Numerical Linear Algebra
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Lecture - 10
Linear Transformation – II

Hello friends, welcome to today's lecture. In this lecture we will continue our study of linear transformation. If you recall in previous lecture, we discussed definition of linear transformation and some properties of linear transformation. Here we will continue our discussion of linear transformation.

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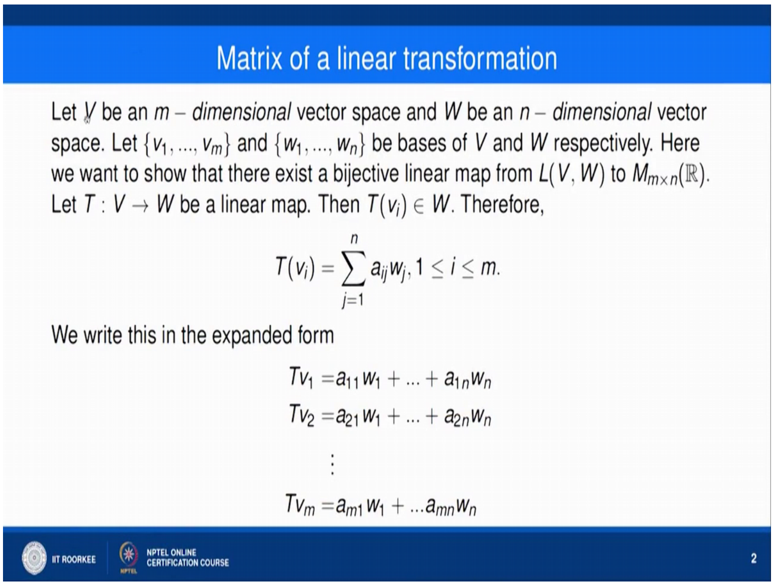

So, let V be an m dimensional vector space and W be an n dimensional vector space, and since we have a and m dimensional vector space and W as n dimensional vector space. So, we can always find out some basis for V and W. So, let v 1 to v m and w 1 to w m be basis of V and W respectively. And here we want to find out that they exist a bijective linear map from L V W to M m cross n to R. So, L V W stand for set of all linear map from V to W, and this capital M m cross n R stand for set of all m cross n matrices over R. So, what we try to find out is, that corresponding to every linear map, here we have a matrix defined on this set m cross n set. So, let T be a linear map which is defined over V and going to W, then you take any element, let us say v i basis element here and then this T of v i belongs to W.

Now, since we already know the basis element of W so, we can write down T of v i as linear combination of j from 1 to n a i j w j, and here this we can do for each i between 1 two m. So, if we write this expression for each v 1 to v m, we can say that T of v 1 is equal to a 1 1 w 1 plus a 1 2 w 1 and so on, a 1 n w n. Similarly we can write down for T of v 2. So, T of v 2 can be written as a of 2 1 w 1 plus a 2 2 w 2 and so on up to a 2 n w n. So, this we can go up to T of v m. So, T of v 1 v m is written as a m 1 w 1 plus a m n w n. So, if we want to write this as in matrix form.

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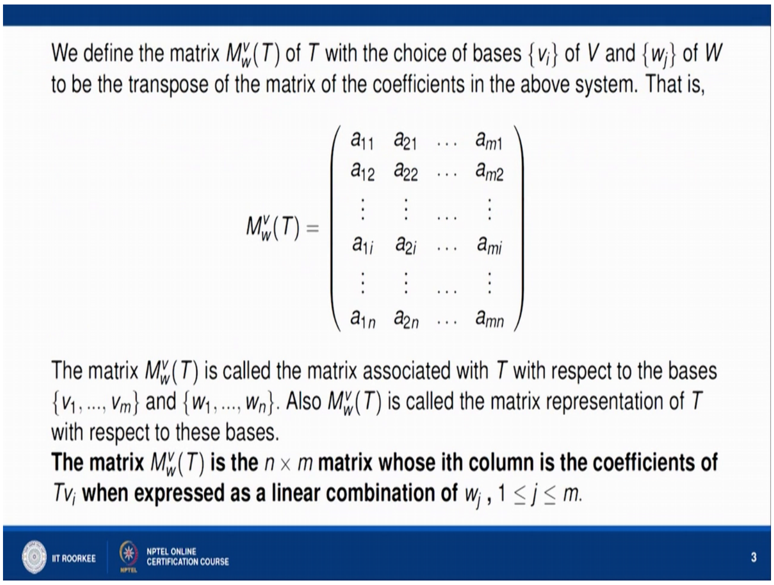

Then we can say that corresponding to this a linear map T we can define the matrix M v w corresponding to this T of T with a choice of basis v i of v and w j of W, and this is defined as the transpose of the matrix of the coefficient in the above system. So, if you look at here, then if you look at the first thing that T of v 1 is given by this. So, if you look at the coefficients here. Coefficient here are a 1 1 a 1 2 up to a 1 n. So, what we try to do here. We take the transpose of the coefficient here and write as a first column. So, first column is what the coefficient of T of v 1 with respect to w 1 and w is,

So, here M v w T is defined as a 1 1 a 1 2 a 1 i a 1 n first column. Second column is the coefficient of T of v 2 in terms of W. So, the matrix M v w T is called the matrix associated with T with respect to the basis v 1 to v m and w 1 to w n, and also M v w T is called the matrix representation of T with respect to these bases. Please note down here that the matrix M v w T is the n cross m matrix, whose ith column, if you look at the ith column is the coefficients of T of v i when expressed as a linear combination of w j for example, ith means let us say the first column. So, first column is basically the coefficients of T of v 1, when expressed as a linear combination of w js. And if you look at the second column, second column will be the coefficients of T of v 2 when expressed as a linear combination of w js. So, let us consider an example of this matrix representation of a linear map.

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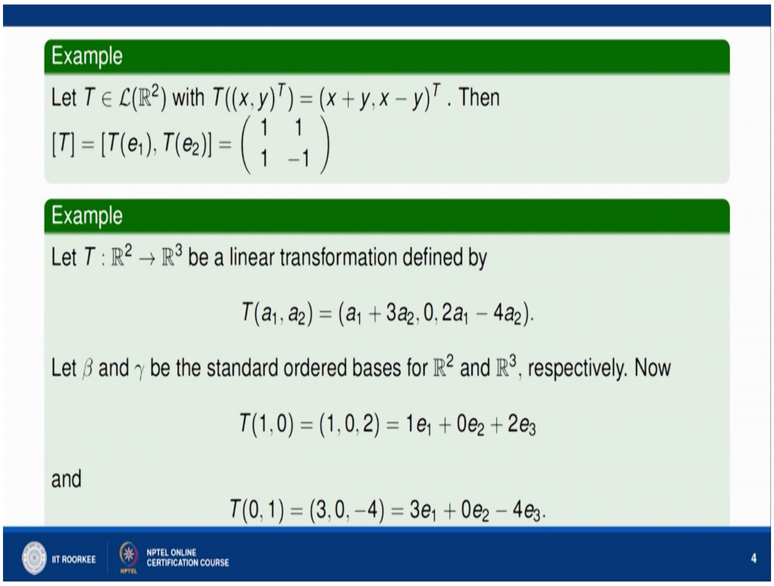

So, let us take very simple example here. Let T be a linear map defined from R 2 to R 2 with this notation that T of x y transpose equal to x plus y comma x minus y transpose. So, and we try to find out say matrix representation of this linear map. So, here without loss of generality I am assuming say standard basis, here you can use any basis given. So, right now I am using only standard basis here. So, we want to find out say basis matrix representation of this linear map. So, let us see how we can find out. So, here this is the linear map; T from R 2 to R 2 which map x y to x plus y and x minus y.

