Numerical Linear Algebra
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Lecture - 01
Matrix Operations and Types of Matrices

Hello friends, I welcome you to my lecture on matrix operations, and types of matrices. So, this is; in this lecture, we are going to review the matrix operations and the various types of matrices. As you know, a matrix is a rectangular area of numbers, which are arranged in the form of rows and columns.

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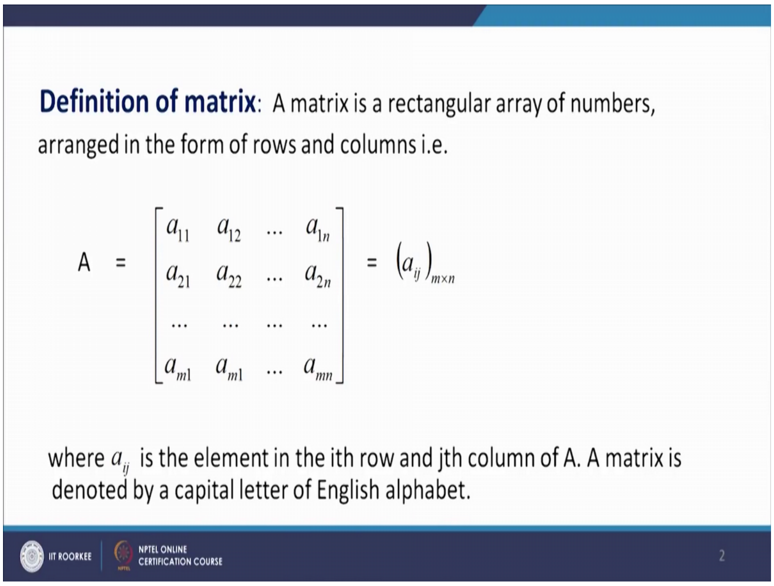

That is A is equal to a i j m by n, where there are m rows and n columns, and a i j is defined as the element which occurs in the ith row and jth column of the matrix A.

A matrix is always denoted by a capital letter of English alphabet. Now matrix occur naturally in many contexts.

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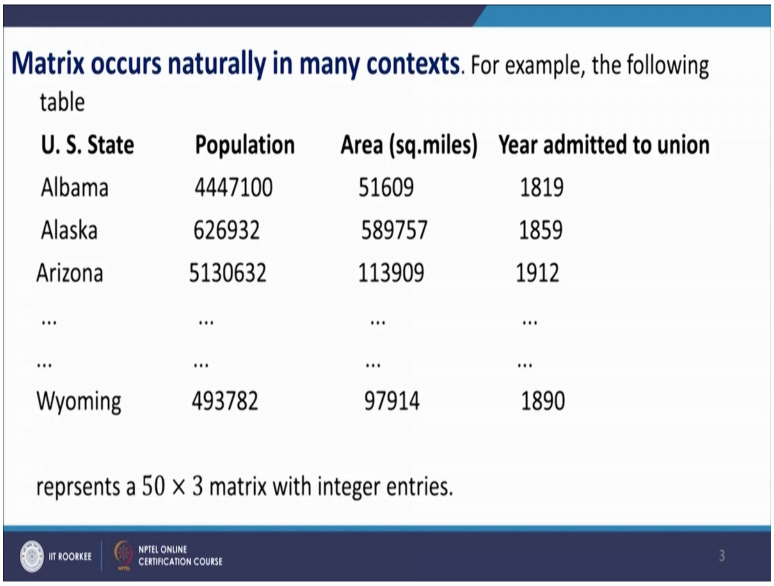

For example, let us look at this following table where we are listed the 50 US states; Alabama, Alaska, Arizona, Wyoming and so on and their populations are given of the area in square, miles is given, and the year; the year; year in which they were admitted to the Union of United States of America. They are also given. So, this is a 3 by 3; this is a matrix, where there are 50 rows and 3 columns.

So, one first column is that of population, second column is that of area in square miles, and third column is the year in which the state was admitted to the United States of America. So, like Alabama was admitted in the year 1819, Alaska was admitted in the year 1859, Arizona was admitted in the year 1912 and fine Wyoming was admitted in the year 1890. So, you can see that this information can be compiled in the form of a matrix which of size 50 by 3. There are 50 rows and 3 columns and the entries of the matrix are all integer.

Now, let us go to the next example, let us consider an electrical network having branches; this electrical network.

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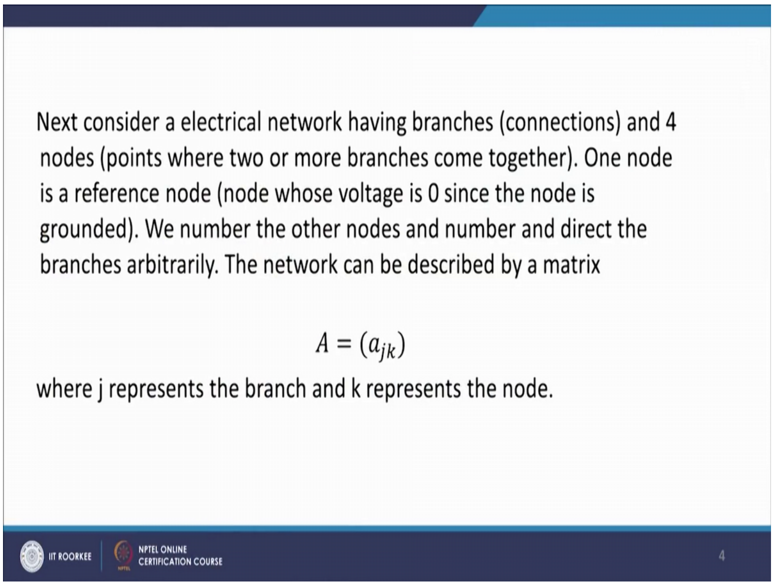

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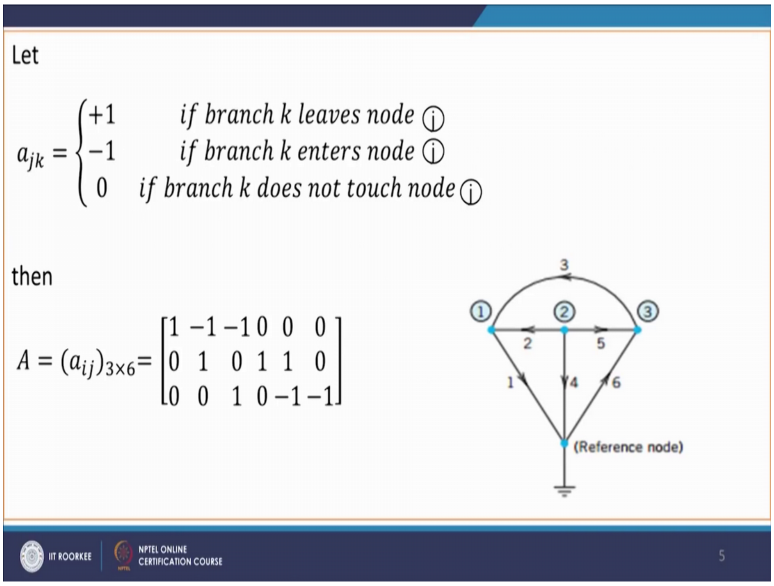

So, in this, you can see there are branches, and branches means their connections and 4 nodes, there is one reference node because it is being earthed, and the other 3 nodes are node number 1, 2 and 3. And so, there are 4 nodes that is the points where 2 or more branches come together, one node is a reference node because that node has voltage 0, since the node is grounded and the number of the other nodes the other nodes are numbered, and the branches are numbered and their directions are taken arbitrarily.

So, we have taken this network, and then this network can be described by a matrix; A equal to a j k where we shall see that, j represents the branch, and k represents the node. So, let us look at this figure, we are given; we have taken the values of a j k as 1, if branch k leaves node j and a j k equal to minus 1, if branch k enters node j, and a j k equal to 0 if branch k does not touch node j.

So, let us see how we get this matrix. So, we have this matrix; this figure.

