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Nonlinear Programming - 1

Lec – 18
Search Techniques-I

Dr. S. K. Gupta
Department of Mathematics
Indian Institute of Technology Roorkee

So hello fiends welcome to lecture series on linear programming so now we come to search techniques you see there are some nonlinear problems there is unique matter to solve any nonlinear problems non linear programming problems however we have studied some problem some specific problems which we can solve using KKT conditions or quartile programming or separable technique or geometric programming okay but all the problems cannot be solved using these techniques.

There is no an y unique matter, so we go to numerical approach or first techniques okay to find at least approximately solution appreciate solution approximate optimal solution of a given non linear problem, so we first start with unconstrained optimization problems.

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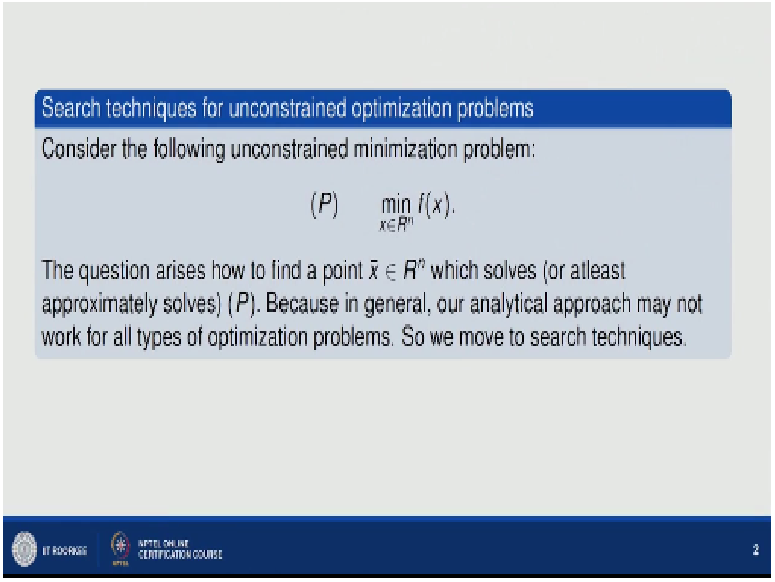

We have already discussed that a problem with without any constrain that is simply minimization of a function fx, x $\in R^n$ such problems are called unconstrained optimization problems, now the question arises how can we find out a point say $\bar{x} \in R^n$ which solves or at least approximately solves the given problem P okay because in general our analytical techniques may not work for all type of optimization problems, so we move to such techniques.

So this is the reason that we should move to such techniques because all the kind of optimization problems cannot be solved using analytical approach.

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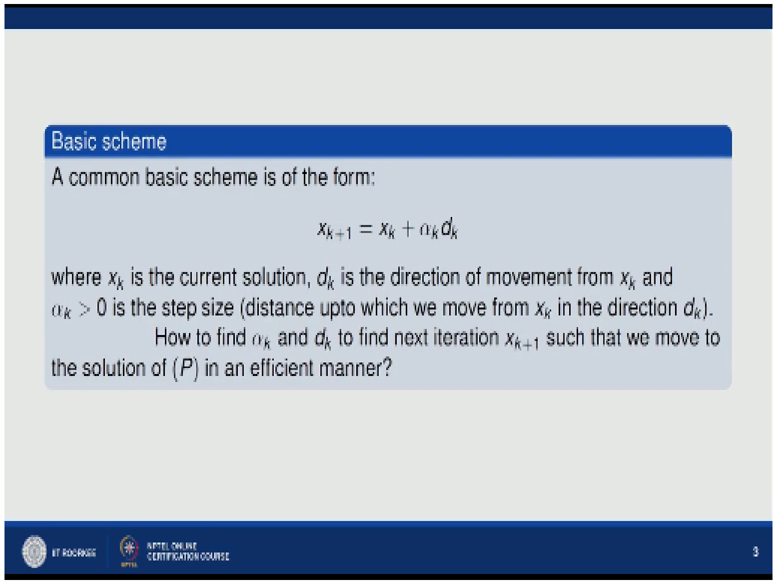

Now what is the basic scheme the common basic scheme is of the form xk+1 is the recursion algorithm basically all the such techniques are recursion algorithms so basic scheme is you take xk+1 that is the iteration at k + 1th point is equals to xk that is the pervious iteration + αk dk now xk is a current solution okay dk is the direction of movement from xk you see that your moving from a point xk to xk+ 1 you have two distingue point xk and xk+1.

