lambda 1.

So, H minus lambda 1 into x is equal to H plus I here lambda 1 I am taking as minus 1, then H plus I operating on x. So, it is nothing, but H of x plus x. Now, we already know; what is H of x? H of x is given as minus x of S plus x of N and x is given as x of S plus x of N. So, it is given as 2 of x N.

Now, it means that H minus lambda 1 x is always coming out to be 2 of x N; so, it means that if this x is an eigenvector corresponding to lambda 1, then H minus lambda 1 x has to be 0. So, it means that x N has to be 0 if x is an eigenvector corresponding to lambda 1. So, this implies that x belongs to null space of H minus lambda 1, I provided that this x N is equal to 0. Now if x N is 0, then your x is nothing, but x of s.

So, it means that the Eigen vector corresponding to lambda 1 equal to minus 1 is nothing, but x which is given in x. So, it means that x belongs to null space of H minus lambda 1 I, it means that x is in S. Similarly, we can prove that x belongs to S implies that x belongs to null space of H minus lambda 1 I. So, it means that Eigen space corresponding rule lambda 1 equal to minus 1 is nothing, but null space of H minus lambda 1 I, this is by definition of Eigen vector and it is coming out to be S. So, it means that Eigen space corresponding to lambda 1 equal to minus 1 is S here. So, we can say that dimension of S is 1. So, it means that geometric multiplicity of lambda 1 which is minus 1 is 1 here.

Now to find out the Eigen space corresponding to lambda 2 equal to minus 1 we repeat this process and we have H minus lambda 2 y x equal to H minus I into x and this is nothing, but H x minus x and when we simplify the H x H S 6 given by minus x of S plus x of N minus x S plus x, it is coming out to be minus 2 of x S. So, it means that if x belongs to null space of H minus lambda 2 y this implies that x of S equal to 0.

Similarly, if x S x of S is equal to 0 then H minus lambda 2 y x equal to 0; so, it means that x belongs to null space of H minus lambda 2 y. So, it means that if x belongs to null space of H minus lambda 2 y implied and implied by that x of S equal to 0. Now f of x of S is equal to 0 means your x is nothing, but x and or we can say that your x is in S perp. So, it means that x belongs to null space of H minus lambda 2 y is implied and implied by that x belongs to S perp.

It means that Eigen space corresponding to lambda 2 equal to 1 is nothing, but null space of H minus lambda 2 I which is nothing, but S perp. So, it means that geometric multiplicity of lambda 2 as one is N minus 1 that is the dimension of S perp. So, here which we have find out we are able to find out the Eigen space corresponding to lambda 1 equal to minus 1 and lambda 2 as 1.

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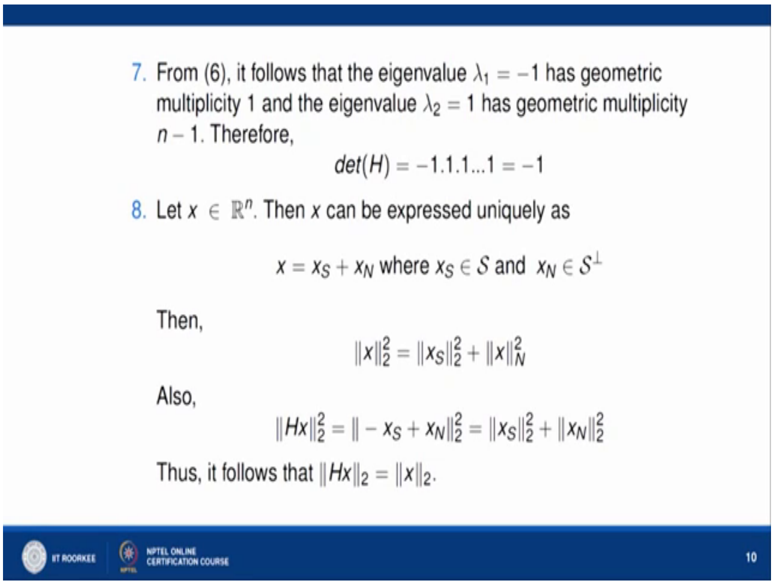

Now, with the help of Eigenvalues; we can easily calculate the determinant of H that is product of Eigenvalues and we already know that geometric multiplicity of lambda 1 is equal to minus 1 and geometric multiplicity of lambda 2 equal to 1 is N minus 1 and we already know that algebraic multiplicity of Eigenvalues is always greater than or equal to geometric multiplicity.

So, here we can say that this one has algebraic multiplicity one and algebraic multiplicity of lambda 2 equal to 1 is N minus 1. So, determinant of H is given by minus 1 into N minus times 1. So, it means it is given by minus 1. So, householder matrix is a nonsingular matrix and determinant of H is given as minus 1.

Now, we want to show that. So, that gives you the determinant of H is equal to minus 1. Now in eighth; we try to show that it preserved the 2 norm of x. So, it means that 2 norm of H x is same as 2 norm of x; So, for that just calculate the 2 norm of H S H f x. So, let us consider the 2 norm of H of x whole square. So, H of x we already know that it is minus of x S plus x N 2 norm of this whole square.

Now, what is this x S N; x N this x has this unique representation x S plus x N. So, if you take the square of this then since x of S and x x of N are orthogonal to each other, then we can simplify this and we can write this as 2 norm of x of S whole square plus 2 norm of x and whole square and if you see that this is nothing, but 2 norm of x where 2 norm of x whole square is given by 2 norm of x S whole square plus 2 2 norm of x and whole square..

So, it means that 2 norm of H x whole square is nothing, but 2 norm of x whole square. So, this implies that 2 norm of H x is equal to 2 norm of x. So, here we have proved that 2 norm of H of x whole square is nothing, but 2 norm of x whole square. So, by taking the square root we have the result which we wanted to prove that it preserve the length under 2 norms. So, H x 2 norm of H x is equal to 2 norm of x.

