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Nonlinear Programming - 1

Lec – 19
Search Techniques-II

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So hello friends welcome to lecture series on nonlinear programming, so we were discussing about first techniques because we know that there are some nonlinear problems which cannot be solved analytically or it is difficult to solve by the analytical approach so we go to numerical techniques we go to some numerical such techniques so that we can solve some nonlinear optimization problems in the last class I have discussed that what is order of convergence what are you model function and one such technique that is dichotomous search technique.

Now in this lecture we will deal with some more such techniques to find out an at least approximate optimal solution of for nonlinear programming problem okay now the next technique is next search technique is Fibonacci is search technique, so before starting Fibonacci search technique what is and what is the Fibonacci series okay.

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$$1, 1, 2, 3, 5, 8, \ldots$$

$$F_0 = F_1 = 1.$$

$$F_K = F_{K-1} + F_{K-2}, \quad K \geq 2.$$

So Fibonacci series is basically as you already know first term is 1 the second term is again 1 and the next term is simply the sum of the previous two terms, so 1 + 1 is 2 then, 2 + 1 is 3 then 3 + 2 is 5, then 5 + 3 is 8 and so on that means in the Fibonacci series the first two term first term is f0 the second term is F1 which are 1 and the k^{th} term any k^{th} term is simply the sum of two previous terms that is simply fK - 1 + fK - 2 where K ≥ 2 okay.

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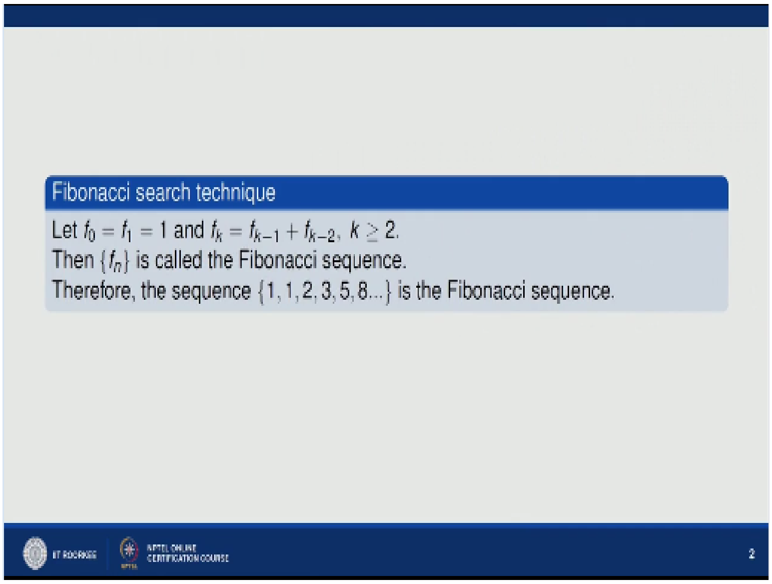

So that that is how we can define the Fibonacci series or Fibonacci sequence, that is simply you take any term if it is the sum of the first two terms and the first term and the second term are 1 then such series are called Fibonacci series okay now based on this we have a search technique called Fibonacci series technique now what is algorithm let us see, so let us suppose what we do in first technique we start with an initial interval of uncertainty it which is given to us a, b and we have we in each iteration we go on reducing the size of the interval of uncertainty such that they next interval of uncertainty which we obtained after performing some experiments will contain the optimal point here we are dealing with unmoral minimum functions.

So I can say we can say that we are trying to find interval of uncertainty which contain a point of minima okay by reducing the size of the interval of uncertainty so let us suppose the start with a comma B which is the initial level of uncertainty okay.

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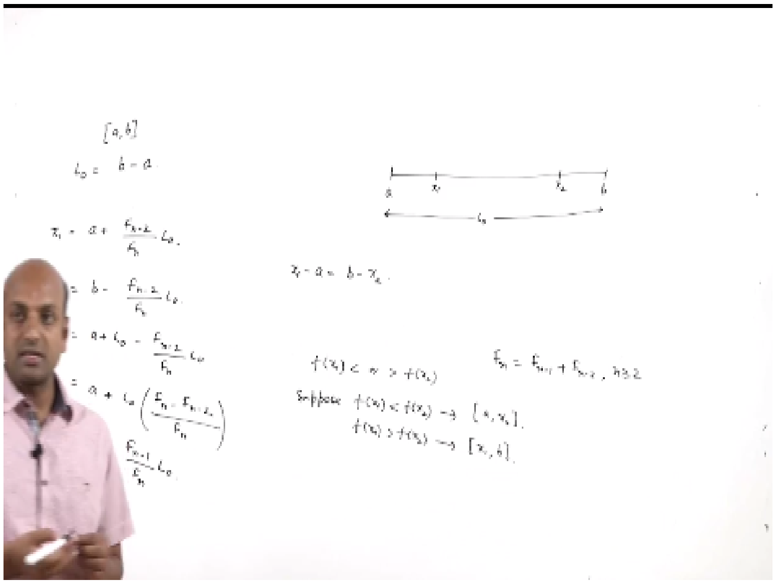

So we start with a, b which is the initial level of uncertainty okay so what is what is the length here what is the length of this interval it is b- a okay now we performed two experiments we fix up two experiments how you fix you two experiments you see we have this interval a, b a to b this length is l_0 okay this Δ l_0 now we place to experiment x_1 here and x_2 here we find two experiment X_1 and X_2 such that now X_1 =a+ F_n - 2 upon Fn into l_0 this is how we can we will find X_1 the first experiment okay.