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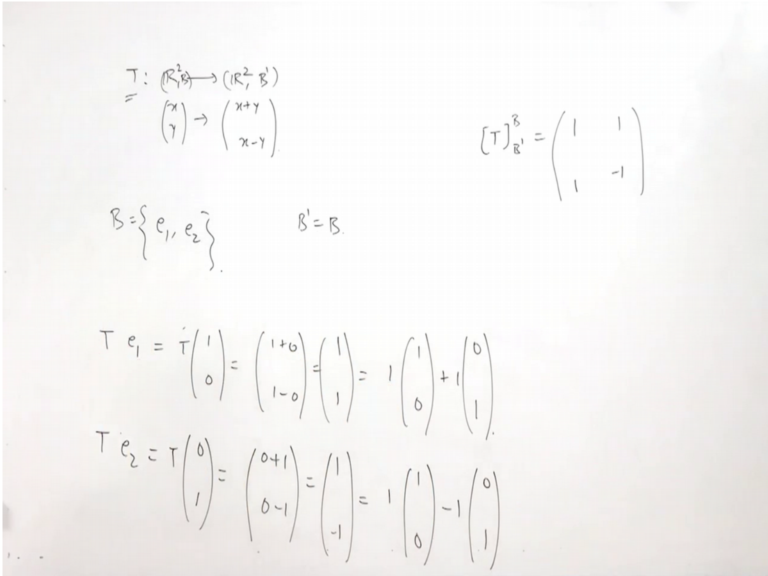

So, here we want to find out say matrix representation of this T. So, here we are taking a standard basis here. So, standard basis means, here B is, we are taking as e 1 and e 2 and similarly your matrix basis for this R 2. We are also taking same as a B dash as same as B here. So, we can write it R 2 B, and R 2 B dash. If you do not write anything we can assume that it is a standard basis. So, to find out say matrix representation here, we operate T on first basis element that is e 1.

So, it is basically what. So, that T operating on 1 0, and according to this definition of T, this is nothing, but 1 plus 0, and 1 minus 0, or you can write it 1 1 and this 1 1 you can write it as 1 of 1 0 plus 1 of 0 1. So, here if you look at the coefficient is nothing, but 1 and 1. So, you can say that matrix representation of T with respect to B and B dash, the first column is basically the coefficient of image in terms of a basis element of this image alright. So, here basis for this image is basically standard basis. So, 1 0 and 0 1. So, you can say that the coefficients are nothing, but 1 1. So, you can write it the first column is basically 1 1 if you look at image of e 2.

So, e 2 will be T of 0 1, and according to the definition T of 0 1 will be what 0 plus 1 and 0 minus 1. So, it is nothing, but 1 n minus 1. So, let us write this 1 n minus 1 in terms of basis of co domain. So, it is nothing, but 1 of 1 0 minus 1 of 0 1. So, here coefficients are 1 and minus 1. So, the second column will be the coefficient corresponding to image of second basis element. So, here you can say that matrix representation of T with respect to B and B dash is nothing, but 1 1 and 1 minus 1

So, that is what is given here, the matrix representation of T is equal to matrix of first column is T of e 1 and second column n here T of e 2. So, here I am not writing anything, because I am using the standard basis. So, this can be written as 1 1 and 1 minus 1. So, first column is coefficient of coefficient of T of e 1 and second column is the coefficient of T of e 2, when it is written in terms of standard basis. Similarly let us define, let us take some other example.

So, let us take this example here T is a map from R 2 to R 3 and it is a linear transformation defined by T of a 1 a 2 equal to a 1 plus 3 a 2 comma 0 comma 2 of a 1 minus 4 of a 2. So, because of space constant this a 1 a 2 transpose, I have written as this simple column vector as a 1 a 2, but this is basically this represent a element of R 2 here. So, here again we let us take this beta and gamma be the standard order basis and we try to find out say matrix representation of this t. So, let us see how it is. So, here our T linear map is given by T from R 2 to R 3 which map a 1 a 2 to a 1 plus 3 a 2 comma 0 comma 2 of a 1 minus 4 a 2.

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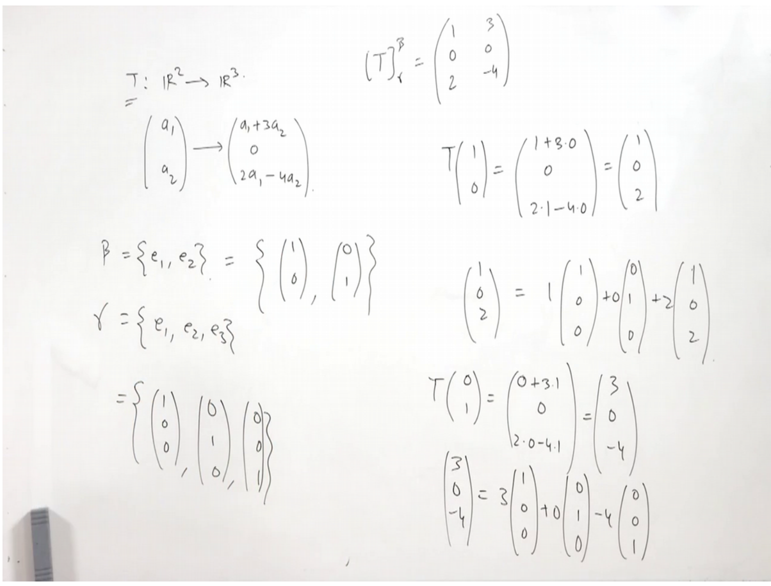

So, here we want to find out a matrix representation of team. Here also we have taken beta and gamma as same as standard basis. So, standard basis means, we are taking as beta as a given as e 1 e 2, because it is two dimension and gamma is a standard basis for

R 3, which is given as e 1 e 2 and e 3, and we have already discussed what is these standard bases. So, e 1 e 2 is basically this; here we have 1 0 and 0 1. So, this is standard basis for R 2, and if you look at the standard basis for R 3 it is given as 1 0 0 comma 0 1 0 comma 0 0 1. So, this is standard basis for R 3.

So, with respect to this beta and gamma, we want to find out; say matrix representation for this team, let us operate T on first basis element; that is on e 1, that is 1 of 0. So, according to the definition of map here, it is 1 plus 3 times 0 comma 0 and here 2 into 1 minus 4 into 0. So, it is nothing, but 1 0, and this is 2 here, so map is 1 0 2. Now let us look at the coefficient of 1 0 2. So, coefficient of 1 0 2 in terms of basis element is nothing, but 1 of 1 0 0 plus 0 times 0 1 0 plus 2 times 1 0 2.