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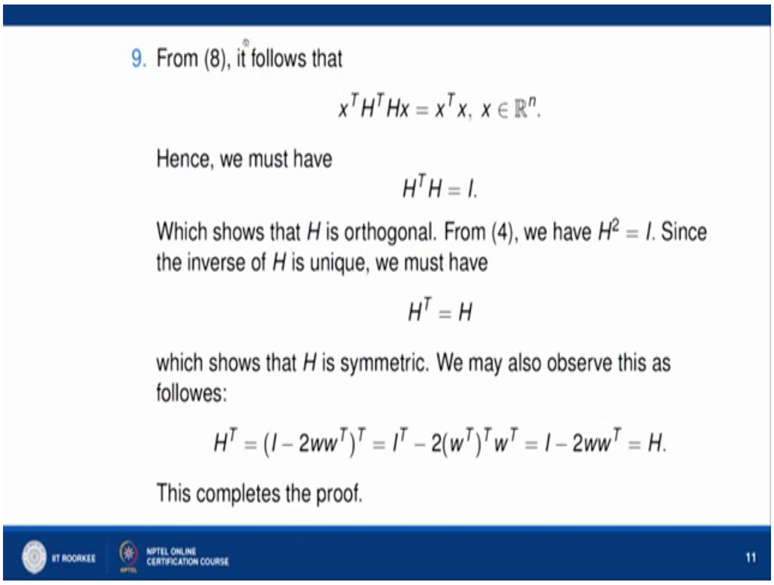

Now, you want to show that H is a symmetric matrix and H is orthogonal matrix. So, to show that H is orthogonal and symmetric matrix we have this that 2 norm of H x is same as 2 norm of x and if we simplify, then we have this in terms of inner product, we have x transpose H transpose H x equal to x transpose x equal to x transpose x where x is any member in R m. Since, this is true for any member in R n then we can say that this implies that H transpose H is nothing, but identity matrix and that shows that H is an orthogonal matrix.

In fact, this you can also prove with the present representation of H that is I minus 2 omega omega transpose. So, we will proved it you know. So, now, to prove that H is symmetric we can start with we are showing that S transpose is equal to H for that we observe that we have already proved that H is H is an involution means H square is equal to I and here we have H transpose H equal to I and we know that the inverse of a matrix if it exists has to be unique. So, it means that S transpose, this simply says that H transpose is the inverse of H and this says that H is be the inverse of H itself. So, this implies that combining these 2 result, we can say that H transpose is equal to H it means that H is a symmetric matrix. So, H is orthogonal H is symmetric matrix.

And this you can prove without going here and you can simply say that you start with H transpose and H transpose, you can say that I minus 2 omega omega transpose whole transpose which is nothing, but I transpose into minus 2 omega omega transpose omega transpose whole transpose into omega transpose and if you simplify; it is coming out to be H. So, you can say that H transpose is coming out to be H. So, this you can prove with the help of this form of matrix H.

So, here we have shown that H is a symmetric matrix. So, here we have done the proof by uniqueness of a inverse matrix and we can show the same by exactly finding the transpose of matrix H and we have shown that it is coming out to be H similarly we can prove that H is orthogonal with the help of the form of H.

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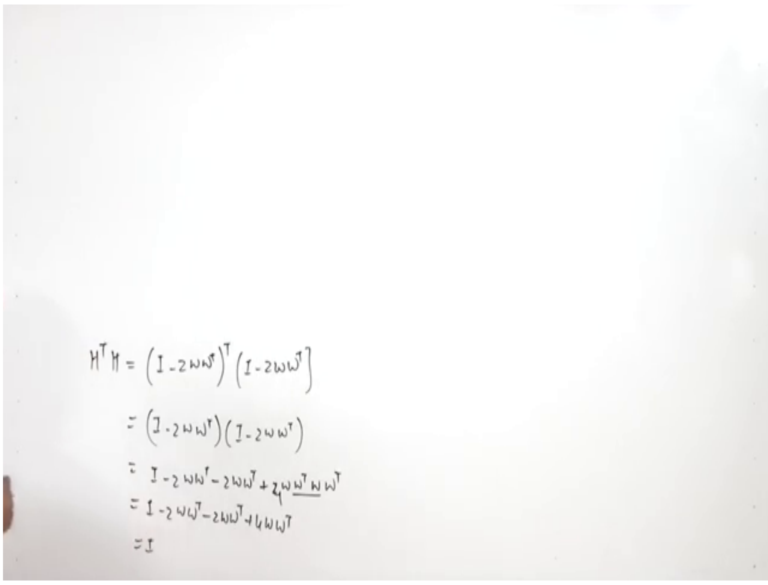

In fact, you can see that H transpose H is what H transpose is nothing, but I minus 2 omega omega transpose whole transpose into I minus 2 omega omega transpose and if you calculate we just have seen that H transpose is same as H. So, we have I minus 2 omega omega transpose into I minus 2 omega omega transpose and if you if you simplify, then it is I minus 2 omega omega transpose minus 2 omega omega transpose plus 4 times omega omega transpose into omega omega transpose.

So, this is what here this omega transpose omega is norm of omega with respect to 2 norm and since, it is a unit vector. So, it is coming out to be 1. So, it is nothing, but I minus 2 omega omega transpose minus 2 omega omega transpose plus 4 times omega omega transpose. So, these will cancel out and it is coming out to be I. So, if you do not want to use the result given in 8; you can directly prove it is quite easy.

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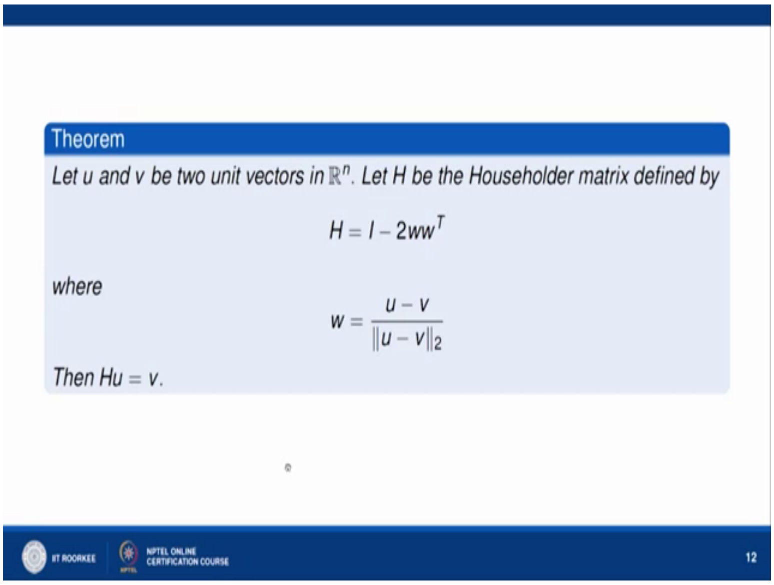

Now, let us coming out to be and the important result by a for which we are discussing this household matrix the important application or very very important application of householder matrix is that given any 2 unit vectors; we can define our matrix householder matrix H such that H of u is going to be v. So, here u and v be any 2 unit vectors then we can define H as householder matrix defined by H as I minus 2 omega omega transpose where omega is nothing, but u minus v divided by v 2 norm of u minus v then such a matrix H which will send u to v. So, it means that H of u equal to v.

Now, the important part of this theorem is that; now we may have very a desired property it means that here I can take v as a E 1 E 2 E 3 and so on. So, it means that is the application of this matrix that you can send any vector u to E 1 same. So, we will see that why it is going to be useful. In fact, by this property only we can say that it is useful in finding the QR factorization of a given matrix. So, let us first prove this theorem.

