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Nonlinear Programming

Lec 15
Dynamic Programming II

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Welcome to lecture 15 in non linear programming. So in the last lectures we have seen that if we have a complicated network how can we solve it using forward or backward correction. That is not a dynamic programming approach. Now how can we solve the same problem, the same network which we have discuss in the last lecture using dynamic programming approach okay. You see that we have already find out the minimum path from the node one to node 10 is to 20 units.

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Stage 1: $f_1(x_1) = 0$

Stage 2: $f_2(x_2) = \min \{ \lambda_1(x_1, x_2) + f_1(x_1) \}$

x_1	x_2	$\lambda_1(x_1, x_2)$	$f_2(x_2)$	\bar{x}_2
1	2	4+0	4	1
2	7	3+0	3	2
3	9	1+0	1	3

Stage 3: $f_3(x_3) = \min \{ \lambda_2(x_2, x_3) + f_2(x_2) \}$

x_2	x_3	$\lambda_2(x_2, x_3)$	$f_3(x_3)$	\bar{x}_3
1	6	4+3	7	1
2	7	3+3	6	2
3	9	1+3	4	3

Stage 4: $f_4(x_4) = \min \{ \lambda_3(x_3, x_4) + f_3(x_3) \}$

x_3	x_4	$\lambda_3(x_3, x_4)$	$f_4(x_4)$	\bar{x}_4
1	10	4+7	11	1
2	10	3+6	9	2
3	10	1+4	5	3

Stage 5: $f_5(x_5) = \min \{ \lambda_4(x_4, x_5) + f_4(x_4) \}$

x_4	x_5	$\lambda_4(x_4, x_5)$	$f_5(x_5)$	\bar{x}_5
1	10	4+11	15	1
2	10	3+9	12	2
3	10	1+5	6	3

Stage 6: $f_6(x_6) = \min \{ \lambda_5(x_5, x_6) + f_5(x_5) \}$

x_5	x_6	$\lambda_5(x_5, x_6)$	$f_6(x_6)$	\bar{x}_6
1	10	4+15	19	1
2	10	3+12	15	2
3	10	1+6	7	3

Stage 7: $f_7(x_7) = \min \{ \lambda_6(x_6, x_7) + f_6(x_6) \}$

x_6	x_7	$\lambda_6(x_6, x_7)$	$f_7(x_7)$	\bar{x}_7
1	10	4+19	23	1
2	10	3+15	18	2
3	10	1+7	8	3

Stage 8: $f_8(x_8) = \min \{ \lambda_7(x_7, x_8) + f_7(x_7) \}$

x_7	x_8	$\lambda_7(x_7, x_8)$	$f_8(x_8)$	\bar{x}_8
1	10	4+23	27	1
2	10	3+18	21	2
3	10	1+8	9	3

Stage 9: $f_9(x_9) = \min \{ \lambda_8(x_8, x_9) + f_8(x_8) \}$

x_8	x_9	$\lambda_8(x_8, x_9)$	$f_9(x_9)$	\bar{x}_9
1	10	4+27	31	1
2	10	3+21	24	2
3	10	1+9	10	3

Stage 10: $f_{10}(x_{10}) = \min \{ \lambda_9(x_9, x_{10}) + f_9(x_9) \}$

x_9	x_{10}	$\lambda_9(x_9, x_{10})$	$f_{10}(x_{10})$	\bar{x}_{10}
1	10	4+31	35	1
2	10	3+24	27	2
3	10	1+10	11	3

Stage 11: $f_{11}(x_{11}) = \min \{ \lambda_{10}(x_{10}, x_{11}) + f_{10}(x_{10}) \}$

x_{10}	x_{11}	$\lambda_{10}(x_{10}, x_{11})$	$f_{11}(x_{11})$	\bar{x}_{11}
1	10	4+35	39	1
2	10	3+27	30	2
3	10	1+11	12	3

Stage 12: $f_{12}(x_{12}) = \min \{ \lambda_{11}(x_{11}, x_{12}) + f_{11}(x_{11}) \}$

x_{11}	x_{12}	$\lambda_{11}(x_{11}, x_{12})$	$f_{12}(x_{12})$	\bar{x}_{12}
1	10	4+39	43	1
2	10	3+30	33	2
3	10	1+12	13	3

