Numerical Methods

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Lecture 21
Interpolation Part VI (Central Difference Formula)

Welcome to lecture series on numerical methods. And in numerical methods we are discussing interpolation. So in interpolation section we have discussed like different operators. So today we will discuss about this central difference formulas. First we will discuss about this Gauss's backwards difference interpolation formula, then Gauss's forward difference formula, then we will just go for Stirling's central difference approximation and Bessel's central difference approximation with examples.

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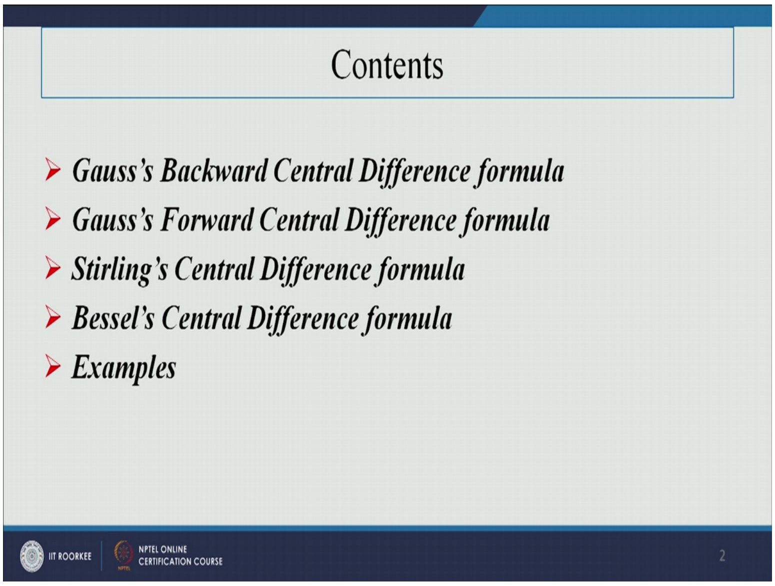

So in the last class also we have given little hints about Gauss's backward difference formula. Here usually we are just using even differences of y 0 and odd differences of y of minus half where the formula for this Gauss backward difference formula is expressed in the form like y p or y of x p can be written as a 0, y 0, a 1, delta, y of minus half, a2, delta square, y 0, a 3, delta cube, y of minus half, so likewise.

If you will just express this left hand side of y of x p as y of x 0 plus p h here then it can be expressed in the form E to the power p of y 0 which can be written as 1 plus delta whole to the power p of y 0 here. And if we will write the right hand side expression here in the coefficient form like a 1 delta y of minus half it can be expressed as a 1 delta E of minus half y 0 here. Similarly if we express here a 3 delta cube y of minus half it can also be expressed as a3 delta cube E to the power minus half y 0 here.

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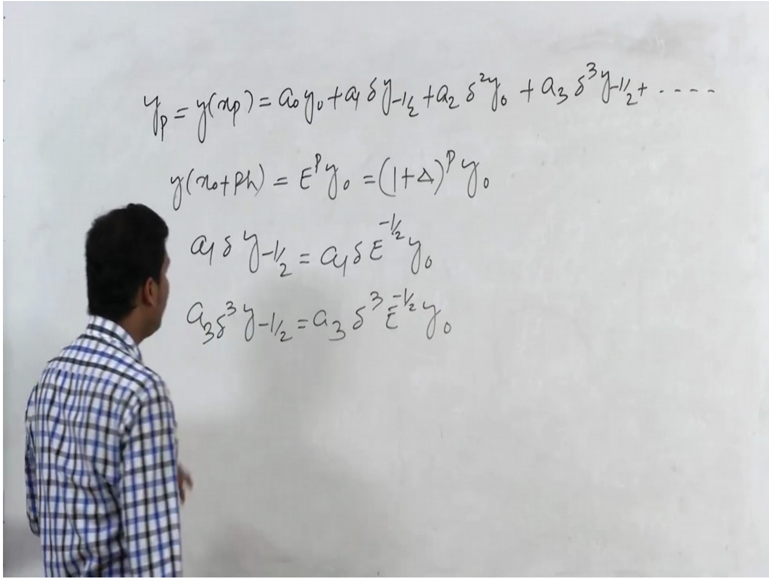

So if you will just express in the coefficients as y 0 here then both the sides can be expressed as operator here. So operator means we can just express this left hand side in the form of 1 plus delta whole to the power p, this can be expressed as right hand side as a 0 plus a 1 delta E power minus half plus a 2 delta square plus a 3 E to the power minus half plus as in this form here.

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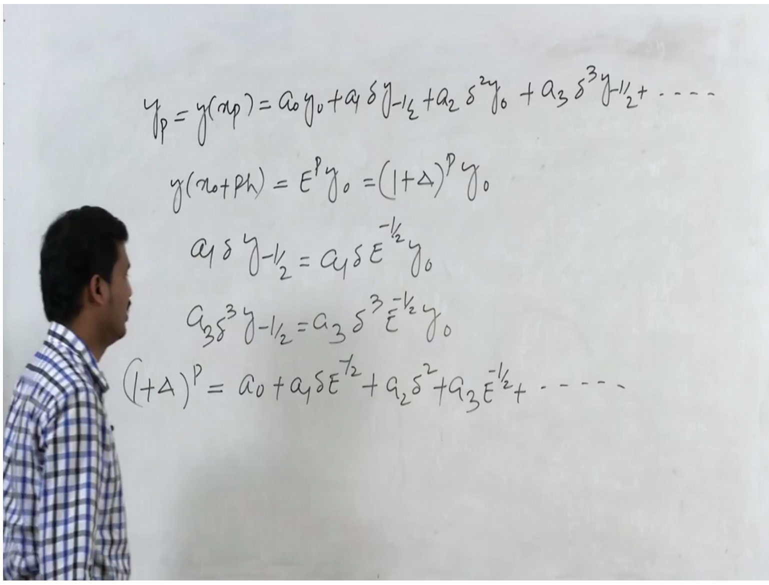

So now we can just express delta E to the power minus half in the form of delta function here and delta cube E to the power minus half also in the form of like delta here then if we will just equate both the sides coefficients then we can obtain the values for a 0, a 1, a 2 and a 3.

