Numerical Methods

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Lecture No 20
Interpolation Part V

Welcome to the lecture series on numerical methods. So now we are discussing about interpolation in various polynomials. So last lecture we have discussed about these errors occurring in a Newton's forward difference formula, backward difference formula and the general error terms what is occurring in polynomials. So today we will discuss about in this lecture will just discuss about various examples associated with this Newton's forward difference formula and backward difference formula.

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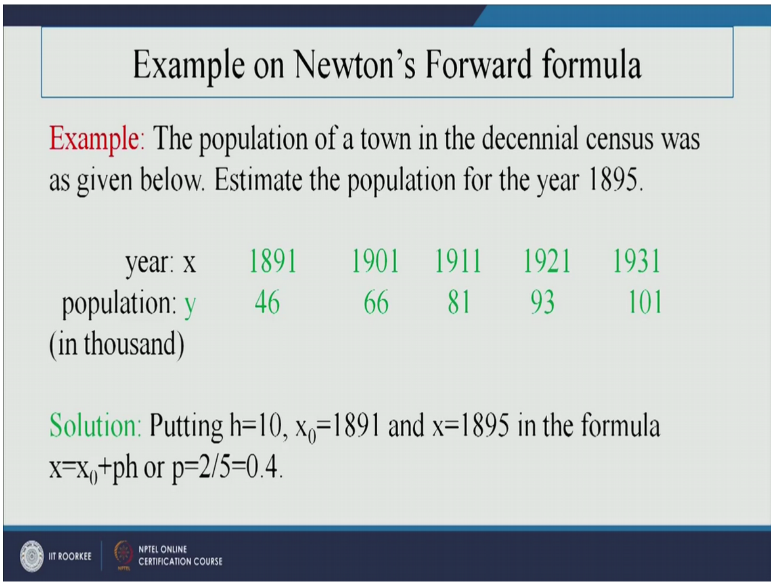

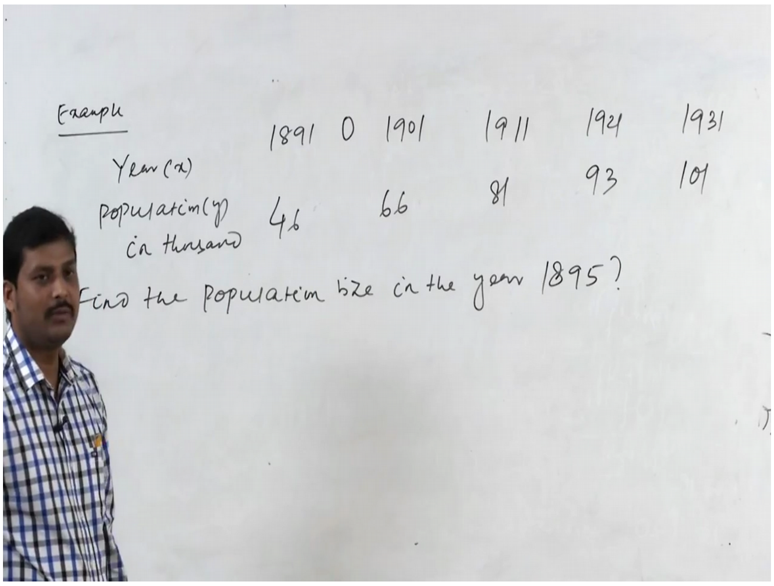

So if you just discuss this example we have to consider certain finite difference points which would be exist at equal spacing's to get the solutions here. So first example if you just consider here is a population model that is given in censual or census in 10 years, suppose we will just consider a problem that first example a population is given for like 5 years here.

So if you are just writing this problem that is year as x and population as y here in thousand that is supposed in 1891 the population size is 46,000 and 1901 the population size is like 66,000 and 1911 the population size is 81,000, 1921 the population size is 93,000 and in 1931 the population size is like 101000. So it is asked to find this population size for the year 1895, find the population size in the year 18 sorry this is 1895.

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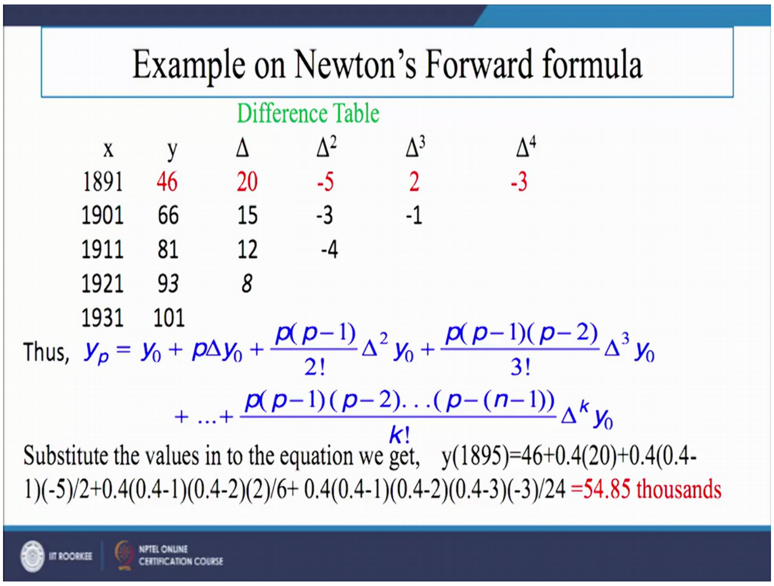

Since if you will just see that this population level is asked in the year 1895 which is existing at the beginning of the table this means that 1895 will lies between 1891 to 9001 here. So that is why we can use Newton's forward difference formula to evaluate this population size at this level here. And for that we will just construct the table and from there itself we can collect the data and then we can use the formula to evaluate this population size in the year 1895 here.