Stage 13: $f_{13}(x_{13}) = \min \{ \lambda_{12}(x_{12}, x_{13}) + f_{12}(x_{12}) \}$

x_{12}	x_{13}	$\lambda_{12}(x_{12}, x_{13})$	$f_{13}(x_{13})$	\bar{x}_{13}
1	10	4+43	47	1
2	10	3+33	36	2
3	10	1+13	14	3

Stage 14: $f_{14}(x_{14}) = \min \{ \lambda_{13}(x_{13}, x_{14}) + f_{13}(x_{13}) \}$

x_{13}	x_{14}	$\lambda_{13}(x_{13}, x_{14})$	$f_{14}(x_{14})$	\bar{x}_{14}
1	10	4+47	51	1
2	10	3+36	39	2
3	10	1+14	15	3

Stage 15: $f_{15}(x_{15}) = \min \{ \lambda_{14}(x_{14}, x_{15}) + f_{14}(x_{14}) \}$

x_{14}	x_{15}	$\lambda_{14}(x_{14}, x_{15})$	$f_{15}(x_{15})$	\bar{x}_{15}
1	10	4+51	55	1
2	10	3+39	42	2
3	10	1+15	16	3

Stage 16: $f_{16}(x_{16}) = \min \{ \lambda_{15}(x_{15}, x_{16}) + f_{15}(x_{15}) \}$

x_{15}	x_{16}	$\lambda_{15}(x_{15}, x_{16})$	$f_{16}(x_{16})$	\bar{x}_{16}
1	10	4+55	59	1
2	10	3+42	45	2
3	10	1+16	17	3

Stage 17: $f_{17}(x_{17}) = \min \{ \lambda_{16}(x_{16}, x_{17}) + f_{16}(x_{16}) \}$

x_{16}	x_{17}	$\lambda_{16}(x_{16}, x_{17})$	$f_{17}(x_{17})$	\bar{x}_{17}
1	10	4+59	63	1
2	10	3+45	48	2
3	10	1+17	18	3

Stage 18: $f_{18}(x_{18}) = \min \{ \lambda_{17}(x_{17}, x_{18}) + f_{17}(x_{17}) \}$

x_{17}	x_{18}	$\lambda_{17}(x_{17}, x_{18})$	$f_{18}(x_{18})$	\bar{x}_{18}
1	10	4+63	67	1
2	10	3+48	51	2
3	10	1+18	19	3

Stage 19: $f_{19}(x_{19}) = \min \{ \lambda_{18}(x_{18}, x_{19}) + f_{18}(x_{18}) \}$

x_{18}	x_{19}	$\lambda_{18}(x_{18}, x_{19})$	$f_{19}(x_{19})$	\bar{x}_{19}
1	10	4+67	71	1
2	10	3+51	54	2
3	10	1+19	20	3

Stage 20: $f_{20}(x_{20}) = \min \{ \lambda_{19}(x_{19}, x_{20}) + f_{19}(x_{19}) \}$

x_{19}	x_{20}	$\lambda_{19}(x_{19}, x_{20})$	$f_{20}(x_{20})$	\bar{x}_{20}
1	10	4+71	75	1
2	10	3+54	57	2
3	10	1+20	21	3

Stage 21: $f_{21}(x_{21}) = \min \{ \lambda_{20}(x_{20}, x_{21}) + f_{20}(x_{20}) \}$

x_{20}	x_{21}	$\lambda_{20}(x_{20}, x_{21})$	$f_{21}(x_{21})$	\bar{x}_{21}
1	10	4+75	79	1
2	10	3+57	60	2
3	10	1+21	22	3

Stage 22: $f_{22}(x_{22}) = \min \{ \lambda_{21}(x_{21}, x_{22}) + f_{21}(x_{21}) \}$

x_{21}	x_{22}	$\lambda_{21}(x_{21}, x_{22})$	$f_{22}(x_{22})$	\bar{x}_{22}
1	10	4+79	83	1
2	10	3+60	63	2
3	10	1+22	23	3

Stage 23: $f_{23}(x_{23}) = \min \{ \lambda_{22}(x_{22}, x_{23}) + f_{22}(x_{22}) \}$

x_{22}	x_{23}	$\lambda_{22}(x_{22}, x_{23})$	$f_{23}(x_{23})$	\bar{x}_{23}
1	10	4+83	87	1
2	10	3+63	66	2
3	10	1+23	24	3

