Numerical Linear Algebra
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Lecture - 20
Computer Representation of Numbers

Hello, friends I welcome you to my lecture on computer representation of numbers

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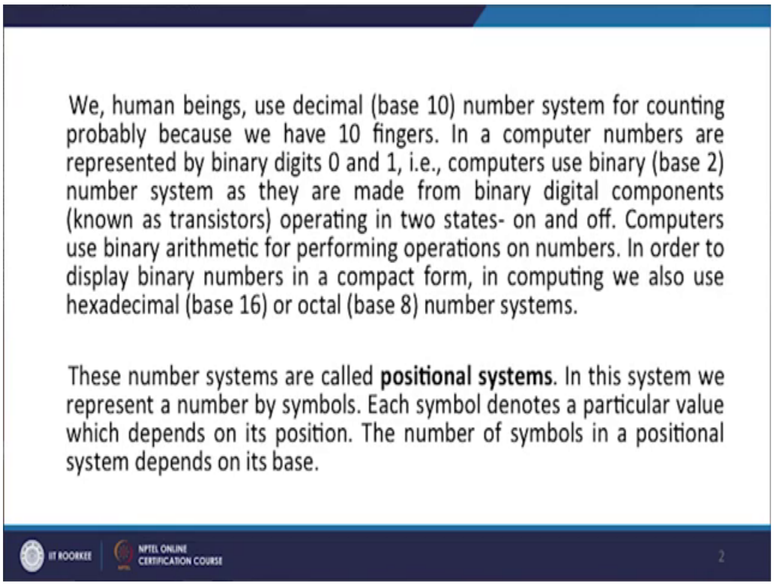

Be human beings use decimal that base 10 as the number system for counting, probably because we have 10 fingers. Now in a computer numbers are represented by binary digits 0 and 1 that is computers use binary base 2 number system, as they are made from binary digital components which operating 2 states on and off.

Computers use binary arithmetic for performing operations on numbers, in order to display binary numbers in a compact form in computing we also use hexadecimal, representation with that is base 10 or octal representation that is with base 8 of number systems. Now, these number systems are called positional systems, in this system we represent a number by symbols, each symbol denotes a particular value which depends on its position the number of symbols in a positional system depends on its base or for example let us consider the decimal system in this system 10 is the base value and symbols are from 0 to 9 to represent numbers.

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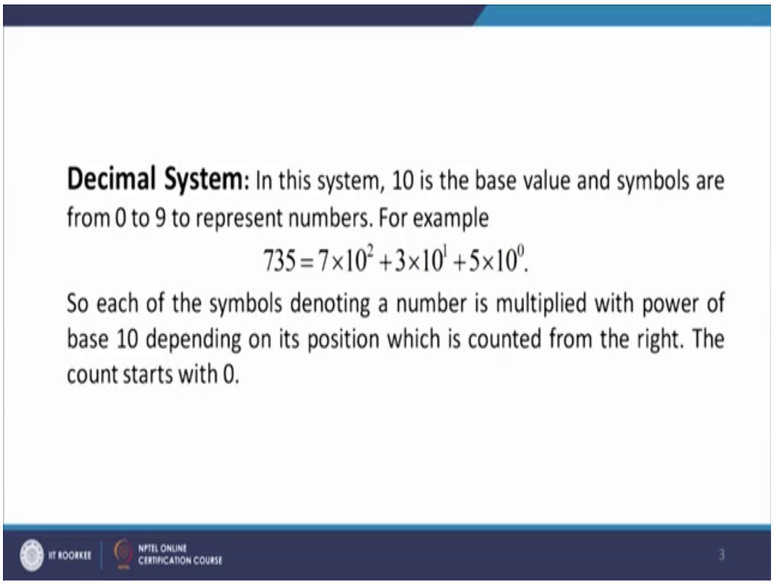

For example if you take the number 735, it can be expressed as 7 into 10 to the power 2, plus 3 into 10 to the power 1 plus 5 into 10 to the power 0. So, each of the symbols 7 0 to 9 denoting a number is multiplied with the power of base 10, depending on its position which is counted from the right, the count starts with 0.

So, 5 is multiplied with 10 to the power 0, then 3 is multiplied with 10 to the power 1 and 7 in multiplied with 10 to the power 2. Now so if you a in general you take a decimal number C n C n minus 1 in so on C 1 C naught.

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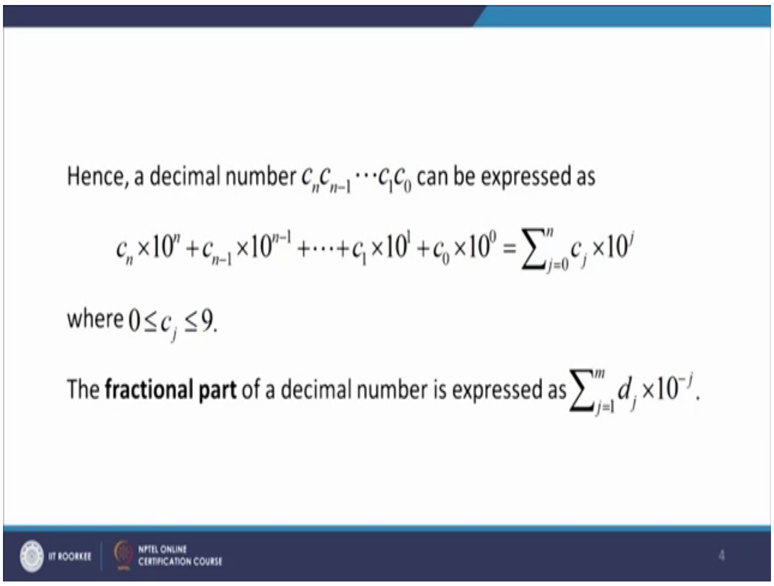

It will be expressed as C n t 10 to the power n plus 10 minus 1 10 to the power n minus 1 and so on, C 1 into 10 to the power 1 plus C naught into 10 to the power 0, which can be written in sigma notation sigma j j equal to 0 to n C j into 10 to the power j where C j is a vary from 0 to 9.