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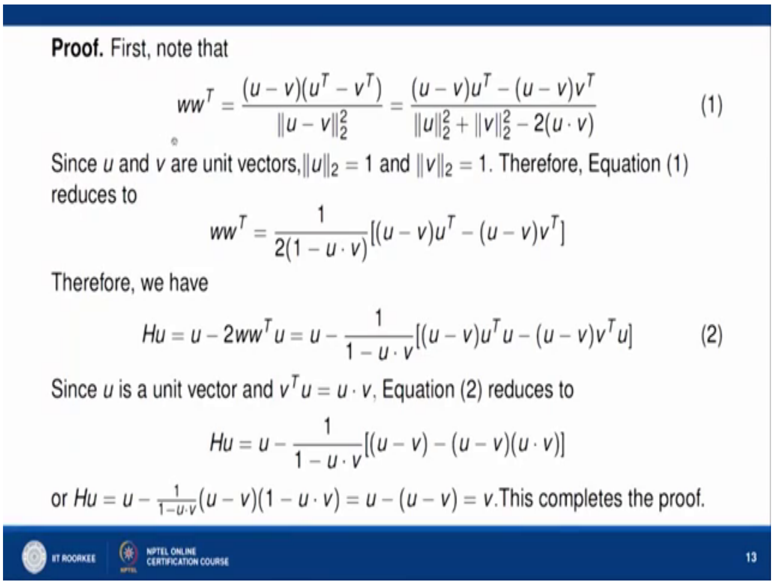

So, to prove this theorem; you simply observe ww transpose; so, w is u minus v divided by 2 norm of u minus v and w transpose is u transpose minus v transpose divided by 2 norm of u minus v. So, in denominator, we have 2 norm of u minus v whole square and in numerator we have this product and if you simplify it is u minus v into u transpose minus u minus v into v transpose and in numerator in denominator, we can simplify this and we can say that it is 2 norm of u whole square plus 2 norm of v whole square minus 2 times of inner product of u and v.

Now, this further can be simplified since u and v are unit factor. So, this is 1 plus 1 minus 2 times u comma and dot v and we can write, we can simplify ww transpose as 1 upon 2 1 minus u dot v into u minus v u transpose minus u minus v v transpose.

Now, let us calculate the H of u. So, H of u is u minus 2 omega omega transpose operating on u. So, u minus now here we are writing 2 times 2; we will be cancelled by this. So, it is one upon one minus u dot v into this operating on u. So, it is u minus v u transpose u minus u minus v transpose u. Now u is a unit vector. So, u transpose u is nothing, but 1 v transpose u is nothing, but u dot v. So, we this is nothing, but u dot v. So, we this can be simplified as H of u is equal to this is u. So, it is u here minus 1 upon 1 minus u dot v and in bracket u minus v; now this is 1. So, I am writing 1 minus u minus v into u dot v.

So, here u minus v you can take it out and what is left here it is 1 minus u dot v which is also in denominator. So, this will cancel out each other and we can say that at H of u is equal to u minus this will cancel out; so, u minus u minus v. So, it is coming out to be v. So, here we say that H of u is coming out to be v.

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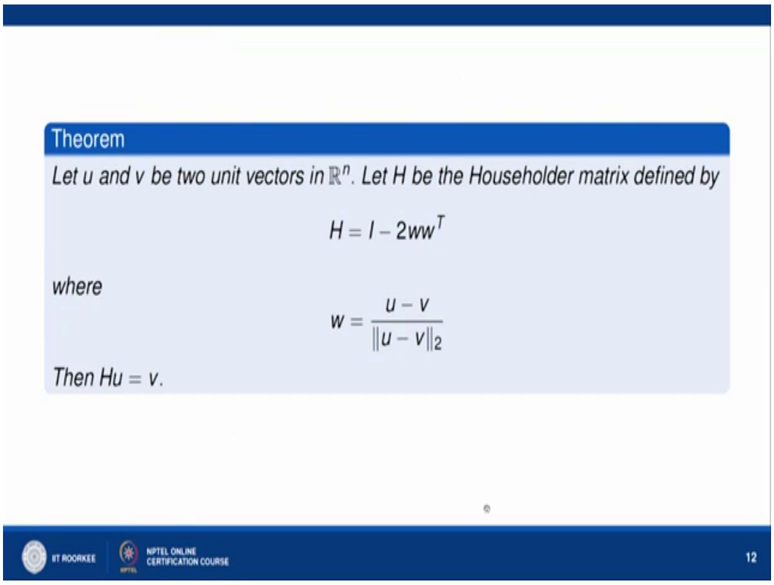

So, if this proves the theorem here that if u and v be any 2 unit vectors in R n, then we can form a matrix H householder matrix H as I minus 2 omega omega transpose where omega is given by u minus v divided by 2 norm of u minus v then such a matrix H will send u to v. So, H of u is going to be v we will see that we will going to utilize this important theorem later on.

So, now, based on this important theorem, since here we have used that u and v are both you unit vectors.

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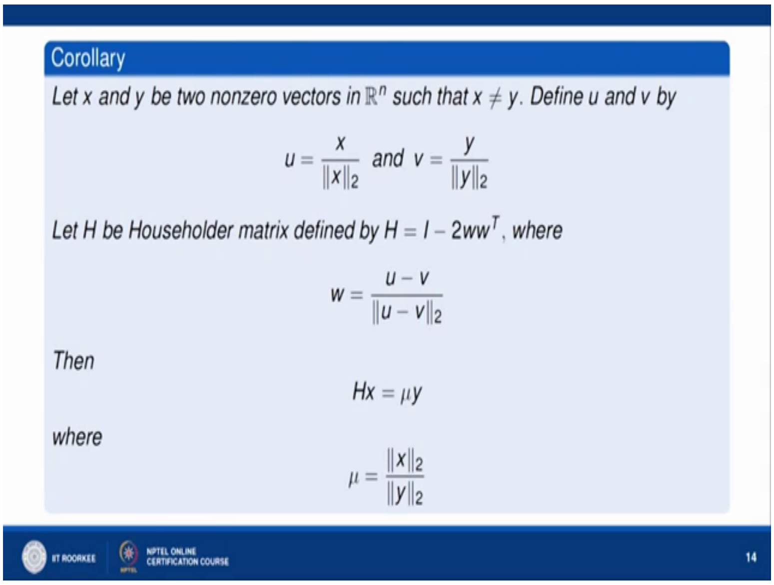

Now, consider a simple corollary based on this that you and we may not be same unit vectors. So, let us say let x and y be any 2 nonzero vectors in R n such that x is not equal to y, then we can apply our theorem by writing u as x upon 2 norm of x and we as y upon 2 norm of y and if we again apply our theorem, then H we can form as I minus 2 omega omega transpose where omega is u minus v upon 2 norm of u minus v then claim is that H of x is nothing, but mu into y where mu is some constant it is given by 2 norm of x divided by 2 norm of y that is our claim; let us prove this claim this is quite simple.

