Numerical Methods
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Lecture No 7
Regula Falsi and Secant Methods

So welcome to the next lecture of the 2nd unit of this course and in this particular lecture I will talk about 2 methods for solving non-linear equations the ones call Regular Falsi method, is also called method of false position and another one which is just similar to this particular method and different a very little different from this one that is called the Secant method. In the last class we have learned about bisection method for solving non-linear equation. In the bisection method we take the next iterate as the midpoint of the 2 endpoints of the initial interval or earlier interval in an iteration.

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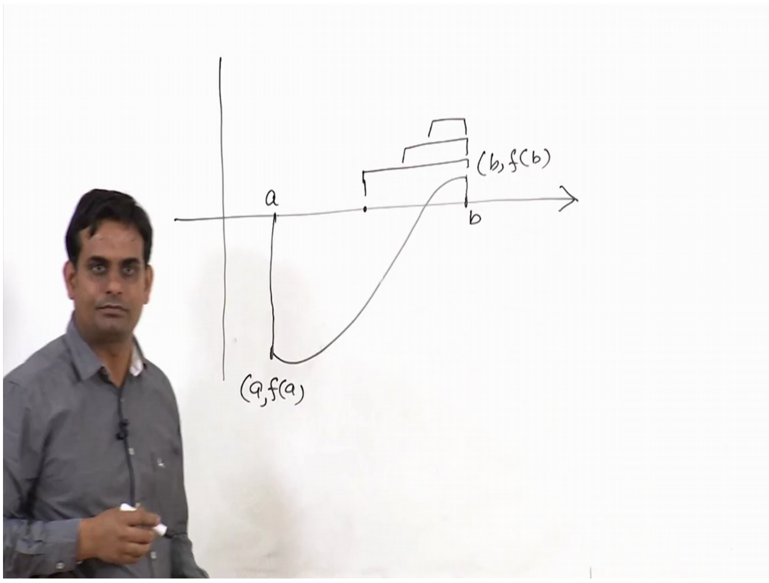

Just consider a particular scenario like we are having a function like this, so here this is my point a this is point b, so a f a, b f b. Now here we can see that the roots are close to b when compared to a, now if I use the bisection method in this particular example or in any other example where root is just close to one of the endpoint in these cases bisection method does not work or having does not work in a efficient manner, efficient means in terms of convergence. The bisection method used to take a lot of iterations to converge to this particular solution, the reason behind it since we are having we are finding the midpoint in each iteration.

So like if root is close to one of the endpoint we have to take quite large number of iterations to reach up to here because first we will take this particular interval then we will go here then again we will take this one then this one and so on. Can we have other methods where we do not use such a midpoint instead of such a midpoint as the next iterate, we use some other idea and we can have if root is close to one of the endpoint, our particular our midpoint or our next iterate comes quite closer to the root, so that we can have a better convergence. So Regular Falsi as well as Secant method use this idea where we do not use the midpoint we will use some other weighted average instead of midpoint to find out the next iterate.

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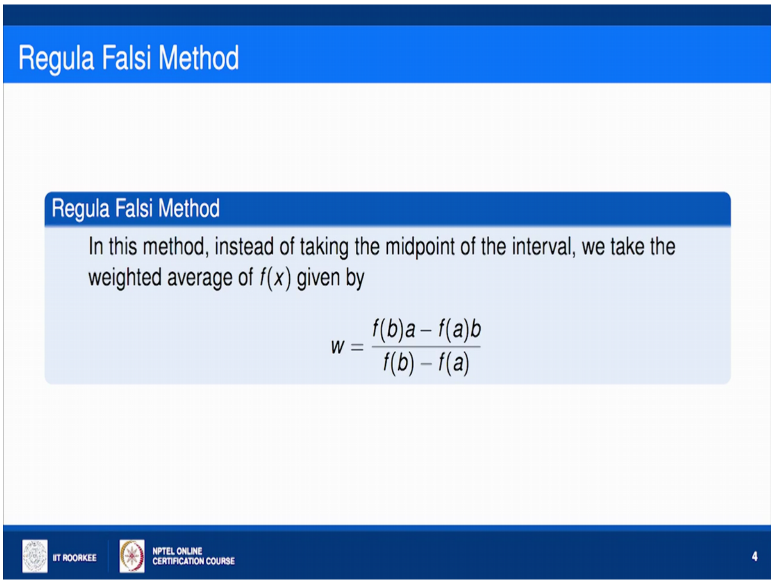

So let me introduce 1st Regular Falsi method and then I will go to the Secant method, so in Regular Falsi method instead of taking the midpoint of the interval, we take the weighted average of fx given by w equals to f of b into a minus f a into b upon f b minus f a.