So, here coefficients are 1 comma 1 0 and 2. So, it means that if you look at the matrix here, matrix with respect to beta and gamma. So, first column is given by 1 0 and 2. So, 1 0 and 2 look at the second thing. So, operate T on second basis element; that is e 2 0 1, and its image will be what 1 times. Sorry 0 0 plus 3 times 1 comma 0, and here it is 2 into 0 minus 4 into a 2; that is 1 right. So, this is what this is 3, 0 and it is minus 4 here, and if you write. Now write down the coefficient of 3 0 minus 4 in terms of gamma here. So, 3 0 minus 4 can be written as 3 times 1 0 0 0 times 0 1 0 and minus 4 times 0 0 1. So, this coefficients of 3 T of 0 1 in terms of basis element of W is given by 3 0 and minus 4. So, you can write 3 0 minus 4 here.

So, this is the matrix representation of T with respect to beta and gamma, and it is given by 1 0 2 and 3 0 minus 4. So, that is what is given here. So, here our matrix representation of T with respect to beta and gamma is given by 1 3 0 0 2 minus 4.

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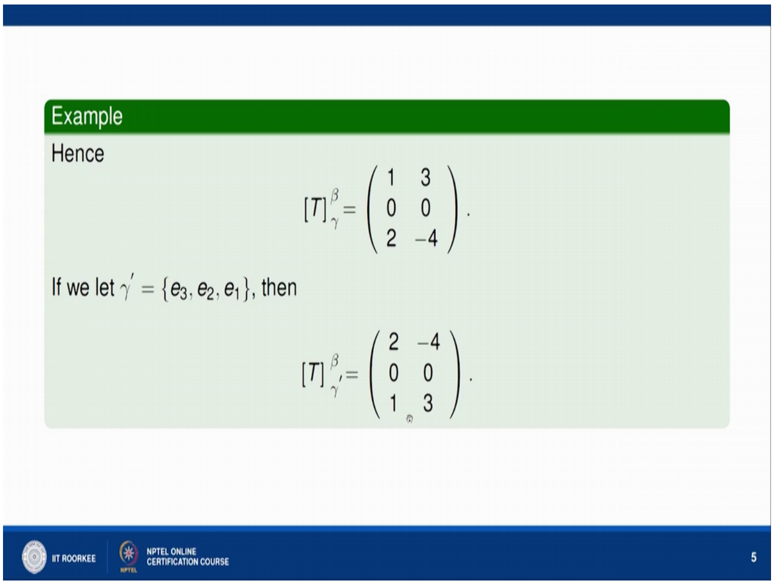

Now if we change our basis here; for example, in place of gamma, if we consider say gamma dash. So, gamma dash, let us take here as this.

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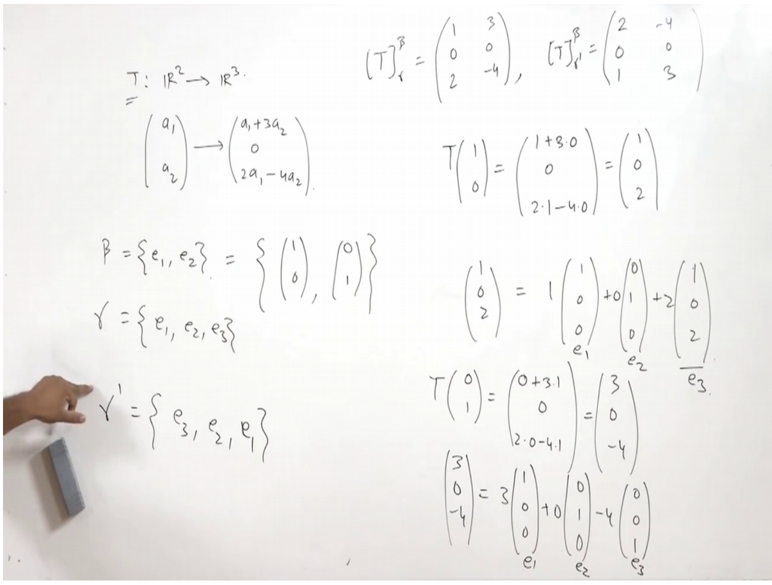

So, here if we take gamma dash as pointed out in slide that it is e 3 e 2 e 1. So, let us say e 3 e 2 and e 1. So, order is changed. So, this order basis is changed, then how we can write down the matrix representation of T with respect to beta and gamma dash. So, if you look at, if we want to find out say matrix representation corresponding to beta and gamma dash, then look at here T of 1 0. If you operate it will be 1 0 2, but now we want to write 1 0 2 in terms of e 3 e 2 e 1. So, e 3 e 2 e 1 means, here it is e 3, this is e three, this is e 2 and this is e 1.

So, we can write down this as; say at first place it is coefficient of e 3, second place the coefficient of e 2 and third place it is coefficient of e 1. So, it means that if you want to write down the matrix representation of T with respect to beta and gamma dash, then the first column will be what? First column will be the coefficient of this order basis. So, order basis means 2 0 and 1. So, it is written as 2 0 and 1.

Similarly, if you want to find out the second column will be what? Image of T image of 0 1 under this T and it is given as 3 0 minus 4. Now we want to find out the coefficient of 3 0 minus 4 with respect to e 3 e 2 and e 1. So, e 3 e 2 e 1 means, here it is e 3 e 2 and e 1. So, the second coefficient will be minus 4 0 and 3. So, it is minus 4 0 and 3 right. So, it means that if we change our basis from gamma to gamma dash, then your matrix representation will also change. In this case your matrix is given by 2 0 1 and minus 4 0 3.

So, if you; so, just look at the difference here that if you change any basis, then your matrix will also change and in this case if you change our basis from standard basis which is given as e 1 e 2 e 3 to gamma dash, which is e 3 e 2 e 1, then your matrix representation will change from this to this. So, it means that this matrix representation is basis dependent. So, if you change your basis, your matrix representation will also change. So, whenever we write any matrix here, it means that we already know what is the basis for domain, and co domain spaces is that.

So, here we have written that if we let gamma dash equal to e 3 e 2 e 1, then matrix representation of T is given by with respect to beta and gamma dash as 2 0 1 and minus 4 0 3, is it ok.

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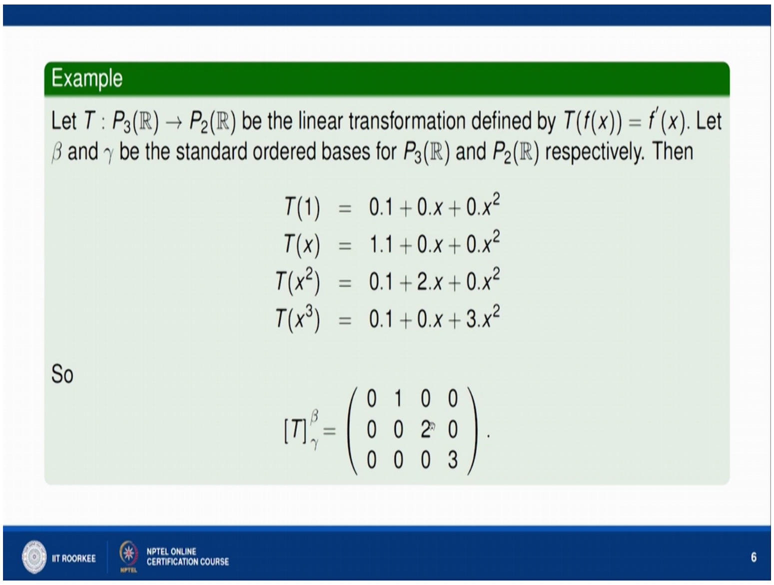

So, now let us move to one more example here, we consider a linear map T from P 3 R 2 P 2 R here, P 2 R represents set of all linear transformation, whose degree is at most 3 and P 2 R is what set of all polynomials whose degree is at most 2, and coefficients are coming from R here, and it is defined as differential operator means T of f of x is given by f dash x. And here also we want to find out; say matrix representation for this team. So, let beta and gamma be the standard order basis for P 3 R and P 2 R. Here also we are assuming standard basis for this P 3 R and P 2 R.