How to move from xk to xk+1 so that we are moving towards the optimal solution if it is a minimization problem then we are towards the minimum point, how can we find a direction in which we should move so that is dk and next is αk which is ≥ o and is called as step size is the distance up to which we move form xk in the direction of k xk that is how much direction from how much distance from xk we should move so that that we can find an we can kind the improved solution okay.

So this is a basic sachem so basic scheme in basic scheme what we have to find basically we have to find two things xk is the current solution it is this is given to us or we can find next is αk αk is a step size which we have to finds and dk is the direction in which we should move so that we can get the improved solution the next iteration, so two things we have to find in all the schemes αk and dk so how to find αk and dk to find the next iteration xk+ 1 such that we move in to the solution of P in the efficient manner okay.

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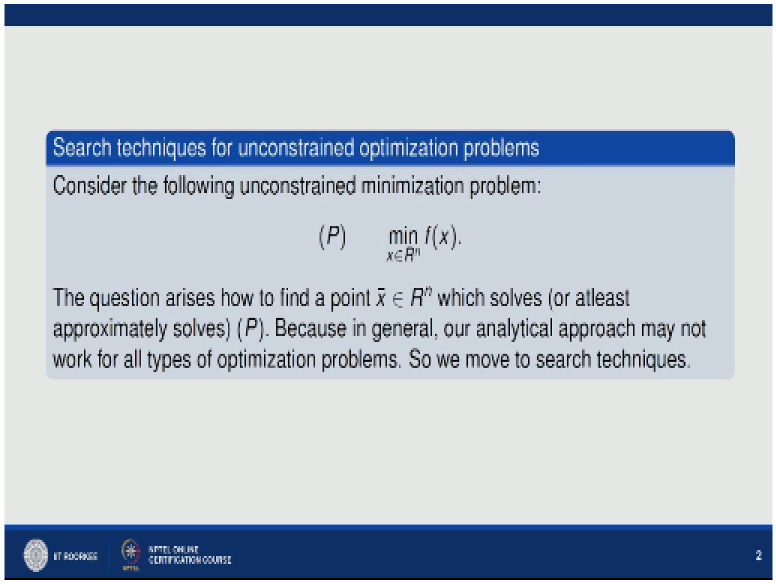

So that is a main phenomena main problem now next is descent property what is decent property an algorithm for solving P use see this is a problem P that is minimization of fx sub to $x \in R^n$ this is a minimization unconstraint optimization problem now we say that it is satisfy it is an property decent property means you see you have find you start from a point current solution is xk and finding αk and dk u comes to αxk + 1 that us iteration at k + 1 at point okay, so xk + 1 will be better point better solution than xk only if the function value at xk + 1 will be lesser than what we obtain at xk okay, so that we say that satisfy that it satisfy.

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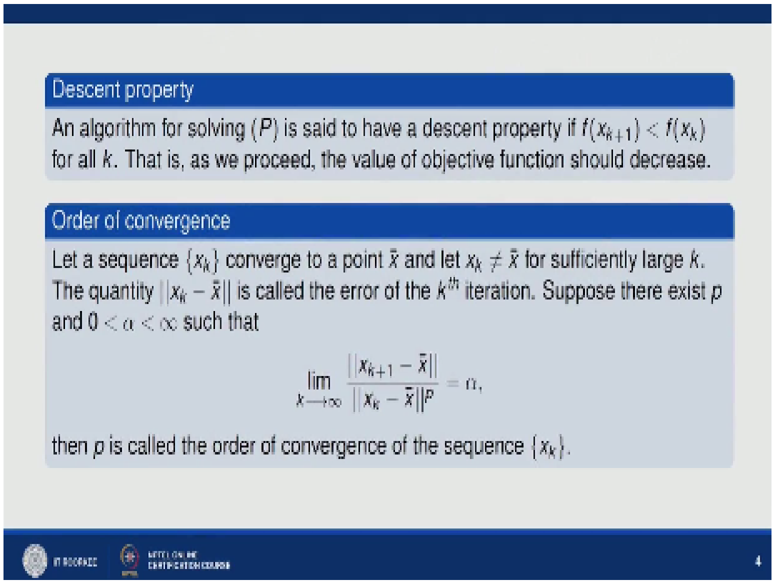

It satisfy descent property because the functional value functional value c functional value at xk = 1 is less than fx k that is we are moving towards the improved solution in the case of minimization we have more than towards the improve point where the value of f are improving are decreasing okay so then we say that it has satisfy descent property okay, now order of convergence now we have different search techniques to find out an optimal solution of a given problem p.