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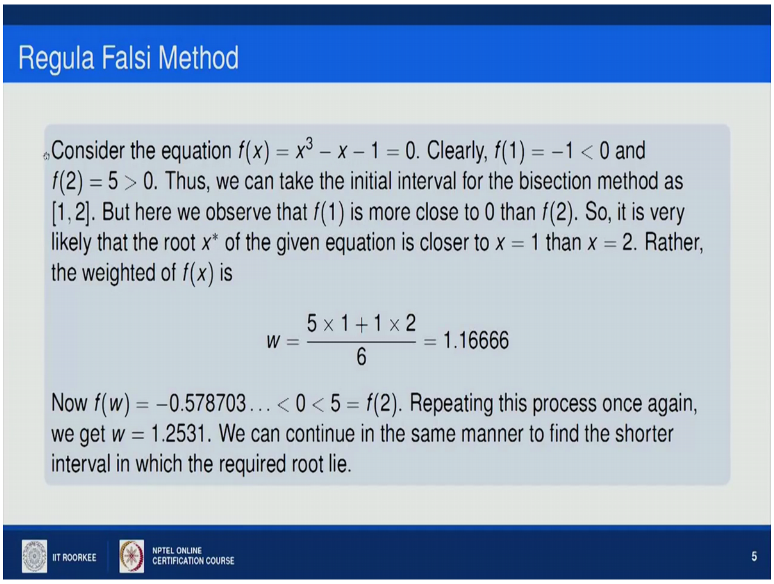

So consider the equation fx equal is to x cube minus x minus 1 equals to 0. If we check f at 1 it is coming out to be minus 1 which is a negative quantity and f2 is coming out to be positive. So hence we can say that a root will lie between 1 and 2 thus we can take the initial interval for the regular Falsi method or like in bisection method as 1, 2 but here you just observe one more thing f 1 is coming minus 1, f 2 is coming 5.

So here I can say that f 1 is quite close to 0 when compared to f 2 means the root is closer to 1 when compared to the 2, so it is very likely that the root of the given equation is closer to 1 then x equals to 2 and if we use the bisection method we will calculate our c 1 as the midpoint of 1 and 2 that is 1.5 but here in the Regular Falsi method we find the weighted of f x w as fb into a minus fa into b upon fb minus fa, so here if I take a equals to 1 and b equals to 2, so f of a will become minus 1, f of b become 5.

So w will come 1.16666. Now if I find out fw it will be minus 0.578703 which is a negative number while f 2 is positive, so it means root will be between 1.16666 and 2. Repeating this process once again I will get next if I assign it as a, w equal to a and b remain as 2. So I get the next iterate as w equals to 1.2531. We can contain the same manner to find the shorter interval in which the required root lies. Please notice that I can the bisection method here again we are reducing the length of interval by taking care that root will always lie in the shorter interval which we are choosing in each iteration.

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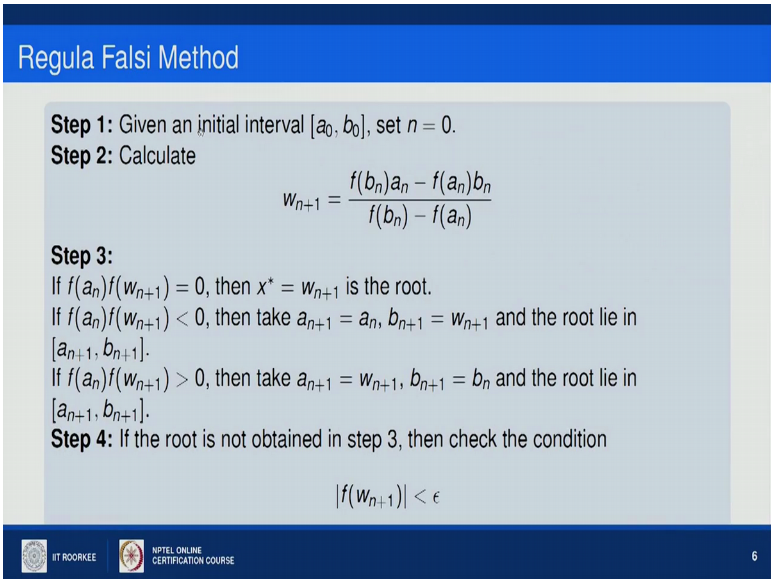

So if we talk about algorithmic way of this method given an initial interval a naught, b naught set n equals to 0. In the step 2 calculate W n plus 1, so like here if n is 0 we will have w 1, so it will become f of b 0 a 0 minus f of a 0 b 0 upon fb 0 minus fa 0. In any inert iteration it will become fb n n minus f of a into b n upon f of b n minus f of a n. Once we calculate this w, we will check in the next type as f of a n into f of w n plus 1.