The fractional part of a decimal number is expressed as sigma j equal to 1 to m, d j into 10 to the power minus j. Now, so in the binary number a system they are the bases 2 in this system there are 2 symbols 0 and 1.

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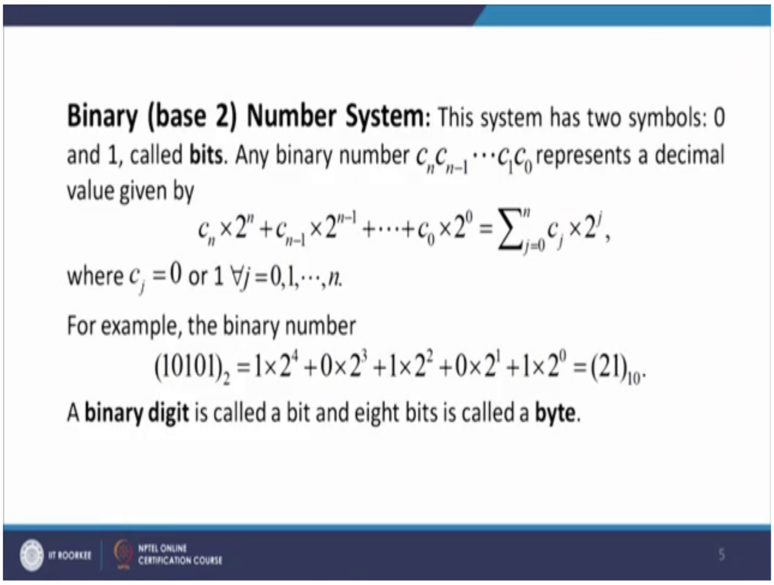

Which are called as bits any binary number C n C n minus 1 and so on, C 1 C naught represent decimal value given by C n into 2 to the power n plus n minus 1 to the power n minus 1 and so on. C naught into 2 to the power 0 which can be expressed in the sigma notation a sigma j equal to 0 to n C j 2 to the power j.

And where j is state value 0 or 1 for all j 0 to n for example, if you take the binary number 10101 now outside the bracket we are writing to because it is in the base 2. So, 10101, when we write in the decimal form will write it as 1, 1 is the rightmost digit 1 into 2 to the power 0 then 0 into 2 to the power 1, plus 1 into 2 to the power 2 plus 0 into 2 to the power 3 plus 1 into 2 to the power 4 and when you calculate the sum, this sum is 21.

So, this 21 is the value in the decimal number system, of this binary number 10101 with base 2. Now binary digit is called a bit and 8 bits is called a byte, now we consider the hexadecimal a system in the system there are 16 symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F.

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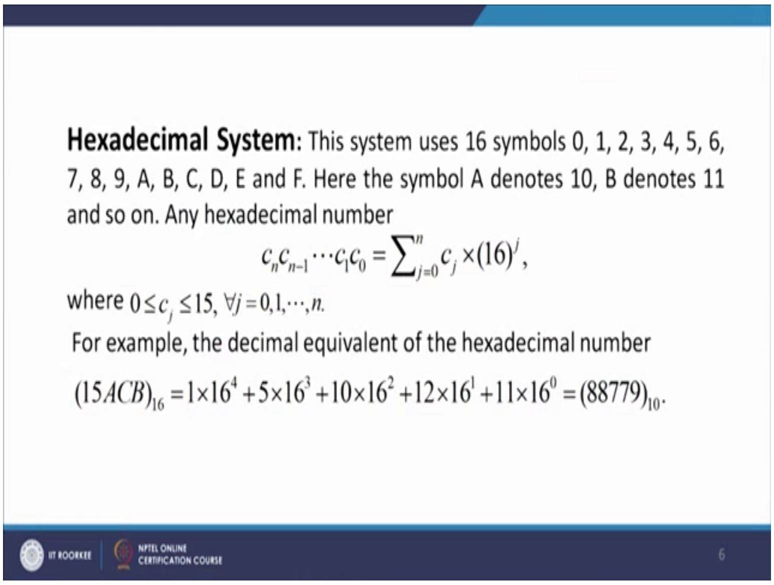

So, A it has value 10, B has value in 11, C has value 12, D 13, E 14 and any hexadecimal number C n C n minus 1 1 and so on, C 1 C naught is equal to sigma j equal to 0 to n, C j into 16 to the power j in a similar manner as we have written in the case of previous functional systems.