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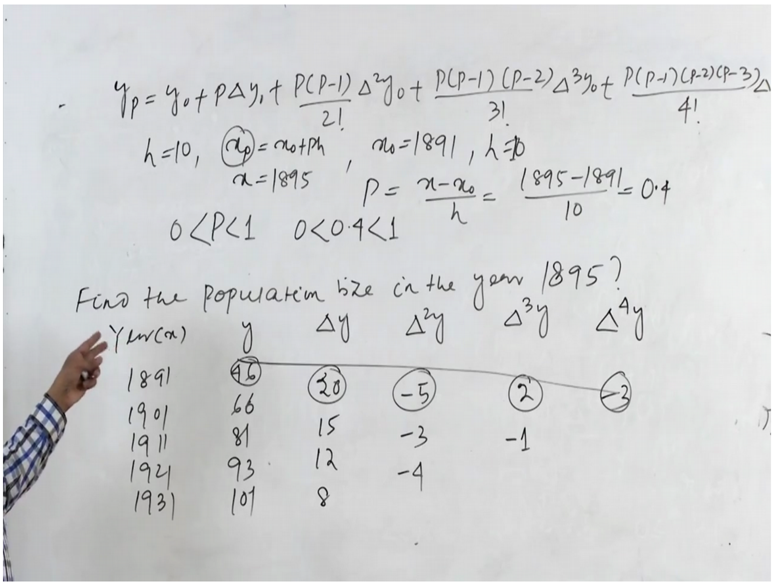

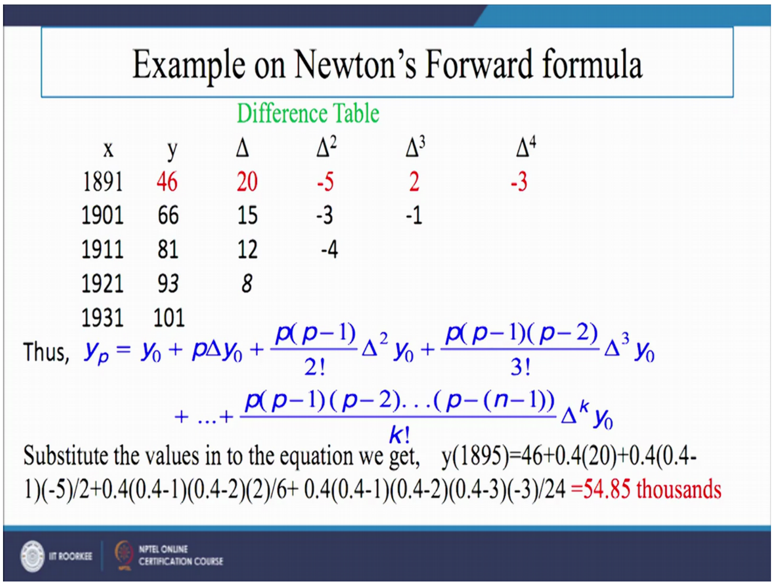

So if you just prepare this table we can just write in a tabular form here first one is here x then the population signees that in the form of y here, so year is given as 1891, 1901, 1911, 1921, 1931 and the population size are like 46, 66, 81, 93, 101 here. And first differences if you will just calculate here del y, 66 minus 46 that will just give you 20 here, and next difference 81 minus 66 this will just give you 15 here, 93 minus 81 that will just give you 12 here and 101 minus 93 that will just give to 8 here.

And for the 2nd differences if you will just compare here that is in the form of del square y here, so this is minus 5 here, then minus 3 here, then minus 4 here. If you will just go for 3rd differences here that is 2 here, then 1 here, so sorry this is minus 1 here, minus 4 so this is plus 3, so then the final difference it is del 4 y, so it will just take like minus 3 here. And if we are just seeing here so the final value will be approaches towards the beginning of people, so that is we can just consider this tabular values are in this form here.

This means that if you will just use this formulation for Newton's forward difference formula always this point is shipping towards the beginning of the table so that is why we can just consider the upper part of the tabular values for the computation here and we have signified that one in like red colour symbols in the data in the slides.

So if you just write the formula in your y p can be written in the form of y 0 plus p delta of y 0, p into p minus 1 by factorial 2, del square of y 0, p into p minus 1 into p minus 2 by factorial 3 del cube of y 0 plus p into p minus 1 into p minus 2 into p minus 3 by 4 factorial delta 4th of y 0 since your tabular value is existing upto fourth order difference here, so we can just consider up to 4th order term there.

So if we want to compute p first since the year it is asked to find in the year 1895 here, so first we have to find the value of p here. So if you will just see this spacing here, the spacing is 10 here and usually the formula is written as x 0 sorry x p equal to x 0 plus ph or the data which is asked to find which can be represented in the form of x here and which can be written as x 0 plus ph and x 0 is exempted as 1891 here and h is 10 here, so we can just compute that is our xp value for the year which we have to compute that is 1895 here.

So from this 3 data points sorry this is 10, so from this 3 data points we can compute p as x minus x 0 by h here, so which can be written as 1895 minus 1891 by 10 here, so this can be the computed value will be 0.4 will. So if you will just see here p value should be lies between 0 and 1 for the application of Newton's forward difference formula, so that condition is satisfied here since p value is lies between 0 and 1 here.

So next if we can just use this formula here, since p value is known to us, y 0 value is this value here and del y 0 value is this one here then del square y 0 here, then del cube y 0 value is this one here and del 4th y 0 value is known to us here and p value in 0.4 here and all other values are known to us. So now you can just compute what is the population size in the year 1895 here.

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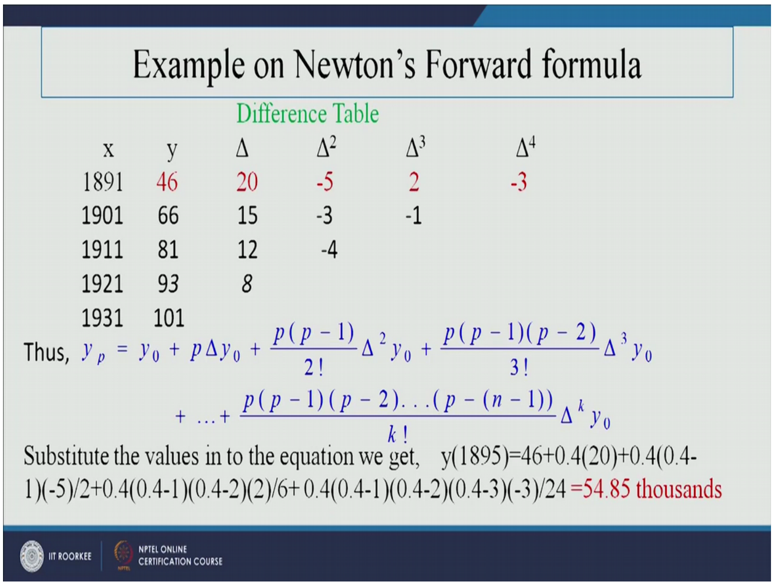