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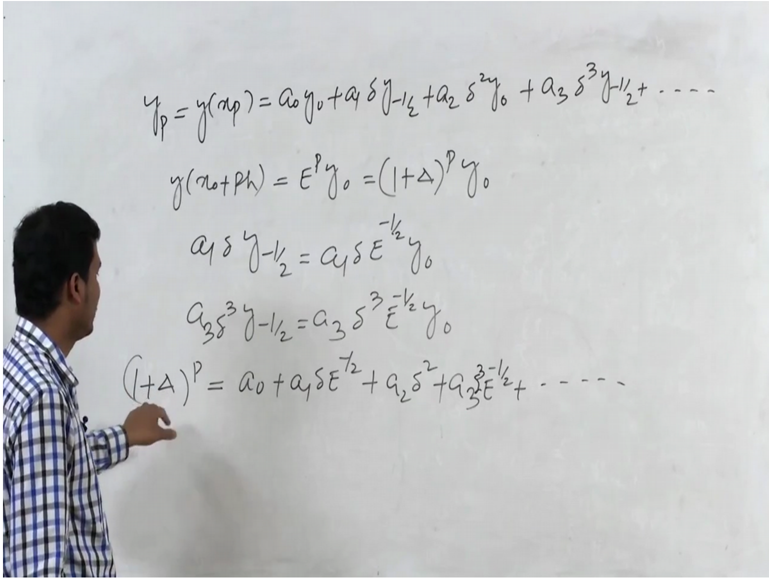

And where we can just find the exact formula for Newton's sorry this is Gauss backward formula. So if you will just expand here 1 plus delta whole to the power p here as lhs side here, so in lhs we can just write 1 plus delta whole power p as your first will be 1, then p delta plus p del square by factorial 2, so likewise we can just express. So this will be like p into p minus 1 here. So like p into p minus 1 by factorial 2 del square here. So likewise we can just express.

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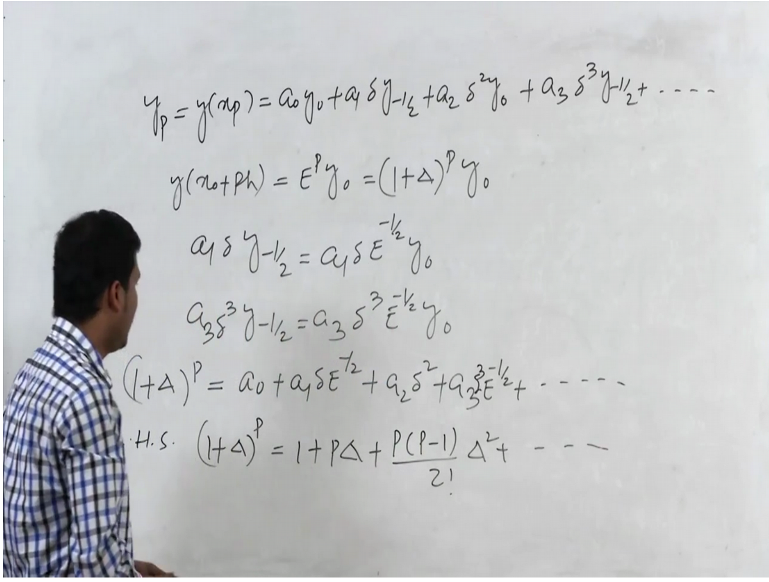

And if you will just express this right hand side terms here delta E to the power minus half in terms of like delta here we can just express that one in the form E to the power half minus E to the power minus half since already we have known that the central difference operator delta can be expressed as E power of half minus E power of minus half. And if we want to write this term delta E power of minus half here we can just write E power of half minus E to the power minus half here into E to the power minus half.

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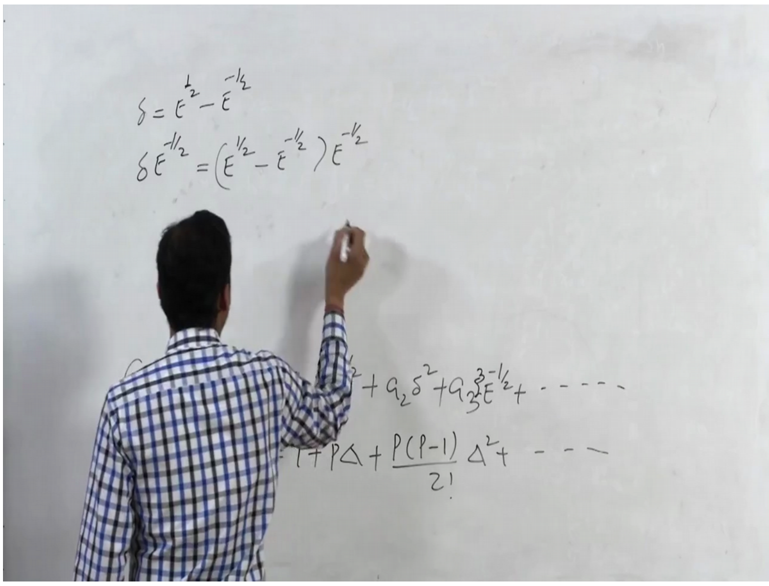

And if we want to express this one in the form of capital delta here so we can just write this term as E to the power half minus E to the power minus half into E to the power half into E to the power minus 1 here. So this mean that if we will just take the product here we can just obtain E minus 1 here and they say this one we can just write E to the power minus 1 which can be expressed in the form of like delta into 1 plus delta full inverse and which can be expressed as delta by 1 plus delta here.

Similarly we can just express rest of the terms like delta cube E to the power minus half in this form of capital delta then all other terms also it can be expressed.

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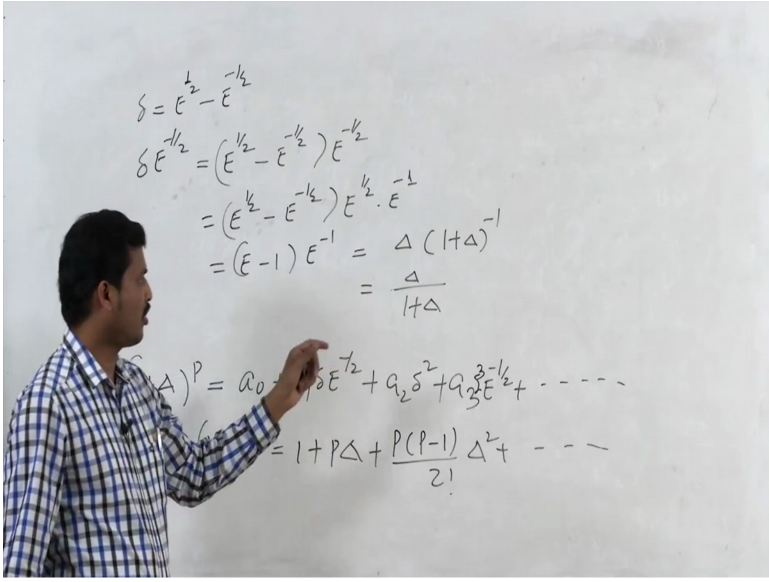

So if you will just express all these terms this can be represented in the form of like delta square can be expressed as delta square by 1 plus delta, delta cube can be expressed as delta cube by 1 plus delta whole square and delta to the power 4 can be expressed as delta to the power 4 by 1 plus delta whole square and delta to the power 5 E to the power minus half, it can be expressed as delta to the power 5 by 1 plus delta whole cube.