If it is 0 this product, then the root will be w n plus 1 if it is negative we will assign a n to n plus 1 and w n plus 1 to b n plus 1 and our root will lie in the interval a n plus 1 to b n plus 1. Basically if you check if there is negative means root will be between a n a n w n plus 1. So we are updating interval in this way. In the step for if root is not obtained in step 3 by this condition we check the condition f of w n plus 1 absolute value of this less than given threshold that is the permissible error in our method. If it is not we will go back to set 2, if this is this particular inequality agreed then w n plus 1 will be the root like that.

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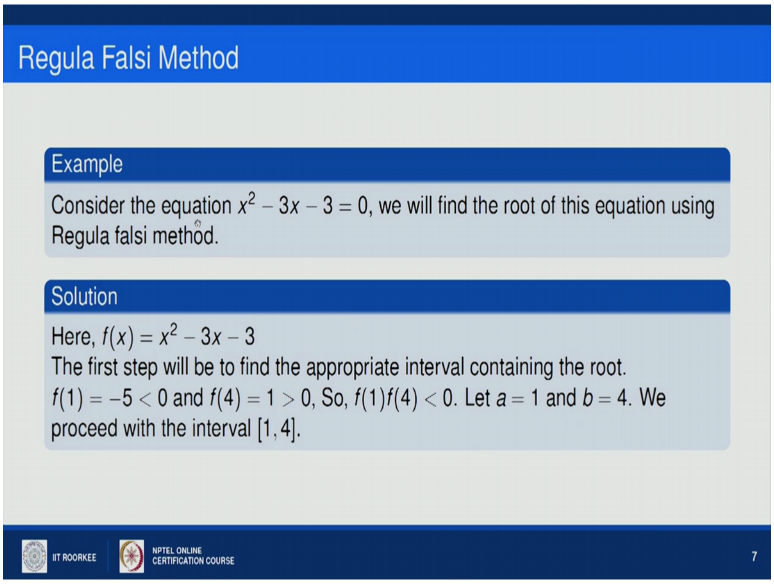

Consider the equation x square minus 3x minus 3 equals to 0, we need to find the root of the equation using Regular Falsi method.

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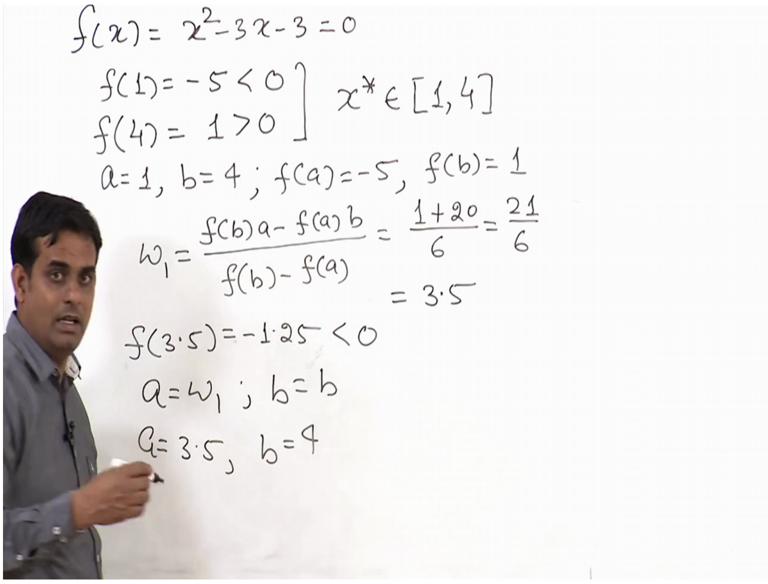

So what we will do or how this method will work let me explain here so equation is fx equals to x square minus 3x minus 3 equals to 0. So I need to find out a root of this equation, so if I check f at 1 it is coming out to be minus 5 which is negative, if I check f at 3, so 9 minus 6 at 3 it is coming exactly 0, so let me take f is 4, so at 4 it is 16 minus 15 so 1 which is positive, it means x star belongs to 1 and 4. Now take a as 1, b as 4, so f a will become minus 5, fb will be 1 calculate w 1.

So w 1 will be fb into a minus fa into b upon fb minus fa, so fb is one minus f into b will be minus 20, so it will be plus 20 upon fb minus fa. So 1 minus minus 5, so 1 plus 5 will become 6, so it is coming out 21 by 6, so 21 by 6 will be somewhere around if I take it 3.5, so now I will check f at 3.5, so if I check f at 3.5 it is minus 1.25 yeah minus 1.25, so which is negative since it is a negative number, so what I will do root will lie between w 1 and b, so I will update a as w 1, b as will remain b, so my a will become 3.5, b will become 4 and then I will repeat this particular process.