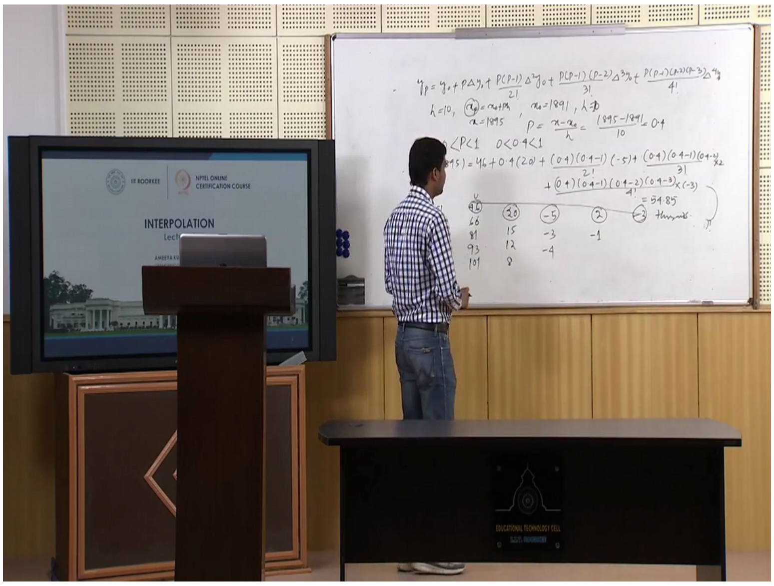

So if we will just go for the computation of this formula here I can just write that one as in the form that is y of 1895. This value first value is 46 year, 46 plus you can just write p value there, p delta of y 0, so i can write here y of 1895 this equal is to y 0 is 46 year plus p value is 0.4 here into del of y 0 is 20 plus p value is 0.4 here, so 0.4 minus 1 here by 2 factorial del square of y 0 is minus 5 here plus p, p minus 1, p minus 2 by 3 factorial into the last tabular value here is 2 here plus your last value here is 0.4, 0.4 minus 1, 0.4 minus 2, 0.4 minus 3 divided by 4 factorial into your last tabular value here minus 3 and the final answer it will just come as 54.85 thousand.

So if you will just see here the population size is whatever is just giving here this should be lies between 46 and 66 here, so that is why this population size is accurate in error of this increment of the values in the preceding years here. So if you will just go for a backward difference formula since all of this values are known to us then we can just use this backward difference formula if the value is asked to compute at the end of the table.

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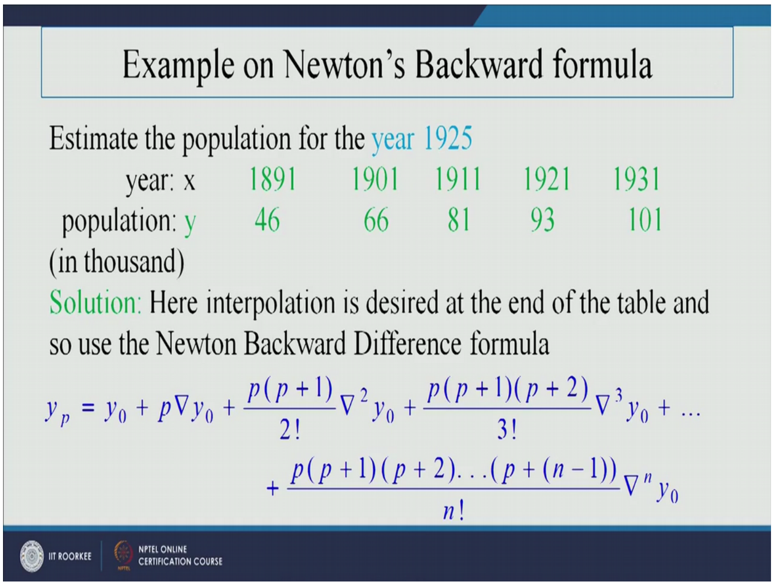

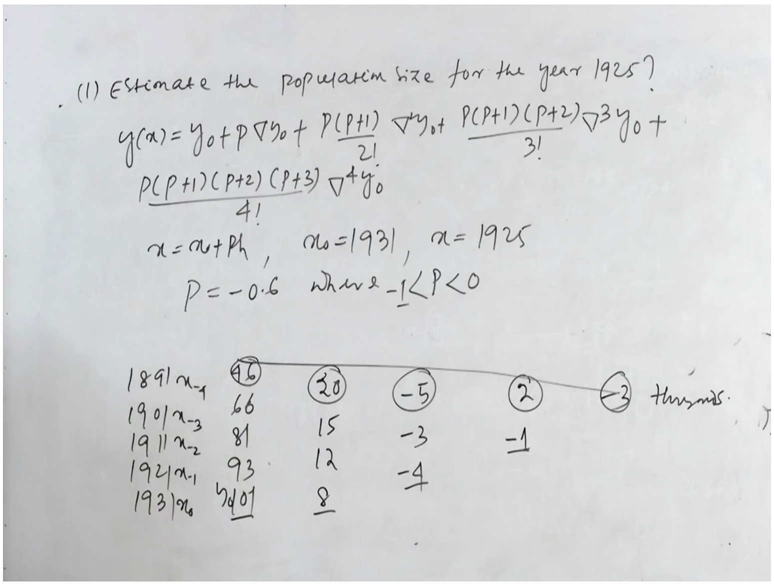

In the 2nd question it is asked that estimate the population for the year 1925 here. Since if the question is asked that estimate the population size for the year 1925, so 1925 if you will see so this is just existing at the lower end of the table here, so that is why we can use Newton's backward difference formula for the competition of the value or to find this population size for the year 1925 here.

So if we can just use this tabular values here then we can write this backward difference formula as y of x as y 0 plus the nabla of y 0, p into p plus 1 by factorial 2, nabla square of y 0 p into p plus 1 into p plus 2 by 3 factorial, nabla cube of y 0 here plus p into p plus 1 into p plus 2 into p plus 3 by 4 factorial, nabla to the power 4 of y 0 here. So now we have to first compute p according to the backward difference formula then we will just go for backward difference tabular values and we can just put that values to obtain the population size for this year 1925 here.