Now, here C j is take value from 0 to 15, j varies from 1 0 to n. Now for example, let us take the decimal equivalent of the hexadecimal number 15 ACB 16. So, this is are 15 ACB in the a 16 is the hexadecimal number. Now this in the decimal represent number system will be 1 into 16 to the power 4, 5 into 16 to the power 3, A means 10; 10 into 16 to the power 2, C mean 12. So, 12 into 16 to the power 1 and B has value 11. So, 11 into 10 16 to the power 0, and when we calculate the sum it comes out to be 88779, between the a decimal number system. Now, each hexadecimal digit is also called a hex digit.

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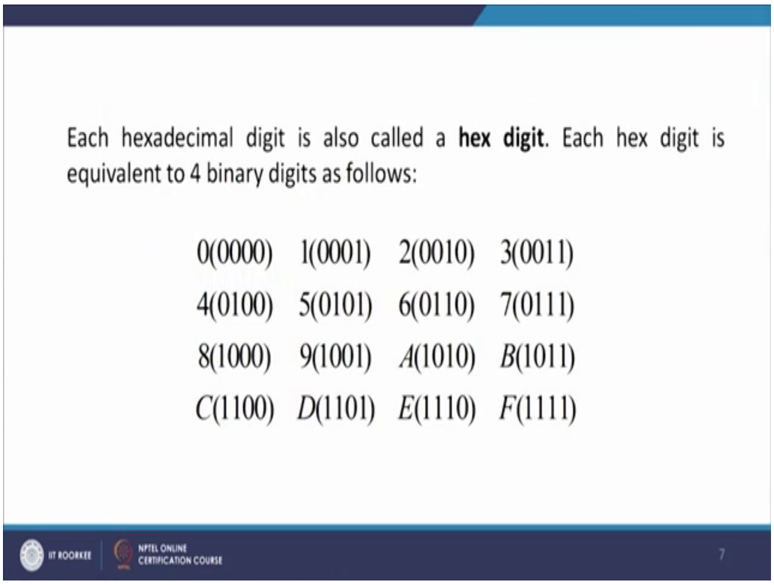

Each hex digit is equivalent to 4 binary digits. So, say for example, 0, 0 in when you write as a binary number in 4 binary digit it will be 0 0 0 0 0, while 1 will be 0 0 0 1; 2 will be 0 0 1 0; 3 will be 0 0 1 1 and 4 will be 0 1 0 0; 5 will be 0 1 0 1; 6 will be 0 1 1 0, and 7 will be 0 1 1 1; 8 will 1 triple 0, 9 will be 1 0 0 1, A will be 1 0 1 0, B we will be 1 0 1 1 B is a 11.

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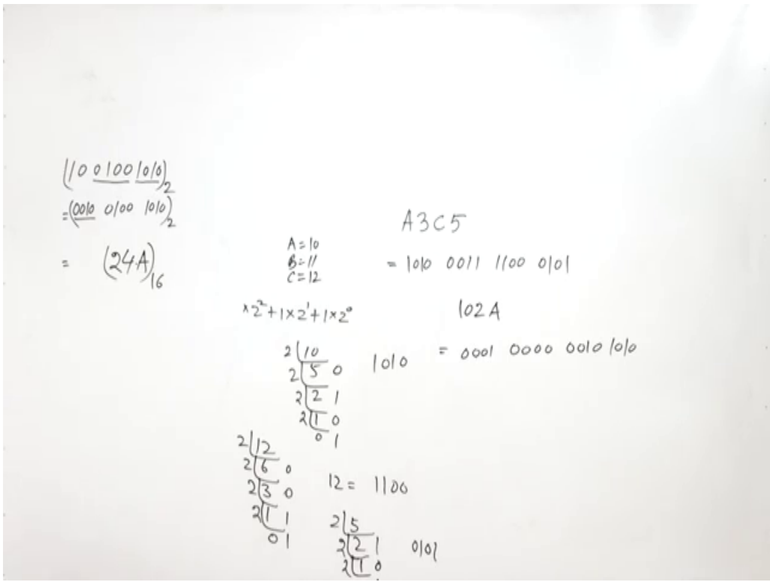

So, it can be written as 1 1 0 1 this is nothing, but 1 into 2 to the power 3, 0 into 2 to the power 2, 1 into 2 to the power 1, 1 into 2 to the power 0, so this is 11. So, B is a 11, so it can be written as a binary number 1 0 1 1 symbol is C is 1 1 0 0, then D is 1 1 0 1, E is 1 1 1 0 and F is 1 1 1 1, F you can also write in a similar manner F is equal to 15.

So, it can be written you can divide so if you want to write it is a binary number what you in the binary representation you divide it by 2. So, you got what it goes for you divided by, so it is remainder is 1 then you again divide by 3 a 2. So, it you get a reminder again as 1 then you again divide it by 2, you get remainder as 1 and when we divide it by 2 you get remainder as 1 . So, what we will do? You will this we start writing from here at the rightmost digit, so we will have it has 1 1 1 1 ok.

So, this is the representation of 15 in the binary number system, we come to that. So, each hex digit is equivalent to 4 binary digits like this, now conversion from hexadecimal system to binary system.