So, let us find out say basis for image of this beta. So, beta consists elements of 1 x x square and x cube. This is the basis for P 3 R. So, P 3 R means set of all polynomials whose degree is at most 3. So, basis element is a standard ordered basis is 1 x x square and x cube. So, let us find out say image of 1. So, T of 1 will be what T of 1 will be nothing, but differentiation of constant polynomials. So, which is coming out to be 0.

So, 0 can be written as in terms of gamma, gamma is 1 x x square. So, 0 can be written as 0 dot 1 plus 0 dot x plus 0 dot x square. So, here we can write down the coefficient as a first column. So, first column will be 0 0 0. So, that is given here 0 0 0. Similarly you take the image of x here. So, T of x will be what? Here if you differentiate, this is coming out to be 1 and 1 can be written as 1 dot 1 plus 0 dot x plus 0 dot x square. So, coefficient will be 1 0 0. So, that is written as second column 1 0 0. Similarly you can find out T of x square T of x square is 2 of x m. So, 2 of x can be written as 0 dot 1 plus 2 dot x plus 0 dot x square.

So, that it written here 0 2 0. So, 0 2 0 is your third column. Similarly T of x cube will be what? If you look at here the definition 3 T of x cube will be 3 x square. So, 3 x square can be written as 0 dot 1 plus 0 dot x plus 3 dot x square. So, coefficient will be 0 0 3 here, so 0 0 3. So, we have find out the coefficient matrix representation of this linear transformation, and it is given by T beta gamma and it is given by 0 0 0 1 0 0 2 0 2 0 0 0 3. So, first column is basically the coefficient of T of 1. So, coefficient of T of 1 is 0 0 0 and second column 1 0 0 is given by the coefficient of T of x in terms of 1 x and x square; that is 1 0 0.

Similarly the third and fourth column, now, here we may ask what is the relation between matrix representation of a linear map and a linear map. So, if you look at here that if we have a matrix linear map from V to W, whose basis is given by say here beta gamma if B, then I can write down the matrix representation as T of beta and gamma.

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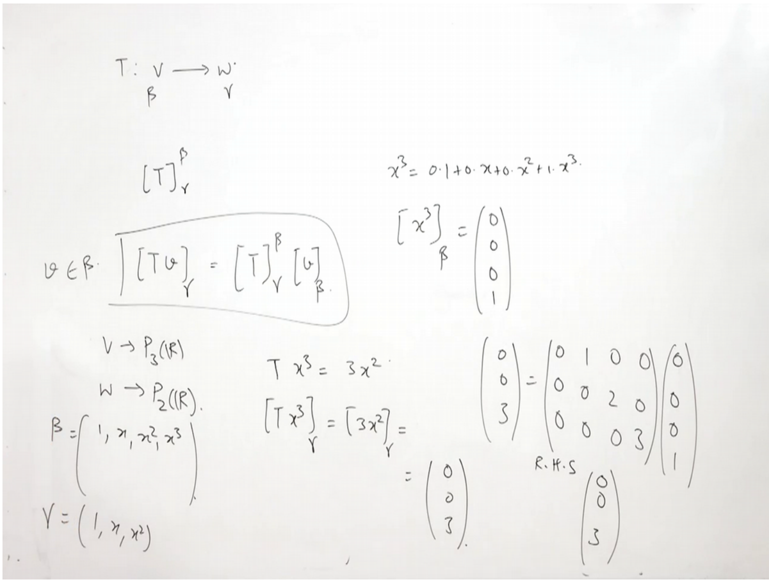

Now, how we can relate this to this? So, it means that if you look at, what we have done is, basically we have taken the image of beta under the map T and written in terms of coefficients in terms of elements of gamma here. So, if you look at here what we have done. We take V V from your beta and then we try to find out coefficients of T of v means coordinates of T v in terms of gamma.

So, this represents, what this represent the coordinates of T v with respect to gamma here. So, this given as matrix representation of T; that is with respect to beta and gamma and here this is coordinates of V with respect to beta. So, this is the how to find out. So, matrix representation, if matrix presentation given by this T with respect to beta and gamma then we can say that coordinates of T v with respect to gamma is given by product of these two matrices. This matrix is matrix representation corresponding to T and this is the coordinates of v here. So, if you look at the previous example here, we have taken V as P 3 R and W is your P 2 R and your beta is given as 1 x x square x cube right and gamma is given as 1 x x square right.

So, let us see how this is true. So, if you look at this x cube. So, let us find out T of x cube. So, T of x cube; since this T is a differential operator, so this is given by 3 x square alright. So, let us find out say coordinates of 3 x square.

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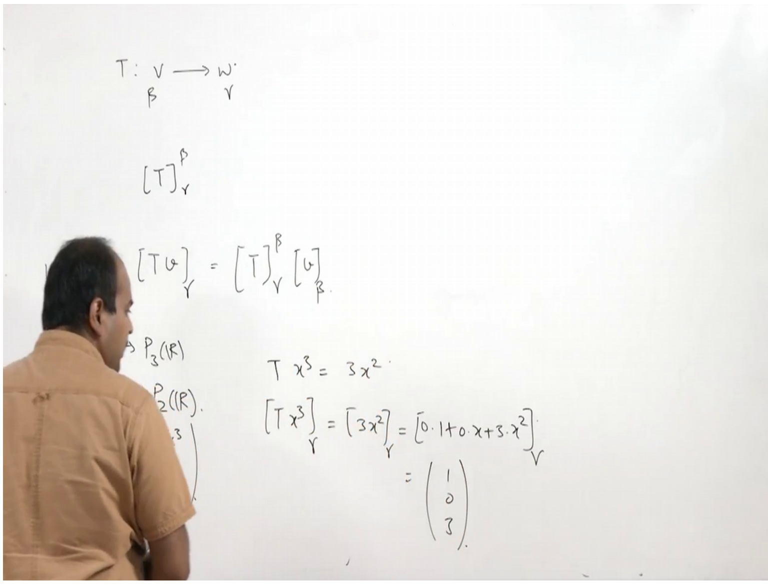

So, T of x cube with respect to gamma. So, with respect to gamma means coordinates of 3 x square with respect to gamma, and it is given by what. This is given by 0 times 1 plus 0 times x plus 3 times x square. So, coordinates is basically given as your 1 0 3 right.

So, coordinates of 3 x square is given by 1 0 3 right. So, and what is the coordinates of x cube. So, coordinate of x cube with respect to beta, beta is here 1 x x square x cube. So, I can write down x cube as 0 times 1 plus 0 times x plus 0 times x square plus 1 times x cube. So, coordinate of x cube with respect to beta, is basically what 0 0 0 1 right.