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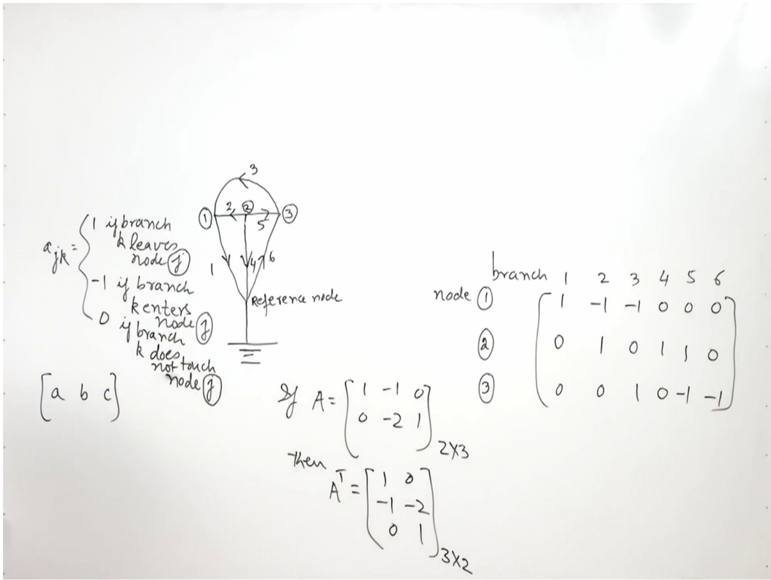

We have this figure we have taken directions like this, this is reference node, and then the directions are taken as, this direction we have for the branch 1, and then we have for the branch 4, we have this direction, and for the branch 6, we have this direction, and then we have taken the nodes like this is node 1, this is node 2 and this is node 3, and the other branches are 2, 5 and 3. Now let us write the matrix. So, we have a j k equal to; a j k; we have taken as 1 plus 1, if branch k leaves node j, and minus 1, if branch k enters node j, and 0 if branch k does not touch node j.

So, let us take the branches here, branch and column wise and then we take nodes row wise, there are 3 nodes and there are 6 branches. So, let us see for the branch 1, the this branch is the branch 1, it you can see the branch 1 is leaving the node 1, and when a branch leaves the node 1, node if a branch k leaves the node j we take it is value as 1. So, when branch 1 leaves the node 1 a 1 1 is equal to 1.

And then let us see this in node 2 the node 2 is here, branch we are the branches the first column branch 1 is this 1 this is branch 1, and this is node 2. So, branch 1 does not touch node 2 and therefore, it is value is 0, and then branch 1 let us see regarding the node 3, node 3 is here and this is branch 1. So, branch 1 does not touch node 3. So, again value is 0.

Now, let us look at branch 2, branch 2 is this one it is entering the load one. So, when a branch enters the load value is minus 1, now branch 2 let us see what happens whether branch 2, leaves the node 2 or enters the load 2. So, branch 2 is leaving the node 2 means it is value will be 1.

Now, branch 2 let us see whether it leaves the node 3, or enters the node 3, or touches the node 3. So, branch 2 is here it does not touch 3. So, we have the value 0, and then us look at branch 3, branch 3 enters the node 1 and therefore, it is value is minus 1.

Branch 3 does not touch node 2. So, it is value is 0, branch 3 leaves the node 3 and therefore it is value is 1, branch 4 does not touch this node 1. So, value is 0, branch 4 leaves the node 2 and therefore, it is value is 1, branch 4 does not touch node 3. So, value is 0.

Now, let us look at branch 5 does not touch node 1 so, value is 0, branch 5 leaves the node 2 and therefore, it is value is 1, and branch 5 enters the node 3 and therefore, it is value will be minus 1. Let us now branch 6 look at branch 6. So, branch 6 does not touch node 1. So, value is 0.

Branch 6 does not touch node 2. So, value is 0, and branch 6 enters node 3 therefore, value will be minus 1. So, we can see this is called as the matrix this matrix we have. So, for this network we have this matrix. Now so, my so, this is an engineering application of matrices.

Now, let us and consider what we mean by the dimension of a matrix, the number of rows and columns of a matrix is called the size or the dimension of a matrix.

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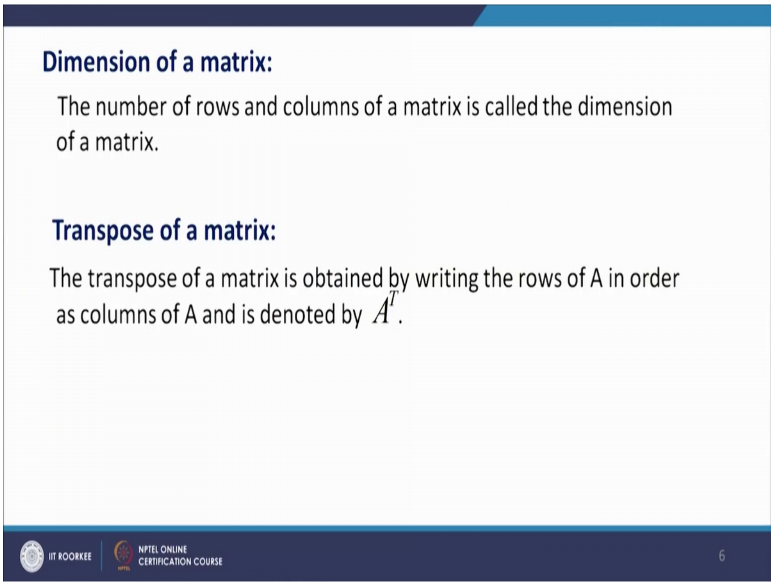

Suppose, we have m rows and n columns in a matrix, then we say that the matrix is of dimension m by n or it is of size m by n. The transpose of a matrix you know we got the transpose of a matrix is uptend by writing the rows of the matrix A in order as columns of A and this denoted by A transpose.

So, if A is equal to I take see for example, 1 minus 1 0 0 minus 2 1, then A transpose, we denote y per suffix t as. So, first row will be written as first column, second row will be written as second column, now you can see here there are 2 rows and 3 columns. So, it is 2 by 3 matrix, when we take it is transpose what we get is 3 rows and 2 columns. So, it becomes 3 by 2 matrix. So, if A is of size m by n A transpose will be of size n by m.

Now, let us discuss various types of matrices that we come across in the literature, first we discuss a row matrix.

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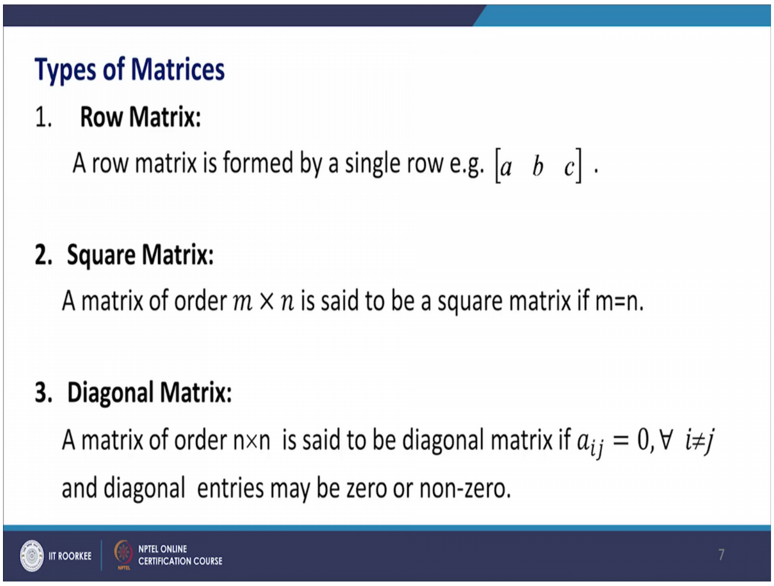

A row matrix is a matrix which is which have which has only one row say for example, the matrix 1 by 3, where you can see there are 3 columns, in a b c and 1 row. So, matrix having only 1 row is called a row matrix.

Similarly, we can define a column matrix, a matrix where we have only 1 column, we call that as a column matrix, now square matrix is 1 where the number of rows and columns are same. So, a matrix of order m by n is said to be a square matrix, if m is equal to n.

Now, let us define a diagonal matrix, a matrix which is of order n by n, that is it must be a a square matrix. So, a square matrix must be is called a diagonal matrix, if a i j is equal to 0 for all i not equal to j.