Stage 24: $f_{24}(x_{24}) = \min \{ \lambda_{23}(x_{23}, x_{24}) + f_{23}(x_{23}) \}$

x_{23}	x_{24}	$\lambda_{23}(x_{23}, x_{24})$	$f_{24}(x_{24})$	\bar{x}_{24}
1	10	4+87	91	1
2	10	3+66	69	2
3	10	1+24	25	3

Stage 25: $f_{25}(x_{25}) = \min \{ \lambda_{24}(x_{24}, x_{25}) + f_{24}(x_{24}) \}$

x_{24}	x_{25}	$\lambda_{24}(x_{24}, x_{25})$	$f_{25}(x_{25})$	\bar{x}_{25}
1	10	4+91	95	1
2	10	3+69	72	2
3	10	1+25	26	3

Stage 26: $f_{26}(x_{26}) = \min \{ \lambda_{25}(x_{25}, x_{26}) + f_{25}(x_{25}) \}$

x_{25}	x_{26}	$\lambda_{25}(x_{25}, x_{26})$	$f_{26}(x_{26})$	\bar{x}_{26}
1	10	4+95	99	1
2	10	3+72	75	2
3	10	1+26	27	3

Stage 27: $f_{27}(x_{27}) = \min \{ \lambda_{26}(x_{26}, x_{27}) + f_{26}(x_{26}) \}$

x_{26}	x_{27}	$\lambda_{26}(x_{26}, x_{27})$	$f_{27}(x_{27})$	\bar{x}_{27}
1	10	4+99	103	1
2	10	3+75	78	2
3	10	1+27	28	3

Stage 28: $f_{28}(x_{28}) = \min \{ \lambda_{27}(x_{27}, x_{28}) + f_{27}(x_{27}) \}$

x_{27}	x_{28}	$\lambda_{27}(x_{27}, x_{28})$	$f_{28}(x_{28})$	\bar{x}_{28}
1	10	4+103	107	1
2	10	3+78	81	2
3	10	1+28	29	3

Stage 29: $f_{29}(x_{29}) = \min \{ \lambda_{28}(x_{28}, x_{29}) + f_{28}(x_{28}) \}$

x_{28}	x_{29}	$\lambda_{28}(x_{28}, x_{29})$	$f_{29}(x_{29})$	\bar{x}_{29}
1	10	4+107	111	1
2	10	3+81	84	2
3	10	1+29	30	3

Stage 30: $f_{30}(x_{30}) = \min \{ \lambda_{29}(x_{29}, x_{30}) + f_{29}(x_{29}) \}$

x_{29}	x_{30}	$\lambda_{29}(x_{29}, x_{30})$	$f_{30}(x_{30})$	\bar{x}_{30}
1	10	4+111	115	1
2	10	3+84	87	2
3	10	1+30	31	3

Stage 31: $f_{31}(x_{31}) = \min \{ \lambda_{30}(x_{30}, x_{31}) + f_{30}(x_{30}) \}$

x_{30}	x_{31}	$\lambda_{30}(x_{30}, x_{31})$	$f_{31}(x_{31})$	\bar{x}_{31}
1	10	4+115	119	1
2	10	3+87	90	2
3	10	1+31	32	3

Stage 32: $f_{32}(x_{32}) = \min \{ \lambda_{31}(x_{31}, x_{32}) + f_{31}(x_{31}) \}$

x_{31}	x_{32}	$\lambda_{31}(x_{31}, x_{32})$	$f_{32}(x_{32})$	\bar{x}_{32}
1	10	4+119	123	1
2	10	3+90	93	2
3	10	1+32	33	3

Stage 33: $f_{33}(x_{33}) = \min \{ \lambda_{32}(x_{32}, x_{33}) + f_{32}(x_{32}) \}$

x_{32}	x_{33}	$\lambda_{32}(x_{32}, x_{33})$	$f_{33}(x_{33})$	\bar{x}_{33}
1	10	4+123	127	1
2	10	3+93	96	2
3	10	1+33	34	3