Which scheme is better okay, so that will be decided by it is order of converges and what is the order of convergence let us see let a sequence xk you see when you start from a initial guess x3 x_0 using any scheme when you come to x_1 then x_2 then x_3 and so on xk xk + 1 and so on okay. So you have a sequence of xk x_0 x_1 x_2 x_3 and so on of other sequence and suppose sequence converge to a point x bar suppose xk will tends to as part okay, and let xk is not equal to x bar for sufficiently large value of k.

Let us suppose then the quantity this of course if you take the distance of xk the point obtain at the k^{th} iteration the difference of the point obtain in the K iteration with $\bar{x}$, that will be the error point that will be error it is K iteration okay, this is simply give the error term at the k^{th} iteration now suppose there exist p and $\alpha > 0$ such that normal xk = 1 that is whatever we obtain at k at k + 1th point – $\bar{x}$ / xk – $\bar{x}^2$ xk tend to ∞ tending to α so then we say there the order of converges the sequences p.

If this expression is tending to α as k tending to ∞ then we say that the order of convergence of the given scheme is p, okay you can simply understand that this norm as a distance of xk + 1 with $\bar{x}$ distance xk with x bar okay now if p = 1.

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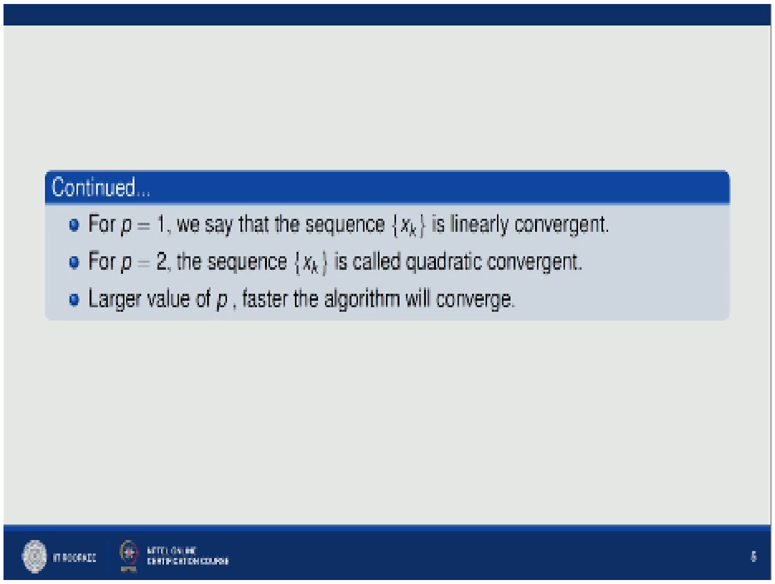

Okay then we say that the convergence that the sequence convergence linearly either p = 2 then we say the sequence xk convergence politically, or it is having quadratic convergence that means if we have a larger value of p the order of then the convergence rate will be faster more the value of p more faster will the scheme so we are always in search of those schemes those techniques which is having larger or order of convergence.

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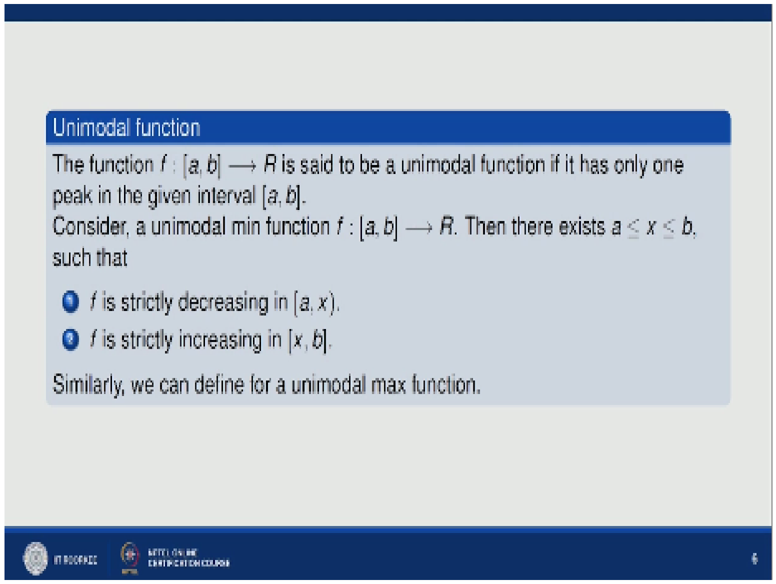

Now next is unimodal function so these are basic terminal logics where should we should know before a starting semi search techniques what is the descent property what is the order of convergence okay, and what is unimodal function we should know then we should go to search techniques okay, now unimodal function what is unimodal function now suppose a function f which is define from a interval a , b close interval a, b to r and it said to be a unimodal function if it is only one peak you see in a to b if in the interval a to b suppose this is interval.