And the thirdly second experiment X_2 is simply b - F_n - 2 upon F_n into l_0 so that is how they find the two experiments okay and n is will be given number of experiments to be performed okay that is given to us suppose n is a number of experiments should be performed so based on this the first to experiment x_1 x_2 will find now from here we can easily say that x_1 - a which is this term okay is equal to b- x2 that is very simple that is very easy to say because s1 - a is same as b - x2 so what does it mean this means that this distance is same as this distance.

So that means we are finite to experiment x1 x 2 such that they are placed symmetrically about the endpoints from the end points we are fired to experiment x1 x2 such that the two experiments are placed symmetrically from the end points okay, now again if we try to solve this thing so it is equal to what is b what is b from here it is L_0 + a that is a + L_0 – fn – 2 / Fn into L_0 so that is equal to a plus you take L_0 not common.

So it is Fn - Fn – 2 / Fn okay that is from the same of these two terms now from the Fibonacci series we know 2x Fn is simply Fn - 1 + Fn – 2 for n> = 2 okay so what is Fn – fn 2 that will be Fn - 1 from here so it is a + Fn – 1 / fn into L_0 so we can also say that x 1 is this or x2 is a + Fn – 1/ fn into n okay, now that is how we will find the two experiments now we will see we will see whether F (x1) is less or greater than F(x2) next we'll find whether F(x1) is less than F (x) or F(x1) is greater than F(x2) so this will decide that what will be the next interval of uncertainty suppose.

Suppose f(x1) is suppose f(x1) is less than f(x2) so next interval of uncertainty will be f(x1) less than f(x2) so this is a, x2 okay and a for fx1 is more than fx2 so next interval iterative will be x1 to be so this is how we can find out the next interval of uncertainty, okay now suppose the next interval of uncertainty is this okay.

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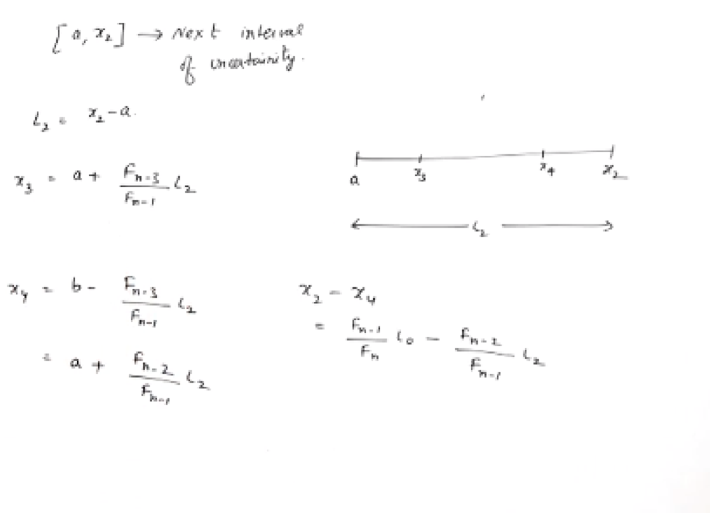

Now let us suppose the next interval of uncertainty is a , x2 it is next interval of uncertainty okay if it is if it is x1 if it is close interval x1 to b then same process will be followed, okay I am discussing here the 1 for the 1 interval the same process will be followed if the interval is x1 to b okay, so now what will be the length is x2 - a okay, and now how we will find a new to experiment the next two experiments suppose next to experiments are x3 and x4 so now that now the interval is now the length is a2 x 2.

And this length is l2 now again we find two is two new experiments say x3 and x4 such that they are placed symmetrically from the endpoints again and for x3 x3 is equal to a plus + fn - 3 upon Fn – 1 into l2 and it is b - 0 Fn – 3 / fn – 1 into L2 okay, simply replace n by n - 1 here and L_0 by L2 we will get that next two experiments okay, now it is also equal to from this expression because x2 is equal to this so from this expression it is also equals to a + you simply replace n by n - n + 1 fn – 2 / fn - 1 into L2 okay.

Okay so this is how we will find the next two experiments now the next thing to see here say now you find say x2 – x4.