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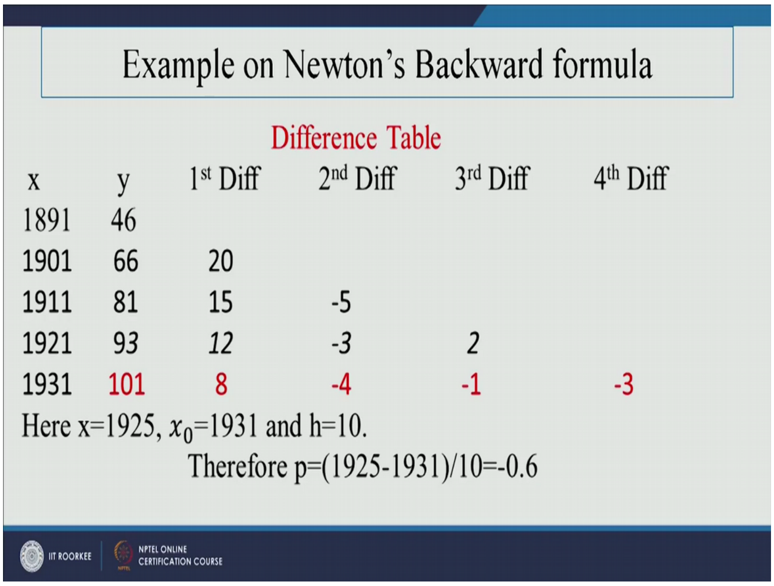

So if you just show this p value here, so p can be written as your x equal to x 0 plus ph here, so if you will just consider x 0 equals to 1931 here, your x value is asked at 1925 years. So p value can be computed as minus 0.6 here, where p is lying between minus 1 to 0 here, hence we can use Newton's backward difference formula since the basic difference is that if P lies between 0 and minus 1 we can use Newton's backward difference formula to compute the tabular values at the end of the table.

So that is why p is just giving a value that is as minus 0.6 which is lying between zero and minus 1, so that is why you can use backward difference formula there. And now we can just put this value y 0 value is starting here, so that is why your x 0 value can be considered as 1931 here, x of minus 1 value can be considered as 1921 here, x of minus 2 can be considered as 1911 here, x of minus 3 can be considered as 1901 here, x of minus 4 can be considered as this value here and the relating values we can just use 101, 8, minus 4, minus 1 and minus 3 there.

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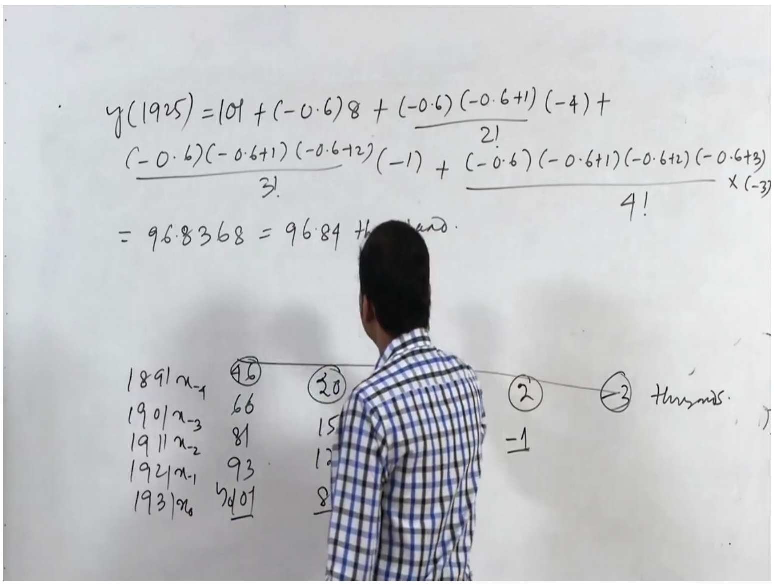

So if you will just write in a complete tabular form here, so that we can just write as, so y of 1925. So first value is y 0 value here 101 so plus minus 0.6 into 8 plus p into p plus 1 divided by 2 factorial, nabla square so that value is minus 4 here, plus minus 0.6 minus 0.6 plus 1, minus 0.6 plus 2 divided by 3 factorial and this value is minus 1 here plus minus 0.6, minus 0.6 plus 1, minus 0.6 plus 2, minus 0.6 plus 3 divided by 4 factorial into last value that is minus 3 here. And the final population size is 96.8368 here and obviously if you will just compare this one upto 2 terms here we can just write this one as 96.84 thousand.

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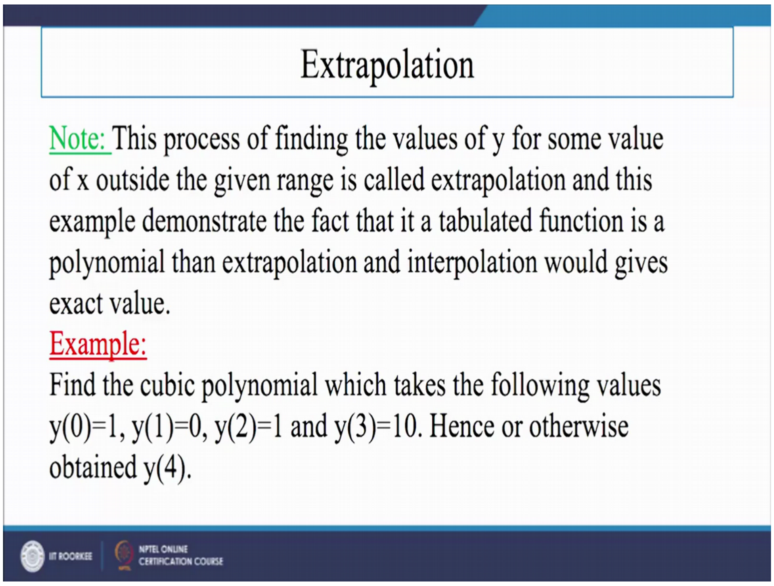

So next we will just go for extrapolation since sometimes we are just finding that if they value is existing outside the table, so that is basically called your extrapolation. The process of finding the values outside the table even if all of this tabular values are given, so by considering all this data we are existing on this tabular values we can just find out the preceding or succeeding values by considering this overall data either it can be a future data or it can be passed data, so that is basically called extrapolation, if the value is computed inside the table that is basically called interpolation here.