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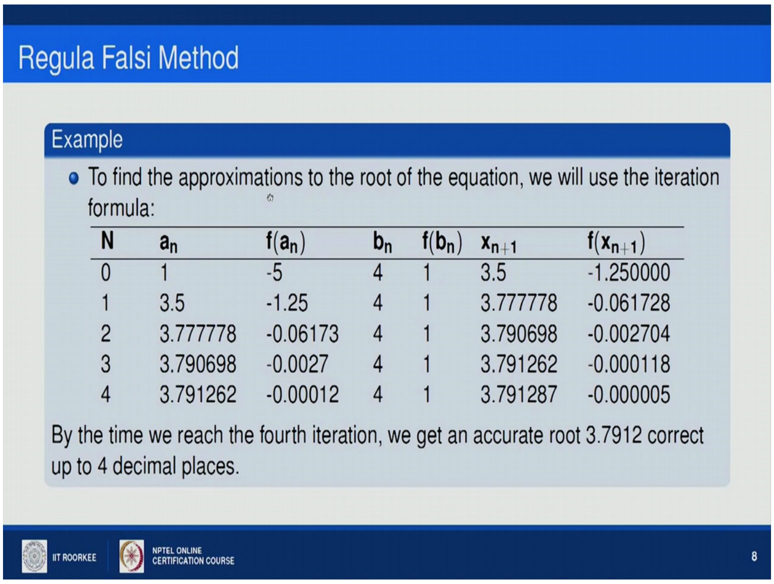

So if I repeat this process using the Regular Falsi method the next iteration will give me W as 3.777778 at this particular point my f n will be 0.06173, b n will remain as 4, fb n is 1, so it means root will lie between 3.777778 and 4 if I use this process I will get the next w as 3.790698 where fx n plus or fw n plus 1 is minus 0.002704. In the next iteration if I use this as my left point and b equals to 4 as the right point next w comes out to be 3.791262 and the value of f and this particular point is given by this particular number.

So hence since it is a negative number again I will say that 3.791262 is my left point of the interval in which root lie and the right interval is 4, so if I continue with with the next iteration I get 3.791287 and here I can see that 3.7912 is correct up to 4 decimal places and hence if I compared with the previous iteration previous value the value of w in the previous iterations and hence the root is numerical solution is 3.7913 12 sorry.

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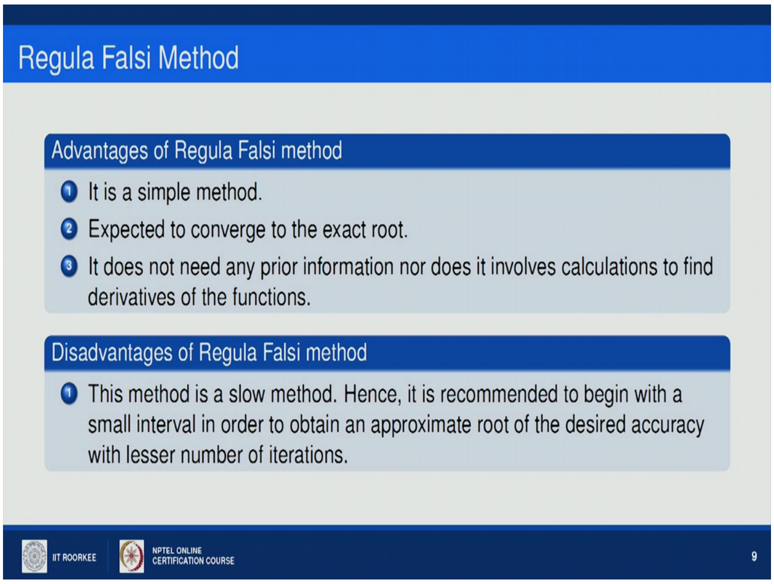

So like in the bisection method we are having some advantage of Regular Falsi method as well as some disadvantages. Advantages is it is a simple method expected to converge to the exact root, it does not need any prior information nor does it involve calculation to find derivatives of the functions which is a very beautiful advantage. We are not calculating any derivative because other we will describe some method is like Newton Raphson and fixed point iterations in the next lectures where you will see that we need to find out their derivatives of a function. Disadvantages are this method is slow method hence it is recommended to begin with a small interval in order to obtain an approximate root of the desired accuracy with lesser number of iterations.