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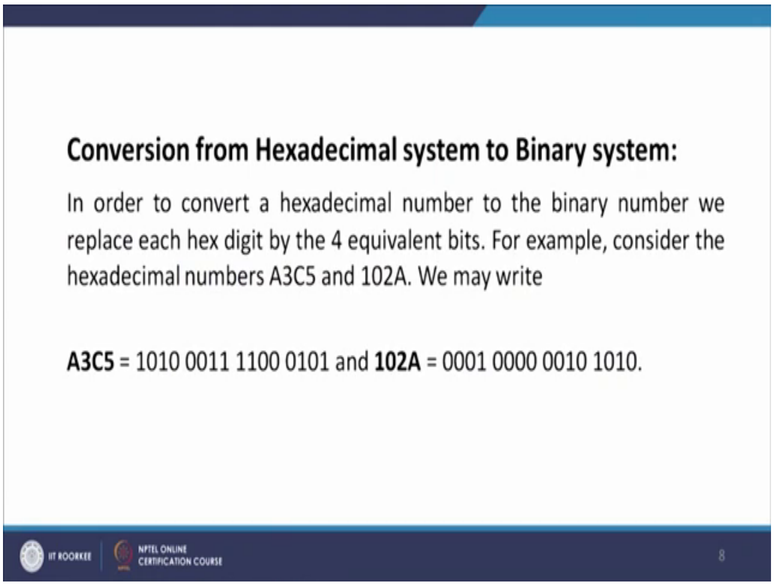

In order to convert hexadecimal number to the binary number, we replace each hex digit by the 4 equivalent bits if for example, consider the hexadecimal number A 3 say C 5. Now a has value 10; a has value 10 means we will divide it by 2 A has value 10. So, we will divide it by 2, so this will this reminder is 0 then we again divided by 2 the remainder is 1 we again divided by 2 remainder is 0 we again divided by 2 remainder is 1

So, we write it as 0 1 0 1, so 1 0 1 0 this is the value of a then 3, 3 is a 0 0 1 1, because this is 1 and 2 to the power 0 this is 1 into 2 to the power 1, and then you have 0 in 2 to the power 2 and then 0 into 2 to the power 3. So, it is 3 and C has a value 12, so 12 also, so we can write 2 time 6 0 2 3 0 2 1 1 and then 2 0 1.

So, 12 can be written as 0 0 1 1, so 0 into 2 to power 0, 0 into 2 to power 1 1 into 2 to the power 2; that means, 4 and 1 into 2 to the power 3; that means, 8. So, 8 plus 4 is equal to 12. So, in a 1 1 0 0 we write for C and then for 5, 5 we can write as 1 0 when we divide 5 by 2 the remainder is 1 when we divide 2 by 2 the remainder is 0, when you divide 1 by 2 the remainder is 1. So, you get 0 1 0 1, so 1 more 1 0 1 0 1 and then you write 0 here, so this is 1 and then 2 to the power 2. So, 0 1 0 1 that is for 5 now, so this is the binary a equivalent of A 3 C 5 1 0 1 0 0 0 1 1 1 1 0 0 and 0 1 0 1 there is the binary equivalent of A 3 C 5.

Now, let us write for 1 0 2 A, so for 1 we write as 0 0 0 1 for 0 we write 0 0 0 0 and then for 2 we write 0 0 1 0 and then for A, A we have written a here 1 0 1 0. So, this is the binary equivalent of these hexadecimal numbers. Now, conversion from binary system to hexadecimal suppose we want to convert a number which is given in binary system, and we want to convert it to hexadecimal number then what we do?

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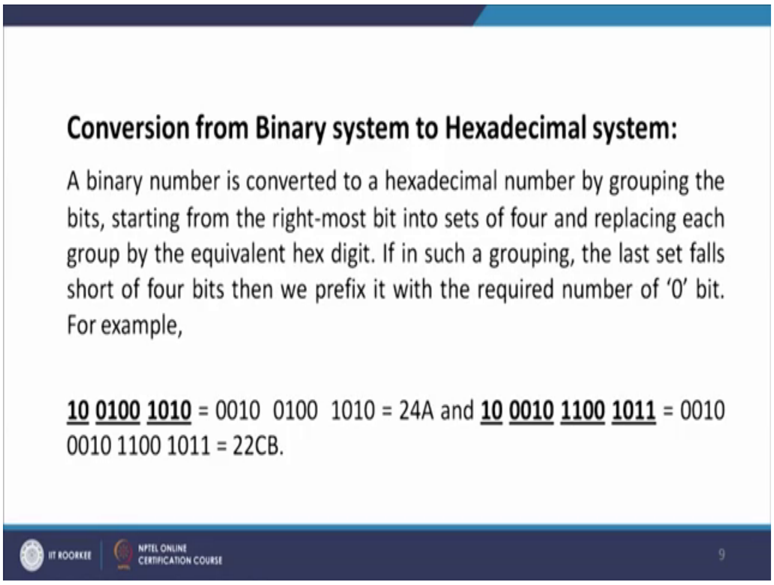

A binary number is converted to hexadecimal number by grouping the bits is starting from the rightmost bit into sets of 4, because we have identified the 15 symbols 0 1 2 3 4

5 6 7 8 9 then A B C D E F into y x digits with 4 binary bits. So, what we do here we start from the rightmost bit and then we make sets of 4 and placing each group by the equivalent x digit, if in such a grouping the last set falls short of 4 bits then we prefix it with the record number of 0 bit.

For example let us take up 10 z, let us take of this 1 10 0 1 we have double 0 then we have 1 0 1 0. Suppose you are given this number in the binary system, and we want to convert it into hexadecimal system. So, then what we do we start from the rightmost digit and may write sets of 4 ok.