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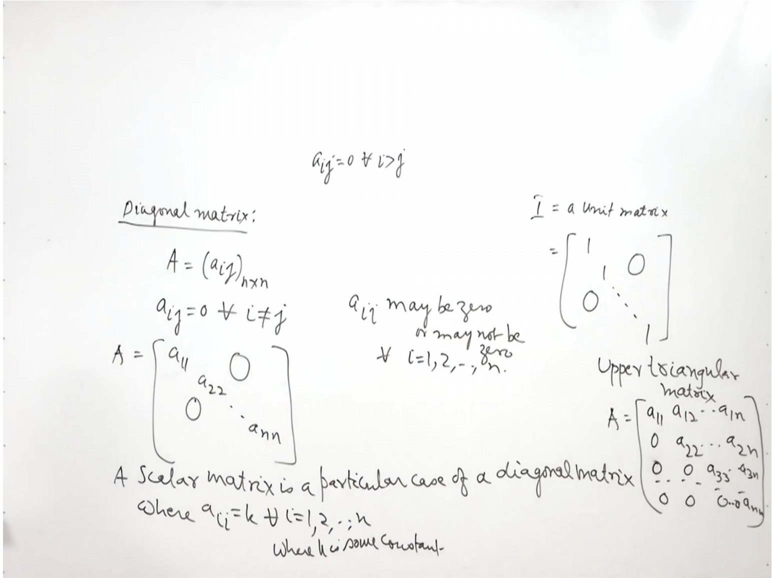

So, a diagonal matrix is a square matrix, say we write it as A equal to a i j n by n, where a i j is equal to 0, for all i not equal to j.

That means A must be of the form, all entries of the matrix above and below the main diagonal are 0s, the main diagonal of the matrix is the diagonal consisting of entries a 1 a 2 and so on a n n, that is a i i where i is equal to 1 2 n, or you can say a i j where i is equal to j. So, the diagonal entries of the matrix may be 0 or non-0, a i js a i i here is equal to 0, or may be 0 or may not be 0 for all i equal to 1 2 n.

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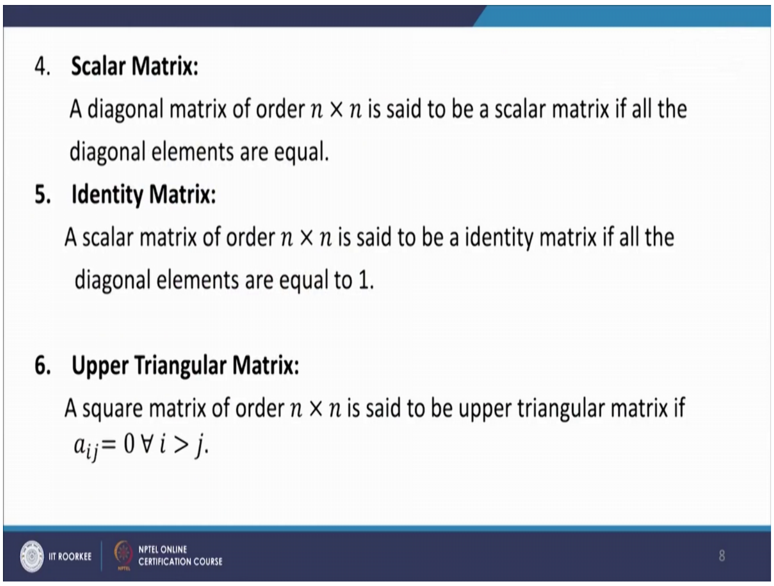

Now, let us go to next one a scalar matrix, a diagonal matrix will be called a scalar matrix if all the diagonal elements are equal. So, here this diagonal matrix, scalar matrix is a particular case of diagonal matrix. A diagonal a scalar matrix is a particular case of a diagonal matrix, where a i i is equal to k some constant k for all i, where k is some constant, now a an identity matrix is a particular case of a scalar matrix, where all the diagonal entries are equal to 1.

So, if k is equal to 1 then the scalar matrix becomes a unit matrix. So, unit matrix be denote by i. So, this will have 1 on the main diagonal and 0 for all the non-diagonal entries, then we have upper triangular matrix, a square matrix of order m by n by n, is called an upper triangular matrix, if all the entries of the matrix below the main diagonal are 0s, that is we can say that a i j is equal to 0. So, in upper triangular matrix will look like this? So, we can see that all the entries below the main diagonal are 0s.

Mathematically we can write that as a i j equal to 0, for all i less than for all i greater than j.

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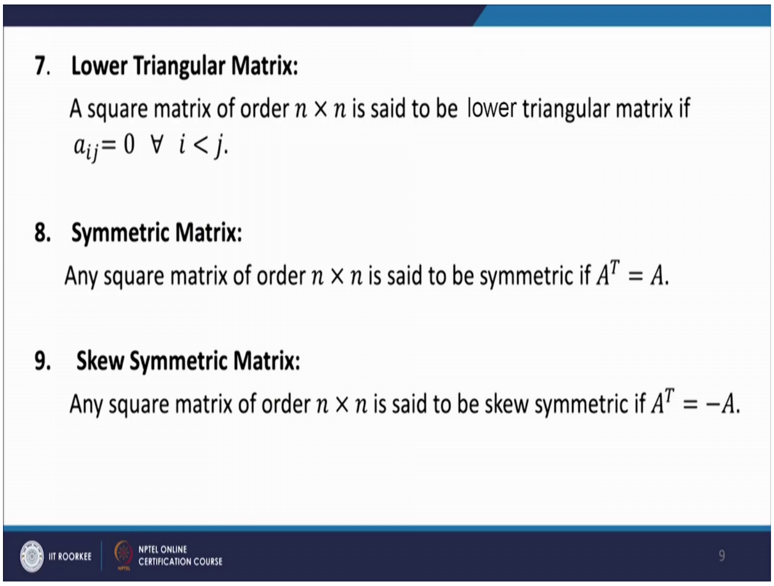

Now let us go to a lower triangular matrix, a lower triangular matrix is similarly defined, a lower triangular matrix is 1 where all the elements of the matrix or entries of the matrix above the main diagonal are 0s.

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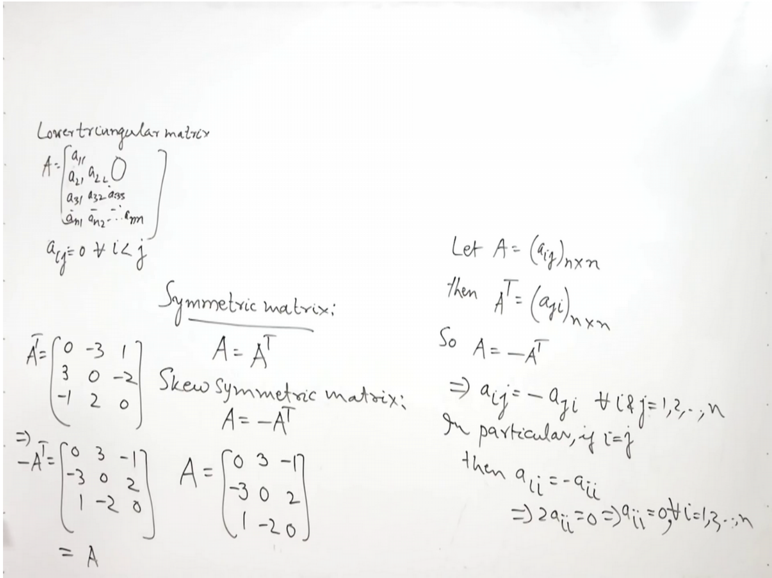

So, an upper lower triangular matrix will look like as, all entries here are 0s, and here you have you may or may not have nonzero entries. So, a 2 1, then you have a 3 1, a 3 2,a 3 3, and here you have a n 1, a n 2 and so on a n n, all the entries above the main diagonal are 0s.

So, here we write a i j equal to 0 for all i less than j. Now let us define a symmetric matrix, a square matrix is called symmetric, if it equals it is transpose. So, a symmetric matrix is a square matrix for which satisfies, the equation a equal to a transpose. Now let us define a skew symmetric matrix, a square matrix is called skew symmetric matrix, if A is equal to minus A transpose, now this skew symmetric matrix has got a very interesting property, you can see here that let us see A B equal to a i j n by n.

Suppose A is a square matrix of order n, then a transpose will be equal to a j i, we write the rows of A in order as columns. So, A equal to minus A transpose, implies that the general element a i j of A will be equal to minus A transpose means a j i, we are multiplying minus A transpose means we are multiplying A transpose by minus 1, and when you multiply a matrix by a scalar all the rows all the entries of the matrix are multiplied by the scalar.

So, minus A transpose means we will get minus a j i, now this valid for all i and j, which vary from 1 to n by the defination. Now in particular if i take here i equal to j, which is true for all the diagonal elements in the case of all diagonal elements i and j are both same. So, what we will get, we get 2 a i i is equal to 0, r a i i is equal to 0 for all i and j, for all I, this means that in the case of skew symmetric matrix all the diagonal elements are taken as 0s. So, if you want to write an example of a skew symmetric matrix, the number to take are diagonal elements as 0s.

