Numerical Methods

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Lecture 27

Numerical differentiation part-III (Differentiation based on Divided difference formula)

Welcome to the Lecture series on numerical methods, currently we are discussing on a numerical differentiation based on different Interpolation formulas. So in the last lecture we have discussed this numerical differentiation based on Lagrange operating polynomial. In the Lagrange interpolating polynomial we have discussed this numerical differentiation at tabular points and in this lecture we will discuss about this numerical differentiation at the non-tabular points, but within that interval only.

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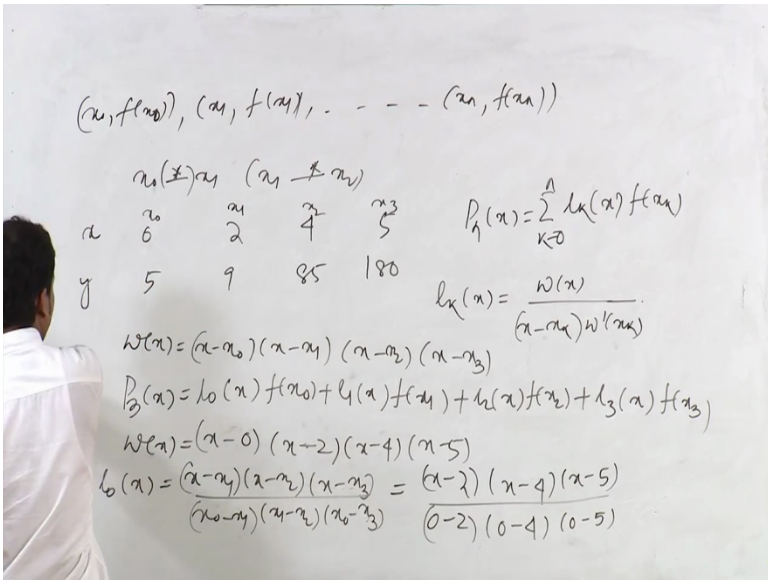

Within that interval means if you will have this tabulated point x 0 f of x 0, x 1 f of x 1, x n f of x n, then if we want to find the derivative at a particular point, it should lies between like x 0 to x 1 within this interval or x 1 to x 2 within this interval, so somewhere maybe it is just lying and at that point we want to evaluate this differentiation.

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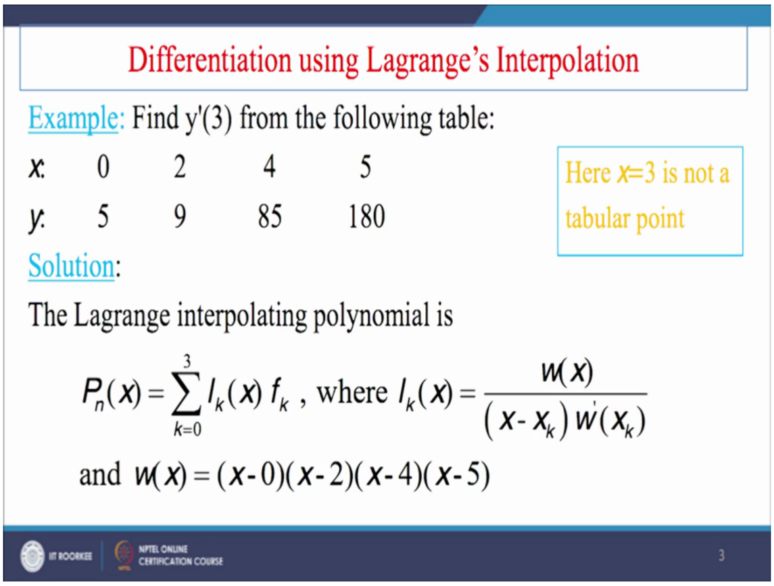

And for that if you just consider first example like suppose you will have this data points are prescribed as like 0, 2, 4, 5 suppose and its functional values are like 5, 9, 85, 180 suppose and corresponding Lagrange interpolating polynomial if we want to write so it can be represented in the form of like P n x = submission k = 0 to n L k x, f of x K, where L k x can be represented as Omega x divided by x – x k Omega dash of x k here. So basically for this problem if you just write here Omega x, so Omega x can be written as – x 0, x – x 1, x – x 2, x – x 3 here, since we will have here 4 points it can construct a polynomial of degree 3 here.