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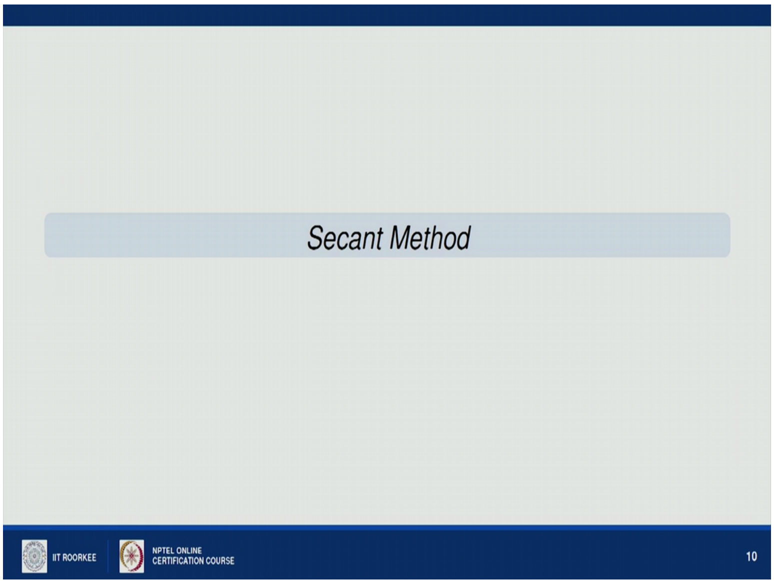

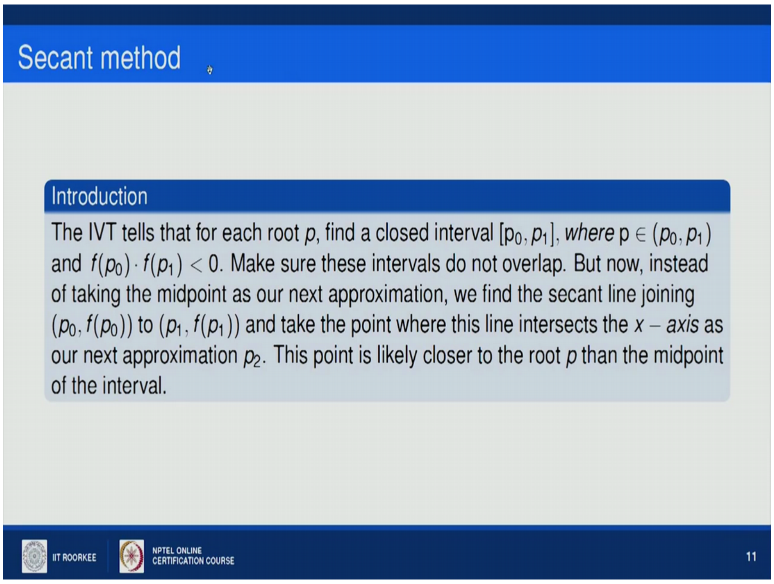

The next method we are going to discuss is Secant method which is a sort of Regular Falsi method only. If we start from the beginning the intermediate value theorem tells us for each root we find a close interval p 0 p 1 where p longs to open interval p 0 p 1 and f of p 0 and f of p 1 is negative. Make sure these intervals do not overlap but now instead of taking the midpoint as our next approximation, we find the 2nd and joining p 0 fb 0 to f p 1 f p 1 and take the point where this line intersect the x axis as our next approximation p 2. This point is likely closer to the root p than the midpoint of the interval.

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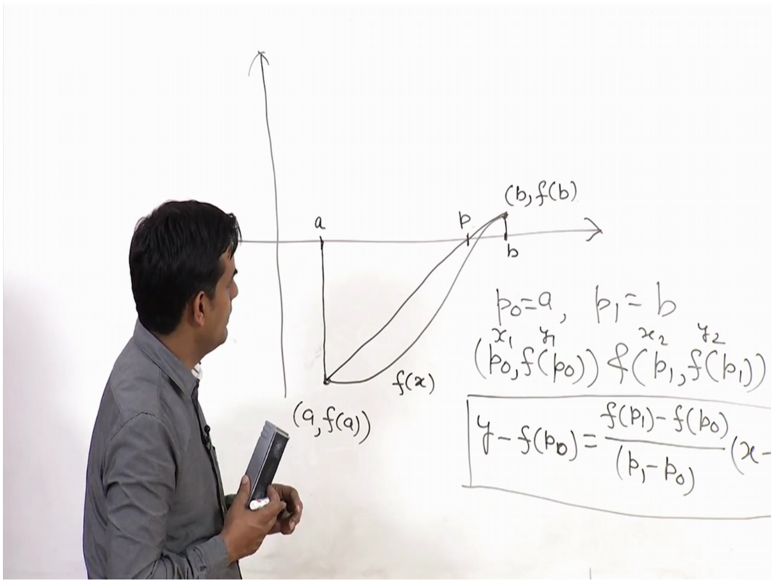

Let me explain this method by taking a graphical example and then I will come to the algorithm of this particular method. So like we are having this particular point a this is the point b, so and this is the function fx, so like earlier in this point will be af of a this will be bf of b. Now in the bisection method what we were doing? We were getting the midpoint of a and b which will be somewhere here as our next iteration however in Secant method what we will do? We will find the line is joining point a fa and b fb as our next iterate.