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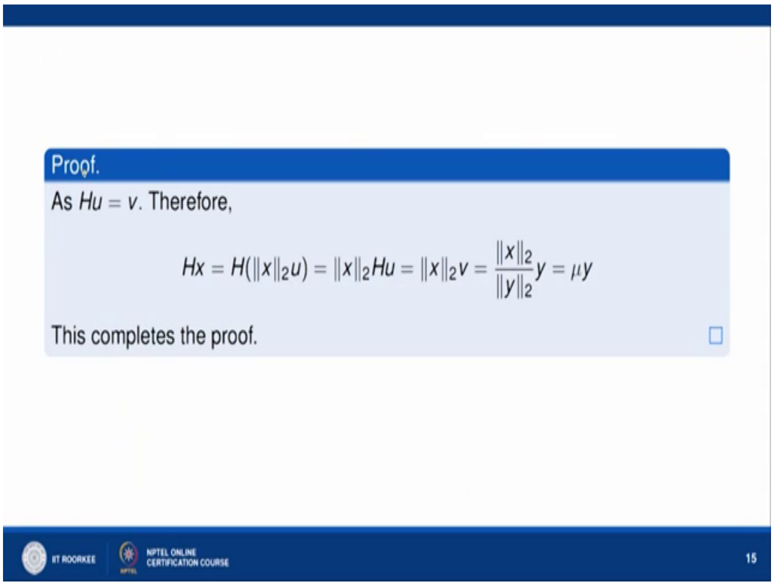

So, here to prove this we already know that H of u is going to be v because u and v are unit vectors and w is defined as u minus v divided by 2 norm of u minus v. So, it means that H of u is going to be v here, now what is u here? U is defined as x u as x upon 2 norm of x.

So, let me write it here H with the help of this if you want to find out H of x where H of x is given by 2 norm of x into u. Now 2 norm of x; we can take it out because it is just a constant. So, 2 norm of x into H of u and H of u is nothing, but v. So, it is given by 2 norm of x into v. Now, what is v here v is nothing, but y divided by 2 norm of y. So, we can write H of x equal to 2 norm of x divided by 2 of y into y and we can call this as some constant call it mu.

So, H of x is equal to mu of y and this complete the proof of the corollary that even if we do not have x and y as nonzero vector, it is not a unit vector, but it still I can send x under this H to some constant multiple of y and this constant is coming out to be 2 norm of x divided by 2 norm of y here.

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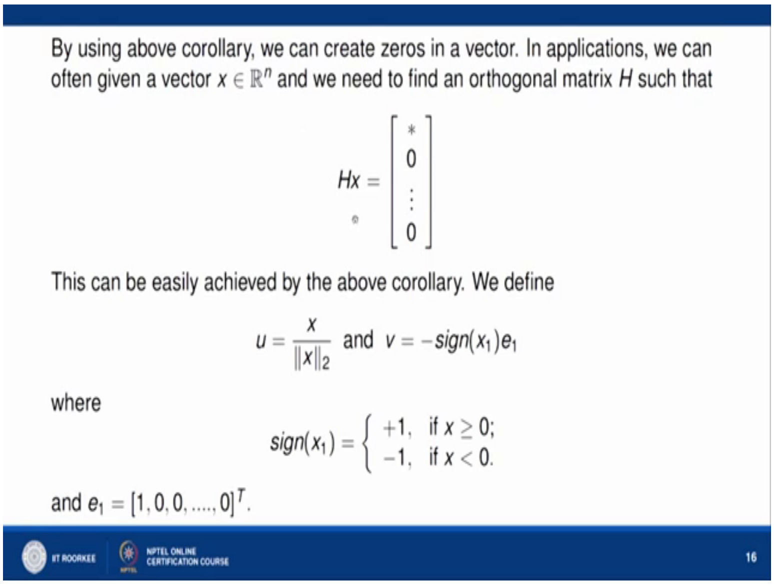

Now, we apply our householder matrix to convert a a given vector x into e 1 or some constant multiple of e 1. So, here we want to find out a householder matrix H such that given a vector x in R n we convert H of x as anything nonzero quantity at first place and rest on it is zero. So, it is constant multiple of e 1.

So, this can be easily done by writing u as x upon 2 norm of x and we as say minus sign of x 1 into e 1. So, here sine function is defined as this that sine of x 1 is equal to 1 if x 1 is greater than or equal to 0 and sine of x 1 is equal to minus 1 if x 1 is less than 0 and e 1 is the unit vector having one at the first place the rest of all 0.

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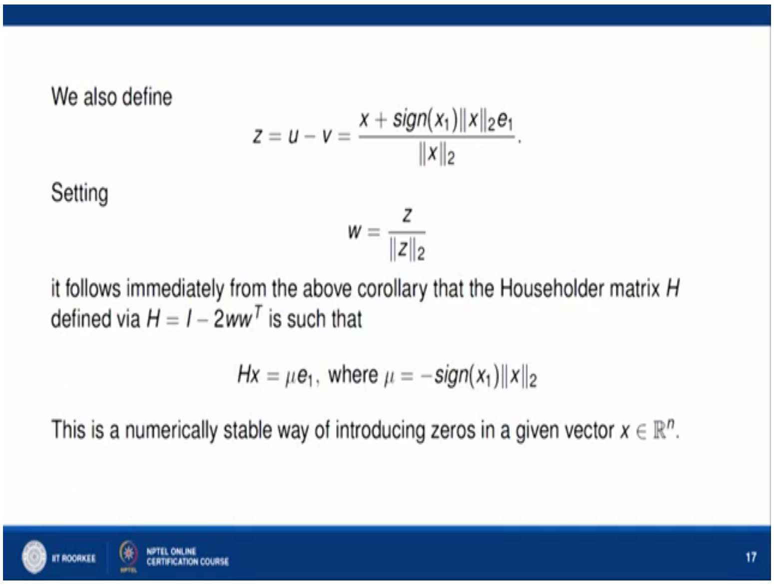

So, how we can do that let us we can define u minus v as z.