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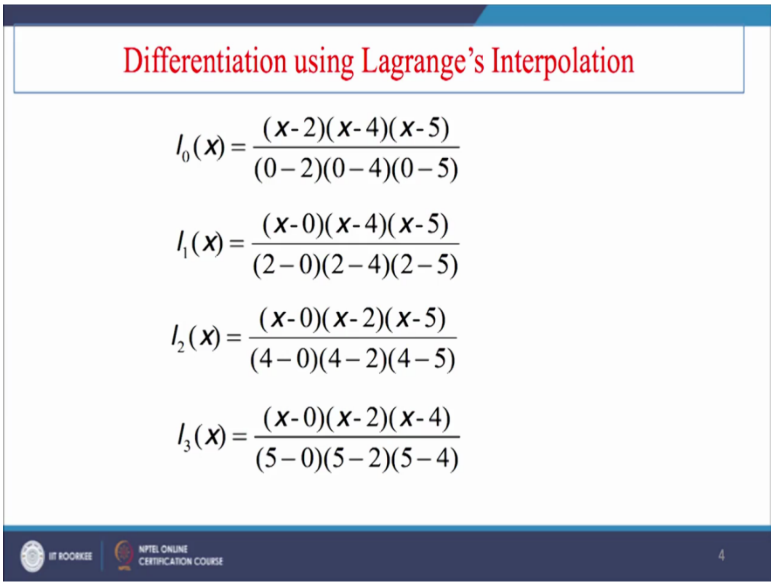

So we can just write P 3 of x that is your L 0 x f of x 0 + L 1 x f of x 1 + L 2 x f of x 2 + L 3 x f of x 3 here, where this Omega x if you just consider this particular problem here this can be written as x – 0, x – 2, x – 4 into x – 5 here. And if you just write here L 0 x and L 0 x can be written in the form like x – x 1, x – x 2, x – x 3, divided by x 0 – x 1, x 0 – x 2, x 0 – x 3 here. And particularly these tabular points we can just write this one as x – x 1 is here since especially we are just defining this is as x 0, this one is x 1, this is as x 2, this is as x 3 here. So if you just write all these points here that is x – 2, x – 4, x – 5 divided by your x 0 is 0 here so 0 – 2, 0 – 4, 0 – 5 here.

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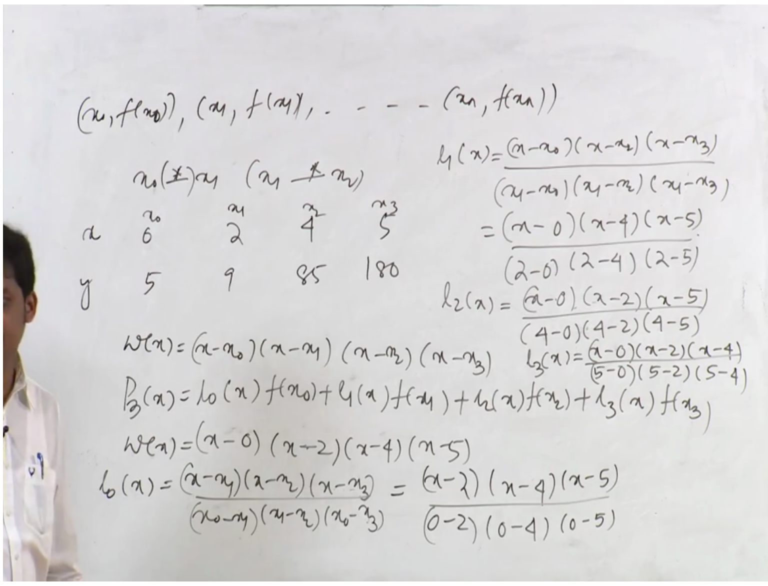