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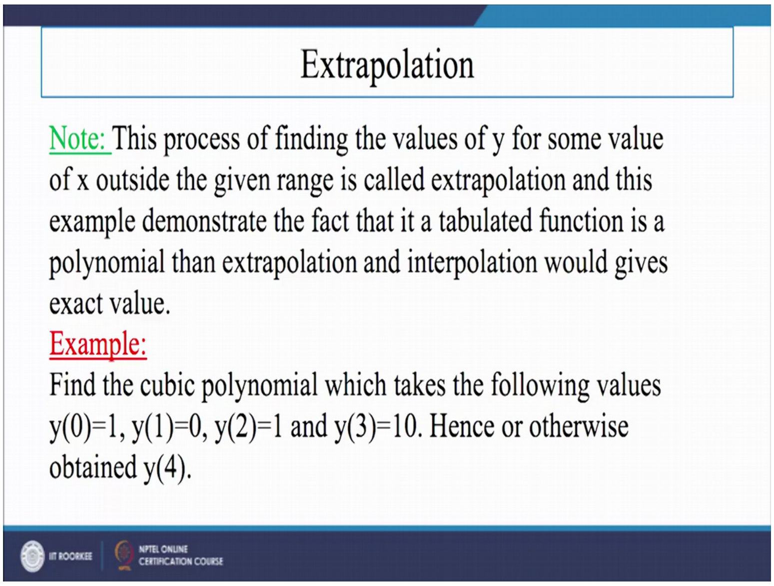

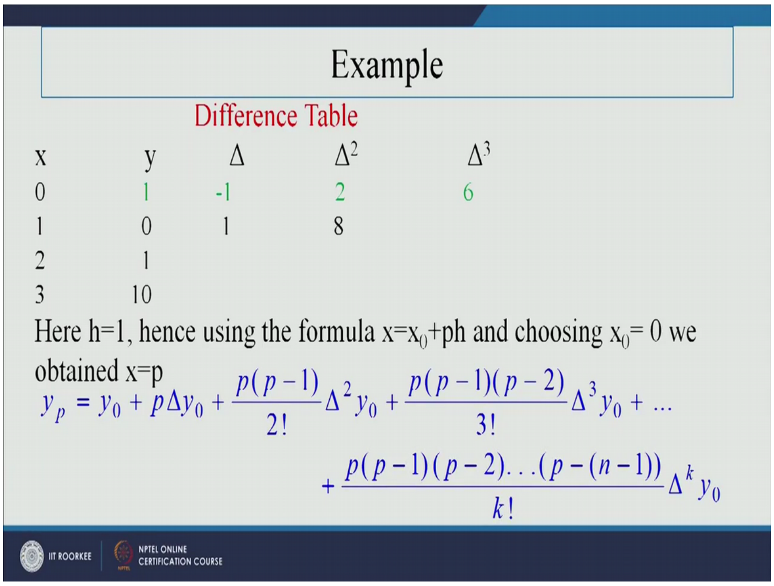

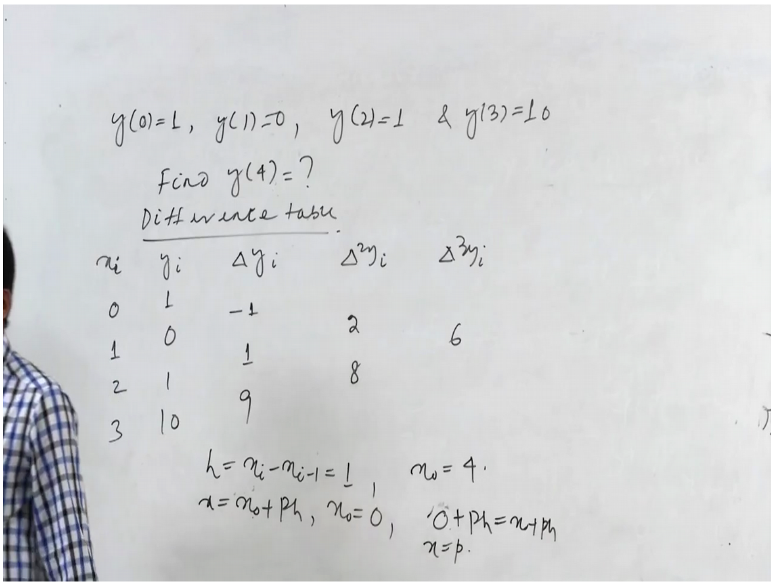

And if you just consider for example that if a value is asked to compute outside the tabular values then it is called speculation and we can use a sample to find these extrapolated values here. So if the question is asked suppose find the cubic polynomial which takes the values like y of 0 equals to 1, y of 1 equal to 0, y of 2 equal to 1 and y of 3 equal to 10 here and it is asked to obtain what is the value of y of 4 here?

Since 3 or 4 preceding values are known to us we can just use this interpolation method to evaluate the value at the point 4 there itself. So if you will just write this difference table here, so that is difference table can be written in the form since X 0 or x i data's then corresponding y i data's. So your x 0 value is 0 here, than 1, then 2, then 3 and y i values are like 1, 0, 1 and 10 here.

So if you will just find this forward difference tabular values here we can just write del of y i, del square y i and the del cube of y i here. So if you just consider this difference, 1st difference is minus 1, 2nd difference is 1, 3rd differences is 9 here and 2nd difference if you can just see here that is 2 here then this is 8 here and last tabular value is 6 here. And if you just see here the spacing that is x can be defined as x i minus x i minus 1 and it can be obviously your value is 1 here and it is asked to compute the value at x 0 equal to 4 here.

So if we can just right here x equal to x 0 plus ph here and x 0 as 0 here, since the value we have to compute that is in the form of x 0 plus ph here and x 0 is 0 it is just given. so that is why we can just write your ph, so x equals to x 0 means we can just write this one as x equals to p there, x equal to x 0 plus ph here. So that is why we can just write this one as x 0 is 0, so that is why x 0 plus ph here, so 1st part if this is 0 here, so 0 plus ph is this one hence we can just right here x as p here.