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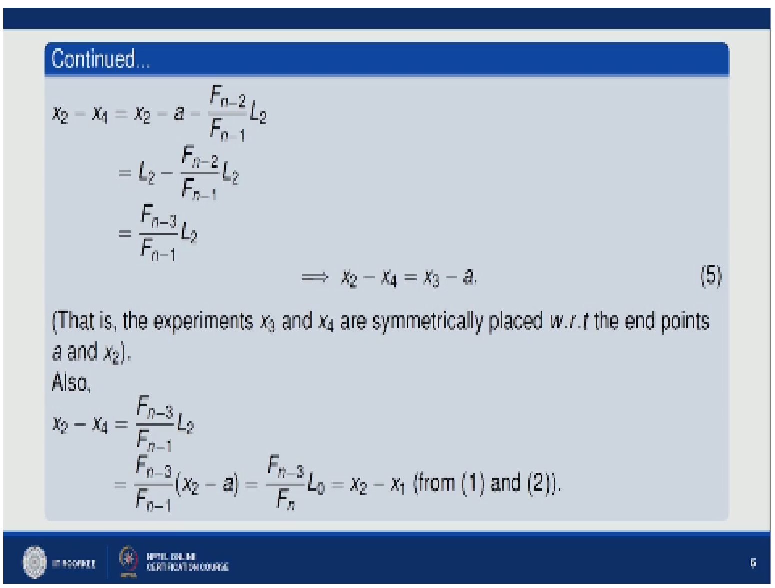

So let us find x2 – x okay so what is x2 now x2 is this quantity okay and x 4 is this quantity so when you subtract these two what we obtain we obtain FN okay it is in L2 it is an L_0 okay so first we will make the same interval first you try to make the same interval okay, so X4 is you can either use this or you can use this so it is X2 is FN – 1/FN, now L0 is now suppose it is L0 - X 4 is it is FN – 2/FN-1x L2 okay.

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$$x_2 - x_4 = x_2 - a - \frac{F_{n-2}}{F_{n-1}} L_2$$

$$= L_2 - \frac{F_{n-2}}{F_{n-1}} L_2$$

$$= \frac{F_{n-3}}{F_{n-1}} L_2$$

$$\implies x_2 - x_4 = x_3 - a. \quad (5)$$

(That is, the experiments x_3 and x_4 are symmetrically placed *w.r.t* the end points a and x_2).

Also,

$$x_2 - x_4 = \frac{F_{n-3}}{F_{n-1}} L_2$$

$$= \frac{F_{n-3}}{F_{n-1}} (x_2 - a) = \frac{F_{n-3}}{F_n} L_0 = x_2 - x_1 \text{ (from (1) and (2)).}$$

So it is it is X4, X 2 is you see from the PPT X2 is simply X2 and X4 is X 4 is a + Fn – 2/ Fn -1 x L_2, now X2 - a is L_2, X2 – a 0s L_2 and minus this a times L2.

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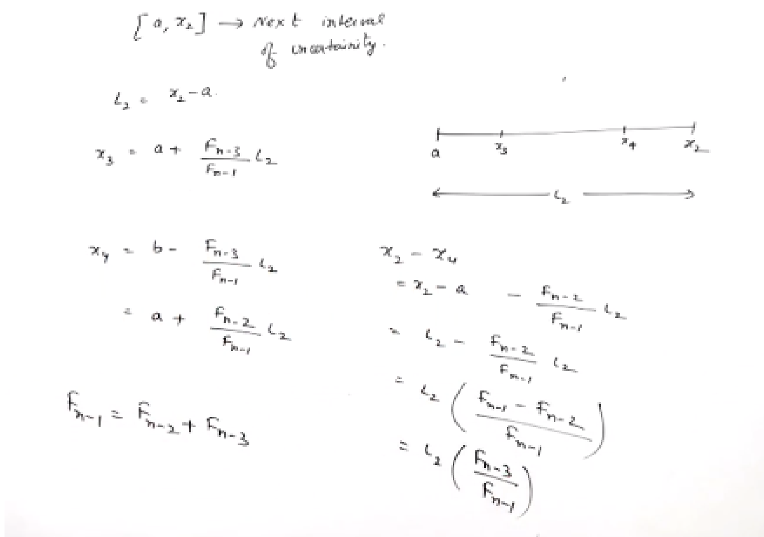

So it is so simply take this x2 okay and x4 you simply replace by this it is –a minus this and x2 – a from here is L2 so it is L2 – Fn – 2/Fn – 1 x L_2 if you take L_2 common so it is Fn – 1, -Fn – 2/Fn – 1 again since it is a Fibonacci series so Fn – 1,Fn – 2 =, if you take Fn – 1 so it is equal to Fn – 2 + Fn – 3, the sum of previous two terms and from this Fn – 1,-Fn – 2 is Fn – 3/ Fn – 1, so that is how we obtain this expression okay.