Now, suppose I take it as 3 then I will write here minus 3, because I take transpose of this and then I multiply by minus 1, here if I take minus 1 here, I will take 1 here, if I take 2 here, I will take minus 2. Now you can see if I take transpose of A if I take transpose of A then A transpose will be equal to write the rows of A in order as columns. So, 0 0 3, minus 1 then minus 3 0 2, and then we have 1 minus 2 0. Now multiply by minus 1. So, this implies minus A transpose equal to 0 minus 3 1 3 0 minus 2 minus 1 2 0.

Which is same as the matrix A, skew symmetric matrix has a very interesting property that it is all diagonal entries are 0s.

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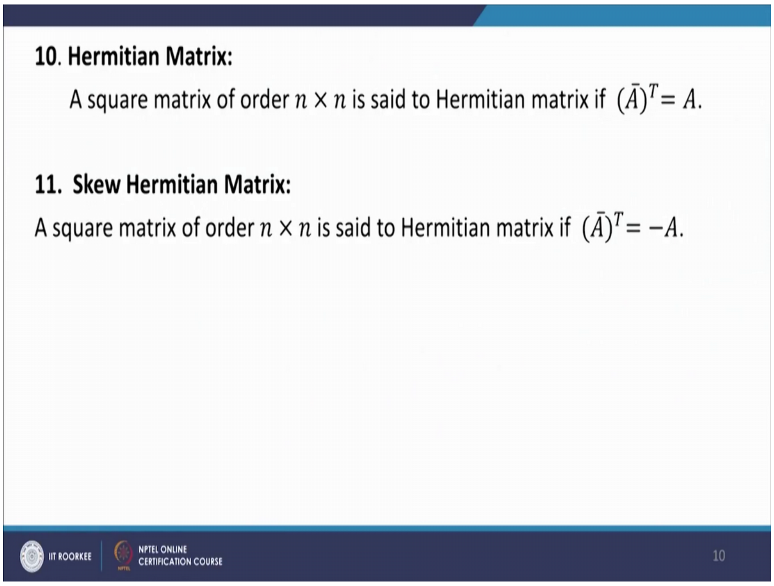

Now, let us go to next one Hermitian matrix.

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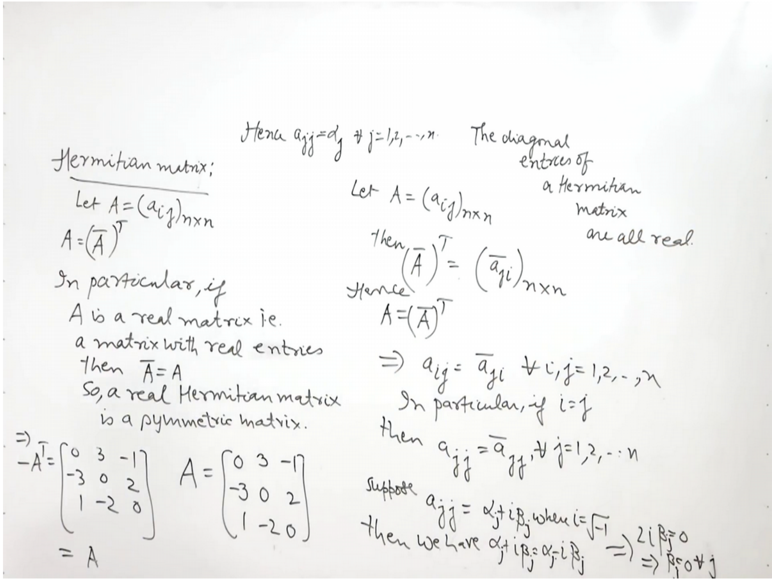

So, suppose the entries of the matrix A maybe complex, if the entries of the matrix are all real we call it a real matrix, if the entries of A are maybe complex then we call it as a complex matrix. So, suppose A is equal to a i j n by n, it is a square matrix of order n, then A is called Hermitian if a is equal to A conjugate transpose.

Conjugate of A means, we take the conjugate complex conjugate of all the entries of the matrix A. So, A is equal to A conjugate transpose if it is satisfied the matrix is called Hermitian. Now from here we can so, see that in particular if A is a real matrix then A equals A transpose. So, a real Hermitian matrix is always a symmetric matrix. So, in particular, if A is a real matrix that is to say a matrix with the real entries, then A conjugate is equal to A then if A is a real matrix then A conjugate is equal to A. So, if. So, real Hermitian matrix is a symmetric matrix.

If the matrix is Hermitian, it is entries are real then A will be equal to A transpose. So, it will be a symmetric matrix, now the Hermitian matrix also has a very important very interesting property, let us look at the definition of Hermitian matrix, let A B equal to a i j n by n, then A conjugate transpose can be written as a j i conjugate, we take the conjugate of the elements of A, and then take transpose. So, a j i conjugate.

So, A equal to A conjugate transpose, implies that a i j is equal to a j i conjugate, for all i j, in particular if i is equal to j, then we have a j j is equal to a j j conjugate, for all j from 1 to n, now suppose a j j is equal to some complex number, say alpha plus i beta, where i is square root minus 1, then we have alpha plus is i beta equal to alpha minus is i beta. So, which will imply that 2 i beta equal to 0 or beta equal to 0, which means that a j j is equal to alpha, for all j a j j is equal to alpha means a no I think I should write it as alpha j plus i beta j to. So, it is dependence on j. So, let us put like this. So, beta j is equal to 0 for all j.

And this means that alpha j j is equal to alpha j for all j. So, in the case of Hermitian matrix all the matrix diagonal entries must be real numbers. So, the diagonal entries of a Hermitian matrix are all real.

So, when you form an example of a Hermitian matrix, remember to take all diagonal entries as real numbers, let us take an example of a Hermitian matrix.

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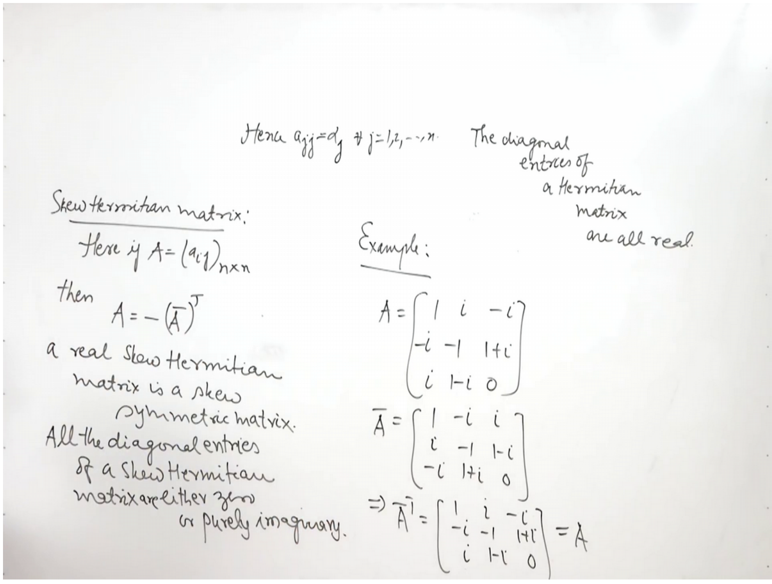

Let me take A equal to so, 1 minus 1 0 then I take here i. So, we want a conjugate transpose is equal to A. So, we have i here, if I take minus i here I take minus i here, if I take 1 plus i here, I take 1 plus i here, there you can see when you take the conjugate and wait if I take conjugate I think I should take here minus i, here I should take plus i, here I take 1 plus i, then here I should take minus i yes.

So you can see here now A conjugate is equal to 1 conjugate of minus i is plus i, conjugate of i is minus i, then conjugate of i is minus i, minus 1 1 plus i, and here we have i, here we have 1 minus i, here we have 0, and which means that A conjugate transpose is equal to 1 minus i i then you have i minus 1, 1 minus i, and then you have minus, i 1 plus i 0.

Which is equal to A. So, A is a Hermitian matrix, now we go to skew Hermitian matrix, in the case of a skew Hermitian matrix, we have A equal to minus A conjugate transpose. So, skew Hermitian matrix here if A is equal to a i j n by n, then A equals minus A conjugate transpose.

So, again here if we take the entries of A to be real numbers, then the skew Hermitian matrix becomes a skew symmetric matrix. So, a real skew Hermitian matrix is a skew symmetric matrix, further we note that all the diagonal entries of the skew Hermitian matrix are either 0 or purely imaginary, all the are either 0 or purely imaginary.

So, this is also very easy to prove like we proved in the case of a Hermitian matrix, that all it is diagonal entries are real.