So, we want to show that this relation holds. So, here we want to show that 1 0 3 is given by matrix representation of T. Now matrix representation of T will be what 0 0 0, and then 1 0 0 0 2 0 0 0 3. So, that is what we have found that the matrix representation of T will be this, and if you look at the coordinate of V v is x cube here. So, coordinate of x cube with respect to beta is basically 0 0 0 1. So, let us find out say right hand side, if you look at the right hand side, right hand side will be, is basically you multiply here and you will see it is coming out to be 0 here, and this is again 0 and this is 3 here right.

So, it is given by this. Here there is a small mistake 3 x square is written as 0 times 1 0 times x and 3 times x square, so 0 0 3. So, that is your left hand side. So, here we can see that left hand side is same as right hand side. So, it means that what is the meaning of matrix representation of a linear map, that it will satisfy this property that coordinates of a image with respect to basis element of W is given by matrix representation here into coordinates of V with respect to beta.

So, in this way we can relate a linear map to matrix representation of linear map. So, here we say that for every linear map, we have a matrix representation. Similarly we can say we can define for every matrix a linear map, and it should define like this, and we know that if we have the coordinates of image space, then we can find out say linear map so, that we may discuss later. So, here this we have discussed here.

Now, let us move to next. Now here will prove very important property of linear transformation that is a rank nullity theorem, but before that we need to understand what is rank, what is nullity. So, here we know that if we take two vector space, let us say V and W be two vector space over a common scalar field.

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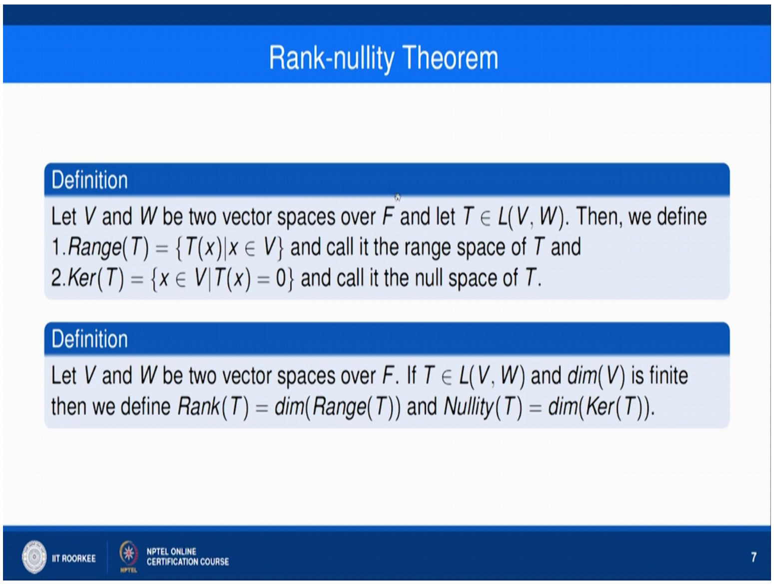

Let us take this as F, it is commonly be taken as R and c R or c. So, let T represent a linear map define from V to W, then we define range of T which represent the range space of T as T of x, where x is coming from vector space V and this we can verify that it is a vector space, vector subspace and we call this vector subspace as range space of T. similarly we can define kernel of T or null space which is given by all those x in V; such that T of x is equal to 0. So, it means all those element of V whose image is 0, and we call this space as null space of T. So, this I am leaving it to you and that proved that range of T is actually a vector space of W and a kernel of T is a vector subspace of V.

So, with the vector space we can define the dimension here. So, let V and W be a two vector space over F, and if T belongs to linear map from V to W and dimension of V is finite, then we can define rank of T as dimension of range of T and nullity of T as dimension of kernel of T. So, rank T means dimension of range space and nullity means dimension of kernel space is that, and then the dimension V and rank nullity are related with this relation that rank of T plus nullity of T is equal to dimension of V.

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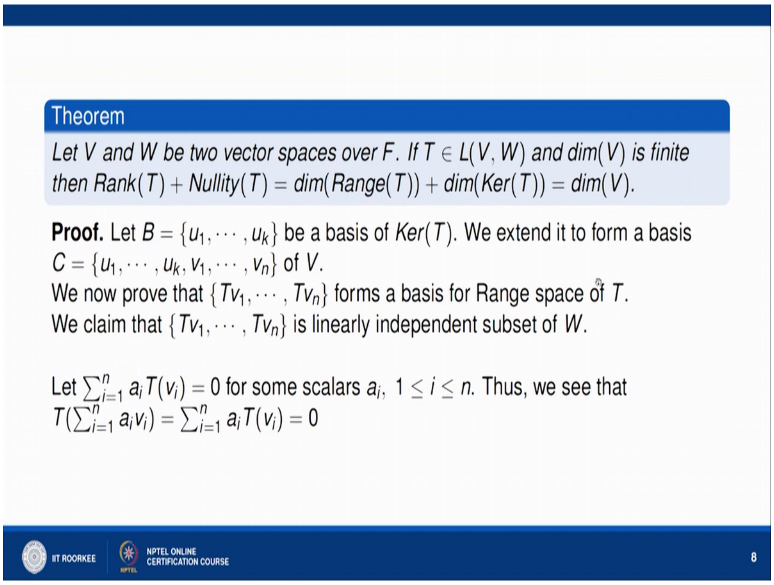

So, let us understand the precise statement of this theorem which say that let v and w be two vector space over f and if T is a linear map from v to w and dimension of v is finite then rank of T plus nullity of T rank of T is nothing, but dimension of range of T plus dimension of kernel of T is equal to dimension of V. So, this we want to prove here

So, to prove this let us take basis for kernel space T, let us say that it is u 1 to u k. So, this B which represent set of these k elements u 1 to u k be a basis of kernel T. So, it means that we are assuming that nullity is basically k. So, dimension of kernel space is k. Now we already know that kernel of T is a subspace of V. So, we can extend the basis for subspace to the basis of whole vector space and let us use this fact, and we can extend the basis of kernel of T to find a basis of whole vector space V, let us call this as C.

So, C consists of u 1 to u k, this is the basis element of kernel T and extended elements are v 1 v n. So, this forms a basis for V. So, it means that we are assuming that dimension of V consisting k plus an element. So, dimension of V is nothing, but n plus k. Now we want to prove that that these element v 1 to v n image of this v 1 to v n forms a basis for range of space of T i, it means that we want to prove that T of v 1 comma T of v 2 up to T of v n forms a basis for range space of T.

So, if you can prove that this forms a basis for range space of T, means we are proving that the dimension of range space is nothing, but n, it means at rank of T will be n and then we can show that this result follows. So, we want to prove only that, that T v 1 T v n forms a basis for range space of T. So, we need to prove two things; first is that this set is linearly independent, second is that this expand the range space of T. So, first let us prove that this T v 1 to T v n is linearly independent substitute of W for that we form a linear combination of this. So, summation i equal to 1 to n a i T of v i is equal to 0, and we want to show that this has only a trivial solution, it means that all the a i s are nothing, but 0.

So, here let us consider these ais are coming from this scalar field F. So, summation i equal to 1 to n a i T of v i equal to 0. Now here using linearity you can take, consider this as summation i equal to 1 to 1 a i T of v i equal to 0 and here I am using linearity and this can be written as T operating on i equal to 1 to 1 a i v i equal to 0, so here I am using. In this system we are using the linearity property and if you look at here, it means what T of this vector is equal to 0, it means that this vector which is given by R equal to 1 to 1 a i v i belongs to kernel space here.