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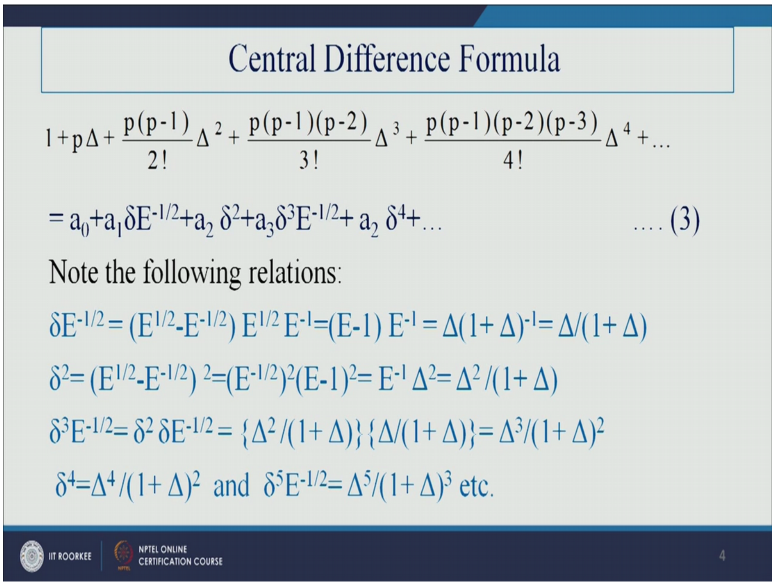

So if you will just put all these coefficients in both the sides then we can just obtain a 0 value as 1 there and to obtain the values of remaining coefficients like a 1 if we will just multiply 1 plus delta in both the sides then we can just obtain these coefficients as in the form of 1 plus p plus 1 into delta plus p into p plus 1 by factorial 2 delta square. So this will be the left hand side and in the right hand side we can just put that one as since a 0 equals to 1 there so this can be expressed as 1 into 1 plus delta plus a 1 delta square plus a 2 delta cube plus likewise.

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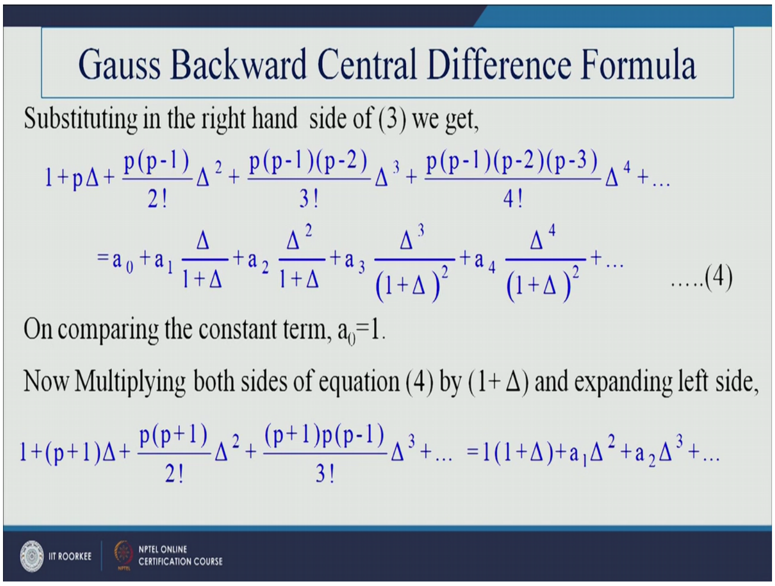

So if you will just see here the complete expression for this Newton's backward difference formula can be written in the form like 1 plus p plus 1 delta plus p in to p plus 1 by factorial 2 delta square. This equals to 1 into 1 plus delta plus a 1 delta square plus a 2 delta cube, likewise we can just write.

And if you will just multiply by this factor like 1 plus delta here so we can just write this coefficient that as 1 plus p plus 1 delta and this can be written as like both the sides if we just take common here like 1 plus p plus 1 delta here and if we just equate the coefficients of delta and delta square we can just obtain a 1 value as p here.

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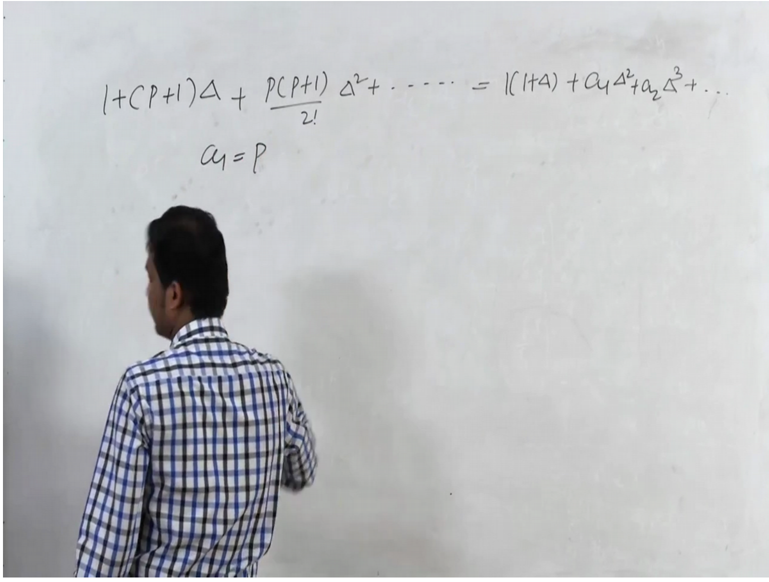

Since if you can just see here like 1 into 1 plus delta and this right hand side and left hand side it can just cancel it out and we can just obtain these values a 1 as p and a 2 equals to like permutation of p plus 1 2 there. And again multiplying like 1 plus delta whole square then some of these coefficients again we can just get it up as a 3 and a 4 there where this a 3 can be expressed as p plus 1 3 and a 4 can be expressed as p plus 2 4. Proceeding in this manner we can just obtain these coefficients for 2 m minus 1 power and 2 m there.

And a 2 m minus 1 can be expressed as since we are just taking this differences here, p plus m 2 m minus 1 minus p plus m minus 1 2 m minus 2, this can be given as p plus m minus 1 2 m minus 1 there. And a 2 m can be represented as p plus m and 2 m there for m equals to 1, 2, 3, likewise we can just consider these values.