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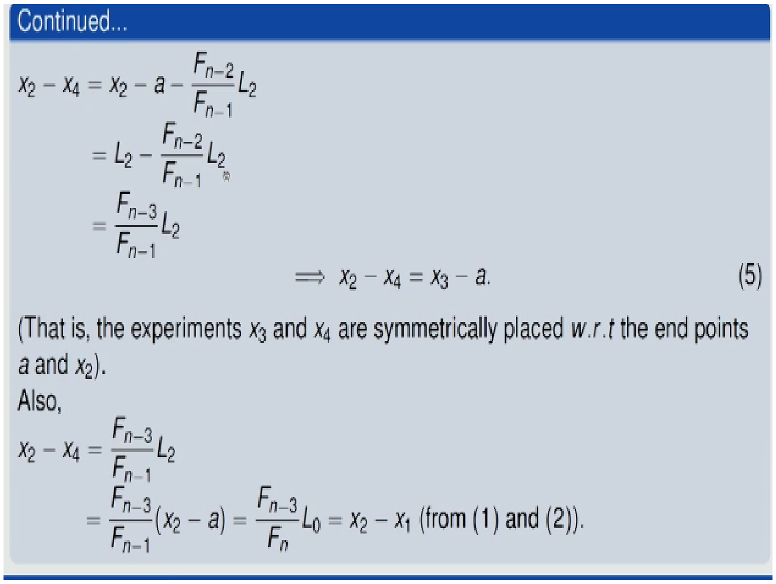

And from here we can say that X 2 - X 4 which is Fn – 3/Fn -1 times L2 is same as X3 - a because what we have seen –a.

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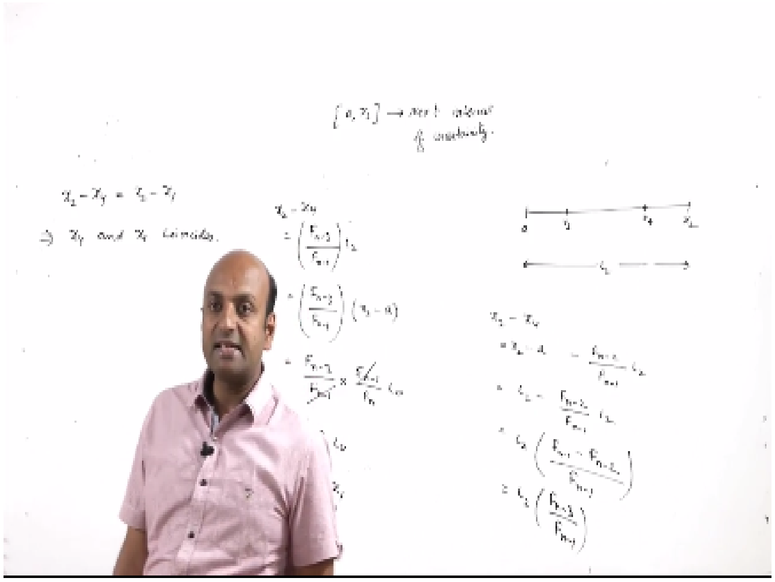

X3 - a is also Fn – 3/ Fn – 1 x L2 which is same as this expression, so from here we can say this thing so hence we can say there are two experiments are placed symmetrically respect to the end points again okay. Now if you find X2 - X4 which is which is this expression again if you see here X 2 - X 4 what we have obtained is equal to FN – 3/Fn – 1x L2 = Fn – 3/Fn -1 and what is L2, L2 is X2 –a, L2 is X2 – a, okay.

And this is this using 1 & 2, now what is what is x2 - a from here, x2 - a from here is Fn – 1/Fn x L0 so this cancels out and it is F n – 3/Fn x L0 and which is Fn – 3/ Fn x L0 and which is equal to x2 - x1 if you find X 2 - X 1 here X2 - X1, so a, a cancels out and it is Fn – 1,-Fn – 2 which is Fn – 3/ Fn x L0 so it is same as x2 - x1okay. Because if you find x2 - x1 so a, a cancels out it is Fn – 1, -Fn - 2 which by the Fibonacci series is equal to Fn -3/Fn x L0. So x2- so what we obtain, we obtain that x2-x4=x2-x1 so this implies x4 and x1coincides okay, so this x4 is nothing but x1 only okay, that means in Fibonacci series in the next iteration one experiment will coincide with the previous one.

So we have to find only one new experiment which is x3, again we will see the value of fx3 and fx4 affects whether fx3 is greater than fx4 or fx3 is lesser fx4 this will decide the next interval of uncertainty and the process repeats, okay. So this is a whole method of Fibonacci series okay, the same steps continued.