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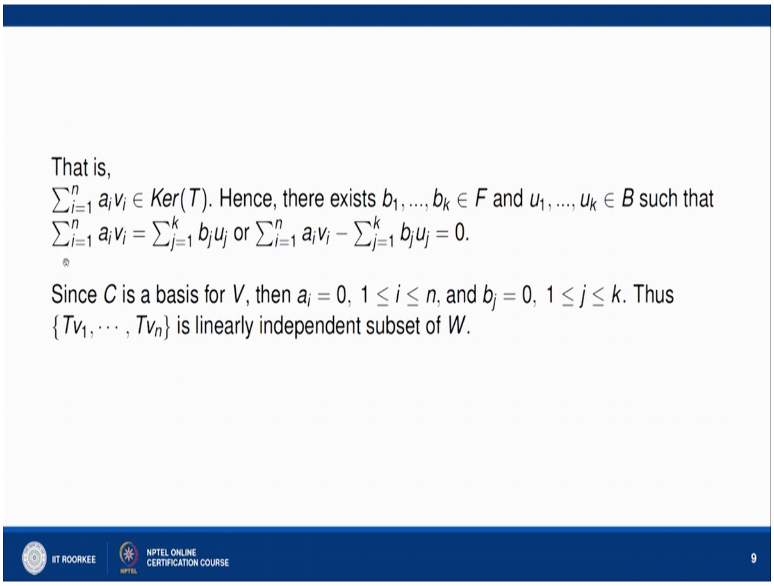

So, here means that the summation i equal to 1 to n a i v i equal to kernel of T. Hence it means that this factor can be written as linear combination of basis element of kernel T. So, it means that we have coefficient scalars b 1 to b k coming from; this is scalar field and u 1 to u k; such that this can be written as linear combination of u i. So, this can be written as j equal to 1 to k b j u j. So, if you write down in a proper format this can be written as i equal to 1 2 1 a i v i minus summation j equal to 1 to k b j u j equal to 0. Now if you look at this, is what this is representation of 0 in terms of v i and u j and, but we already know that C consisting elements v i and u j is a basis for V. So, it means that 0 must have only trivial representation, it means that all this a is and b js must be equal to 0. So, a i equal to 0, for all i between 1 to 1 and b j equal to 0 for all j from 1 2 k. So, it means that 0 have only trivial representation, means that T v 1 to T v n is linearly independent subset of W, because we have started with this representation that summation i equal to 1 to n a i T of v i equal to 0 and here we are able to prove that all these ais are nothing, but 0. So, here we have shown that all a is are 0 for the all i

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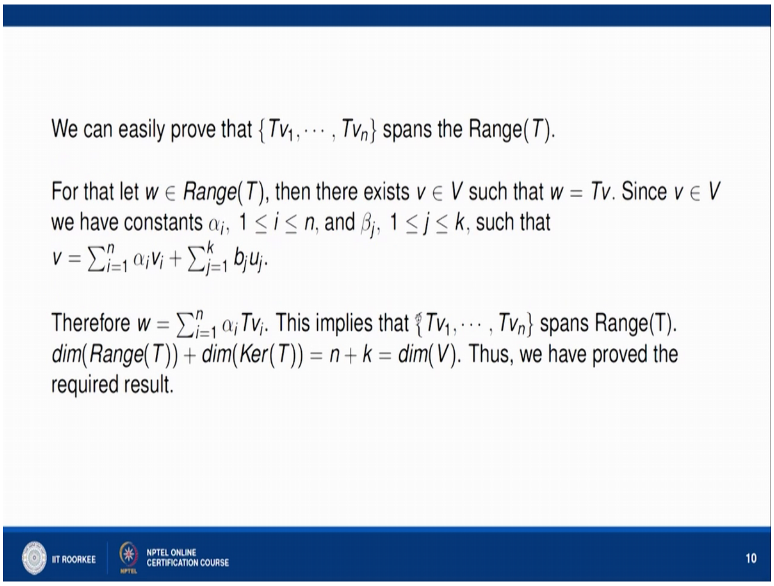

Now, and the second step we want to prove that T v 1 to T v n expand the range space of T. For that let us take any element in range space of T. So, W belongs to range T. Now W belongs to range of T, means there exit a v in vector space V such that w is written as T of v. Now since v belongs to factor space V. So, v can be written as linear combination of basis element of V. So, basis element is given as v i and u j. So, it means at we can find out alpha i and beta j scalars such that v can be written as summation i equal to 1 to n alpha i v i plus summation j equal to 1 to k B j u j.

So, it means that w can be written as. So, it means if we operate T on this. So, T of v will be W. So, W can be written as T operating on this expression. Now if you operate on this expression, then it is what i equal to 1 to n alpha i T of v i plus summation j equal to 1 to k b j of T of u j, but u j are basically what u j or coming from kernel space. So, it means that T of u j is going to 0.

So, there is no contribution from this part and we can write W as T of v as summation i equal to 1 to n alpha i T of v i. So, this means that W can written as linear combination of these T of v is. So, it this implies that T of v 1 comma T of v 2 up to T of v n spans the range of T. So, it means that this set is a basis for range of T. So, it means dimension of range of T will what it must be n here. So, it must dimension of range T is a n dimension of kernel of T is k. So, this is summation of these two number is n plus k which is nothing, but the dimension of V. So, that is clear from the fact here that C consists k plus n element. So, it means that here this is a statement follows. So, this is quite important result here, so which is very often used.

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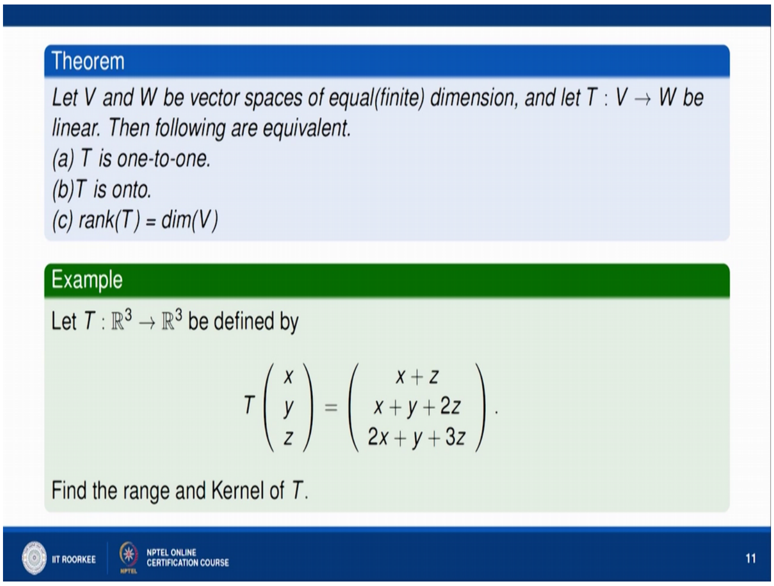

So, now with the help of this let us consider another small result which says that let V and W be a vector space of equal dimension, and let T linear map from V to W be linear then following are equivalent, it means that if T is 1 1, then it must be onto and tank of T must be equal to dimension of V. So, let us consider small proof of this. So, let us consider here.