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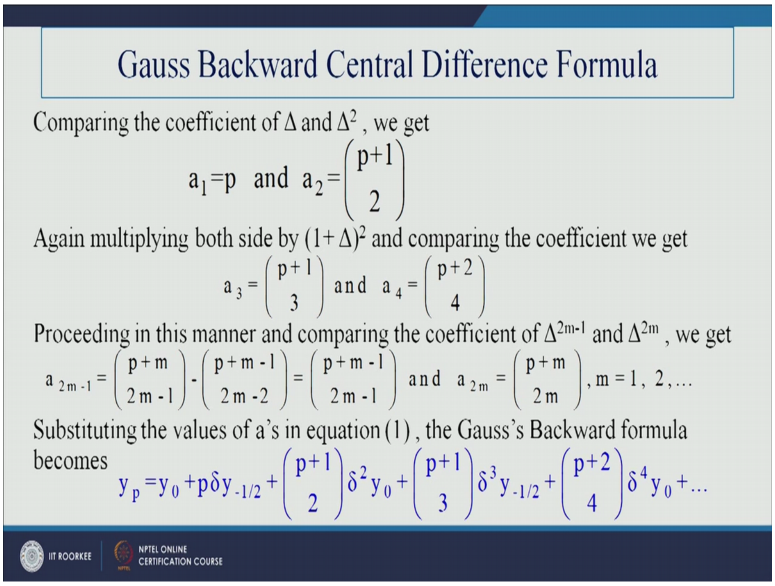

Finally if you just put all these coefficients a 1, a 2, a 3, then we can just obtain this formula as in the form of y p equals to y 0 plus p delta y of minus half plus p plus 1 2 delta square of y 0. So this will be the formula for Gauss's backward difference formula.

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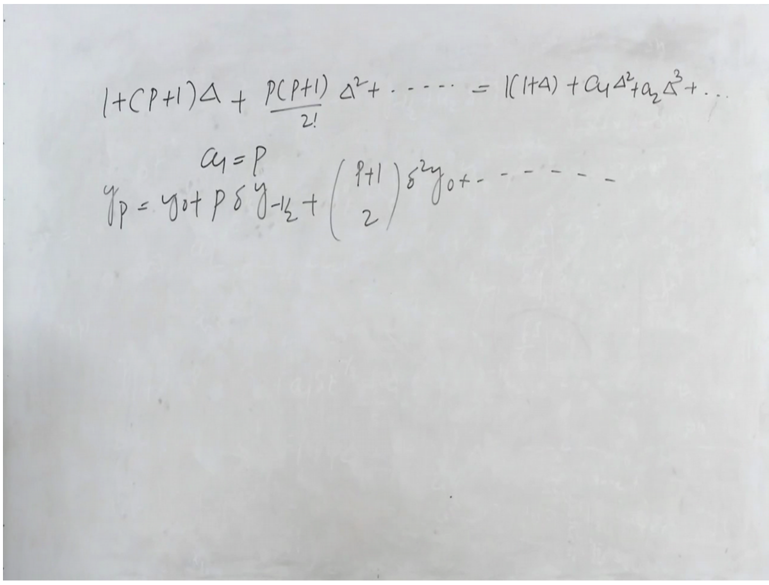

And if we are just going for this Gauss's forward central difference formula then we can just choose these coefficients as these forward marching steps. Like if you will just write this coefficients that we can just write y p as a 0 y 0 a 1 delta y of half a 2 delta square y 0 a 3 delta cube y of half, so likewise we can just write.

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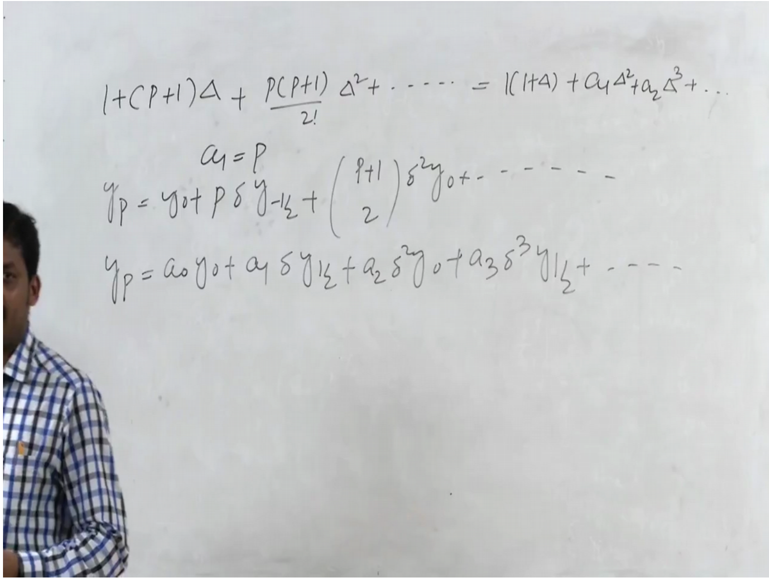

Where these coefficients a 0, a 1, a 2, a 3 are to be determined. And if you will just use this same relationship here like this coefficient we can just replace here delta E to the power half of y 0 and this coefficient it can be taken as delta cube E to the power half of y 0, so then we can just express both the sides in the form of like forward difference operator as the coefficients operated on y 0 there.

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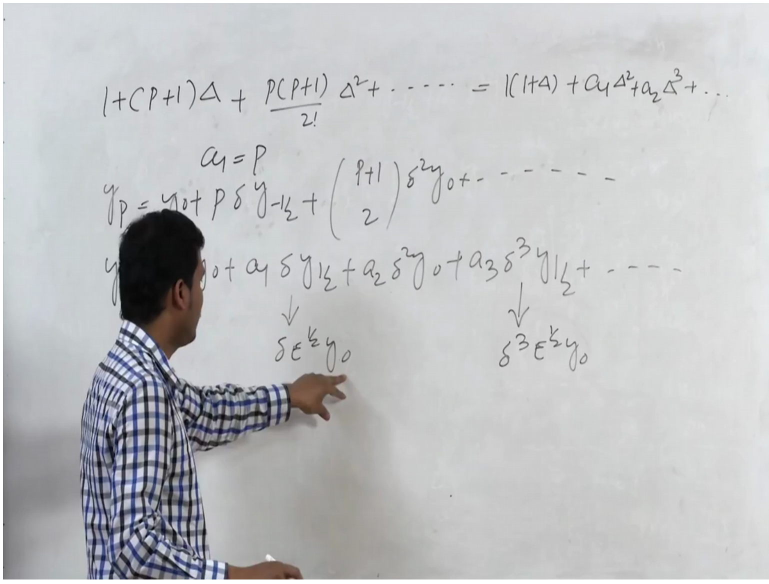

So if you will just write it in operator form we can just write this left hand side that as 1 plus p delta plus p into p minus 1 by factorial 2 delta square plus p into p minus 1 into p minus 2 by 3 factorial delta cube. This equals to a 0 plus a 1 delta plus a 2 delta square by 1 plus delta a 3 delta cube by 1 plus delta a 4 delta to the power 4 by 1 plus delta whole square plus up to like finite number of terms. If you will just compare these coefficients on both the sides like delta to the power 0 and delta to the power 1 here then we can just obtain a 0 as 1 and a 1 as p.