Similarly we can just write L 1 x also so if you just write L 1 x here this can be written as L 1 x as x – x 0, x – x 2, x – x 3 divided by x 1 – x 0, x 1 – x 2, x 1 – x 3, so directly if we will put all these values; x is 0, x – 4, x – 5 divided by x 1 is given as 2 here, 2 – 0, 2 – 4, 2 – 5 here. Similarly L 2 x can be written as x – x 0 that is as 0 here, x – x 1, x – x 3 divided by 4 – 0, 4 – 2, 4 – 5 here. Similarly if you will just write L 3 also here, so L 3 x can be written as x – 0, x – 2, x – 4 divided by 5 – 0, 5 – 2, 5 – 4 here.

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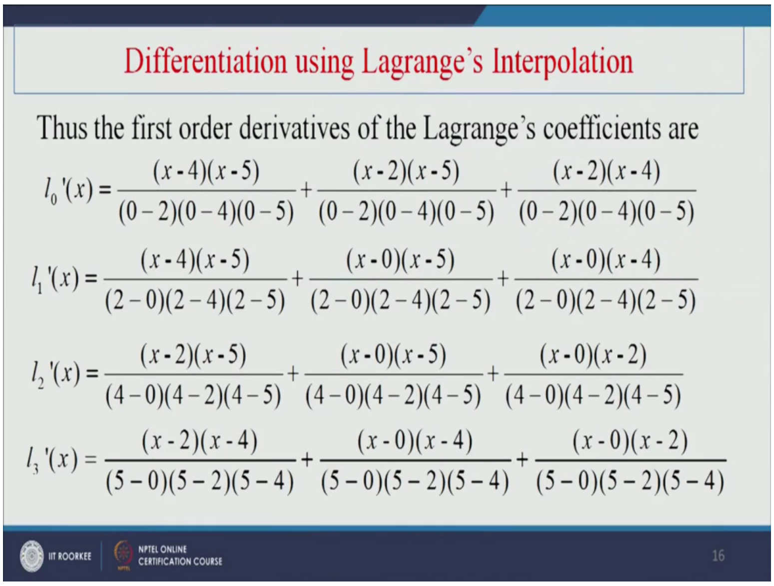

And particularly if you have determined this L 1 x, L 2 x, L 3 x and L 0 x on these tabular values so then we can just obtain the first order derivative of this Lagrange coefficient as L 0 dash x can be written as so first product if you just consider here that is in the form of like x – 4, x – 5 divided by all of these product terms that is – 2 into – 4 into – 5 here so directly we can just write that one and – 2, – 4 into – 5 + second term if you just differentiate here x – 2 into x – 5 divided by – 2 into – 4 into – 5 here.

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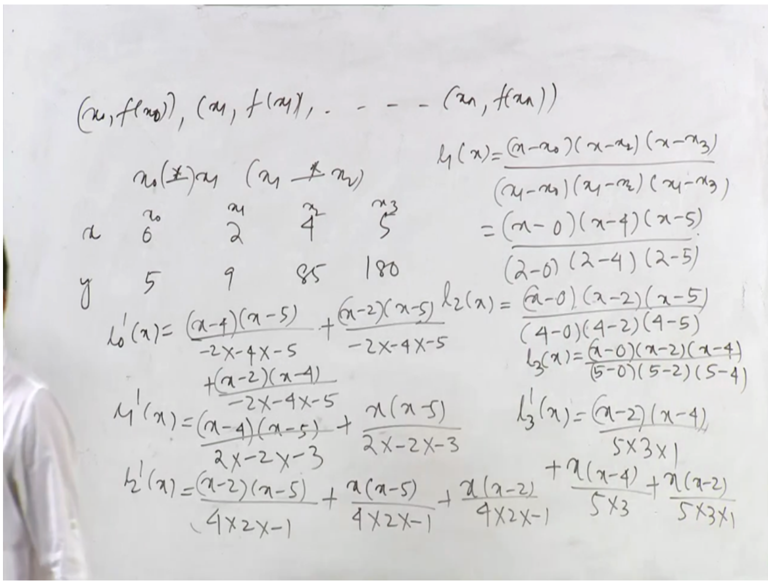

Similarly the last if you just differentiate here, x – 2, x – 4 divided by – 2 into – 4 into – 5 here and similarly if you just differentiate L 1 dash of x here, L 1 dash of x can be written as like first-term if you just differentiate here so x – 4, x – 5 divided by 2 into – 2 into – 3 here +

again x into x – 5 divided by 2 into – 2 into – 3 + last term if you just differentiate here, x into x – 4 divided by 2 into – 2 into – 3 here.