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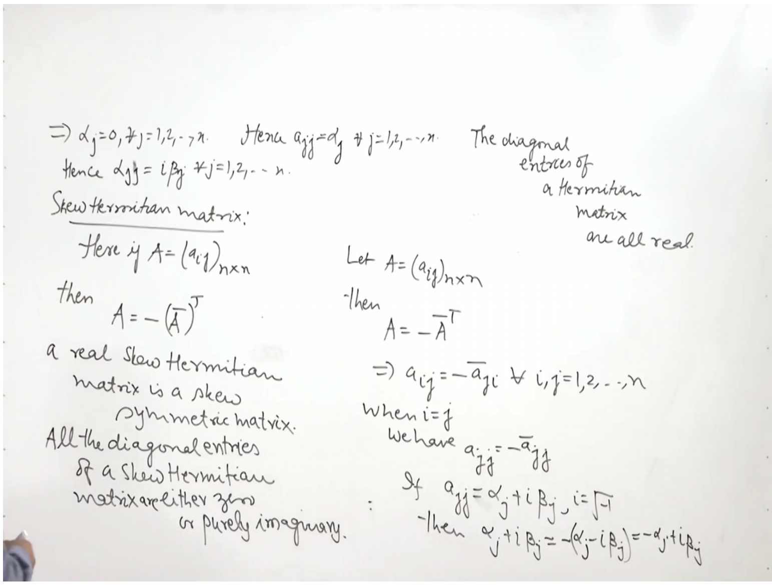

So, in order to prove this statement, let us see let A equal to a i j n by n, then A equal to minus A conjugate transpose, implies that a i j is equal to minus a j i conjugate. Now so, if when i is equal to j what happens when i is equal to j, we have a j j is equal to minus a j j conjugate. So, if a j j j is equal to alpha j plus i beta j, where i is equal to square root minus 1, then then what we will get alpha j plus i beta j is equals minus alpha j minus i beta j, because we have to take the conjugate of a j j which is alpha j minus i beta j.

So, what will happen this is minus alpha j plus i beta j, which will mean that 2 alpha j equal to 0 that is alpha j equal to 0 for all j. And so, a j j becomes i times beta j, alpha j j is equal to i beta j. So, alpha j j is either 0 or it is purely imaginary for all js. Now let us go about to addition matrices, when we have 2 matrices A and B of the same size.

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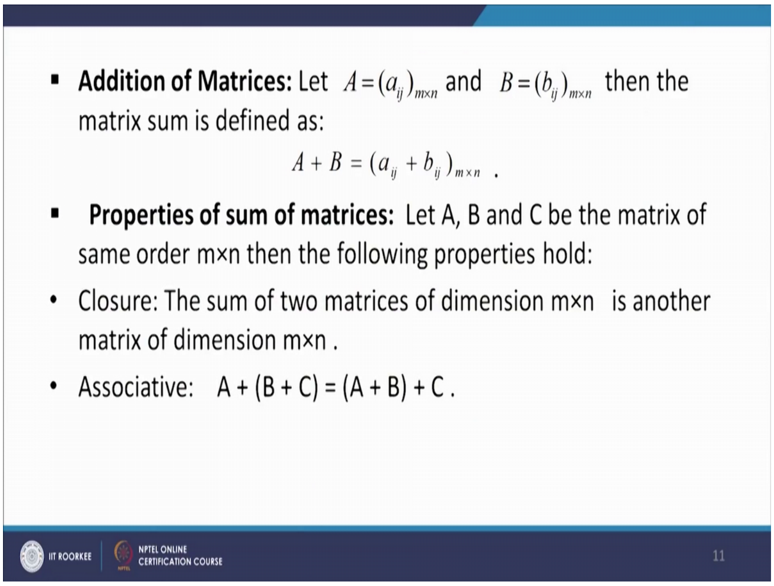

Suppose, A is of size m by n and B is of size m by n, then their matrix sum that is A plus B is defined as the component wise addition that is a i j plus b i j m by n. So, we add the corresponding elements of the 2 matrices to obtain A plus B. Now there are properties of sum of matrices let A B and C B matrix matrices of the same order, then the following properties hold. First thing is that the closure the sum of 2 matrices of the same size, in another matrix of the same size if the matrices are of the size m by n, then the sum of the 2 matrices will also be a matrix of the size m by n, now the then the property of associativity if A B C are of size m by n, then A plus B plus C is same as A plus B plus C whether you add B and C first and then add A or you add A and B first and then add C what you will get is the same matrix.

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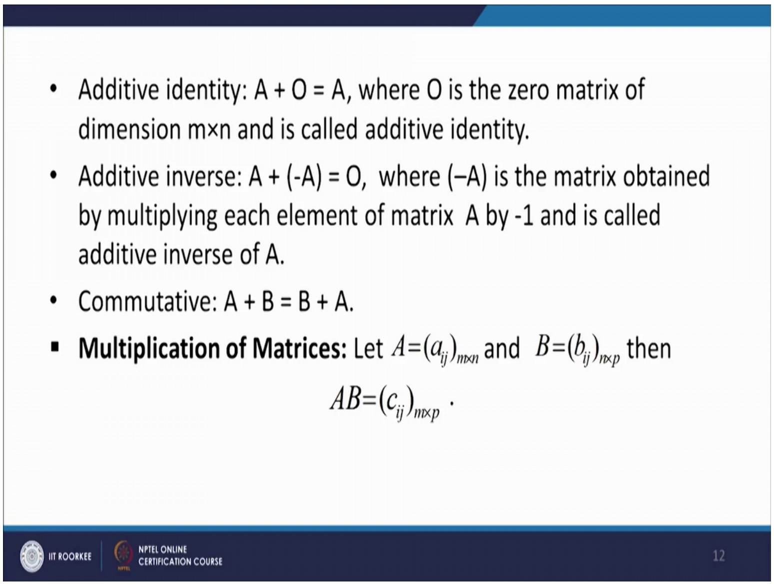

So, that is associativity, then we have additive identity if you add 0 matrix, here 0 represents the 0 matrix, 0 matrix means the matrix where all the entries are 0s. So, if A is m by n matrix and 0 is 0 0 matrix of order m by n size m by n then a plus 0 will be equal to A.

So, A plus 0 will be equal to a where 0, 0 is the 0 matrix of size m by n additive inverse if A is matrix of size m by n, then minus A is the matrix where the entries of a r multiplied by minus 1, and when you add a and minus A matrice what we get is the 0 matrix.

So, minus a is called the additive inverse of the matrix A, then we have commutativity of addition, if we add A and B then we get the same matrix as we get by adding B to A. So, A plus B is equal to B plus A now multiplication of matrices for the multiplication of matrix A, A and B suppose we want A B. So, then the number of columns of A must be equal to number of rows of B.

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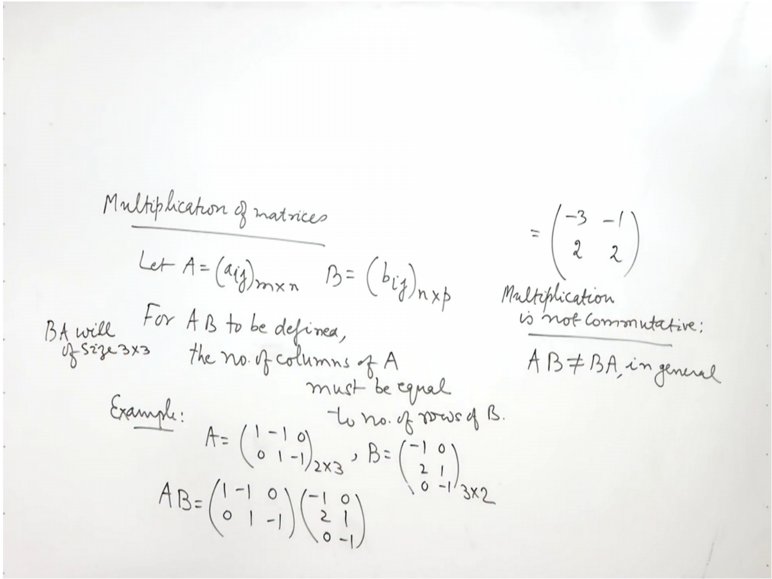

So, multiplication of matrices. So, let us see let A B equal to a i j m by n and B be equal to b i j n by p, now you can see here where number of columns of A is n, and number of columns of B is number of rows of B is n.

So, for A B to be defined, the number of columns of B must be equal to number of rows of B, number of columns of A must be equal to number of rows of B for example, suppose I take A equal to A to be equal to see 1 minus 1 0, and 0 1 minus 1, which is a 2 by 3 matrix then, I can take B as see a matrix with 3 rows. So, 3 rows means minus 1 2 0 and 0 1 minus 1, suppose I take like this.