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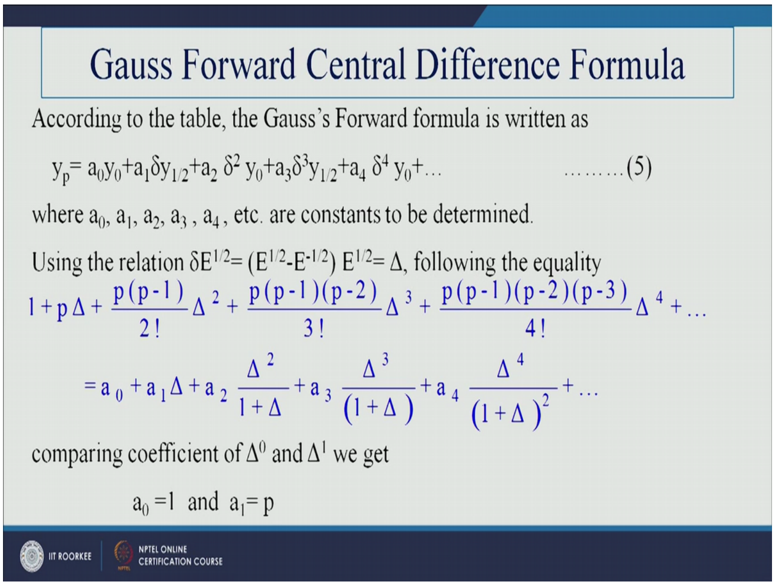

And if you will just multiply both the sides by 1 plus capital delta then we can just obtain the values of a 2 as the permutation of p 2 and a 3 as permutation of p plus 1 3 there. So likewise if you will just multiply one by one we can obtain rest of the coefficients like a 2 m, a 2 m plus 1 and we can just write this Gauss's forward formula as y p equals to y 0 plus p delta y of half plus p plus 1 sorry p 2 here delta square of y 0 plus p plus 1 3 delta cube of y of half, so this one.

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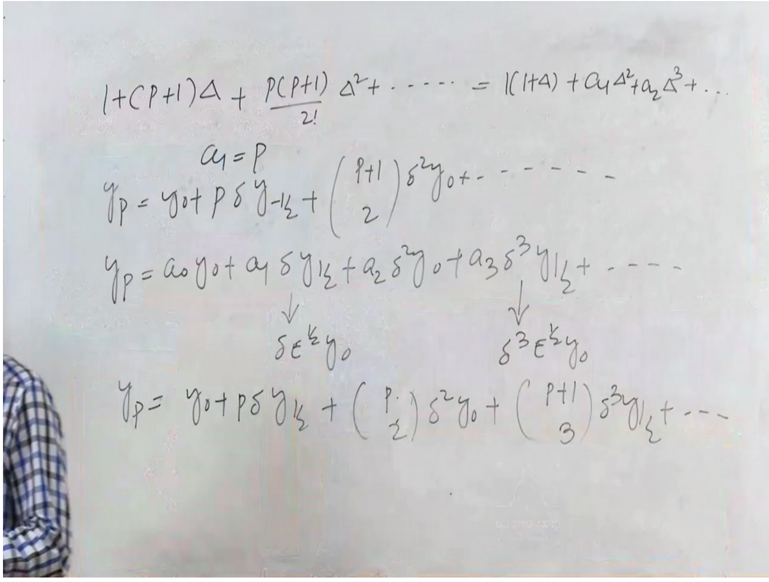

So if you will take the average of these two scales then we can just obtain this Stirling's central difference formula. This means that this formula uses y 0 and its even differences plus average of the odd difference of y of minus half and y of half. Hence the formula can be expressed in the form like y p equals a 0 y 0 plus a 1 mu delta y 0. Since we are just taking the average of this odd terms here so that is why this mu operator is assigned in the odd terms here and even terms are remained in that scale only.

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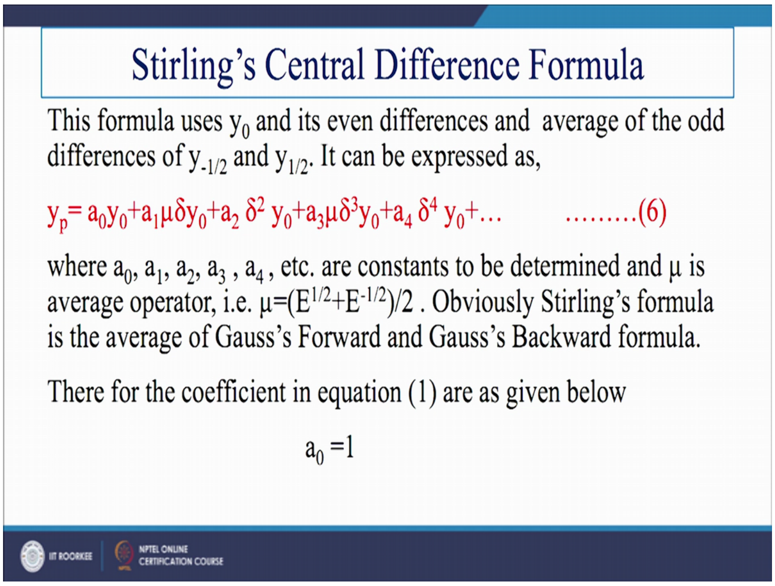

So where these coefficients like a 0, a 1, a 2, a 3 are the constants to be determined and mu is the average operator here where mu can be expressed as if E to the power half plus E to the power minus half by 2.

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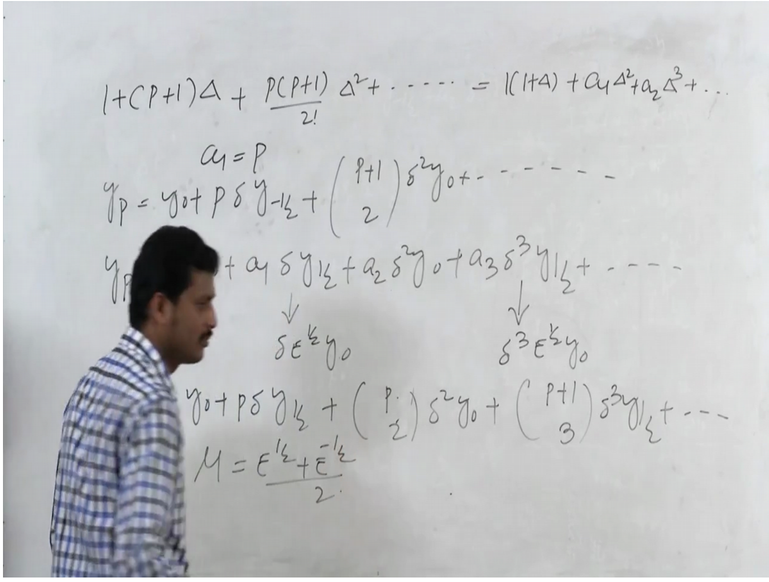

If we will just express this one in constant coefficients form then we can just obtain these values of a 0 as 1 here and rest of the coefficients will be expressed in the form of like a 2 m minus 1 as permutation of p plus m minus 1 by 2 m minus 1 and a 2 m as half of like average terms we are just considering for this odd terms here, so that is why we can just consider p plus m 2 m plus p plus m plus 1 2 m where m will vary from 1 to like finite number of terms.