Similarly L 2 x if you just write it down so L 2 dash of x can be written as your first-term if you just differentiate then x – 2 into x – 5 divided by 4 into 2 into – 1 here + second term if you just differentiate x into x – 5 divided by 4 into 2 into – 1, last term if you just differentiate x into x – 2 divided by 4 into 2 into – 1 here. And similarly we can just obtain this derivative for L 3 dash of x also here so if we just write L 3 dash of x here so L 3 dash of x it can be written as x – 2 into x – 4 divided by 5 into 3 into 1 + second differentiation if you just take, x into x – 4 divided by 5 into 3 + last differentiation x into x – 2 divided by 5 into 3 to 1 here.

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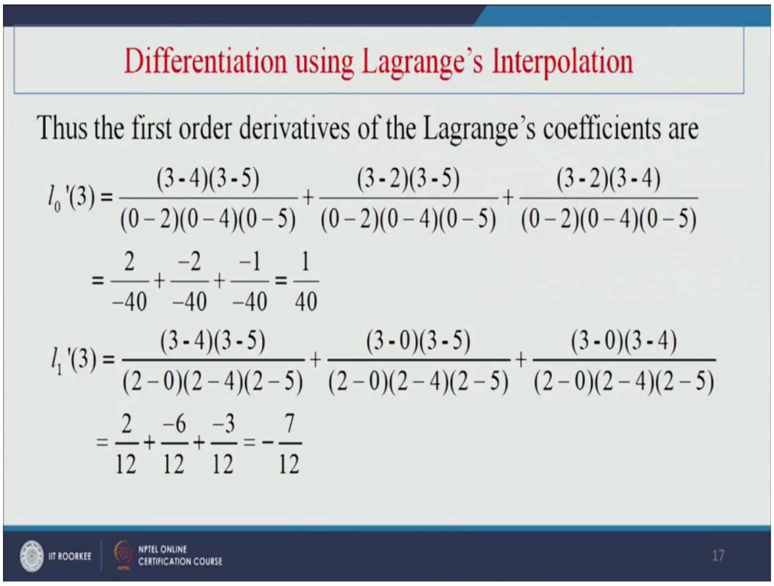

So if you just consider here x as 3 then we can obtain L 0 dash of 3 here then if you just consider L 1 dash of 3 here then we can just obtain the value of L 1 dash of 3. L 2 dash of 3 we can just put x as 3 here and we can obtain the value and L 3 dash of 3 we can just obtain the values by putting x as 3 there. So if you will just put all these values then we can just obtain that L 0 dash of 3 as 1 by 40, L 1 dash of 3 as – 7 by 12, L 2 dash of 3 as 5 by 8 and L 3 dash of 3 as – 1 by 15 here. And once these values are known to us like if you just write here this value is just giving you like that specially we are just writing here y dash of 3 since it has asked you to find here.

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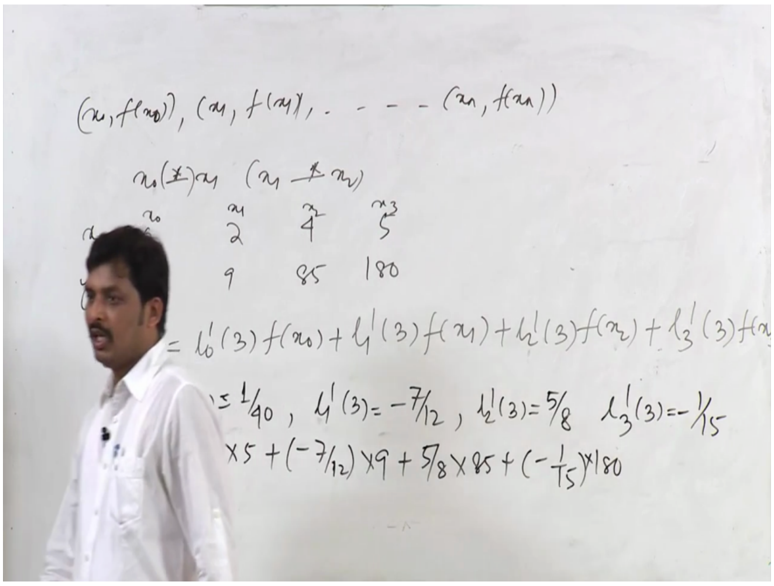