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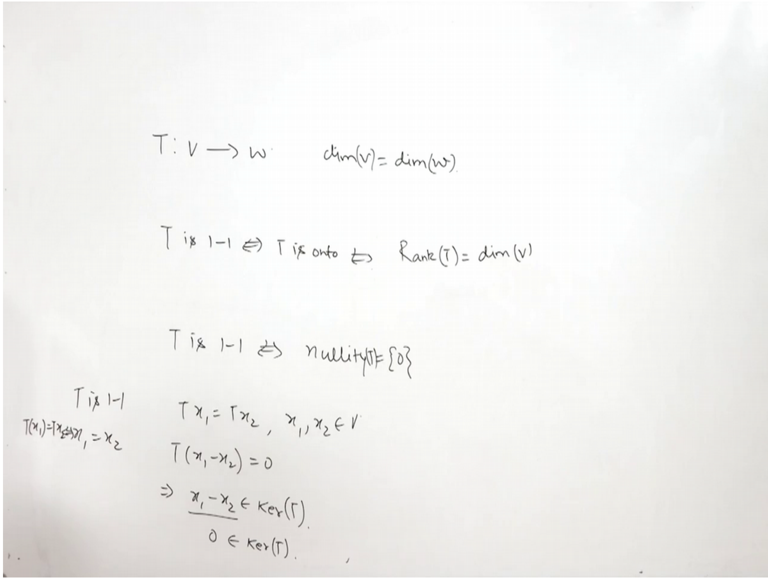

So, what we want to prove here that T map from V to W here and dimension of V is given as dimension of W here, and we want to prove that in this case T is T is 1 1 implies, and implied that T is onto and this implies by that rank of T rank of is equal to rank of dimension of V. Sorry here we have not assumed that dimension of V equal to dimension. Here we have assumed that they have equal finding a dimension. So, we want to prove this. So, I think how we can prove.

Now if we look at T is 1 1 implies what T is 1 1, means we can easily prove that this implies that nullity is basically 0, nullity of T is basically 0, how it if T is 1 1 means T of x 1 equal to T of x 2, where x 1 and x 2 coming from vector space V. So, this implies that T of x 1 minus x 2 is equal to 0. So, this implies that. So, T of x 1 equal to T of x 2 means this. So, this implies that x 1 minus x 2 belongs to kernel space of T here right. Now if I am assume that T is 1 1.

So, if I assume that T is 1 1 means that images are equal implies that x 1 is equal to x 2. So, T is 1 1 means T of x 1 equal to T of x 2 imply and implied by that x 1 is equal to x 2. So, in this case when T of x 1 equal T of x 2, then we know that 0 belongs to kernel of T. So, it means that 1 2 1 implies that that kernel T will contain only 0 element. And in this case nullity is nothing, but 0 and if nullity is nothing, but 0 then it has to be 1 1. So, this statement is equivalent statement that T is 1 1, T is a linear map provided that T is linear map, then T is 1 1 implied, and implied by the nullity of T is equal to 0. Now, so this imply what?

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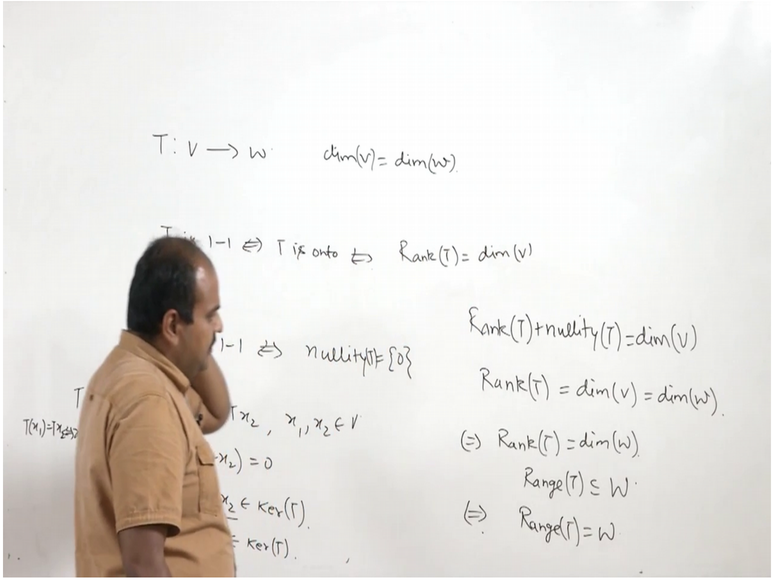

Now, look at rank nullity theorem. So, rank of T plus nullity of T equal to dimension of V. Now here since nullity of T is 0. So, we can say that rank of T is equal to dimension of V. Now we already know that dimension of V is nothing, but dimension of W. So, it is nothing, but dimension of W. So, we can say that rank of T is equal to dimension of W, we already know that range space is a subspace range of T is subspace of W. So, we have range space is a subset of W and their dimension is same. So, it means that your range of T is nothing, but range of T is nothing, but whole of W.

So, it means that in this case we can say that range of T is same as W. So, it means that in this case we can say that T is in on two map. So, this is clear and everything is you can say that this is implies an implied by; so, here we can say that T is 1 1 implies that T is on 2 right, and this implies that T is on 2 means rank of T is equal to dimension of V right. So, that is follow from this is it. So, how we can say that this follows T is on 2 means that rank of T is same as dimension of W, and we already know that dimension of is W is same as dimension of V. So, we can say that rank of T is nothing, but dimension of V.

So, example says at we have a linear map from R 3 to R 3 defined by T of x comma y comma z is equal to x plus z x plus y plus 2 z 2 x plus y plus 3 z, and we want to find out the range space and kernel space of T. So, let us write here. So, here map is defined by R 3 to R 3 defined by that x y z maps to x plus z x plus y plus 2 z 2 x plus y plus 3 z.

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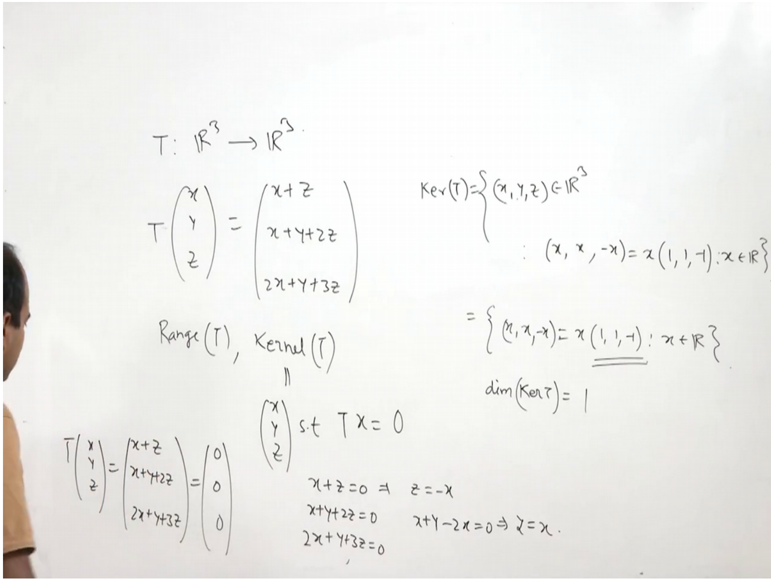

So, we want to find out range space of T and kernel space of T. So, first let us find out say kernel space of T. So, kernel space of T means all those x y z; such that T of x is equal to 0 here. So, it means that if you find out say image of T x y z which is nothing, but x plus z x plus y plus 2 z and 2 x plus y plus 3 z to 0 0 0. So, this we need to find out x y z. So, if you look at this implies that x plus z equal to 0, and x plus y plus 2 z equal to 0 and 2 x plus y plus 3 z equal to 0. If you look at this implies, what this implies that z equal to minus of x and if you use this z equal to minus x here, then your x plus y and 2 z. I am writing as minus 2 of x, here 2 of x is equal to 0. So, this implies that y is equal to x here.