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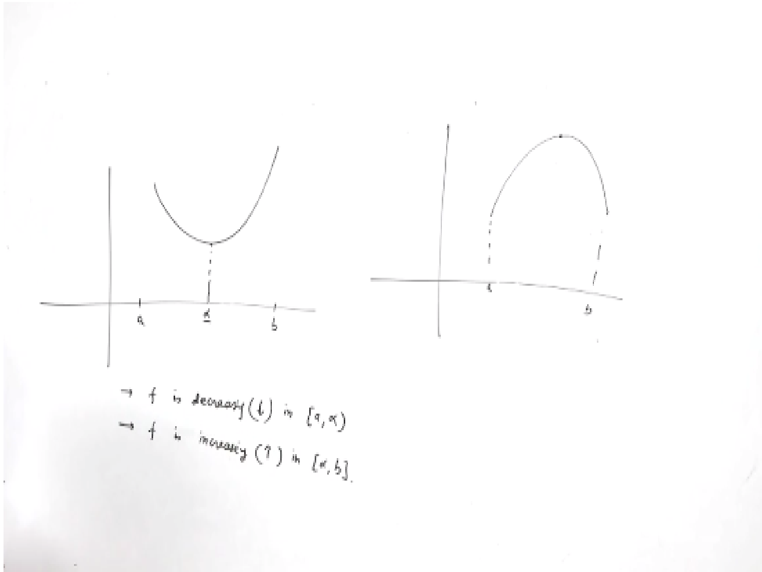

a , b if in this interval the function is only one peak here one peak only one minimum either minimum or maximum peak means if it is only one minima or one maxima in the entire interval a to b then we say this is a function is an unimodal function, okay. Function may be like this also in a to b okay it is only one peak okay, so such functions are called unimodel function, now here we are considering minimum objective function I mean optimization problem of minimization type and similarly we can derive the expressions for optimization problems of maximization types.

So we are dealing with minimization over here, now consider a unimodel minimum function, minimum function means objective function is of minimization type that means the peak in the objective function is of minimum type of this type of problem we are considering okay. So if it is minimum type problem to some α is there in-between a and b such that it has only one peak now that means if it has only one peak I mean only one minimum in the interval a to b.

That means before this α it must decrease and after this α upto b it must increase if it is unimodel function because according to definition unimodel function it must have only one peak I mean only one minima in-between a and b that means if the point of minimize α that means f is decreasing in a to α and f is increasing in α to b, so that is the main concept of unimodel function, if it is unimodel maximum function then if it is a unimodel function and it suppose maxima is obtained at ß. Then it first increase and then decrease it increases up to ß and after ß to b it decrease okay.

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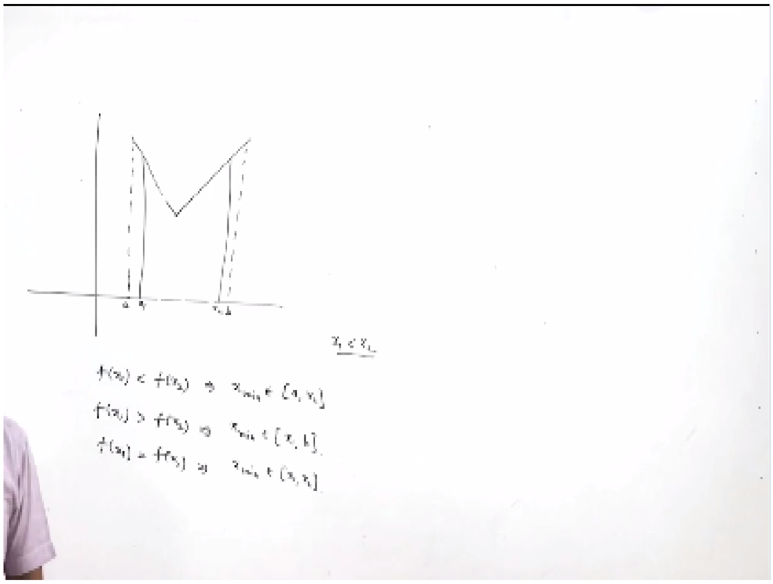

In linear model minimum function suppose we are having this type of problem, okay any problem this way problem suppose we are having, now minima for minima we are not discussing about the continuity or differentiability or the function, okay function may or may not be continuous or may or may not be differentiable. We are only thinking that function is a unimodel function that is it has only one minimum point in a to b, suppose it is a and it is b it is a minimum point okay. It first decreases and then increases, okay.