So we can just write this one as L 0 dash of 3 into f of x 0 + L 1 dash of 3 f of x 1 L 2 dash 3 into f of x 2 + L 3 dash of 3 f of x 3 here. So if you will just evaluate or put all these values here then first value if you just try write so L 0 dash of 3 we just found it as 1 by 40 here and L 1 dash of 3 it is just found as – 7 by 12 here, L 2 dash of 3 it is just found to be 5 by 8 here and L 3 dash of 3 it is just found to be like – 1 by 15 here. And directly all of these functional values that is given as f of x 0 as 5 here so if you put all of these values we can just write this one as 1 by 40 into f of x 0 here 5 + – 7 by 12 into 9 + 5 by 8 into 85 – 1 by 15 into 180 here.

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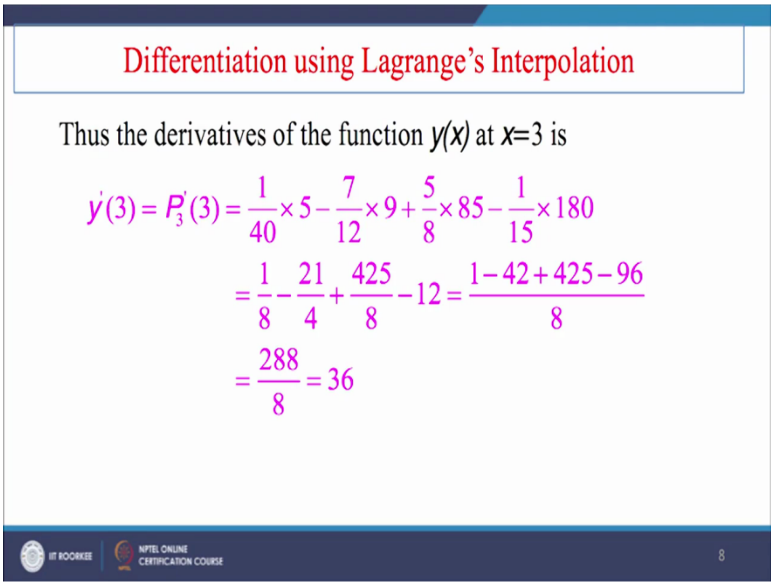

And if you will just do this computation specially you can just get the value as 36 here. So then we will just go for this differentiation using Newton's divided different formula.

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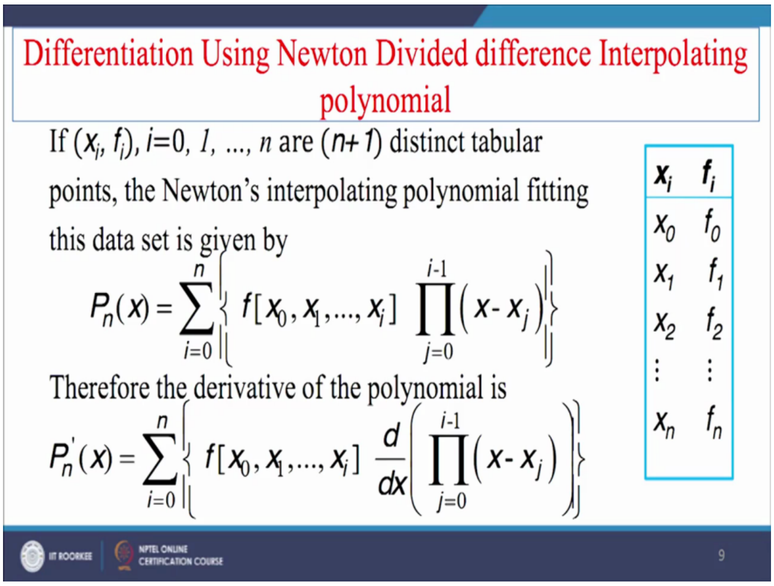