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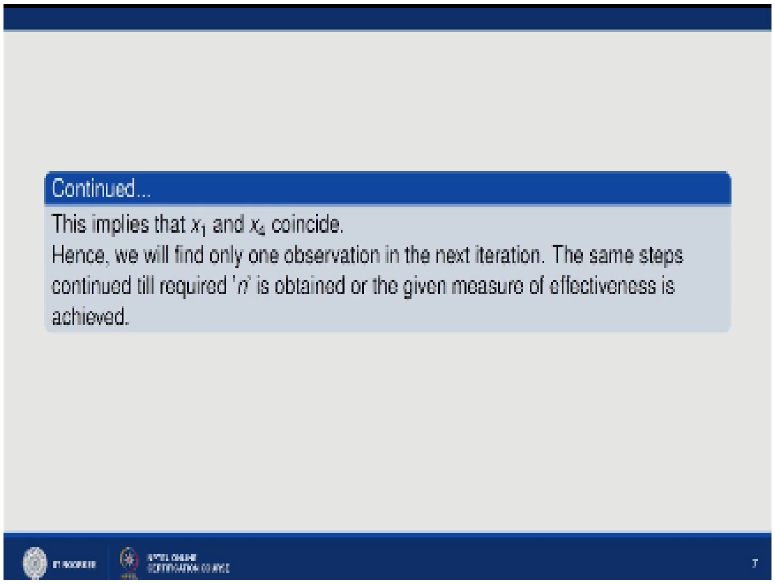

Till required n is obtained or the given measure of effectiveness is achieved, so let us start to solve one problem based on Fibonacci series technique, so let us take n=4.

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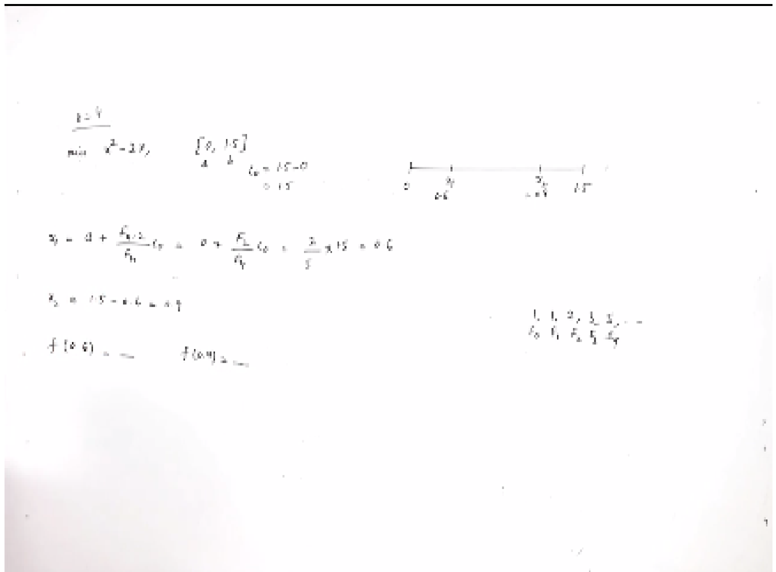

Problem is minimizing of x^2 minimizing of x^2-2x and the initial interval is 0 to 1.5 so interval is 0 to 1.5 okay, we have to find 2 experiments x1 and x2 so how to find x1 and x2, so x1 is given by a+fn-2/fL0 which is equal to so this is a this is b, and n is 4 so it is 0+f2/f4 and L0 is 1.5-0 which is 1.5 so it is and what is f,2 now if I write Fibonacci series it is 1, 1, 2,3, 5 and so on so it is f0,f1, f2 and so on so f2 is 2, it is f3 f4 and f4 is 5 and L0 is 1.5 so it is 0.6, so this experiment is 0.6 okay.

Now we know that if you find out two experiments so two experiments are always placed symmetrically from the endpoints, so if this distance is 0.6 so this must be 0.6 so what will be x2, x2 will be 1.5-0.6 which is 0.9 so this experimentally 0.9 this is by the Fibonacci first method as they already discussed that we have to find a two experiments x1 and x2 such that they are placed symmetrically from the endpoints.

If x1 is 0.6 which we obtain here so what will be x2, x2 will be the same distance here it is 0.6 over here it also a 0.6 which is 1.5 - 0.6 that is 0.9. Now you find f(0.6) and f(0.9) okay, so this we have obtained f(0.6) and f(0.9).