So, we have a set of 4 this is 0 1 0 1 here 0 0 1 0 1 0, and then we are have left with 2 digits. So, we prefix it with to 0 digit, so 0 0 1 0 be in the bit 4, 0 1 0 0 1 0 1 0, now 1 0 1 0 in the hexadecimal system represents A. So, we will write A for this 0 1 0 0, so this represents 4 ok. So, we write 4 and 0 1 0 0 0 1 0 represents this represents 2, so 24 A and then we write 16 for hexadecimal system, and then let us take another example, in this example we have the number 10 0 0 1 0.

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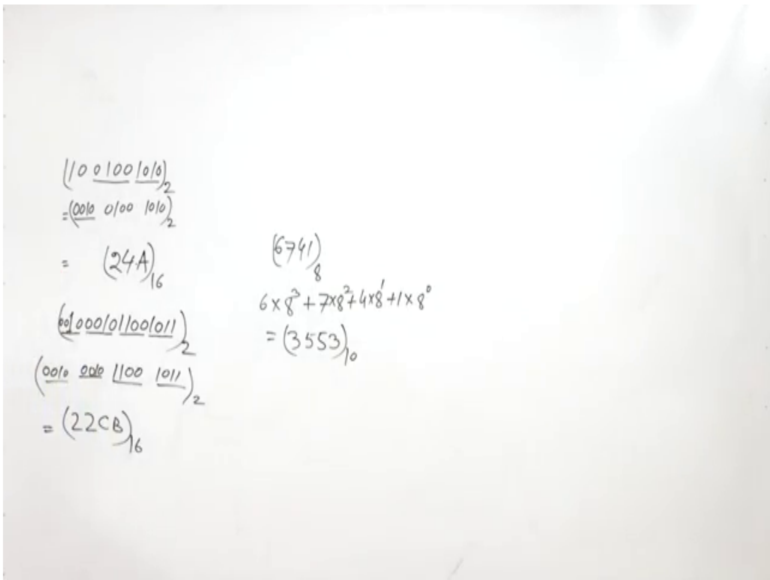

10 0 0 1 0 1 1 double 0, and then we have 1 0 1 1 suppose we have this binary number ok. To make it to write it in a compact form as we said in the beginning to write a binary number in which is a very combustion. So, to write it in a compact form we use hexadecimal system. So, what we do we make a sets of 4 starting with the rightmost digit like this, now we are left with 2 digits should be prefix 2 0 digit here 0 here.

So, this is what we have. Now we have 1 0 1 1 1 0 1 1 this is 1 set 1100, this is 1 set and 0 0 1 0 this one and then 0 0 1 0 this one. So, now 1 0 1 1 what it represents 1 into 2 to the power 0, so 1 1 into 2 to the power 1, so to 2 plus 1 is 3 and then 8 here. So, it means it is 11, 11 means it is a B and then here we have 0 into 2 the power 0, 0 into 2 to the power 1, and then 1 into 2 to the power 2 so; that means 4, 1 into 2 to the power 3 means 8 8 plus 4 is 12, 12 means C and then here we have 0 into 2 to the power 0 1 into 2 to the power 0, so we have 2 here and here we have again 2, so 22 CB we have in the hexadecimal system. Now suppose in now let us consider octal system, here the base is 8 in the octal system base is 8 and the symbols are 0 1 2 3 4 5 6 and 7.

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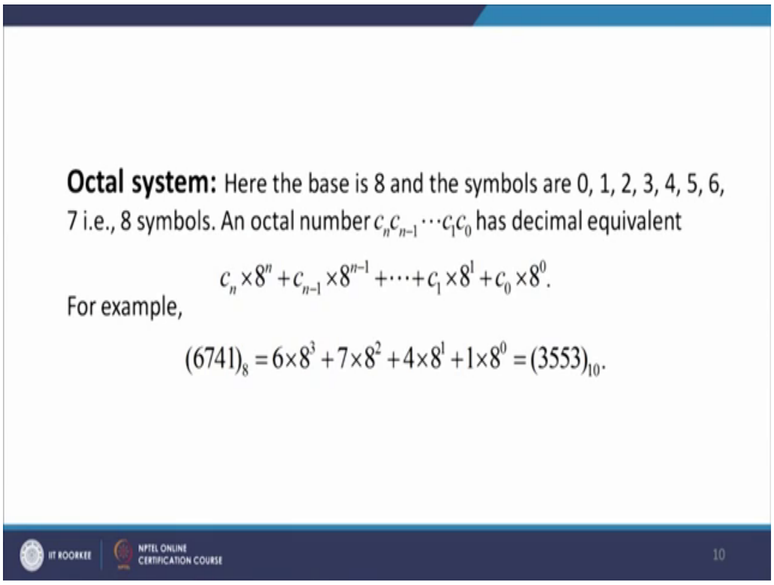

So, there are 8 symbols now in octal number C n C n minus 1 and so on C 1 C naught can be represented as in the has a decimal equivalent C n into 8 to the power n plus C n minus 1 8 to the power n minus 1 and so on, C 1 into 8 to the power 1 plus C naught into 8 to the power 0.

For example 6 7 4 1 within base 8, this is equal to 6 into let me write with 1, 1 into 8 to the power 0 plus 4 into 8 to the power 1 plus 7 into 4 to the 8 to the power 2 plus 6 into 8 to the power 3. So, this is equal to 3553 in the decimal number system, now let us discuss the general conversion rule between 2 base systems with the fractional part. Now, if you have a number which is also given with the fractional part, then what we will do if we will convert the number from one system to another system by considering integer and fractional parts separately.