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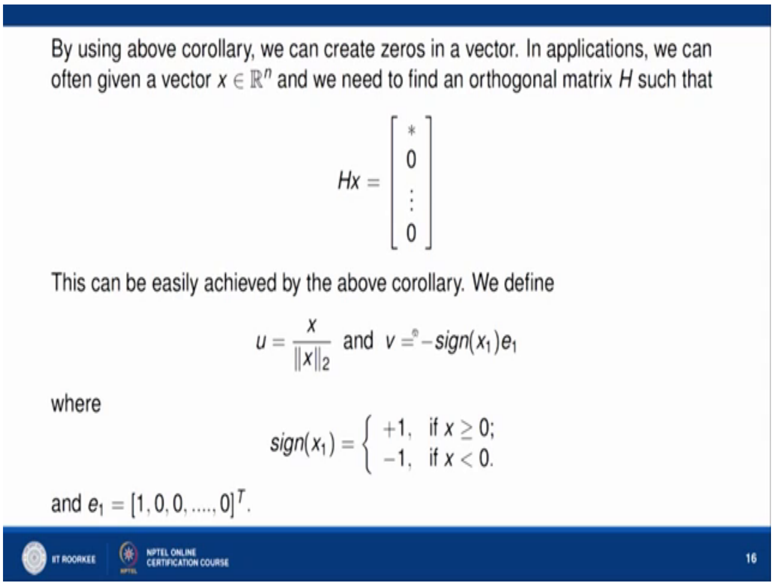

So, z can be written as x plus sine function of x 1 where x 1 is the first component of x into 2 norm of x into e 1 divided by 2 norm of x m and we already know that w is what u minus v divided by norm of v. So, here we can write w in terms of z as z divided by 2 norm of z, it is just simplification of the procedure. So, once we have w we can define H as I minus 2 omega omega transpose.

Then we know that by above corollary that previous corollary that H of x is equal to mu of E one where mu is nothing, but minus sign of x one into 2 norm of x m. So, that we are going to see and this is numerically stable way to introducing zeros in a given vector x. So, it means that if you look at this, then H of x is now map do some constant multiple of e 1 where mu is given as minus sign of x 1 into 2 norm of x.

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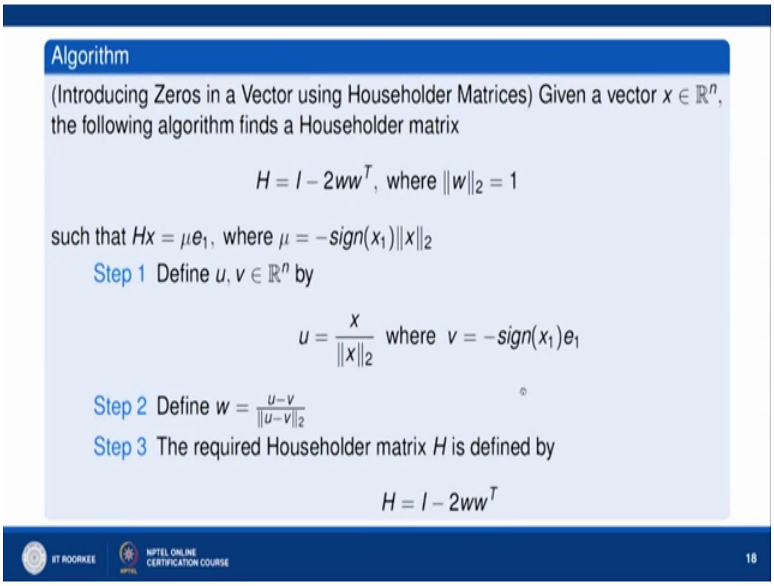

So, we can summarize whatever we have discussed as an algorithm and this algorithm do this that introducing zeroes in a vector using householder matrices. So, what it is given a vector x in R n the following algorithm finds a householder matrix H which is given as I minus 2 omega omega transpose where omega is a unit vectors. So, 2 norm of omega is equal to 1 such that H x is equal to mu of e 1 where mu is given as minus sign of x 1 into 2 norm of x where x 1 is the first component of x.

So, to find out householder matrix we define u and v as u as x divided by 2 norm of x and we as minus sign of x one into E one. So, once u and we are defined then we define w as u minus v divided by 2 norm of u minus v and once w is known to us our householder matrix is given as H as I minus 2 omega omega transpose. So, this is an algorithm which finds a householder matrix which map a any non zero vector x to mu times e 1 where mu is some constant and e 1 is the first unit vector having first position as one and rest of all zeros.

So, this is the algorithm now based on this algorithm let us find out some example.

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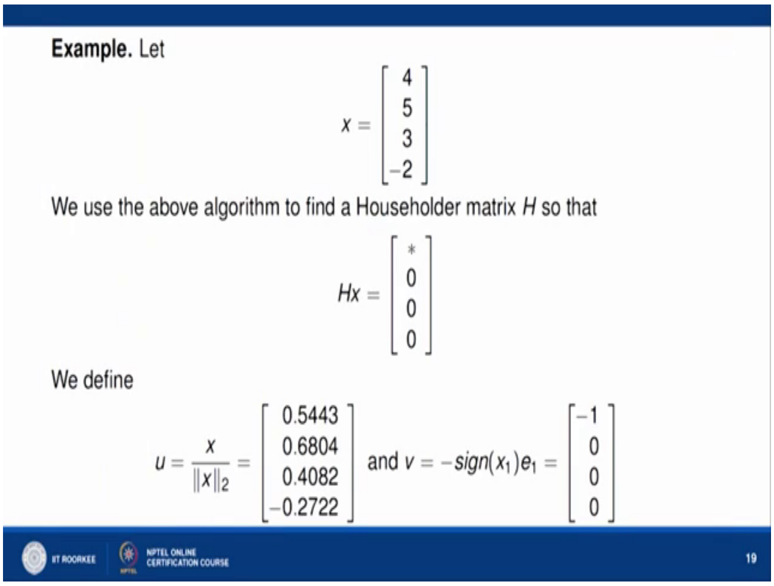

So, first example is this that we have a vector say 4 5 3 minus 2; we have taken any example. So, we let us take this example and we try to find out the householder matrix which map; this x into this thing where this first place is nonzero and rest all zero; So, here we that we have tried to find out this H such that H of x is equal to this.

So, for that you define u that is x divided by norm of x into norm. So, it is given by this quantity and v as minus sign of x y into E one now look at the first component that is x one x one is positive. So, sine of x 1 is given as one. So, this is going to be minus 1 0 0 0 right and now. So, u is known to us v is not known.