So it means this will be our next point and then we will continue with this one, so basically if I write let us say a is p 0 or if I denote in a p 0 as a, p 1 as b so how to find out next iterate? So if I find out the equation of this line that is basically p 0 fb 0 and p 1 f of p1, so line joining these 2 points and be given by y minus so this is point x 1 y 1, x 2 y 2, so fp 0 equals to y 2 minus y 1 that is f of p 1 minus f of p 0 upon x 2 minus x 1 that is p 1 minus p naught in x minus x 1 that is x minus p 0. So this is the equation of the line joining point this point and this point.

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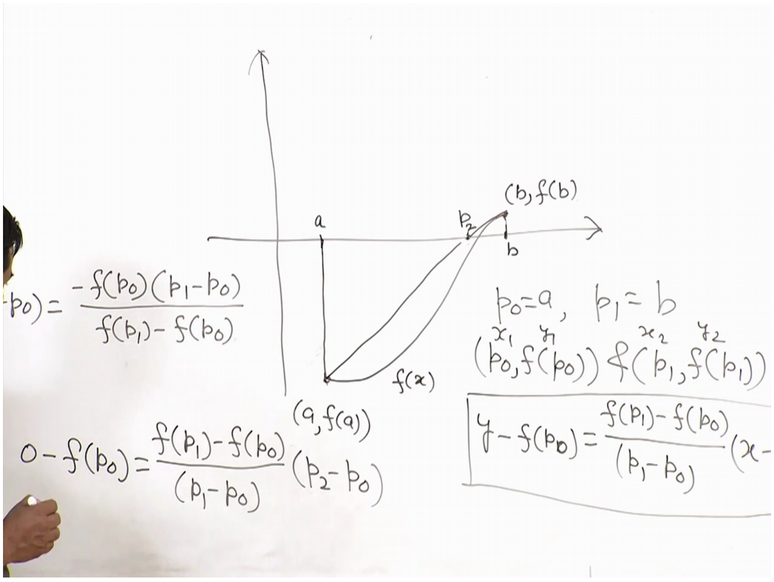

Now what we will do, we will find out this particular point let us say p 2 is the point where this particular line intersect x axis. So this is the equation of the line joining these 2 points. Now if I define this point p 2 where this particular line intersect x axis then I can write this equation as 0 since at this point y 0 minus f of p 0 equals to f of p 1 minus f of p 0 upon p 1 minus p 0 p 2 minus p 0. Please note that I have replaced x by p 2, so now if I want to modify this equation furthermore, I can write p 2 minus p 0 equals to minus fp 0 into p1 minus p 0 up on fp 1 minus fp 0.

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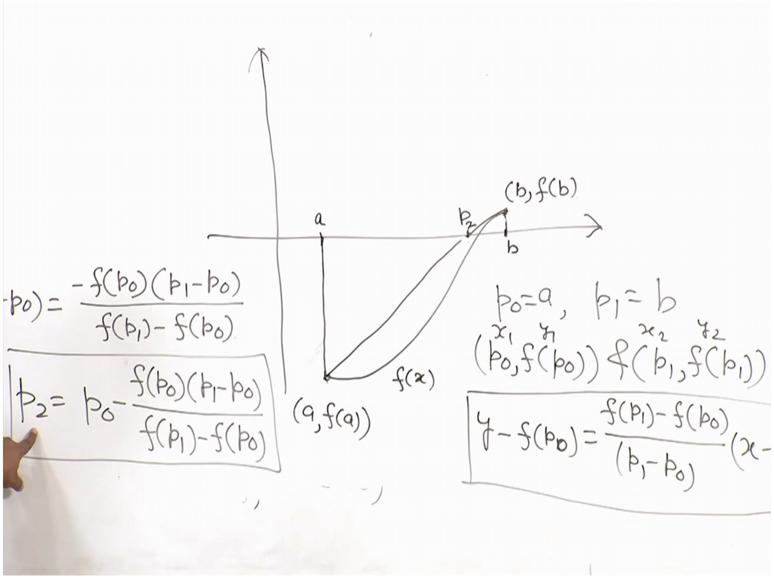