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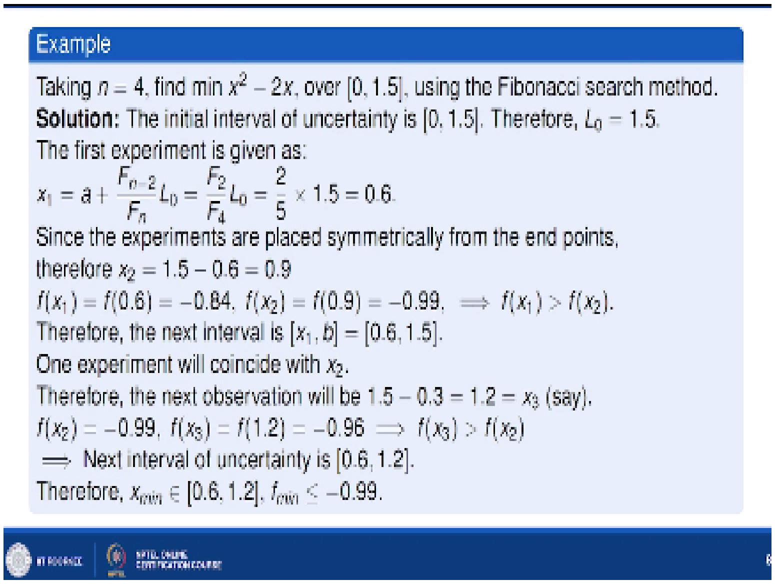

And we have observed that f(x1) >f(x2) so what is the next interval of uncertainty, next interval uncertainty will be x1 to b okay, because the value of x_2 is lesser than the value of x F_X 1. So next interval will be 0.6 to 1.5 so next experiments is 0.6 to 1.5 from here to here, so we are reducing the entered of a uncertainty now we know that always in the next interval for uncertainty one experiment coincides with the initial one.

So which experiment left the experiment which left here is H_2 which is 0.9 so 0.9 will coincide because in the Fibonacci search method if in the next interval uncertainty one experiment always coincide with the previous one, which is left here. So the experiment left here which it will coincide with the new experiment of uncertainty, now the two experimental always play symmetrical from the endpoints.

So this distance is here this distance is okay so this will be this will be 0.9 okay this is this would be 0.9 one experiment coincide the previous one okay, so this is 0.3 so this must be 0.3 so it is 1.2 okay. Now you will find the value of the function of the two experiments again, again you will find F of 0.9 and F of 1.2 again you will see that whether F x1 is this value is less than this or this value is greater than this will decide the next interval of uncertainty and the same process continues till n =4.

Till we find the 4 experiments here we have find the three, so one more one more iteration is required to complete the problem okay. So in this way we are going we are reducing the size of the interval we are reduce eyes of interval such that the point of minimal always lies inside the interval okay. So this is Fibonacci search technique.

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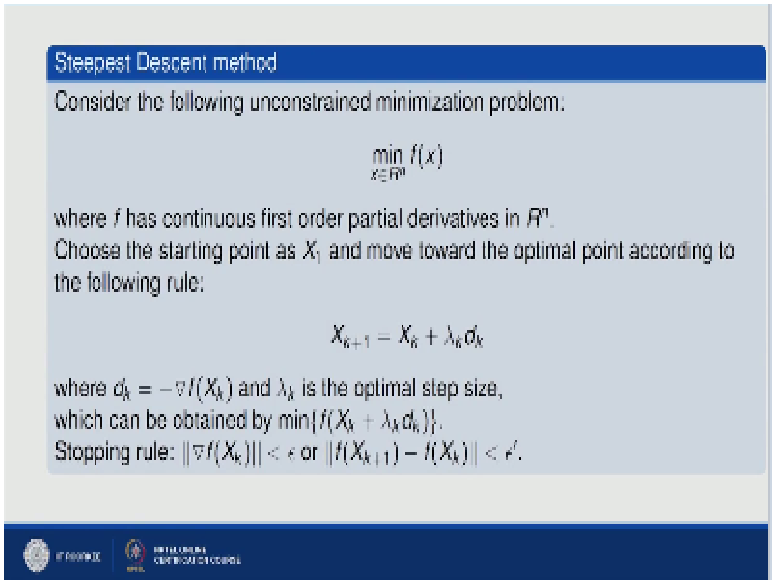

Now the next method is steepest decent method now what is steepest decent, now suppose you have an unconstrained optimization problem which we want to minimize okay. So as we have already discussed that there are two things involved in any such technique one is the direction in which we should move and the next other one is optimal step size okay suppose you start with this initial point x1 okay.

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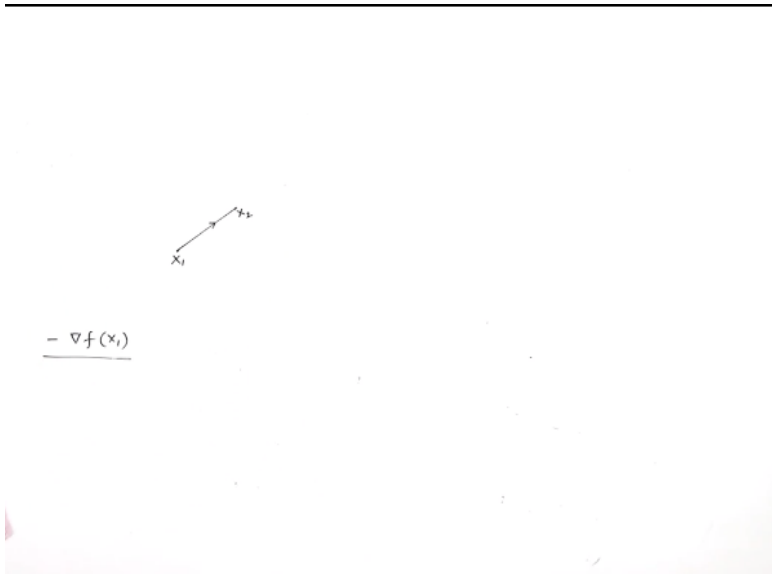