So if you will just expand these terms here then we can just obtained this coefficients as a 2 m equals to p square into p square minus 1 square into p square minus 2 square up to p square minus m minus 1 whole square by 2 m factorial here.

So putting all these values we can obtain this Stirling's central difference formula as y p equals to y 0 plus p mu delta of y 0 plus p square by 2 factorial delta square of y 0 plus p plus 1 3 mu delta cube of y 0 plus p square into p square minus 1 square by 4 factorial delta to the power 4 y 0 here plus up to the finite number of terms. Whatever these terms where we want to truncate this series, so up to that term we can just consider in the series expansion.

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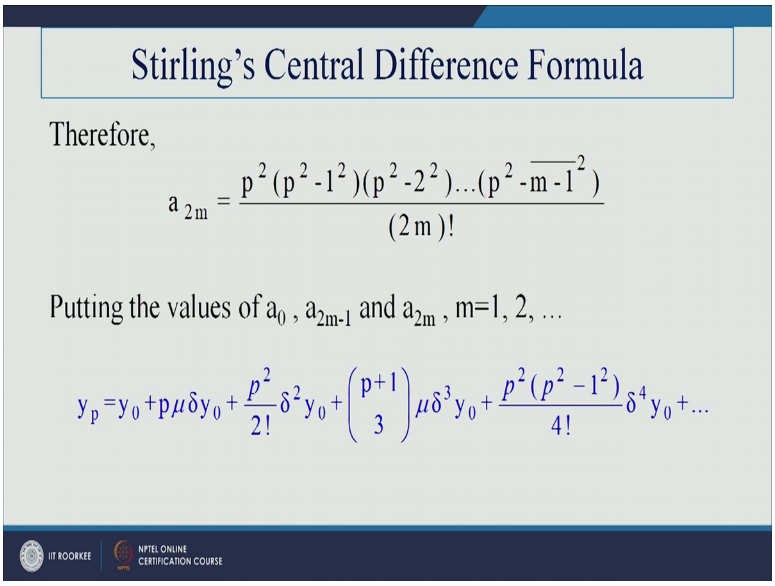

So next we will just go for Bessel's central difference formula where we will just use the opposite of this Stirling's formula. This means that we can just consider the average of odd differences is used in Stirling's formula but here we will just use this average of the even differences.

This means that in the a 0 term, a 2 term, a 4 term we can just use this average operators there. Hence this formula can be represented in the form of y 0 equals to a 0 mu y of half plus a 1 delta y of half plus a 2 mu delta square y of half, so likewise we can just express.

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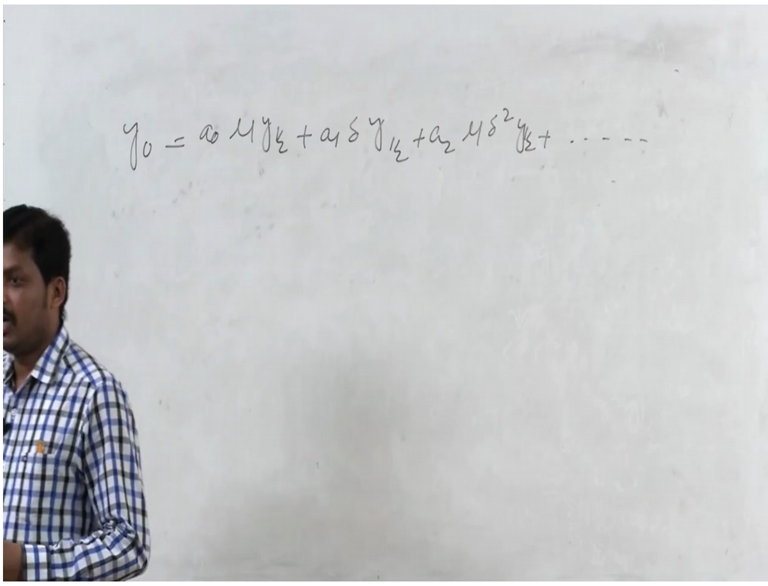

And where these coefficients like a 0, a 1, a 2, a 3, a 4, etc are constants and we can just determine by expanding both the (se) series expansions like if you will just consider like y of x p equals to as 1 plus delta whole to the power p and we can just equate both the sides coefficients then we can obtain the values for this a 0, a 1, a 2, a 3, a 4, etc.

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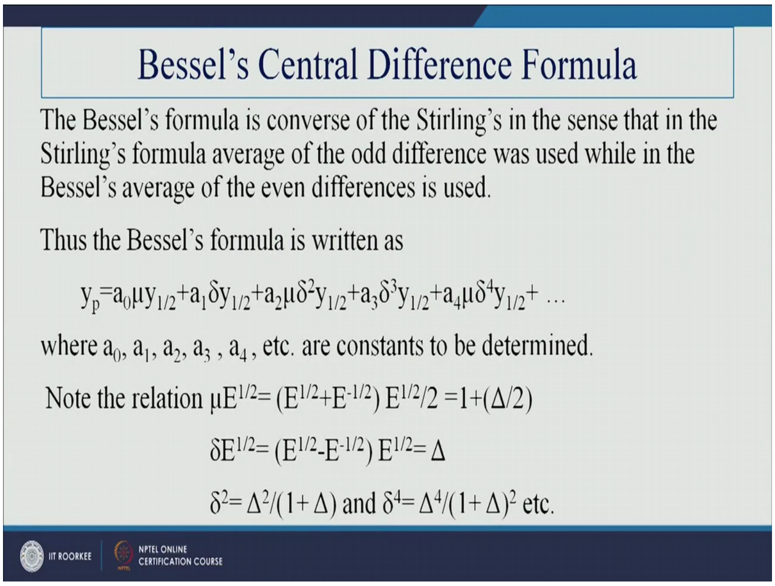

So if we want to expand this one so we have to keep in mind that this average operators like mu E of half it can be written in the form. Since this term here we can just write mu E of half of y 0 and this term also it can be written as mu delta square of E of half here. So if we want to expand this term here we can just write since we are just operating mu operator on E to the power half here, so we can just write this one as E to the power half plus E to the power minus half by 2 into E to the power half here.

Here also the same thing we can just use since we are just using this operator on delta square E to the power half here. So first we can just write this one as E to the power half plus E to the power minus half by 2 into your central difference operator delta square sorry this central difference operator delta square then E to the power half here. So if you will just multiply these terms here, first we will just get E then plus 1 by 2 here.