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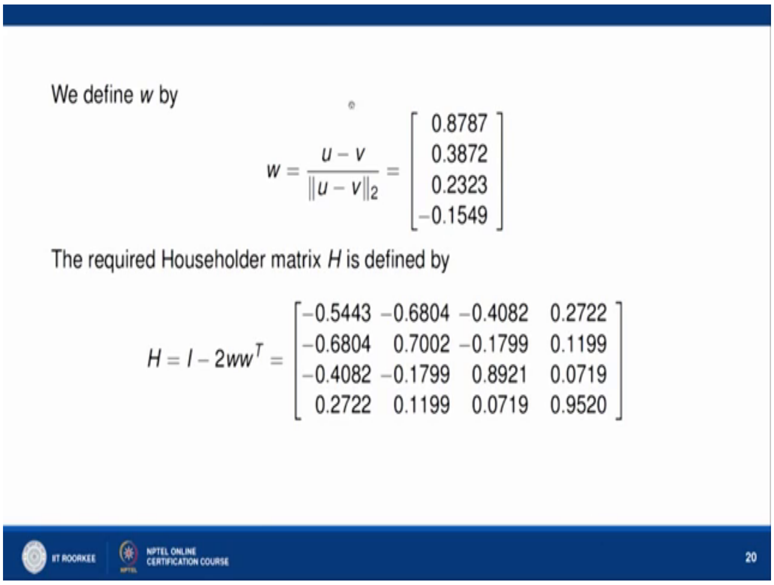

Now, we can find out w by u minus v divided by 2 norm of u minus v which is given by this quantity and once w is known to as, then we can find out H as I minus 2 omega omega transpose that is given by this quantity. and we can see that H of x.

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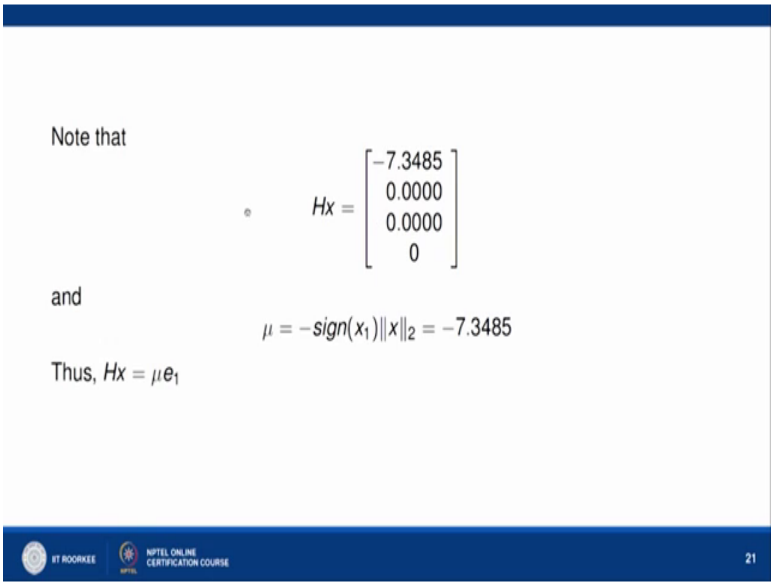

So, if we operate this matrix H on this x it is coming out to be minus 7.3485 and rest all zeros. So, it means that H of x send x to a vector which is having nonzero entries in the first place and rest it is all zeros and if you look at the first place is what minus seven point three point minus 7.3485. So, if you look at mu is what minus sign of x 1 into norm of x 2 2 norm of x and that is coming out to be minus 7.3485 and which verifies the algorithm which we have discussed in earlier.

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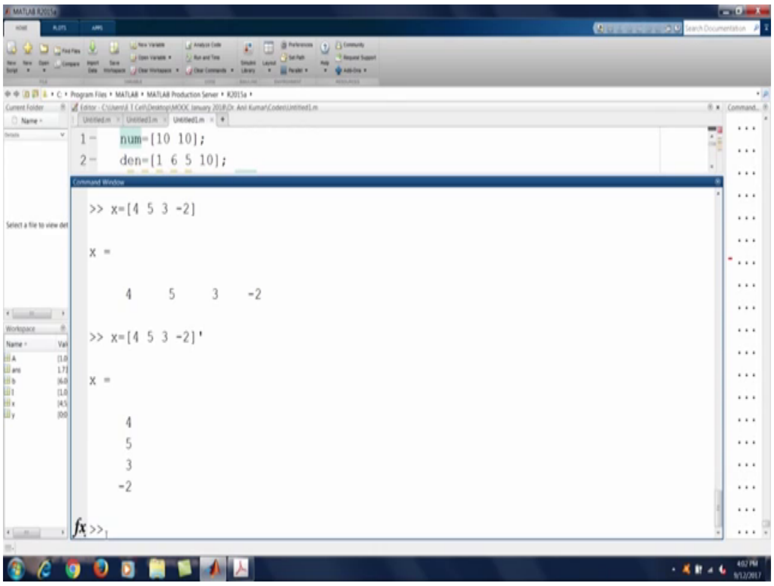

So, let us do the same working in MATLAB also. So, let us first you take this you define x. So, x is x is given as 4 5 minus 3 and 2; I hope it is let us ; so, it is given here 4 or 5 3 2, sorry, 4 5 3 2. So, it is given here.

So, now, we have x as this in fact, how to take the transpose of this. So, let us define as this. So, x is now; this vector we need to find out u and v. So, we find norm of x here.

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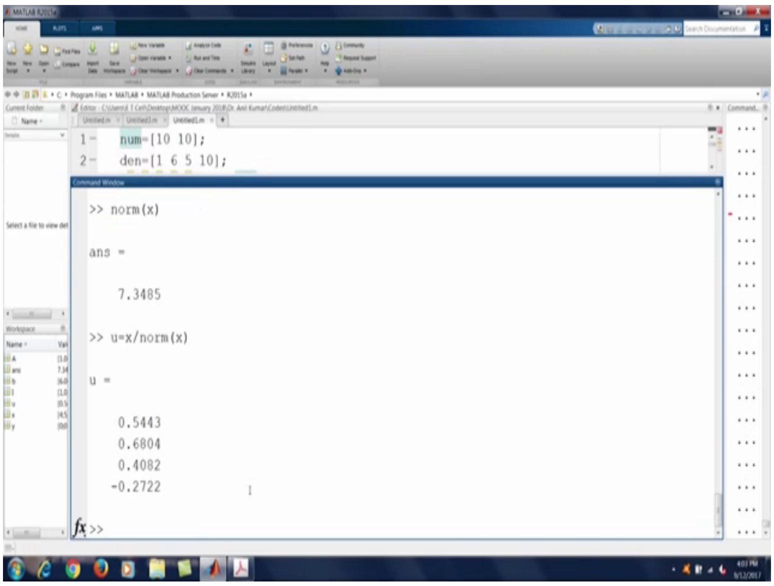

So, norm of x is given as 7.3485. So, here finding the norm with respect to 2 norm if you do not write anything then it is only with respect to 2 norm then. So, u is basically what u is x divided by norm of x norm of x. So, if it is coming out to be this my this thing and you can verify that it is 5.544; 3.68204 that is coming out to be here. So, it is coming out to be same.

Now, v is quite easy because sine of x one is already known to us. So, we is nothing, but minus 1 0 0 0.