So, we get we have A is as a matrix of size 2 by 3, B is a matrix of 3 by 2 then A B will be a matrix of size 2 by 2, that is it will be a square matrix. So, I can we can multiply it easily 1 minus 1 0, 0 1 minus 1, and then we get minus 1 0 2 1 0 minus 1. So, when you get a when you want to get A B, but you do is the first column of B is multiplied to all the rows of A, we will get the first column of A B. So, this will be equal to suppose we multiply first column of B to the first row of A here, what we get minus 1 into minus 1 minus 1, 2 into minus 1 minus 2, and then 0.

So, we get minus 3, and then minus 1 into 0 2 into 1 is 2 and then here we get 0. So, we get 2 here and then we. So, we have now taken all the rows of A now we take the second column of B. So, 0 1 minus 1 we multiply to first row of A here. So, 1 into 0 is 0, 1 into minus 1 is minus 1, minus 1 into 0 is 0. So, we get minus 1, an then we multiply the second column to the second row. So, 0 into 0, 1 into 1 1, and then minus 1 minus 1 is 1. So, 1 1 is 2. So, this matrix we get now you can see here if you want to find B A here.

If you want to find B A here then we you should have number of columns of B, should be equal to number of rows of A. So, that B A is also possible and B A here will be a matrix of size 3 by 3. So, B A will is will be of size 3 by 3 in this example.

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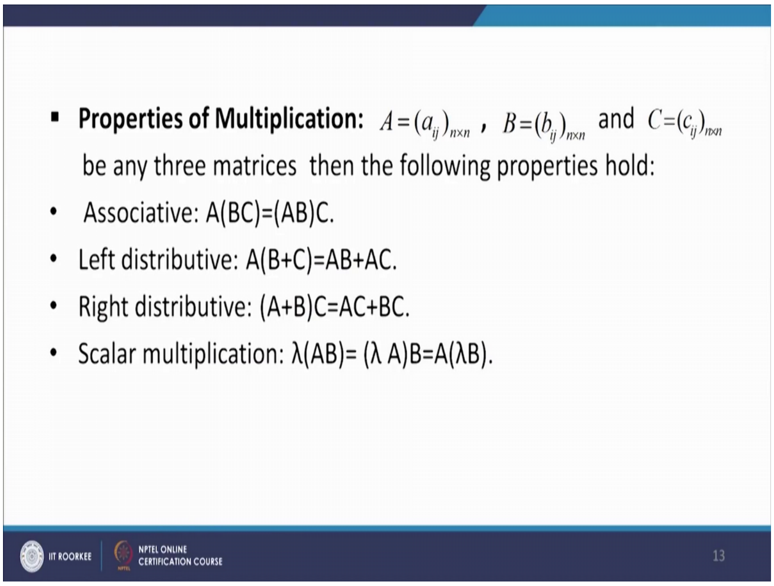

Now there are properties of multiplication the first is associativity. So, let us see A B C are all of size of m n by n, then A into B C is equal to A B into C, left distributive A times B plus C is equal to A B plus A C, then A plus B into C equal to A C plus B C, and then scalar multiplication lambda is real or complex number lambda is a scalar number, then lambda times A B is equal to lambda A into B, which is also equal to a times lambda b.

Now, then we go about and then let us further note here, that multiplication of matrices is not commutative multiplication is not commutative, we have seen that addition is commutative, addition of matrices, but multiplication is not commutative. So, A B is not equal to B A in general, if the sizes of A B such that A B, and B A both are possible then we can form examples where A B is not equal to B A.

Now, properties of transpose if r is a scalar, A and B represents matrices then A transpose, transpose if you take the transpose of the transpose of A, you will get back the matrix A, A plus B transpose is equal to A transpose plus B transpose, and when you take the transpose of A, A B matrix then you get B transpose into A transpose and then r A transpose equal to r times A transpose.

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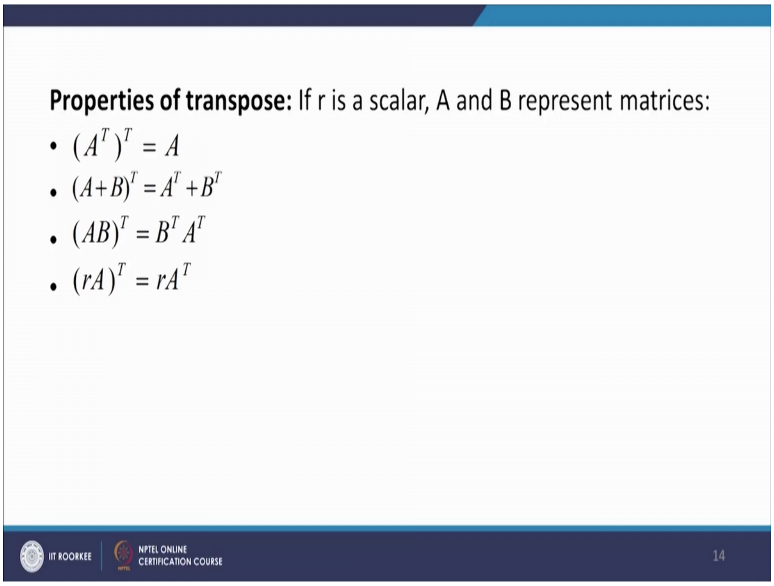

So, these things we have then we have adjoint of a square matrix.

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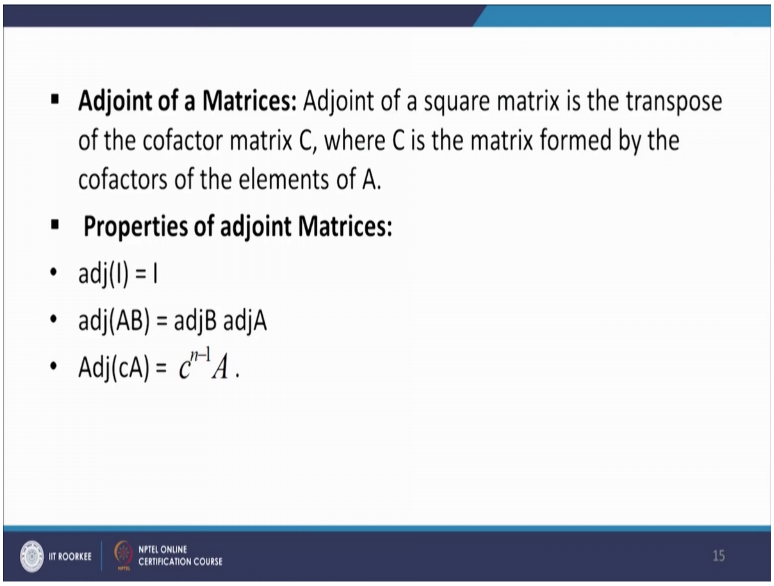

Adjoint of a square matrix is the transpose of the co factor matrix, let us see if we denote the co factor matrix by C, then C is the co-factor matrix C is found by the co factors of the elements of A.

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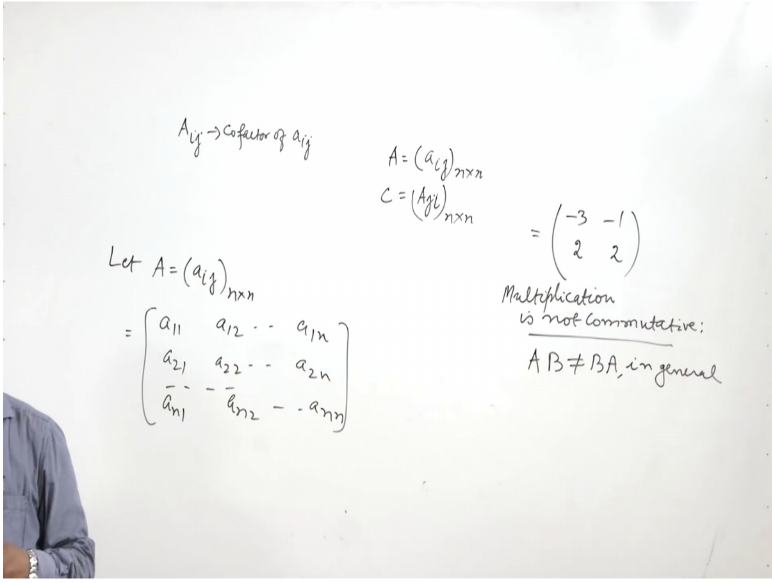

So, suppose you take A equal to a i j n by n, then the co factor matrix of C is equal to transpose of the matrix of co factors of A.