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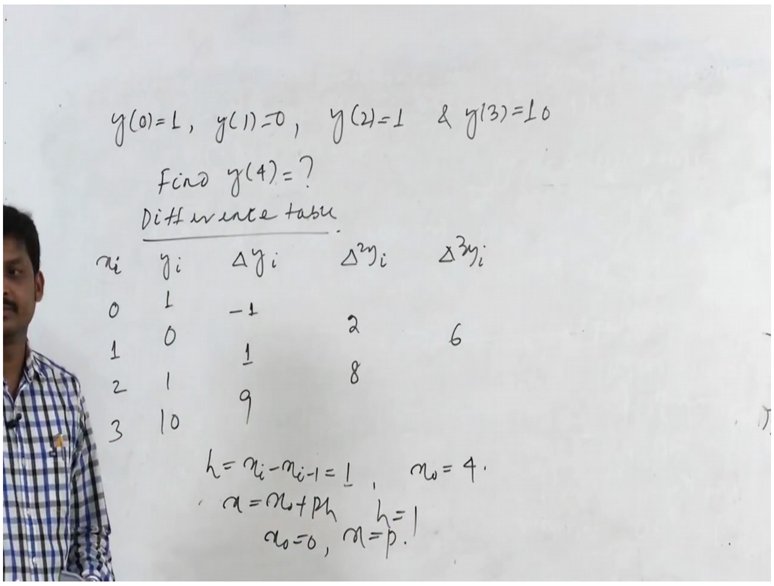

So if we want to compute this value at p equals to x here this means that h equals to 1, first it is just given. I want to clear this things here since x is asked to compute, so x equal to x 0 plus ph here and h is a 1, so we can just write x 0 equal to 0 here, so that is why x equals to p here. So if h equals to 1 and p equals to x here we can just obtain this function y of x in the form of a polynomial that can be expressed in the form of x here.

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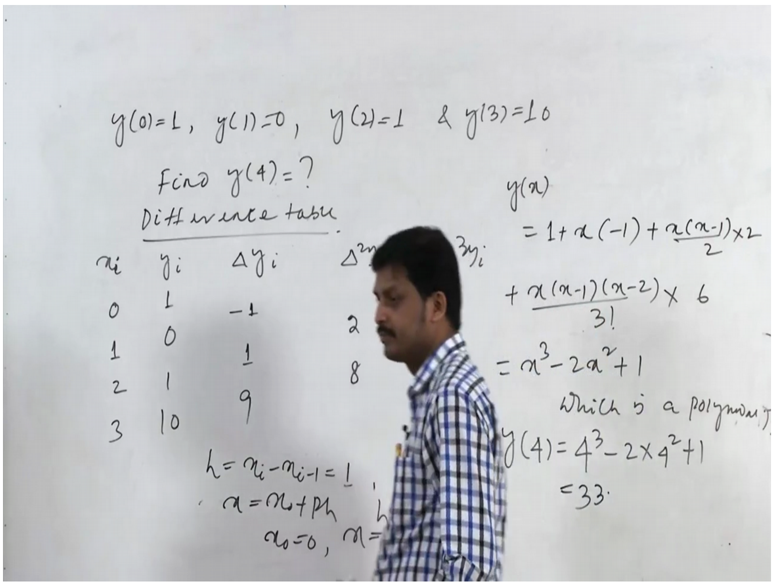

Since 1st value if you just consider here y 0 as one here plus p, p is expressed as x here, so p delta of y 0 that is minus 1 here plus p into p minus 1 and by 2 factorial into next tabular value that is 2 here plus x into x minus 1 into x minus 2 by 3 factorial into last tabular value that is 6 here. So which can be expressed in the form of your x to the power 3 minus 2 x square plus 1 here which is a polynomial and if we want to find the value of y at 4 here we can just write 4 to the power 3 minus 2 into 4 square plus 1 here, so it can be written as 33.

So next we will just go for this Central difference table. Now we can just discuss about Central difference approximations. So if sometimes the value is asked to compute at the middle of the table then you can use this Central difference approximations. So basically this Central difference approximations includes Gauss forward difference formula, Gauss backward difference formula and Vessel's formula and Stirling's formula.

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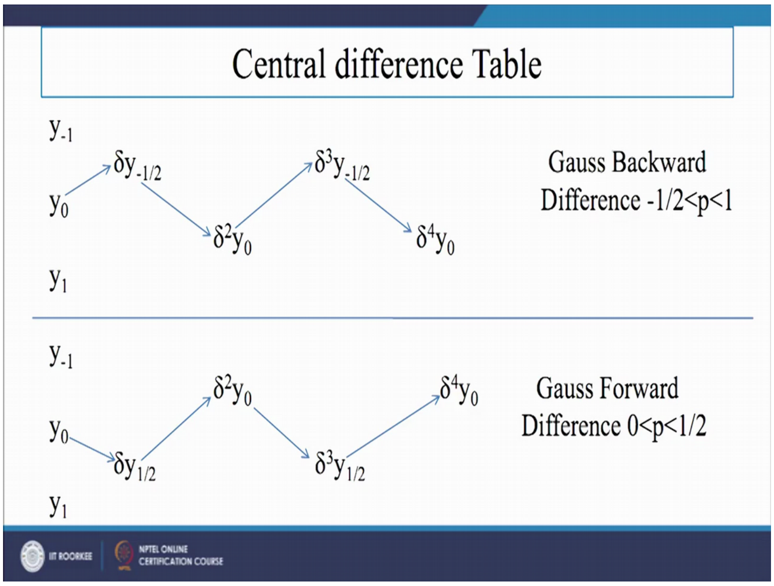

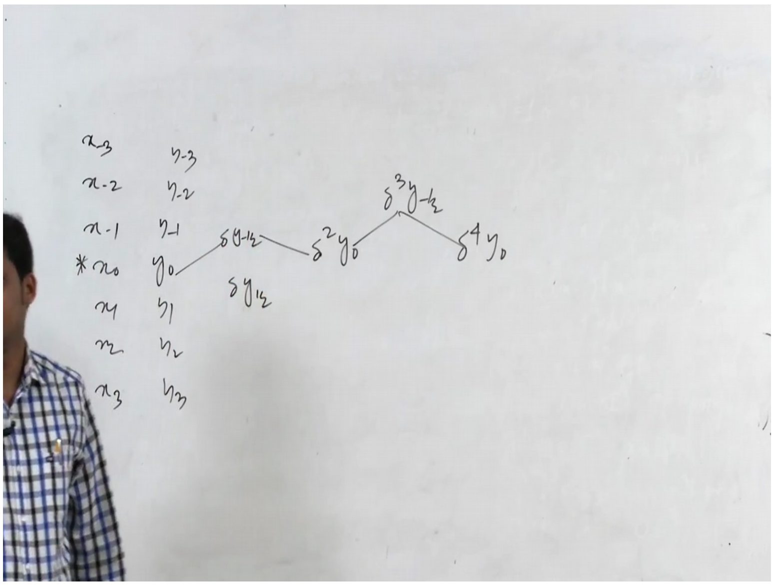

So in Central difference table if you will just use Gauss backward difference formula, so the P value should be lies between minus half to 1 and if you just use this forward difference formula and then the p value should lies between 0 and half there. So if we will write this Central difference table in Gauss backward difference form then this table can be written in the form since we are just approaching here the values like x of minus 3, x minus 2, x of minus 1, x 0, x 1, x 2, x 3 and we are just comparing this value at the middle of the table here.