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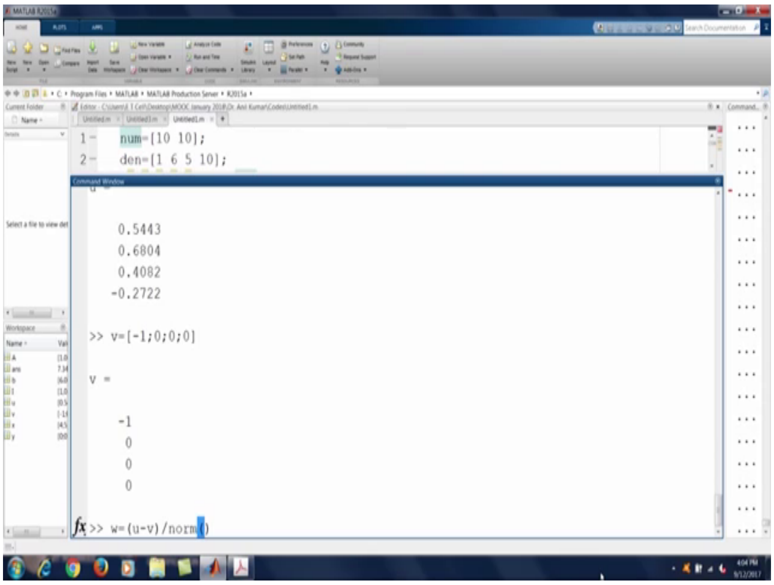

So, we can define as my minus 1 0 sorry 0 0 and 0. So, v is this; Now define w; w is what w is u minus v we can write it divided by norm of u minus v; So, divided by norm of u minus v right.

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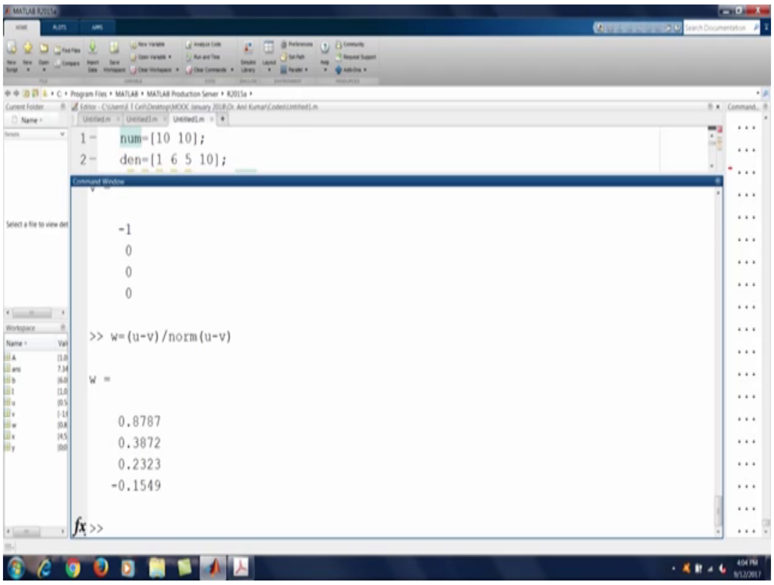

So, it is coming out to be this quantity 0.8787 and that you can verify here it is this w as u minus v divided by 2 norm of u minus v as this right.

So, this we can verify now to find out H. So, H is given as I.

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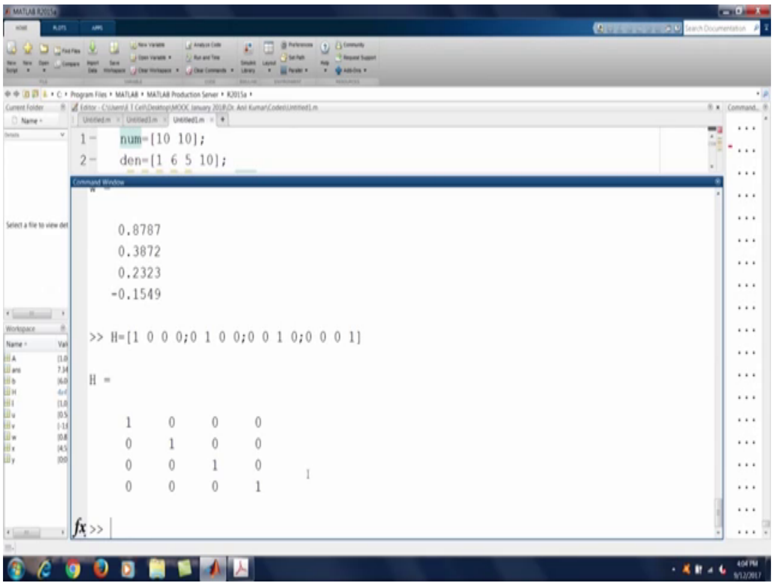

Now, here I have to give what is I because. So, I can write it 1 0 0 0 and 0 1 0 0 0 0 0 1 and I think here, I have to write here I did some kind of mistake one here and 0 here and it is 0 0 0 and 1.

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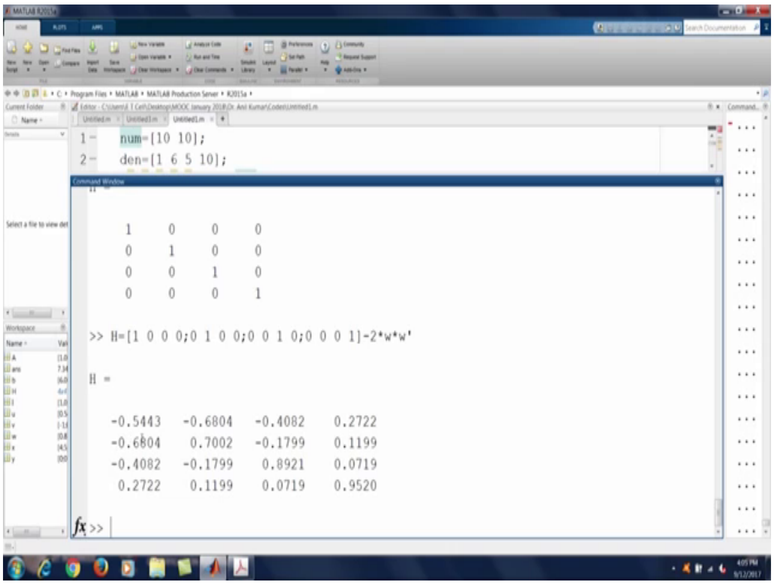

So, that is your H sorry this is identity matrix. So, H I can write it minus minus 2 into omega omega into omega transpose omega dash that is your H. So, H is given by minus 0.5443 and so on.