So, here z is equal to minus x and y is given as x. So, we can write down this kernel of T. So, kernel of T will be basically what. Although x y z from R 3; such that x is given as x y is given as x and z is given as minus of x. So, this I can write as, this is nothing, but x and we can write 1 1 and minus 1 here, x is coming from R. So, we can say that this kernel of T can be written as say x y z and this x y z will be written as x time equal to x into 1 1 and minus 1 x is coming from R here. So, we can say that this kernel of T is expand by this vector 1 1 minus 1 and we can say that here y is equal. So, it means that your x y z will be what? it will be of this kind x x and minus of x.

So, it means that here dimension of kernel T will be what, number of basis element in basis of this which is given by 1 and basis element is 1 1 and minus 1. So, dimension of kernel T is 1 and kernel T is given by x comma x comma minus of x and x is coming from R. So, we want to find out now range of T. So, range of T will be what. We already know by rank nullity theorem.

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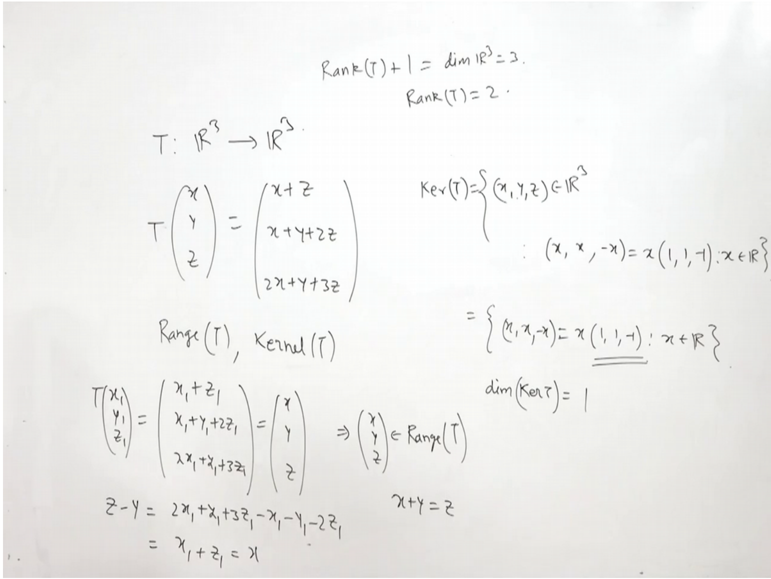

Rank nullity theorem means rank of T plus nullity of T, and nullity of T will be 1 equal to dimension of V. Dimension of V means dimension of R 3 is equal to 3. So, it means that rank of T will be what rank of T is going to be 2 only and we find out range of T. So, range of T will be what. So, basically range of T will be, we need to find out image of x y z

So, here will be what, let us write it x 1 y 1 z 1. So, it is basically what x 1 plus z 1 x 1 plus y 1 plus 2 z 1 and 2 x 1 plus y 1 plus 3 z 1. If you call this as x y z, you want to find out say what is x y z. So, basically x y z is coming from range space of T. So, if you look at here from this if you subtract this z minus y. So, here if you subtract this set z minus y, z minus y will be what? 2 x 1 plus y 1 plus 3 z 1 minus x 1 minus y 1 minus 2 z 1. So, this will be what x 1 plus z 1 which is nothing, but your first component; that is x here. So, it means that here you can say that this x y z will belongs to range space of T, if your x plus y is equal to x plus y equal to z. So, here with the help of this you can define another subspace W which is x y z this belongs to R 3; such that it satisfy the relation x plus y equal to z here.

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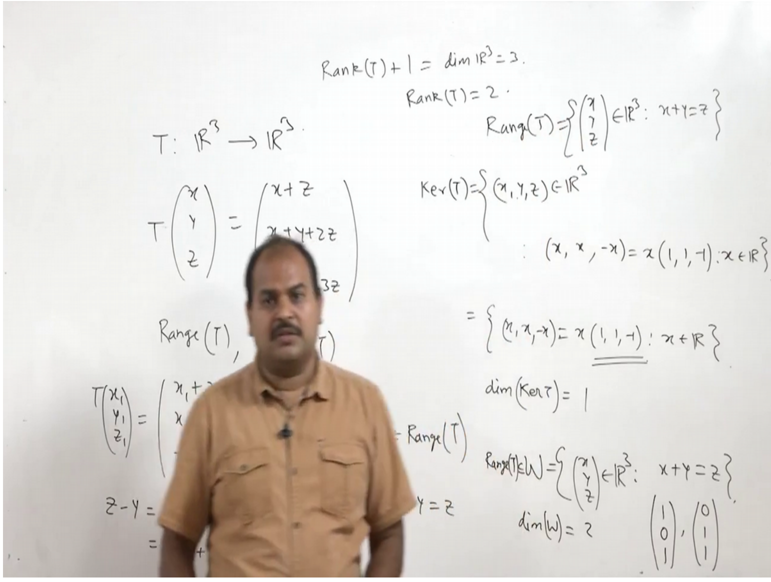

So, we can say that your x y z will belongs to range space of T, it means that it is a subset of W. So, we can say that range of T will be a subset of this W. Now if we look at, what is the dimension of this W.? So, dimension of W will be what dimension of W will be the number of element in this basis of this subspace. Now basis of this subspace will be what? My claim is that it is 2.

So, we want to create a basis for this vector subspace. So, basis of this vector space will be what? You take 1 and you take 0 here and this will be 1. So, 1 0 1 is 1 basis element for this. We need to find out element of R 3. We satisfy this equation and all linearly independent. So, let us take 0 1 and 1. So, here if you look at this also satisfy this relation. So, we can say that w is span by this element. Now it is up to, I am giving you this exercise that you just prove that these two element will generate this vector space W.

So, dimension of W is basically 2 and already know that rank of T is equal to 2, means dimension of range of T is also 2. So, it means that range of T is a subset of W, and dimension of range T and dimension of W is same. So, it means that range of T has to be equal to W. So, this implies that range of T is given by x y z belongs to R 3; such that x plus y equal to z here right.

So, here range of T is given by all those x y z, which satisfy this relation; that is x plus y equal to z, and kernel of T is basically what all those x y z, which can be written as x comma x comma minus of x, where x is coming from R. So, kernel T is given by this range of T is given by this. In this lecture we have discussed the matrix representation of a linear map and rank nullity theorem and some properties of matrix presentation and rank nullity theorem. So, we will continue our study in next lecture. So, here we stop.

Thank you for listening us thank you.