Since already we know that the formula that is Gauss forward formula which can be used at beginning of the table, backward formula which can be used at the end of the table. So that is why we are just trying to evaluate this formula which should be used at the middle of the table here. So that is why we can just express this tabular values as x 0 as in the middle of the table and all other points preceding and succeeding are in the form of minus and plus 1 here.

So the corresponding y values or these functional values can be expressed in the form of y 0, y of minus 1, y of minus 2, y of minus 3, y 1, y 2, y 3 here. And if you just use this Central difference approximations here we can just write this Central difference approximation as del of y of minus half here and del of y of 0 half here. So similarly we can just write del square of y of 0 then del cube of y of minus half then del to the power 4 of y 0.

So likewise we can just consider this tabular values for the Newton's backward difference, we are just going one step back to evaluate this tabular values there, so that is why it is called Gauss backward difference formula and if we will just go forward one step here then we can just say that that is a Gauss forward difference formula here.

So for the forward difference formula if you will just go here so that can be represented in the form of first function is y 0 here then del of y of half then we can just go like del square of y 0 here then we can just go like del cube of y of half here, then del to the power 4 of y 0, so likewise we can just go.

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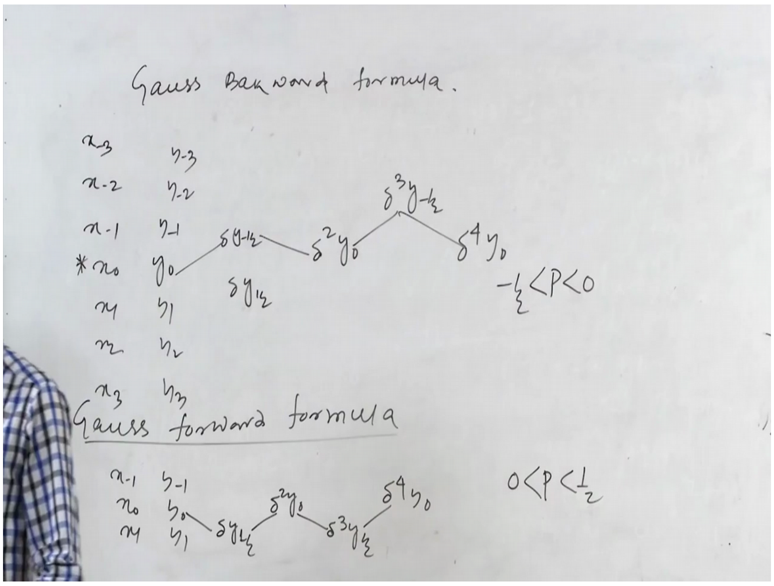

So this is for a like a Gauss backward formula if you will just go for the forward formula here we can just write Gauss forward formula as that we can just consider the Central difference approximations that is in the form of x of minus 1, x 0, x 1 here, so y of minus 1, y 0, y 1 here. So this difference is 1st will just go for here delta y of half here, then del square of y 0 here, then del cube of y of half here, then del to the power 4 of y 0 here. This means that a forward marching step we are just approaching here to get this forward difference formula. So first case we are just telling that p should be lies between minus half to 0 here and in this case p should be lies between 0 and half here this is the condition.

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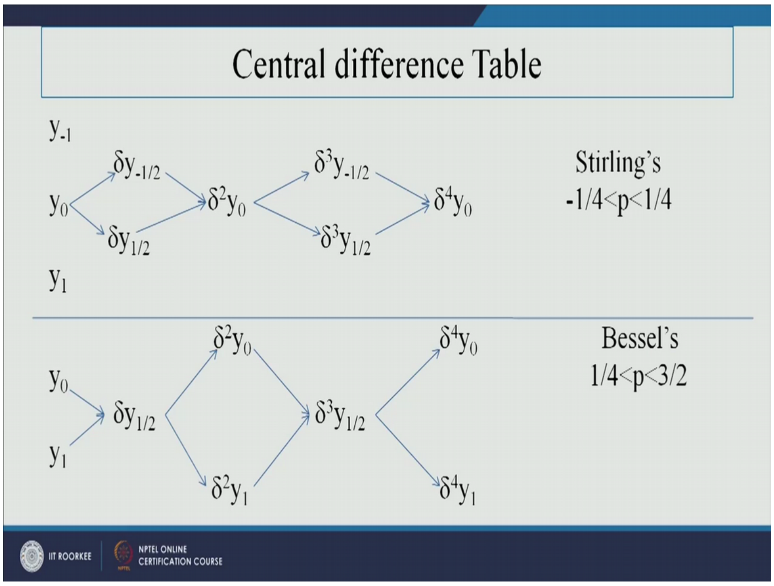

If we will take the average of these 2, so then we can just obtain this Stirling's formula where p should be lies between minus 1 by 4 to 1 by 4 there. And if you will just take this even differences like average of even terms there then we can just obtain Vessel's formula, where this p value should be lies between minus 1 sorry 1 by 4 to 3 by 2 there.

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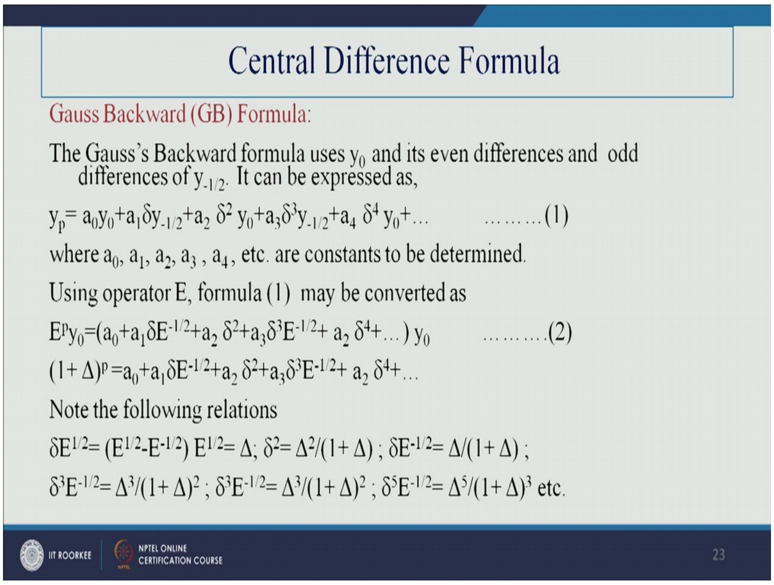

So now we will just go for a complete elevation of this Central difference formula. In the Gauss backward difference formula and we can use y 0 then it's even differences, even differences if you will just see like y 0, del square of y 0, del to the power of y 0 we are just using here. And odd differences of Y of minus half, odd differences means this power if you will just see here that is delta of y of minus half here, del cube of y of minus half here.