And in Newton's divided difference Interpolation formula already we have known that the advantage for this Newton's divided difference formula is that in case of Lagrange Interpolation formula if a small quantified quantity it should be associated with this Interpolation formula then always we will just go for a large multiplication of these vectors, but in divided differences that we can just overrule that factors. So specially if the term is asked you to associate in that terms, directly we can just write a multiplication and we can just evaluate that one.

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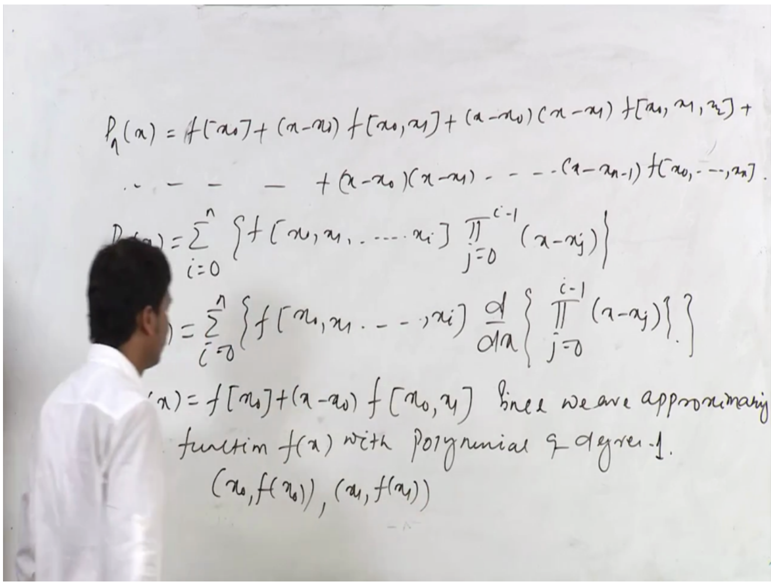

So here in like Newton's divided difference Interpolation polynomial we will have like the set of tabular points that is x i and f of x i defined at n + 1 points like x 0, x 1 up to x n points. So if you just write this Newton's divided difference interpolating polynomial that can be represented as P n x = f of x 0 + x – x 0 f of x 0, x 1 + x – x 0, x – x 1 f of x 0, x 1, x 2 up to x – x 0, x – x 1, up to x – x n – 1 f of x 0 to x n here. So if we may complete form we want to write or in a product form if you want to express or in other sum form if we want to express then P n x can be written as submission of i = 0 to n f of x 0, x 1 to x i product of J = 0 to i – 1, x – x j here.

So if we want to find the derivative for this polynomial here, this polynomial derivative can be written as P n dash x here, this = submission i = 0 to n since all the values are constant here like f of x 0, x 1 to x i then we can just write this derivative as here d by d x of product of j = 0 to i – 1, x – x j. And all of these tabular values it is known to us and associated functional values are also known to us then we can just use these functional values to calculate this derivative with respect to x here, first for this we will consider a linear interpolation here. In case of linear interpolation we have 2 consider only 2 points where these tabular points may be x 0 f of x 0, x 1 f of x 1.

And if we will just consider this linear interpolator polynomial using Newton's divided difference Interpolation here then we can just write this polynomial as P 1 x this = f of x 0 + x – x 0 f of x 0, x 1 here since we are approximating the function f of x with polynomial of degree 1 here. So always we can just remember that if we will have a polynomial of degree n always we have to consider n + 1 terms there. So if we are just approximating this function f of x with a polynomial of degree 1 here then we have to consider 2 points that is in the form of like x 0, f of x 0, x 1, f of x 1 here.