You have you have unconstrained optimization problem which you want to minimize suppose okay you are starting with this initial guess which is x1 and you want to find out the minimum optimal solution of the unconstrained problem okay. Now suppose you start with this initial guess from this initial guess there are infinite directions in which you we should we can move so in which direction we should move so that the value of the function decreases most rapidly.

So that direction is simply negative of gradient of F of F at x1 if in this direction we show if you move in this direction as we already know from the calculus that that if we move along this direction which is negative of gradient of F of F at x1 then the value of then a decrease rate of decrease the value of F in this direction decreases more rapidly okay.

So out of the infinite direction from the x1 we will move along this direction now up to how much okay suppose, suppose this is, this is the direction okay suppose this is this direction up to how much we should move so that we get a new experiment which is x_2 so that is basically that is basically the step size optimal step size.

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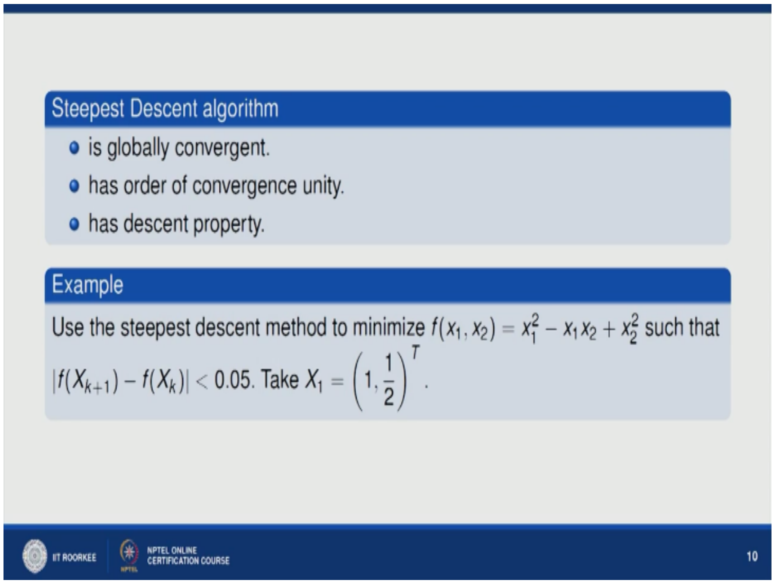

So that we can obtain by simply minimizing the value of XK+ λ KT okay this is with this we can obtain so this will be more clear by an example so we discuss one example so it has some very important properties one is it is globally convergent okay second is it has order of convergence 1 and it has a decent property, decent property means in every stage whatever, whatever experiments whatever X we find this will always decrease the value.

This will always give the value less than the previous one all this so this has a decent property suppose we start with this problem let us discuss this problem so things will be more clear so the main concept of this problem is business method is we start with the initial guess say x_1 from the infinite direction we will move along the negative gradient of F which gives at in this direction the value of F this is most rapidly find the optimal step size come to the next interval come, come to the next experiment again from that experiment.

We move along negative of gradient of F find the optimal step size come to the next iteration say $x_3$and again, again we will find the gradient of F negative of gradient of F and optimal size to find out the next experiment and this process continues till we get the required condition given the problem required tolerance limit or whatever okay.

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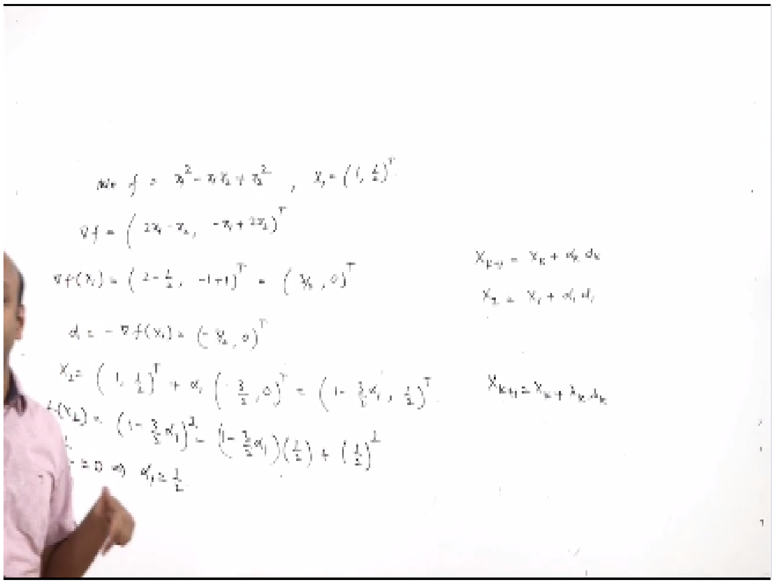