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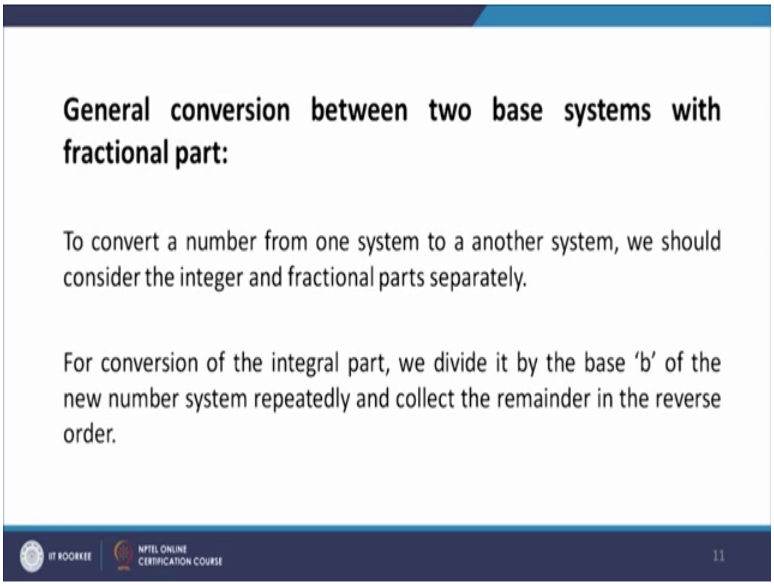

So, for conversion of the integral part b divide the number by its base say base is b, by the new base is b. So, with the base b of the new number system repeatedly and collect the reminder in the reverse order for the conversion of the fractional part we multiplied by the base b of the new number system repeatedly, and collect the integral part in the same order until we get a 0 fractional part are a duplicate fractional part these integer part of this last product.

In the case of duplicate fractional part the integer part of the last product, will be the rightmost digit of the fractional part of the new number are win in the case of 0 fractional part. In integer part of the last product will be the rightmost digit. Now for example, let us say the we have 18.6875.

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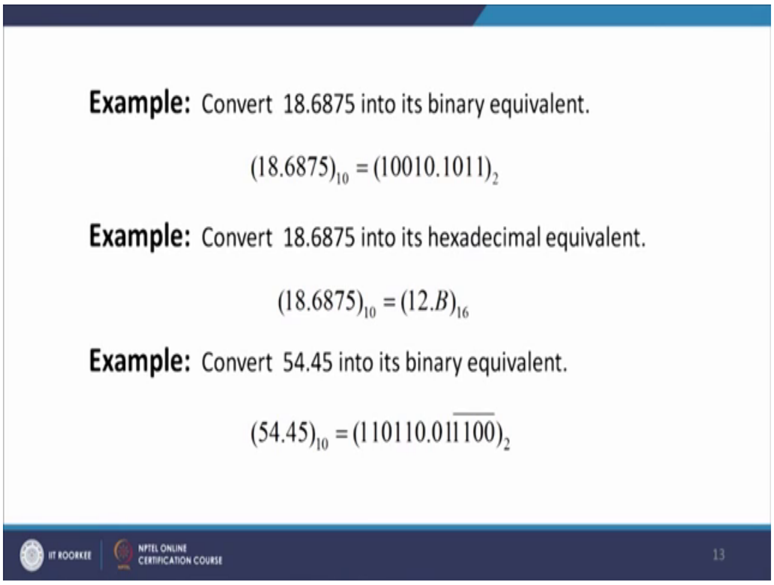

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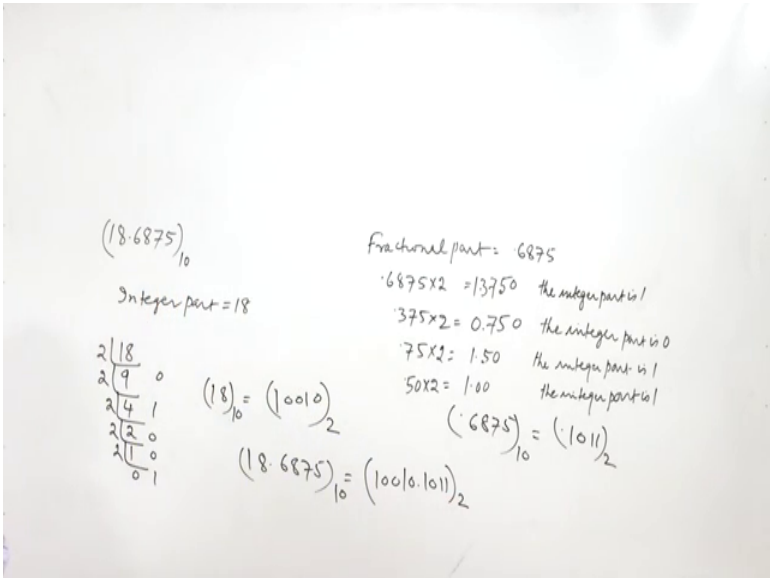

Suppose we have this number in the this decimal number, then we want to write its binary equivalent. So, what will do? This is integer part, integer part is equal to 18, we divide 18 by this base of the new number system, in the new number system that is in the binary number system base is 2.