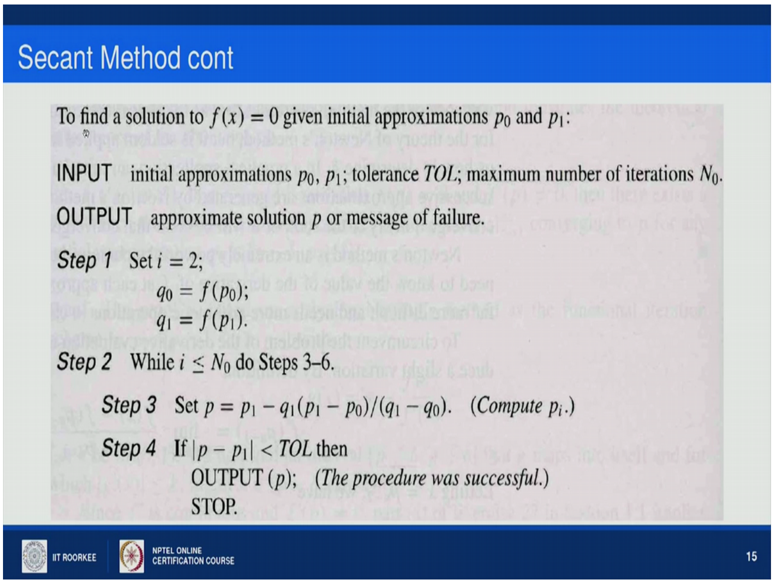

Or if I want to simplify it further then I can write p 2 equals to p 0 minus f of p 0, p 1 minus p 0 upon f of p 1 minus f of p 0. It means our next iterate p 2 can be calculated using this formula and this by continuing this we can get the iterations of Secant method. We got this formula and now the approximation p n plus 1 for n greater than 1, to root of f x equals to 0 is computed from the approximation p n and p n minus 1 using this particular equation as I have drive on the board if I explain this method using an algorithmic manner the input will be initial approximation p 0 p 1 at tolerance TOL and maximum number of iterations N naught, output approximate solution p or a message of failure. In step 1 set i equals to 2, q 0 is f of p 0, q 1 is f of p 1, while i less than equal to N naught do step 3 to 6. In step 3, p can be calculated as p 1 minus q 1 p 1 minus p 0 upon q 1 minus q 0.

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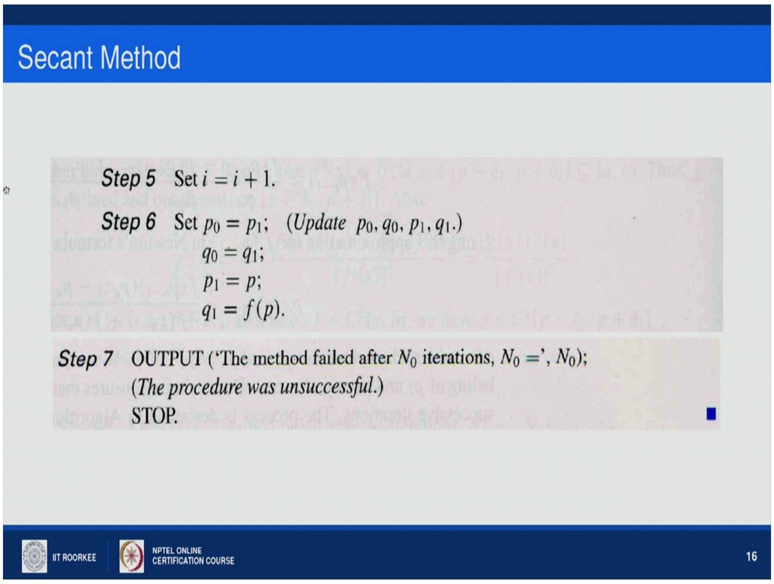

If p minus p 1 is less than tolerance than output becomes p and the method will stop otherwise set i equals to i plus 1, set p 0 is p 1, q 0 is q 1, p 1 as p, q 1 as f of p and step 7 of if you will repeat these steps 3 to 6 if you have achieve the maximum number of iterations that is N naught in step 7 it will give a message of failure that is the method fail after N naught iterations, okay and N naught equals to N naught. The process was unsuccessful means and stop.

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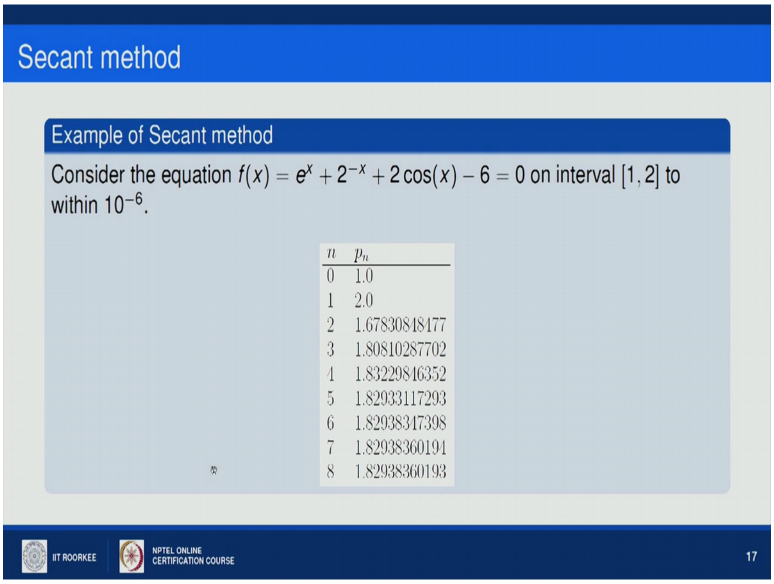