Now the first interval a, b is called first interval of uncertainty what is our aim, our aim is to find out the minimum point off the given problem P we have a some given problem P some given affects which we have to minimize subject 2S belongs to RN our aim is to find out the minimum value of the objective function f(x) okay. Now for that we are in search of we are always try to reduce this interval okay this is the first interval okay we are always try to find out some experiments in this intervals such that the minimum point always lies in-between that interval lies in the interior of that interval.

And we are always try to reduce the size of the interval the width off the interval and such intervals are called interval of uncertainty okay in every two in every like iterations we are trying to find out a new intervals of uncertainties such that the interval of width of the intervals uncertainty becomes smaller and smaller okay and in each interval uncertainty minimum point will lie, okay.

Now suppose in any scheme you find out two experiments yes two experiments means two iterations you perform two iterations okay you find out two interactions suppose you find out first iterations over here x1 and next iteration you find out here if suppose it is x2 okay, you perform any such techniques which we discuss afterwards you find two experiments suppose experiments are x1 and x2 okay. It may be possible it in this case f(x1) < f(x2).

So minimum will belongs to x minimum or minimum point which is this point will belongs to which interval, will belongs to a2x2 why a2x2 why not s1 to x2 because this x1 may be here also, because here also f(x1) is less than f(x2) if we say in between x1 and x2 then here x1 to x2 does not contain the point of minima.

However, a2x2 always cover the point of minima because we are choosing x1<x2 we are taking two experiments such that x1<x2 okay, if x1<x2 and value of function f(x1) is less than function of value f(x2) so of course minimum will lie in between a2x2. Now suppose f(x1) is bigger than f(x2) suppose this is the case, suppose x1 is here okay, if x1 is here so this implies minimum will belongs to which point which interval x2 to b, because minimum is here okay.

Again you may say thus the minimum should lie in from x1 to x2 but it is not true, x2 may be here also, if x2 is here because x1 is less than x2 and f(x1) is greater than f(x2) holds here also, and if it is here and you are saying that minimum lies in between x1 and x2 which is not true because minimum is here may x2 and x1 are the arbitrary experiments which we are performing, okay. So do not know where minimum is, so but minimum always lies in this interval and if f(x1)=f(x2) if it is equal suppose here so in this case minimum will be lying belongs to x1x2.

So from any scheme from any basic scheme from any such technique we first find to two experiments okay, x1 and x2 we find the next interval of one certainty which maybe this, this or this. This case really happens because f(x1) most of the cases really happens that f(x1)=f(x2) so in most of the cases we are having either this case or this case so next interval certainty will be either this or this, so definitely the width of this interval will be x2-a band width of this interval b-x1 which is less than b-c.

So in each step in each iteration we are trying to find out the next interval certainties such that the minimum point will be lying in that interval and we are making interval smaller and smaller, so that is the basic scheme that is the basic like property which we are using in these any such technique, okay. Now next is measure of effectiveness.

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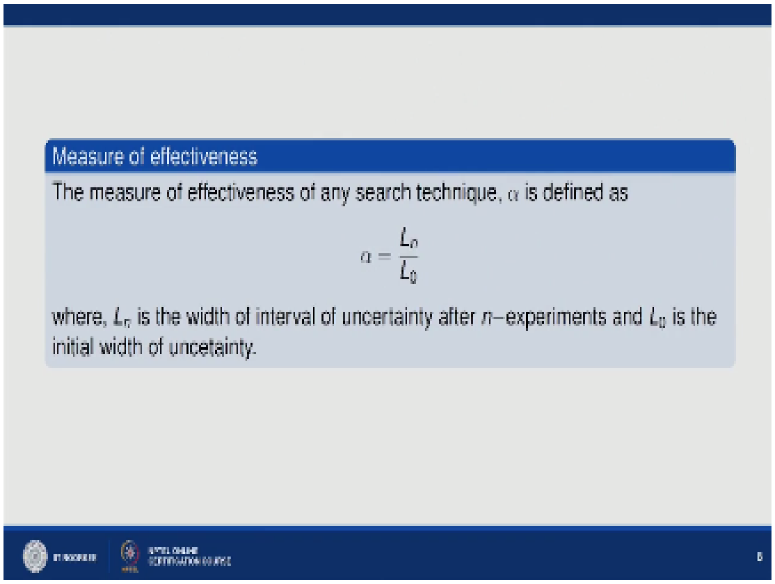

What it is, it is denoted by α and it is defined as L_n/L_0, L_n is simply width of the interval of uncertainty after n experiments suppose it perform n number of experiments, so L_n we are defined as width of interval of uncertainty after n experiments and L_0 is the initial width of uncertainty, initial width means b-a over here. So definitely α will be less than equal to 1 because in each iteration we are trying to reduce the size of the width of the interval so L_n will be definitely $\leq L_0$ so L_n / l0 we definitely ≤ 1 so smaller value of α that means we are making the inter unsatanity smaller and smaller okay. Now the first technique is Dichotomy search technique now what it is let us discussed.