And if you just consider these 2 points then the polynomial approximation can be written as P 1 x as f of x 0 + x – x 0 f of x 0, x 1 here. And if you just try to evaluate this first order derivative for this polynomial here so P 1 dash of x can be written as since first-term if you just see that this argument contains only a constant point here. Hence we can just write P 1 dash of x as f of x 0, x 1 here. And if we just consider like a quadratic interpolation suppose so then we will have like 3 points we have to consider for quadratic interpolation since it represents a polynomial of degree 2 so 3 terms (())(18:14) it is required.

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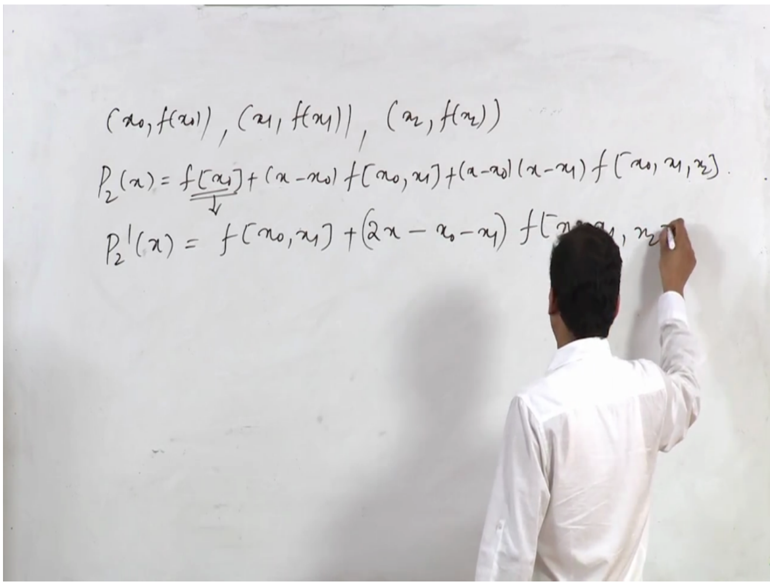

So if you just consider is quadratic polynomial here, the quadratic polynomial associates the points like x 0 f of x 0, x 1 f of x 1, x 2 f of x 2 suppose, since 3 points are required that is why we have just considered here 3 points here as x 0, x 1 and x 2. And if you will just approximate this one with a polynomial of a degree 2 here, this polynomial can be written as P 2 x as f of x 0 + x – x 0 f of x 0, x 1 + x – x 0, x – x 1 f of x 0, x 1, x 2 here. If we want to find this first order derivative for this function P 2 of x here so this can be written as P 2 dash x as first-term this can just give you 0 value and second term if you just consider here that can be represented as f of x 0, x 1 here and immediate next term if you just consider that can be written as 2 x – x 0 – x 1 into f of x 0, x 1, x 2 here.

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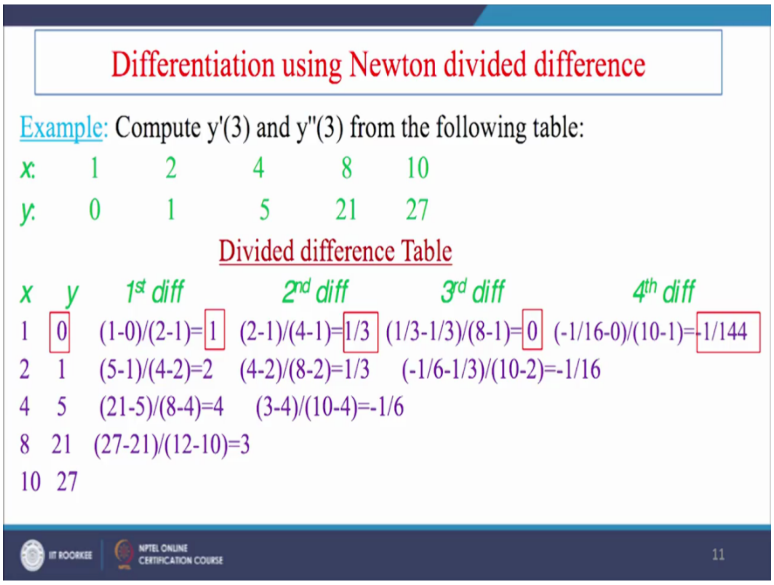

So based on this if you will just go for an example here then we can just write this tabular values as like compute suppose y dash 3, y double dash 3 from the tabular data like 1, 2, 4, 8, 10 suppose the functional data is given and sorry this tabular data is given and the functional data are given like 0, 1, 5, 21, 27 suppose.