If we consider a example of this method, so for example if we take fx equals to e raise to power x plus 2 raise to power minus x plus twice of cos x minus 6 equals to 0 on interval 1, 2 within (())(24:28) of 10 raise to power minus 6. Then as a told you in the beginning my p 0 will become 1, p 1 will become 2 and from here I will get p 2 as 1.67830848477 by continuing this by using p 1 and p 2 I will get my p 3 by using p 2 and p 3 I will get my p 4 by continuing this in 8 iteration I will get the desired accuracy of 10 raise to power minus 6 and the correct root will be 1.829383 up to 6 decimal places.

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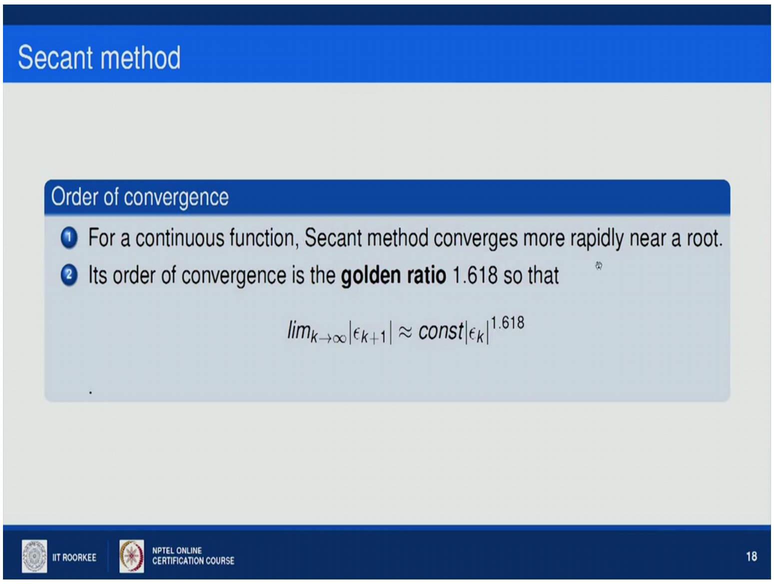

Order of convergence for a continuous function, Secant method converge more rapidly near a root. Its order of convergence is the golden ratio that is 1.618 so that limit k tends to infinity epsilon k plus 1 equals to constant epsilon k race to power 1.618. Hence you can notice in bisection method we were having it as 1 but here it is 1.618 hence this method is having better convergence when compared to the bisection method hence we told earlier at bisection method is having linear convergence so here I will say that the method is having super linear convergence.

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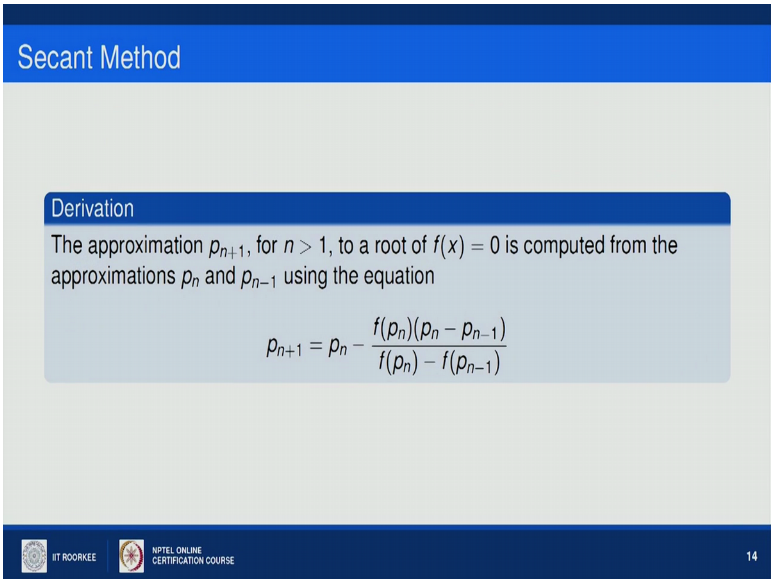

As you can see when we are finding the next iterates in Secant method that is similarly to the Regular Falsi method available these 2 methods are different up to the assignments of the atoms only like in Regular Falsi method what we are doing, we are taking the in each iteration the interval in such a way that root will lie in that particular interval however in this particular method what we are doing? We are updating our p using previous 2 iterations like if I want to find out p 3 I will use p 1 and p 2, I am not making any assignment of p 1 and p 2 to any other variables. So hence these 2 methods are different only up to an assignment. Thank you very much.