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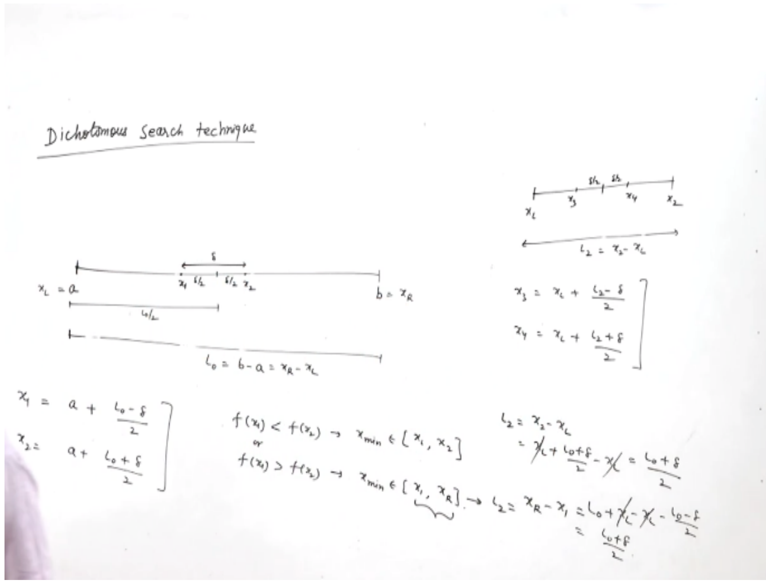

So first technique is dichotomous search technique you see her to experiment you have the initial interval is given to you interval is a, b close interval a, b you have find out the experiments and the next interval of unsatanity okay how to find that so in this search technique in this scheme what we do we basically so this is l_0 we first find l_0, l_0 is b – a the initial length of unsatanity that is b – a okay.

Now you find two experiment here x_1 and x_2 such that x_1x_2 are suppose δ this is a part, now we take the two point at equal distance from the mind point of this that is so this is $l_0/2$ this length is l_0 / 2, and we are making it δ/2 we are shifting one point δ/2 on the left side of the interval and we are making one point δ/2 on the right side of this interval. This is δ/2 and this is δ/2 so x_1 will be equal to a +you see the left point a + this is l_0 – δ/2 this is from here to here it is l_0 /2 and $l_0/2$ – δ/2 +a will give this point x_1.

If you are add l_0 – δ/2 on this a we will get x_1 okay and how to find the next experiment x_2, x_2 is $l_0/2$ + δ/2 so a + l_0+ δ/2 that will give x_2, so this is how we can find x_1 and x_2, in this PPT we have denoted a /x_l that is a left point and this by x_r okay. So we are placing two experiments such that the experiment are lying δ/2 on the left side of this line and δ/2 on the right side on this line okay such that.

So next we find out the sign f_x1 we find $f_x1 < f_x2$ or $f_x1 > f_x2$ okay next we find out whether $< f_x2$ or $f_x1 > f_x2$, if f_x1 1 $< f_x2$ this means minimum will belongs because $f_x1 < f_x2$ this means minimum will belongs to xl to x_2 and if this is the case then this means minimum will belongs to

x_1 to x_r okay. Now this will our next interval of unsatanity if suppose this is the, our next of our unsatanity so this will be x_l and this will be new x_r, again we divide we take the midpoint and place the two next two experiments δ/2 on the left side and δ /2 on the right side okay suppose this will be our next experiment so next experiment will be now x_l to x_2 again this length is suppose l_2 which is x_2-x_l okay now we again take the midpoint of this place one point Δ/2 on the left side of this which is x_3 and one point here which is x_4 Δ/2 on right side.

So what will be the x_3 now x_3 will $x_l+l_2-\Delta/2$ and x_4 will be $x_l+l_2+\Delta/2$ again we take the sign of fx_3 and fx_4 whether fx_3 is less than fx_4 fx_3 is greater than fx_4 on the basis of this we can decide the next interval of certainty okay so this is the main scheme we place that two experiments starting from midpoint Δ/2 on left side and Δ/2 on right side so if you find that next length l_2, l_2 is x_2-x_l x_2 is what x_2 is this value

That is a+ a or x_l a means x_l okay that is $x_l+\Delta/2$ x_l so it is l_2 and from here also if you find L_2 it is same because this is are the same x_r or –x1 what is x_r, x_r is a lot plus xl and xl is x1 is this quantity a is x_l so it is $-x_L$ and –L Δ/2 which is equal to x_l, x_l is cancel out it is so both are same okay.