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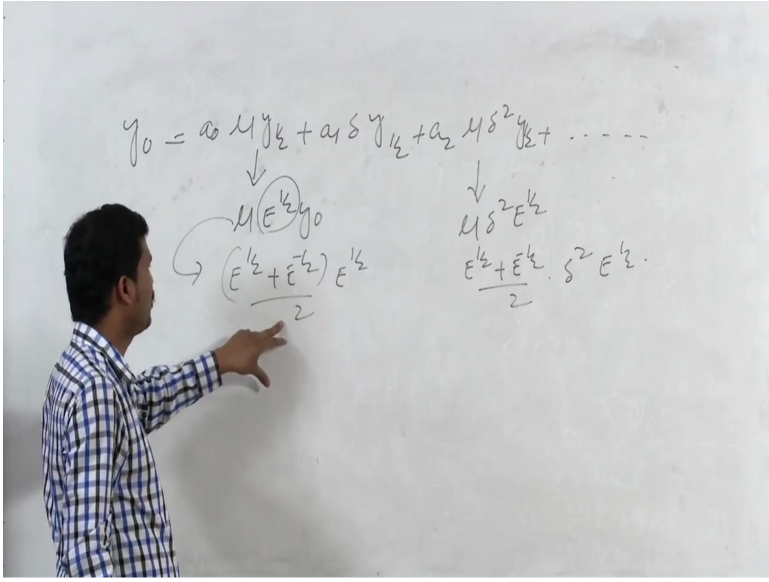

And rest of the terms we can just (equa) equate with both the sides and we can just obtain these coefficients that in the form of like delta E to the power half, it can be written as delta there capital delta and delta square can be written as delta square by 1 plus delta. Already we have discussed these things in the earlier slides and delta to the power 4 can be expressed as delta to the power 4 by 1 plus delta whole square there.

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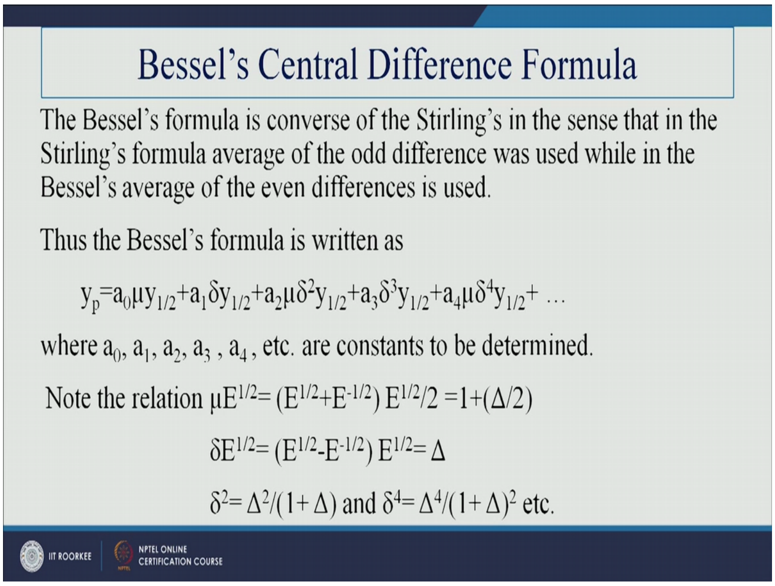

And if you will just put all these terms in terms of capital delta here and equate both the sides then we can just obtain this coefficients a 0 as 1, a 0 by 2 plus a1 as p then we can just get since a 0 is 1 there so a 1 can be computed as p minus half here. And if you will just multiply both the sides 1 plus capital delta throughout the equation and comparing the coefficients for delta square and delta cube we can just obtain p plus a 2 as p into p plus 1 by 2 factorial and a2 as the permutation of p plus 1 2 minus p 1, this equals to permutation of p 2.

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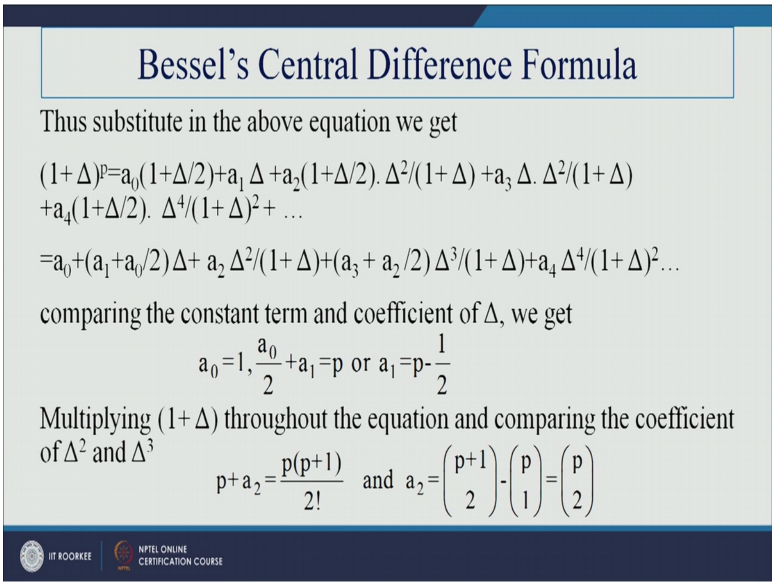

Similarly we can just obtain these coefficients for a 3 and a 4. In general term if we want to write a 2 m that can be written as p plus m plus 1 2 m there. And a 2 m plus 1 this can be written as permutation of p plus m minus 1 2 m into p minus half by 2 m plus 1. And if we will just put all these coefficients and rewrite this formula, the formula can be written in the form of y p equals to mu y of half plus p minus half delta y of half plus p 2 mu delta square y of half, so likewise we can just write.

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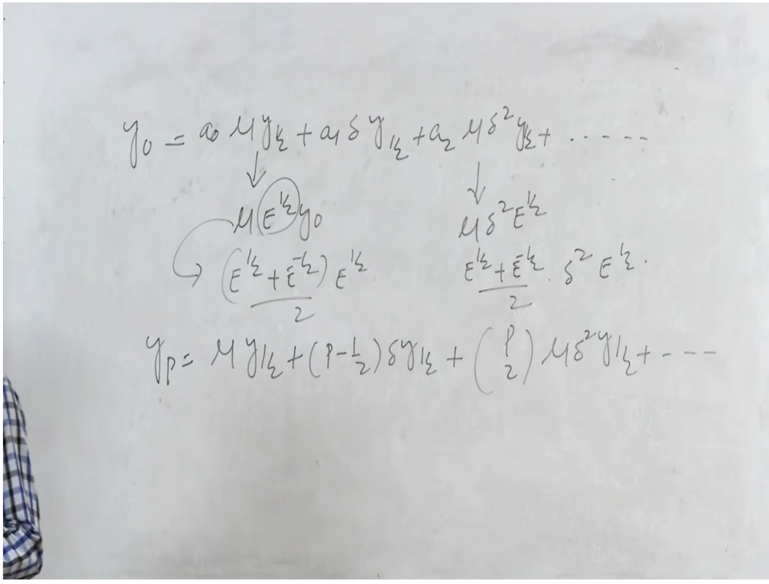

So with this formulas we can just try to solve some of these problems like for defined example it is just given that suppose the values of E to the power x are tabulated at different points starting from 1 point 00 to 2 point 0 with the incremental size point 20 and it is asked to evaluate this value at 1 point 44 here. So if we want to evaluate this value at 1 point 44, we have to consider this value along the centre of the table here since the tabular values are expressed as if you will just see.