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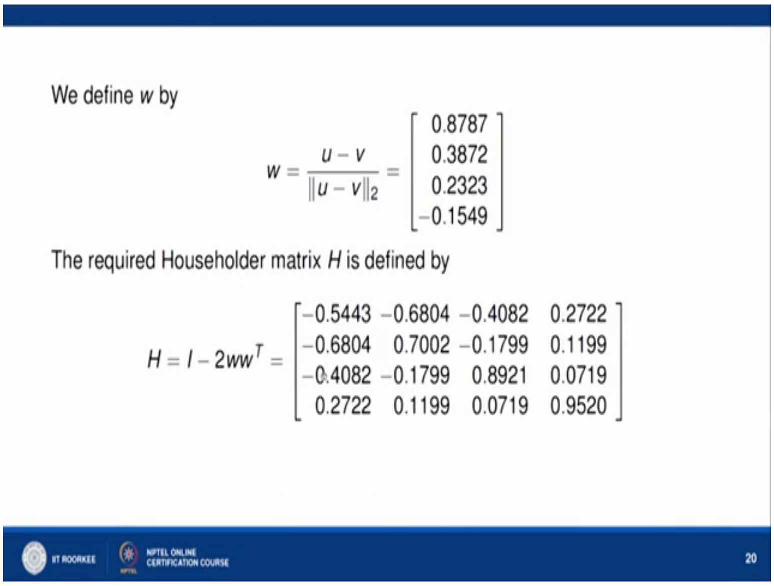

And that is given here. So, H is coming out to be this. Now if we operate H on x.

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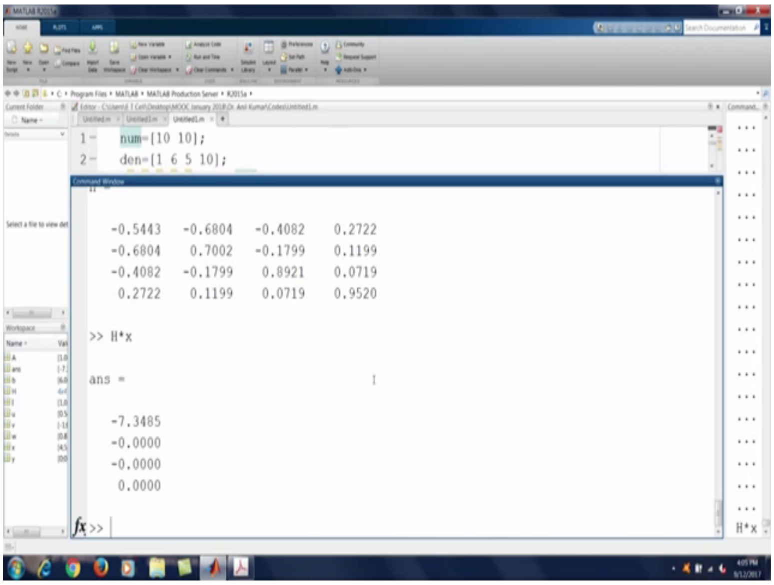

So, H operating on x we will see what it is. So, it is minus 7.3485 and all these are entries as 0 and which is given here.

So, and the first entry is nothing, but minus we already know that what is minus norm of x.

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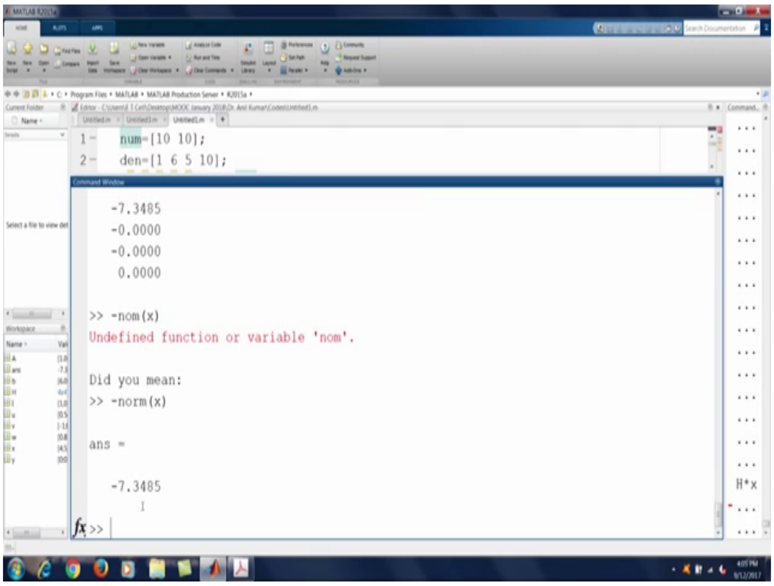

So, that is coming out to be oh sorry it is norm of x yeah.

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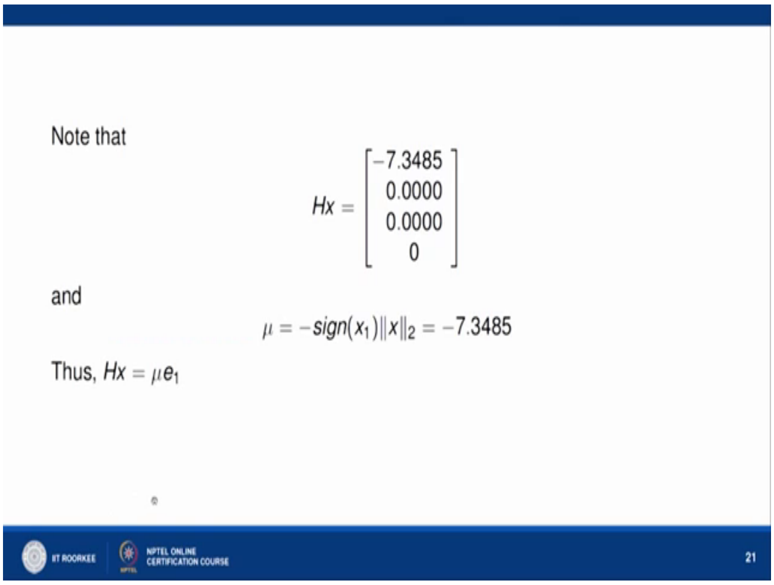

So, it is coming out to be minus 7.3485. So, it is also matching.

So, that verifies the corollary and algorithm. So, we have verified our algorithm and corollary and we stop here. So, in this lecture what we have done we have defined householder matrix and discuss some elementary properties of householder matrix and with the help of householder matrix.

We have discussed one important theorem which says that if we have a to any unit vectors then we can find out a householder matrix H such that H send u into v where u and v are any 2 unit vectors and also we have discussed the corollary where u and we may not be unit vectors and we have seen one example and based on this corollary an algorithm.

So, we will continue this study for a matrix. So, it means that if we start with a matrix a and how we can find out a householder matrix as that its send say first column to say multiple of e 1 and so on. So, that we are going to discuss in next lecture. So, here we conclude our lecture thank you for listening us.

Thank you.