Whether we have this next interval certainty or this it will remain same okay so what we are obtained now the number of experiments and always even because here any iteration we are calculating two experiments x_1 and x_2 x_3 x_4 and so on so number of experiments are the 2,4,6 and so on okay now the interval of certainty I mean width of interval of certainty will be if you perform two experiments.

So we are seen that interval certainty is that we have seen Δ/2 okay basically what we are doing we are taking the half of the length here what we are doing we are taking half of the length plus Δ/2 in the next experiments x3 and x4 if you find the length of the interval of certainty or width of the interval of certainty that will be simply half of this because now the next length is this if you take length as in the initial interval so it is Δ/2 now in the next iteration now the length is this quantity.

So this by 2 plus Δ/2 will be the next length of uncertainty that you can easily show here you can easily see also okay now in this 6^{th} experiment this will be half of this plus Δ/2 and so on because in each step we are making Δ/2 now this is the next interval length so this by 2 Δ/2 now this is next length this by 2 by Δ/2 so what happens after say 2k number of iterations say n= 2k if n= 2k so it is you see ½ okay so it is L_0 2^k it is 2 x 3 so it is 2^3 if it 2 x k, so it is 2k + δ^k + δ^k -1 and so on, if it is 6 so δ^{is} up on 2^3 + δ^2 + δ / 2.

If it is 2k it is δ /2k and δ /2k -1 δ /2 so that will be = $L_0/2^k$+ it is geometric progression having k number of terms, so it is a 1 – r^n r is 2, is 2^k and 1- 2 that will be $L_0/2^k$+ δ /2^k -1 it is a 1- r^n r is 2 okay, so it is 2^k 1- 2, that is this is correct okay. So it is L_0/2 n /2 because k is n/2 + δ -1, so this will be the length ln, so if you see here L_0 Ln will be.

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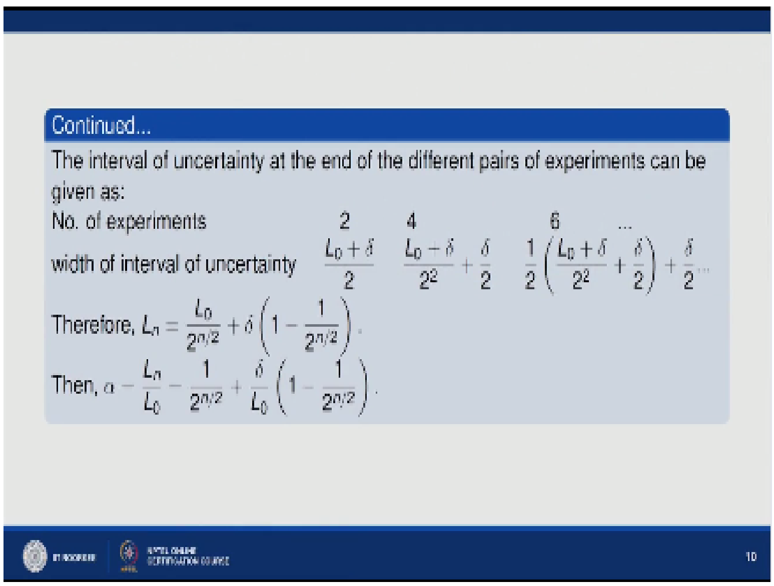

L_0 Ln will be L_0 + 2 + δ -1/2, and α which is the major effectiveness is Ln upon L_0 so it will be 1 up on L_0 + 2 + δ -1/2 , so this is the main scheme let us discuss it by an example again okay, now we have to find the maximum value of function fx which is.