So likewise all the even powers Y 0 we are just using square here, 4^{th} here, then 6^{th} again and if you will just go for this odd differences here, so odd differences means it can be taken y of minus half here, so this is 1 here, then this is 3 here then again this is 5 there, so likewise it will just continue. And our aim is that for the derivation of this formula we have to consider y p as in the form of a 0, y 0, a 1, delta of y of minus half there, a 2 del square y of 0 there, a 3 del cube of y of minus half plus a 4 del to the power 4 of y 0.

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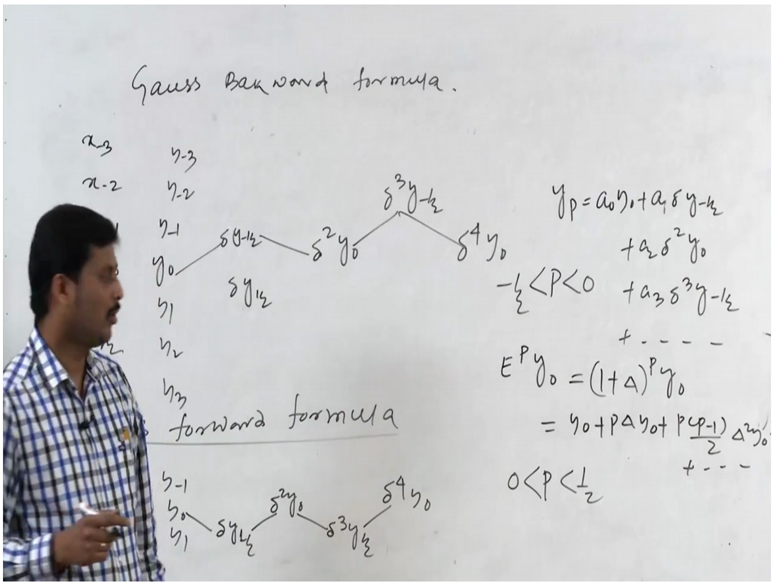

So if we can just express in this form here then y p can be written as a 0 first-term we can just write as a 0 y 0, a 1 delta of y of minus half here, a 2 delta square y 0 here, a 3 del cube y of minus half here, so likewise we can just express. Since we are just expressing y p as in this form here if we can just express y p in terms of delta and if we will just compare both sides the coefficients we obtain the value of a 0, a 1, a 2, a 3 all the coefficient there.

So if we want to compute this coefficient first we have to express y p in terms of Newton's forward difference formula here. So if we just express y p in terms of Newton's forward difference formula and we can just write that as E to the power of p of y 0 here and which can be written as 1 plus delta whole to the power p of y 0 here. And if you will just expand this coefficient in terms of delta here which can be expressed also in the form of Central difference operators we can just find all coefficients there over.

so if you will just use this operator then we can just write that in the form of y 0 plus p delta of y 0, p into p minus 1 by factorial 2 del square of y 0, so likewise we can just right here. So it can also be expressed in the form of like Central difference form that is if you will just write delta p to the power sorry delta y of minus half here, so it can be written as delta means y of half, we can just write E to the power minus half of y 0 there. Then we can just use this Central difference operator E to the power half minus E to the power minus half into product of the terms.

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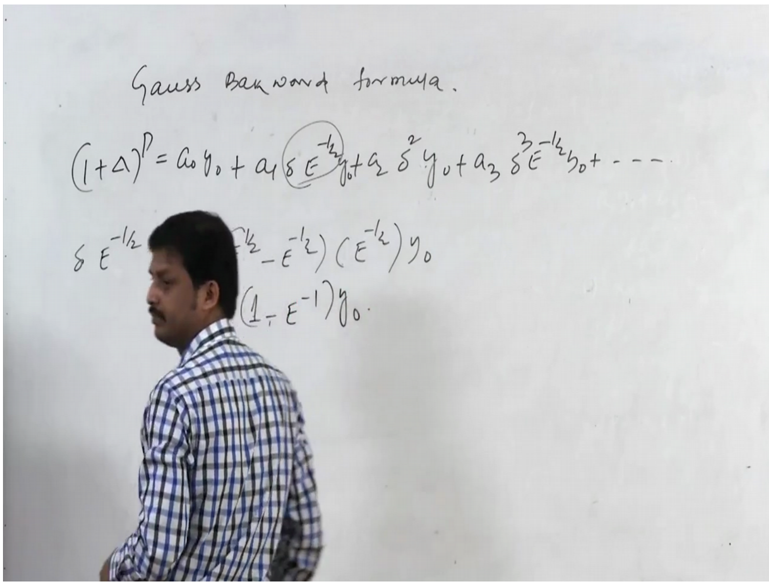

So if you will just write that one we can just write that one as 1 plus delta whole to the power p this can be written as a 0 y 0, plus a 1 delta E power of minus half, a 2 delta square of y 0 here, plus a 3 delta E power of delta cube E to the power of minus half of y 0, so likewise we can just write. And if will just try to find this coefficient here, so it can be written as delta to the power minus half of y 0, so which can be expressed as E to the power half minus E to the power minus half into E to the power minus half here of y 0. And if you just take this product here this can be expressed as E minus 1 or if I am just adding it appear so I can just write E to the power half this is one minus E inverse of y 0 here.

Till now we have discussed this Newton's forward difference formula and backward difference formula and the examples based on it. And in this lecture also we have discussed about this Central difference operators and in tabular form how it can be related and in the next lecture we will just continue this Gauss forward difference formula and backward difference formula, thank you for listening this lecture.