So, if I denote a i j to be the co factor of a i j element, then it will be a i j a j i you can write it as a j i. So, transpose of the matrix of co factors and if you want to find the co factor of a mat of an element, what we do is be following.

So, let us say A B equal to a i j n by n, I can write it in the (Refer Time 45:54) from of this matrix. So, then if I want the co factor of any elements suppose I want the co factor of A 1, I omit the row in which A 1 occurs and the column in which A 1 occurs, that is first column first column first row because A 1 element occurs in the first column first row.

So, I omit that and then the take the determinant of the sub matrix that we get by omiting the first row first column. So, the determinant of that multiplied by minus 1 to the power i minus 1 to the power 1 plus 1.

So, that we do in order to find the co factor, this we do for each element when we do the determinents, there we shall discuss this how to find the co factor. So, properties of adjoint matrices adjoint of the identity matrix i is i, adjoint of A into B is adjoint of B into adjoint of A now inverse of A matrix A, matrix A is called invertible.

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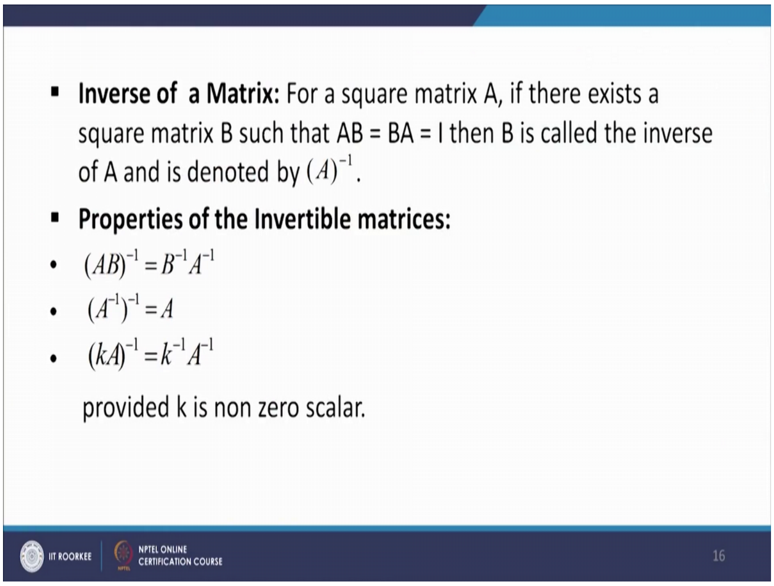

Or it is inverse adjust provided there exists a matrix B, such that A B is equal to B A equal to i.

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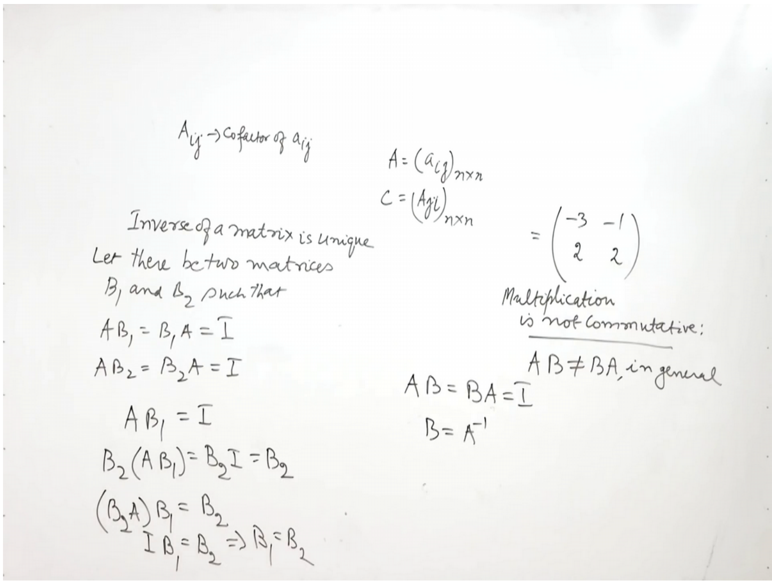

So, if the matrix B exists if a matrix B exist such that, A B is equal to b A equal to i, then B is called inverse of A, and it is denoted by A inverse.

And we can easily prove that we have written here B is called B inverse, because the inverse is always unique we can easily prove this. So, let say inverse of a matrix is unique, let there be 2 matrices B 1 and B 2 such that, A B 1 equal to B 1 A is equal to i, A B 2 is equal to B 2 A is equal to i.

So, let there be 2 matrices which are the inverses of A, then we will show that B 1 is equal to B 2. So, what we do is, let us look at this, A B 1 A B 1 is equal to i let us pre-multiply this by B 2. So, B 2 A B 1 is equal to B 2 i.

When you multiply a matrix by a identity matrix you get the same matrix. So, this is B 2 and a matrix multiplication is associative. So, I can write it as B 2 A, into B 1 equal to B 2, but B 2 A is identity matrix. So, identity matrix into B 1 is equal to B 2 which imply that B 1 equal to B 2. So, the inverse of the matrix is always unique.

Now, then we have orthogonal matrix, a square matrix of size n by n is called orthogonal matrix. If A into A transpose is equal to identity matrix.

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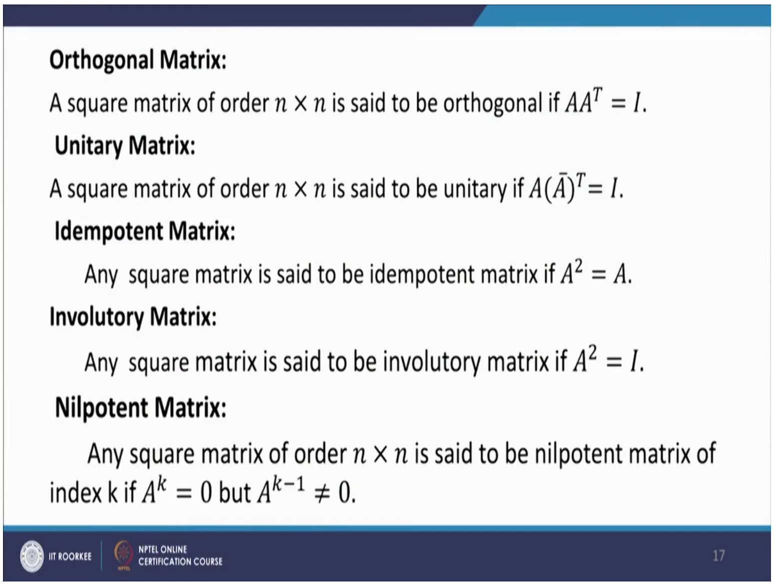

Square matrix of size n by n is called unitary, if A into A conjugate transpose is equal to identity matrix, and in idempotent matrix is 1 where we get a square equal to A, when you take multiply the matrix A by A what you get is the matrix A.

Then we come to involutory matrices a square matrix is called a involutory matrix, if a square is equal to i, and lastly, we have nilpotent matrix, nilpotent matrix is a any square matrix of size n by n is called nilpotent.

If of index k if A k equal to 0, but A k minus 1 is not equal to 0, let us take the examples of the such matrices 1 by 1 we can easily do this. So, in the case of an orthogonal matrix, we can take the following example.

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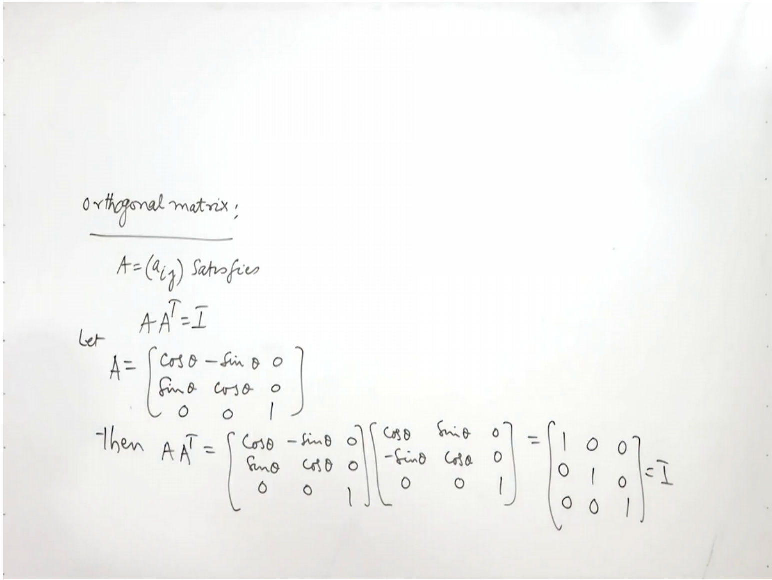

A equal to a i j satisfies a A conjugate A transpose equal to identity matrix.