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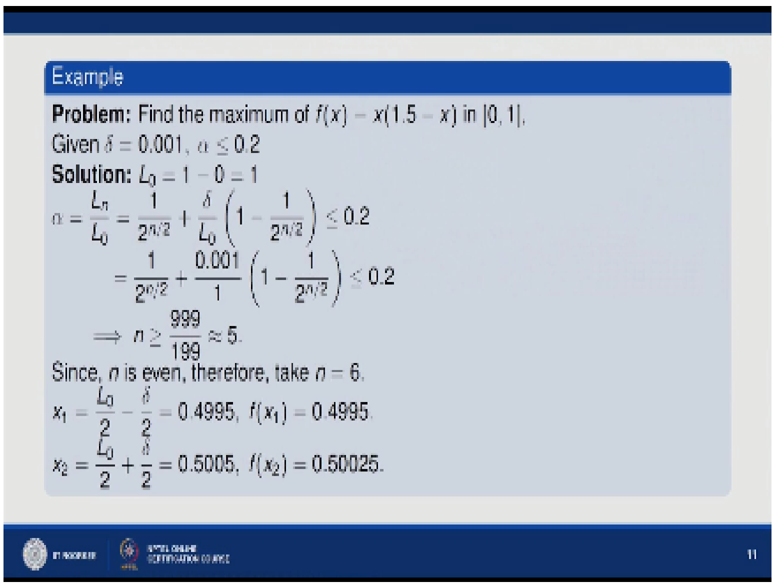

X x 2.15 – x in 0 to 1 so xl is 0 and x* is 1 so what is the 1st internal on certainty that is 1- 0 which is 1 okay δ is given to us it is 0. 001 and α which is the major effectiveness also given to us it should be ≤ 0.2. So we know that α in the expression called as dicosomatic technique is given by this expression, this we have already derived, so this must be ≤ 0.2 now if you simply δ is 0.001 and this L_0 – 1 -0 it is 1, you simplify this so you will get n ≥ approximately 5, but n must be even for dichotomies.

So we take n = 6 that means we have to find 6 number of iterations so that α will be ≤ 0.2, now how to find that experiments, so this we have already seen that x1 will be a + xl + L_0-δ/2 and x2 xl+ L_0 δ/2 so this xl is 0 + L_0 is 1 –δ is 0.001 /2 and it is again 0 + 1+ 0.001/2 so in this way we find x1 and x2. There you will see the sign of fx and fx2 find the values of fx and fx2, so we find that fx < fx2, so therefore the next internal of certainty will be x1 2 xr that is

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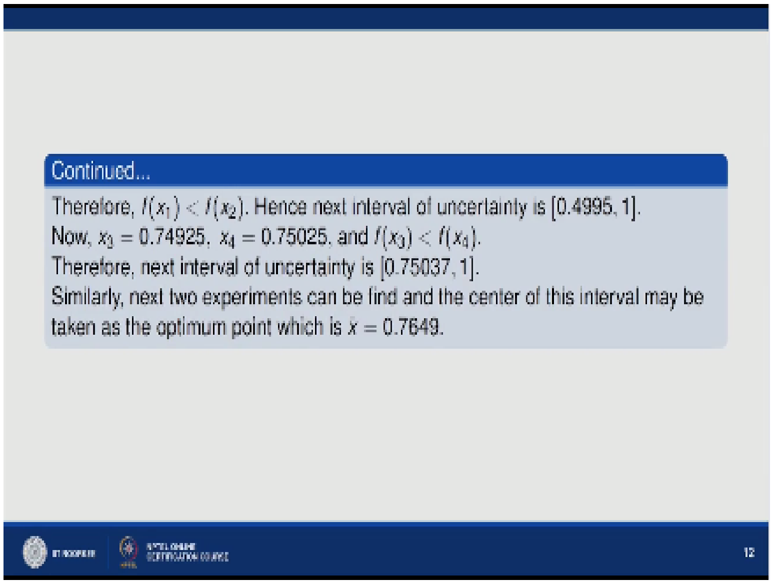

0.499521 okay again using this interval as the new interval of our certainty again we find the two experiments x3 and x4 which is given by this expression okay, because now you can take xl as this xr is this again you will take the midpoint and place δ/2 on the left side and in one experiment δ/2 on right side okay. You find x3 and x4 again you see the sign fx3 fx4 whether fx3 is < fx4 so the next interval certainty will be 0.7503721.

So in this way we will perform the next two experiments also and finally the midpoint of final interval that will be the approximate solution of the given problem so this is the simple illustration that how can we use the technique by using the search technique so thank you very much.

For Further Details Contact
Coordinator, Educational Technology Cell
Indian Institute of Technology Roorkee
Roorkee – 247667
E Mail: etcell.iitrke@gmail.com. etcell@iitr.ernet.in
Website: www.iitr.ac.in/centers/ETC, www.nptel.ac.in

Camera
Jithin. K

Graphics
Binoy. V. P
Online & Video Editing

Mohan Raj. S

Arun. S
Pankaj Saini
Neetesh Kumar
Jitender Kumar
Nibedita Bisoyi

Production Team
Sarath Koovery

An Educational Technology cell
IIT Roorkee Production