So if we want to discuss about this central difference approximations so we have to consider this value along the centre of the table only. So that is why the value it is asked here it is 1 point 44 which is lying between 1 point 40 and 1 point 60 in the tabular values. So if this is the value then first we can just find this divided difference from here like forward differences form. If we will just take the differences so we can just find first difference, second difference, third difference, fourth difference and fifth difference here.

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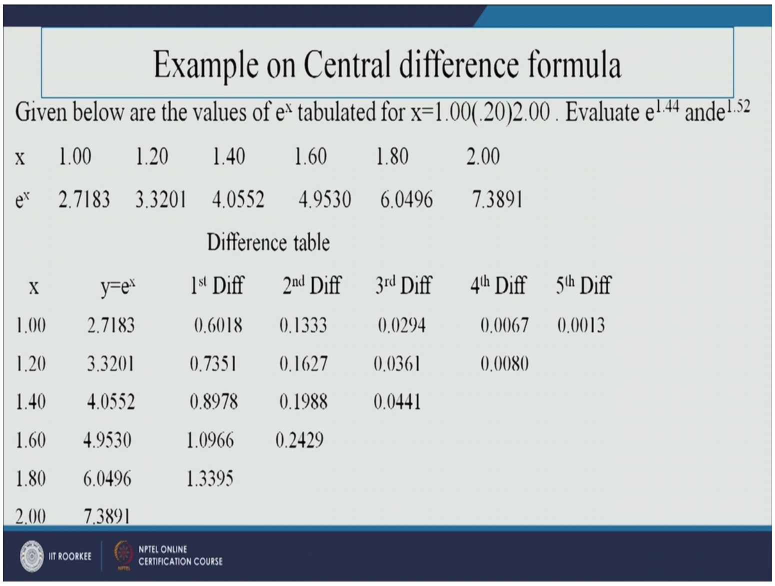

Since the points are in the middle of the table we will use this central difference formula like Stirling's formula or Bessel's formula here. And if we will just consider x equals to 1 point 44 by choosing x 0 equals to 1 point 40 we can just obtain these values of p that is form of like 1 point 44 minus 1 point 40 by 2 point 0 where this formula for p can be used as x equals to x not plus p h where x can be chosen as like 1 point 44 here. And from that formula itself we are just getting p as 0 point 2.

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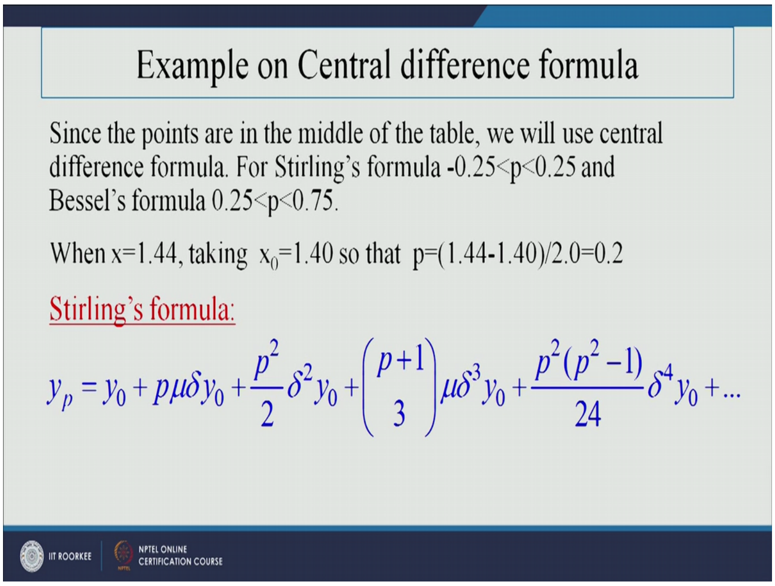

And if you will just use this formula we can just obtain this value as 4 point 22068 and especially if you will see here we are just using this values that is delta y of half plus delta y of minus half since we are just using the central difference approximations. And in the central difference approximation if you will just see that this average operator is operating on delta operator here. Delta of y of half means it can just take the values of y 1 minus y 0. Y of minus half means it can just take y 0 minus y of minus 1.

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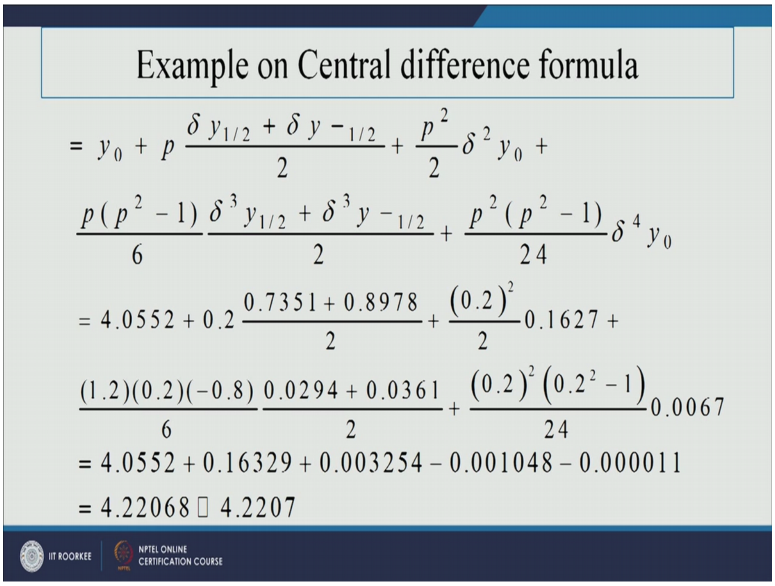

Since the tabular values are associated like y of minus 3, y of minus 2, y of minus 1, y of minus 0 then y 1, y 2, y 3, so as I have discussed in the earlier classes the similar fashion we can just choose this data values and if we will just put in a proper form then we can just use this forward difference table to get the central difference values here easily and in a comfortable form.

Till now we have ended of all these formulations that formed by the central difference approximations and we have solved the examples based on this central difference approximation also. In the next lecture we will just go for some other interpolation formulas to compute these functional values and the polynomial approximations. Thank you for listen this lecture.