So, let us take the matrix as cos theta minus sin theta 0, let A B equal to first row is cos theta minus sin theta equal 0, then sin theta cos theta sin theta 0, and the third row let us take as 0 0 1.

Then A A transpose, you can see it will be cos theta minus sin theta 0, sin theta cos theta 0 0 0 1 multiplied by it is transpose. So, cos theta minus sin theta 0, then we have sin theta cos theta 0, and then we have 0 0 1.

Now, we can multiply. So, when we multiply first column to the first row, we get cos square theta plus sin square theta plus 0 which is 1. So, this will be equal to 1 here, then first column we multiply second row sin theta cos theta minus sin theta cos theta 0. So, we get 0 then we multiply first column to the third row we get 0 into this 0 into this 0 into this so, we get 0, similarly we can multiply second column and third column to the rows of the matrix A you will see that what we get is this. So, a is an orthogonal matrix.

We can then take the example of a unitary matrix, let us sy we can take unitary ma unitary matrix let us take.

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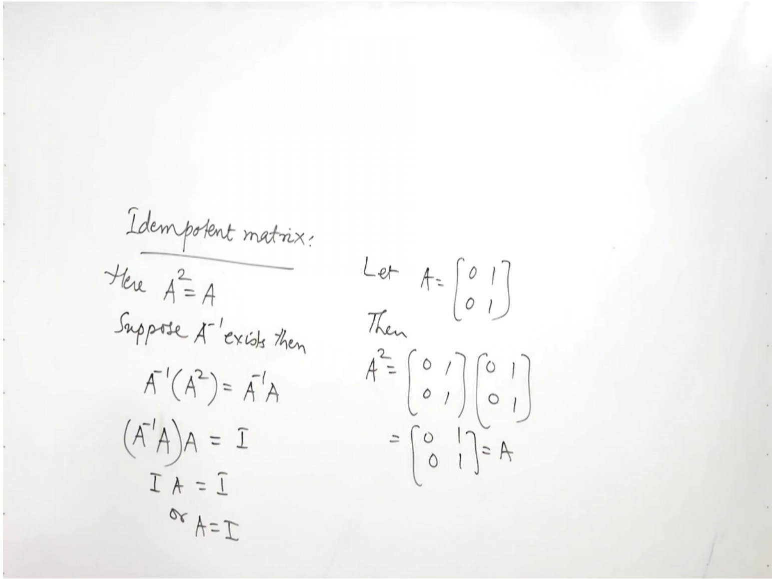

So, let us take let A B equal to we take it as 1 by 2, i by 2 root 3 by 2 and then we take it as root 3 by 2 i by 2, see if I take A equal to i by 2 root 3 by 2, root 3 by 2 i by 2 then a conjugate transpose.

So, A transpose will be retake conjugate. So, minus i by 2 and then root 3 by 2, root 3 by 2 minus i by 2. So, then if you take a then A A conjugate transpose you multiply this column to this row here. So, minus i by 2 into i by 2 will be 1 by 4, plus 3 by 4. So, you get 1 here then you multiply this column to this second row minus i root 3 by 2 4 plus i root 3 by 4. So, you get 0 similarly you get 0 1 here. So, you get i.

So, this is a unitary matrix, and now let us take example of a idempotent matrix. So, in the case of idempotent matrix, we must have. So, here we should have a square equal to A, now remember one thing here that if you take the matrix A to b in vertible then what happens, suppose you take an example of a matrix A, whose inverse adjust suppose A inverse adjust then what will happen.

You multiply by A inverse this is A inverse A I can write a square as A into A. So, A inverse A and right-hand side will be identity matrix, here A inverse A will be identity. So, what you get is. So, if you take a matrix A whose inverse adjust, then that matrix must be identity matrix.

This means that if you do not want to take an identity matrix, the write the example of idempotent matrix, you should always take a matrix which is which is not invertible, if you take it is invertible matrix it will have to be identity matrix.

So, what we do is let us take the following example suppose I take A equal to, let us see let us take a equal to 0 1 0 1 then a square. So, you multiply 0 0 first column to this here. So, this will give you 0 0, when you multiply second column to 0 here, then 0 1 1 1 is multiplied to this you get 1, 1 1 is multiplied to this you get 1. So, you get a square equal to A. So, this is a idempotent matrix and then we can take an involutory matrix. Let us say involutory matrix this means A square equal to A.

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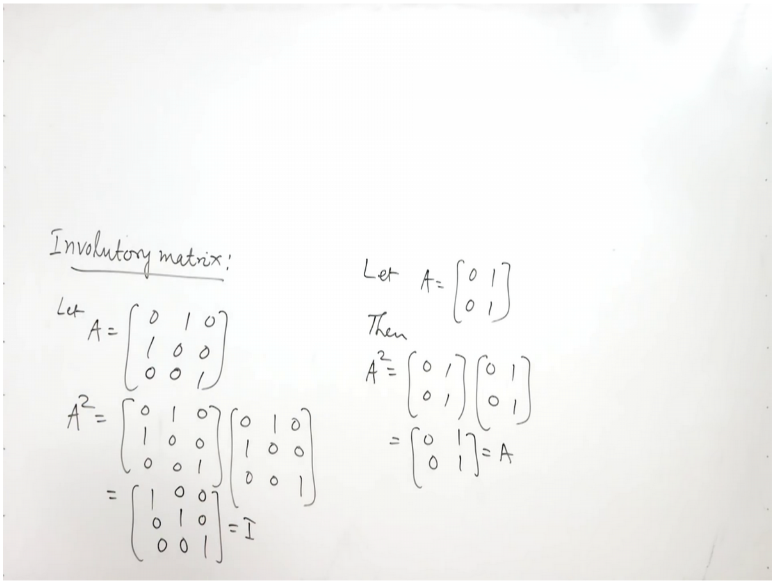

So, we can take a to b, 0 1 0 then 1 0 0, and then we take 0 0 1. So, here you can see a square if you find this. So, multiply first column to the first row, what you get is 1, what you when you multiply first column to the second row you get 0, when you multiply first column to the third row you get 0, and when you multiply second column to the first row what you get is 0, when you multiply second column to the yeah second row you get 1, similarly you get you can multiply the this thing, other you can similarly do the other multiplications you get a square is equal to i.

So, this is an involutory matrix, and lastly, we take the example of a nil potent matrix. So, let us take a nil potent matrix.

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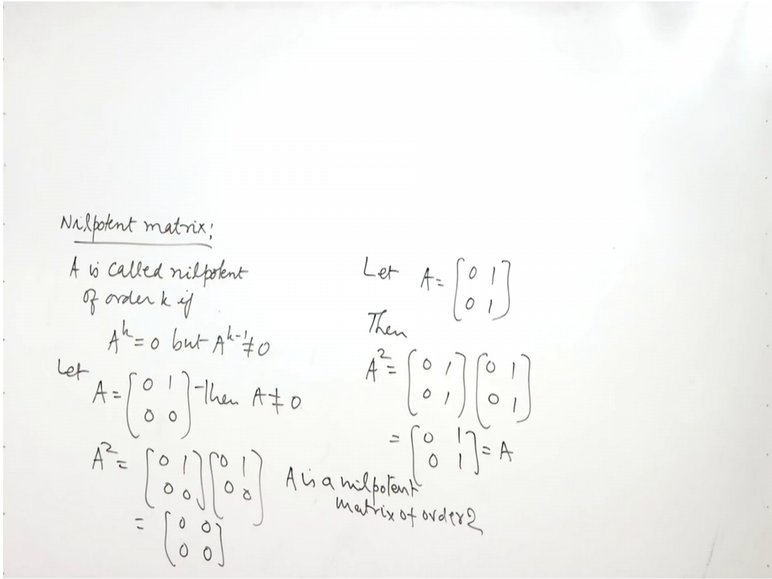

So, a nil potent matrix is a square matrix it will called of index A, if a take a equal to 0 A is called nil potent of order k, if A k equal to 0, but A k minus 1 is non-0, A k minus 1 is a non-0 matrix.

So, let us take an example let us see let, a b equal to 0 0 1 0, then it A is not equal to 0, A is not a 0 matrix, let us find A square. So, A square is equal to 0 1 0 0. So, 0 0 multiplied to second first and second row will give you 0 0, when you multiply first second column to the this 1 you get 0 this you multiply here you get 0. So, A square is a 0 matrix and therefore, A is a nil potent matrix of order 2. So, A is a nil potent matrix of order 2. So, with that I would like to conclude my lecture.Thank you very much.