So, be divided by 2 we get 9 the remainder is 0 we again divided by 2, we get the remainder as 1, we again divide by 2 the remainder is the 0, we again divided by 2 the remainder is 0, we again divided by 2 the remainder is 1 and then we write this digit as the rightmost digit so, 18 in the base 10 is equal to 01001 with base 2.

Now, point 6 fractional part this is equal to point 6875. So, 0.6875 is multiplied by the base of the new number system. So, 0.6875 multiplied by 2 and this gives you 0. So, the now the integer part here is integer part is 1, now we multiply 0.3 again the fractional part 0.375 by 2 and what do we get? No this is0. 750. So, here the integer part is 0 then 0.75 be multiplied by 2, we get 1.50 the integer part is 1.

We multiply by 0.50 by 2 and we get 1, so the integer part is 1 and we stop here because there is no fractional part the fractional part is 0. So, we have now we write it in the same order. So, we write it as a 0.6875 this is in the base 10 is written as point 1 1 0 1 1 ok. So, we write together, so 18.6875 will be written as 1 double 010.1011, in the binary number system like this. Now convert 18.6875 into its hexadecimal equivalent.

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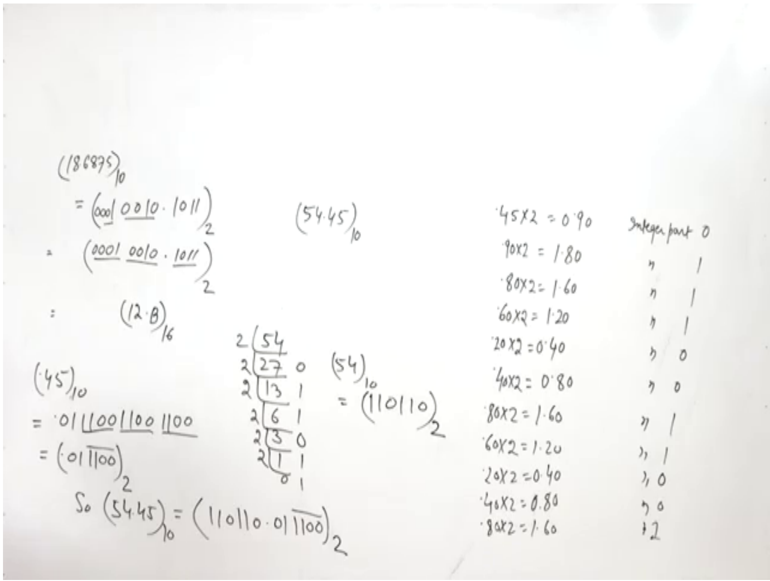

So, 18.685 we want to convert to its hexadecimal equivalent. So, what we will do? If we will first right in the binary number system in the binary number system we have already written this is equal to 1 double 010.1011, and then in the fractional part we start from the rightmost digit and make groups of 4 binary digits. So, 0 0 0 0 1 0 that is 1 and then this 1 we pre prefix here 3 0s, so 0 0 0 1. So, like this and in the decimal part we start from the leftmost digit and so 1 0 1 1 that is one group like this and then we write the corresponding hex digit.

So, what is the corresponding hex digit? So, here 0 1 that is 2 and here we have 1, so 1 point 1 2 point and here we have 1 into 2 to the power 0 that is 1 then 2 here 3 then 8 here, so we have 11 so 11 means we have B. So, 12 point B and we have 16, so this is hexadecimal equivalent, so here it is the best 10. Now, let us go to next question, so we have 54.45, and we want to write it in the binary number system. So, 54 we divided by 2 we get 27 with remainder 0 we divide by 2, 13 remainder 1 be divide by 2 remainder is 1 we divide by 2 remainder is 0 we divide by 2, remainder is 1 and we divide by 2 remainder is 1. So, we have 54 equal to 011011 in the binary number system ok.

Then we have 0.45 let us take the this is integer part, integer part has been converted to the new binary new number system and that where the basis 2, now here point for the fractional part 0.45 and let us write the corresponding binary equivalent. So, be multiplied by 2 we get 0.90. So, 0 is the integer part integer part is 0, then we multiply 0.90 by 2 we get 1.80 so integer part is 1. Then we multiply 0.80 by 2 we get 1.60 integer part is 1, with multiply 0.60 by 2 and we get is 1.20 integer part is 1, then we multiplied 0.20 by 2 and what we get is 0.40 integer part is 0, then we multiply 0.40 by 2 and what we get is 0.80 so integer part is 0.

Now, once we have 0.80 here and we multiply with these numbers will repeat; that means, the integer part will also repeat. So, 0.80 into 2, 1.60 integer part is 1, then 0.60 into 2, 1.20 integer part is 1, and then 1.20 multiplied by 2, 1.40 integer part is 1, and then 0.40 multiplied by 2, we get 0.80 integer part is 0, and then we have 0.80 multiplied by 2 we have 1.60. So, multi integer part is 1 so like this ok.

Now, let us write it in the same order, we can see 0.45 is equal to 0111 double 01110, we have 0 then we have 1 1 1 then we have double 0, then we have 1 1 1 then we have 0, let us see from here no I think here something is there somewhere, so this is 0, so this is 0 here, so this will be 0 ok.

So, this is yes, so this will be 0011, and then here 0. Now you can see this 1100 is repeating again also again we will have 1100. So, what we do we will write it as 0.01 and then 1 1 0 0 and we put a war here over 1100 to show that it this set of 11 1 1 0 0 is repeating. So, then this, and this is in the binary system 2 and so we will write 54.45 as a 110110.01 and then 11002 in the. So, this wills the binary equivalent of this number. Now suppose we have another problem where we want to convert number from the octal.