So say we have this problem minimizing of F which is X_1^2-X_1 X_2 + X_2^2 so this is a problem and stopping rule is then we should stop a stopping rule is given the problem that mode of F XK plus 1 minus F XK should be less than .05 that a difference of the two consecutive value other function must be less than .5 them as well as in .05 okay and the initial guess is given to us it is 1 2 1,0.5 initial guess is this.

So first you find gradient of F gradient of F is 2x_1 - x_2 ΔF uponΔx_1 ΔF uponΔx_2 now gradient of F at $x_1$which is, which is 2 minus 1/2 and it is minus 1 plus 1 okay that is nothing but 3 by 2, 0 okay what is the recursion recursive algorithm it is xk +1 so it goes to xk +αkdk or sk dk is simply d_1 now the now, now X_2 is X_1+α1 d_1 okay.

So what will be $d_{1,}$ d_1 is simply negative of gradient of Fx_1 along this direction so it is -3/2, 0 so what x_2 we obtain from here x_2 is x_1 x_1 is 1,1/2 +α1d_1 is - 3 /2 $^{>}$ 0 which is =1 - 3 /2 α1 and it is 1/ 2, now to find our α 1 which is a step size and what should be the optimal step size, we simply substitute this X $_2$ in this expression and find out the optimal value of α for which α 1 for which α 1 F is minimum okay, so what will be f of X_2 it is 1 – 3/ 2 α 1^2 - 1- 3/ 2 α1 x 1/ 2 + 1/ 2 ^{2} and simply df upon dl α 1 should be 0 for maximum minima of this function and this gives α 1, as α1 we are here we are calling as λ 1 both are same.

So here this λ 1 is 1/ 2 or L4 1 is 1 / 2 okay it hardly matters whether we take λ 1 or we can also take this as X + 1 = X_K + λ $_K$S $_K$ so these are only notations okay. So this is how we can find λ 1 or α 1 which is half and the next interval next experiment is 1/ 2, 1 / 4, 1 / 2 when you substitute λ 1 in α 1 over here okay. So here they are observed at the difference of the.

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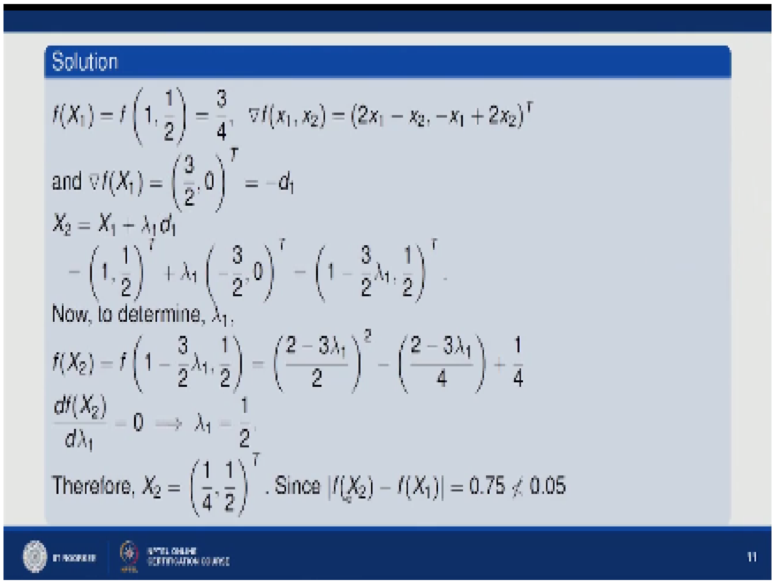

Two functional values is not <0.05, so we will not stop here we will perform the next experiment again we will form the next experiment okay, here it is ¼, 1/ $_2$ + λ $_2$ times d_2 which is negative of gradient of f at X $_2$ which is this thing okay and again we will find the function value at X $_3$ put it put it here X $_3$ put it here find the optimal size optimal step length which is λ $_2$ and we obtain λ $_2$ = ½. So next experiment is ¼, 1 / 8 and the difference now we have achieved that it is less than 0.05.

So this top here okay, so this is how we can we can apply we can apply C position method to solve on unconstrained optimization problem okay. So basically in this method we start with the initial guess okay goes to a direction of negative gradient of f find optimal size by putting this new experiment in the function and find the minimum that will give the optimal step size and the next experiment the same process continues till we get the tolerance level okay, so that is all in this lecture so thank you very much.

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