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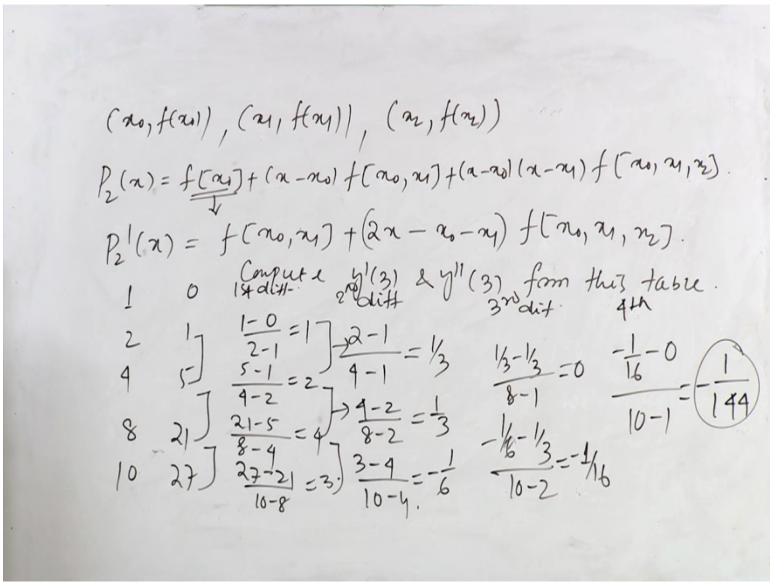

If you just write this tabular data that is in the form of like 1, 2, 4, 8, 10 suppose and the functional values are like 0, 1, 5, 21, 27 and it has asked you to compute y dash of 3 and y double dash of 3 from the following table. Compute y dash of 3 and y double dash of 3 from this table, so first we will just use divided difference table formula to get all this difference data there. So like first difference if you just find here, this can be represented as 1 – 0 by 2 – 1 since it is formula specially, so the first order difference we can just write here 1 – 0 by 2 – 1, second one if you just consider 5 – 1 divided by 4 – 2 here, third one if you just consider 21 – 5 divided by 8 – 4 and last 2 points if you just consider, 27 – 21 divided by 10 – 8 here.

Obviously we can just obtain the data as 1, 2, 4, 3 here and in the second difference or second divided difference we can just write this data as 2 – 1 divided by 4 – 1 then we will have like 4 – 2 divided by like 8 – 2 here then last value if you just consider here 3 – 4 divided by 10 – 4 here . So obviously you can just obtain these values as 1 by 3, 1 by 3 and last value this will be – 1 by 6 here. And next third difference table you can just obtain these values as 1 by 3 – 1 by 3 divided by sum of this data like 8 – 1 and then – 1 by 6, – 1 by 3, 10 – 2 here.

So this can first value you can just get it as 0 here and second value can just get it as – 1 by 16 here and forth divided difference you can just find – 1 by 16 – 0 divided by 10 – 1 here and you can just obtain this one as – 1 by 144. And if you will just use this Newton's divided difference Interpolation formula for this calculation of these values here so we can you just use these values that is 0, 1, 1 by 3, 0, – 1 by 144 as the tabular values here.

So maybe in next lecture we can just continue that how we can just use this Newton's divided difference formula for finding this derivative, since I have just given you recount of this Newton's divided difference formula and directly we can just use these formulas for the derivation of the data and I can just consider the same example in the next lecture and I can explain that how we can just put this data in the tabular form and we can obtain the first order derivative of second order derivative in a complete form, thank you for listening the lecture.