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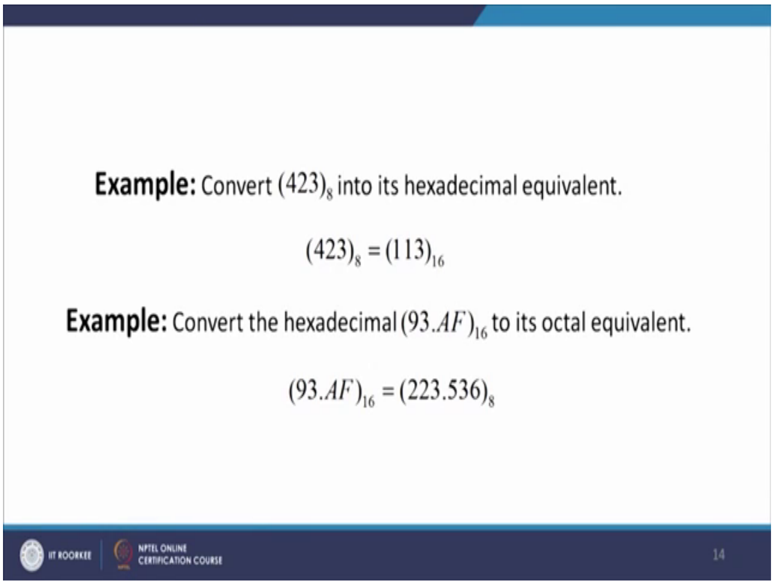

Octal base to the hexadecimal number that is 16, so let us see how we do that.

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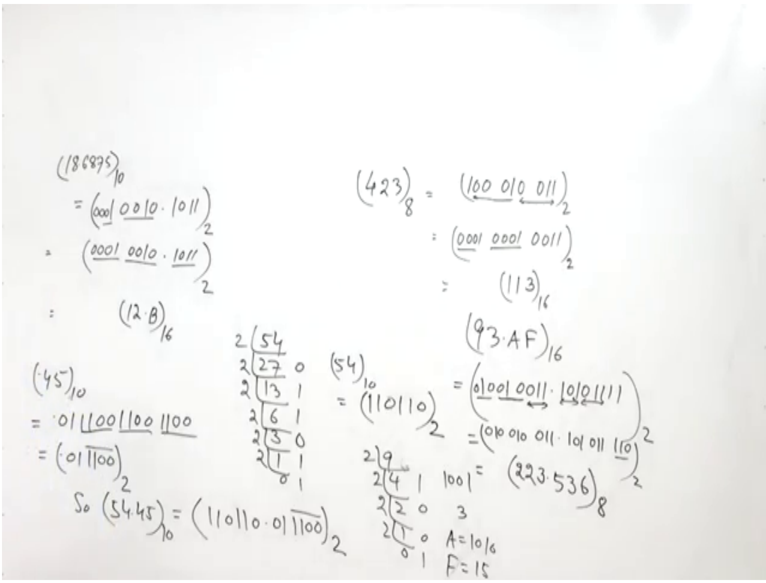

So, let us write the binary equivalent 4 means we write it as 1 0 0 a group of 3 bits, so 100 then 010, then 011. So, we convert from 8 to the base with 8 2 with base 8 2 with base 2 and then we write it as a group of we make groups of 4 digits bits. So, we have 4 here, and we have 4 here, and we are 1 is left, so 0 0 0 1 then 0 0 0 1 0, so then we have 0 0 1 1. Now, we write it as 3 this is 1 this is 1, so this is 16. So, we hexadecimal equivalent will be 1 1 3 16 then the last one is when we convert from hexadecimal to its octal equivalent 93.AF. So, first we write 93 so 93, 93 will be written as when we hexadecimal 9 is written as when we divided by 2.

So, we get 1 when divided by 2, we get 2 0 we divide by 2 we get 1, so we get 0 we divide by 2 again, so we have 1, so 1 0 0 1, so 1 0 0 1 and 3 will be written as 0 1 0 0 1 1 ok. Then we have AF, A is equal to 10, so A is given as 1 0 1 0 and then we have F, F is equal to 15. So, this is 1111 and then we make when we want to convert into the corresponding 8 with base 8. So, this will be 2, so with base 2 with base 8 what we will do we will make sets of 3.

So, 0 1 1 then we have 0 1 0 this 1, so 3 here, 3 here, and then we are left with 2, so we prefix 0 here, so like this. So, this will be equal to 0 0 0 1 1 0 1 0 and then 0 1 0 and here 1 0 1 this one, we start from left and then 0 1 1 and then 1 1 we prefix we so at 0 here like this. So, let us see what does it represent in octal number system, so this is 1 plus 2 3 this is 2 here, this is 2 here point and here it is 1 plus 1 plus 4, so 5 and here it is 1 plus 2 so 3 here it is 0 into 2 to the power 0 then 1 into 2 to the power 1, so 2 plus 4 that is 6.

So, these how we convert from one system to the another system, the hexadecimal system we have converted to its octal equivalent, we have converted the octal number in octal system, octal number system to the hexadecimal system and so on. So, this how we convert number from one system to the other with that I would like to conclude my lecture.

Thanks very much.