Stage 34: $f_{34}(x_{34}) = \min \{ \lambda_{33}(x_{33}, x_{34}) + f_{33}(x_{33}) \}$

x_{33}	x_{34}	$\lambda_{33}(x_{33}, x_{34})$	$f_{34}(x_{34})$	\bar{x}_{34}
1	10	4+127	131	1
2	10	3+96	99	2
3	10	1+34	35	3

Stage 35: $f_{35}(x_{35}) = \min \{ \lambda_{34}(x_{34}, x_{35}) + f_{34}(x_{34}) \}$

x_{34}	x_{35}	$\lambda_{34}(x_{34}, x_{35})$	$f_{35}(x_{35})$	\bar{x}_{35}
1	10	4+131	135	1
2	10	3+99	102	2
3	10	1+35	36	3

Stage 36: $f_{36}(x_{36}) = \min \{ \lambda_{35}(x_{35}, x_{36}) + f_{35}(x_{35}) \}$

x_{35}	x_{36}	$\lambda_{35}(x_{35}, x_{36})$	$f_{36}(x_{36})$	\bar{x}_{36}
1	10	4+135	139	1
2	10	3+102	105	2
3	10	1+36	37	3

Stage 37: $f_{37}(x_{37}) = \min \{ \lambda_{36}(x_{36}, x_{37}) + f_{36}(x_{36}) \}$

x_{36}	x_{37}	$\lambda_{36}(x_{36}, x_{37})$	$f_{37}(x_{37})$	\bar{x}_{37}
1	10	4+139	143	1
2	10	3+105	108	2
3	10	1+37	38	3

Stage 38: $f_{38}(x_{38}) = \min \{ \lambda_{37}(x_{37}, x_{38}) + f_{37}(x_{37}) \}$

x_{37}	x_{38}	$\lambda_{37}(x_{37}, x_{38})$	$f_{38}(x_{38})$	\bar{x}_{38}
1	10	4+143	147	1
2	10	3+108	111	2
3	10	1+38	39	3

Stage 39: $f_{39}(x_{39}) = \min \{ \lambda_{38}(x_{38}, x_{39}) + f_{38}(x_{38}) \}$

x_{38}	x_{39}	$\lambda_{38}(x_{38}, x_{39})$	$f_{39}(x_{39})$	\bar{x}_{39}
1	10	4+147	151	1
2	10	3+111	114	2
3	10	1+39	40	3

Stage 40: $f_{40}(x_{40}) = \min \{ \lambda_{39}(x_{39}, x_{$

dynamic programming we divide the larger problem into sub problem which we are calling is stages. We divide this problem into stages, we find out the solution of each stage. And use the solution of the previous stage in next stage till we get the last stage.

And whatever we obtain the last stage, that will give the optimal solution okay. So how can we solve this problem, let us see. In a dynamic programming we defined some terms, some variables first. First variables is stage variable okay. We are calling this as stage 0 suppose for which we are denoting by j . stage variable we are denoting by j , so these are the stage variables. The first stage we are calling as 0 stage okay.

Now this stage we are calling as stage 1, this is stage we are calling as stage 2 and this stage we are calling as stage 3, and this is stage 4 okay. Now in this stage we are having only one variable because we are having only one node. So we are calling that is state variable okay. So here x is 1. What are x is—what are x size are nothing but state variables okay. Now for—for this corresponding stage how many nodes we are having three.

So we will be having 3 state variable cross points to this stage. So that is x_1 , so x_1 may be 1, 2, 3 so we are calling it 1, we are calling it as 2, and we are calling it as 3 okay. This is stage 1 and then in this stage there are three nodes, so we are calling each node as $1x_1$ I mean $x_1=1$ means this node $x_1=2$ means this node $x_1=3$ mean this node. Now for this particular stage x_2 we already having only 2 nodes, so 1 and 2, so this is 1 and this is 2.

For this stage we are again having 3 are nodes, I mean 3 node so we are calling it 1, 2, 3. So it is 1, it is 2, it is 3 and this only having 1 okay. So first we defining a stage variable a stage variables are 0, 1, 2, 3 and 4 corners point to each stage we are defining a state variables here only one node, so we called as 0 is 1. Here we are having 3 stage 3 node so x_1 is 1, 2 or 3 it is 1 or 2, 1, 2 or 3 and then it is 1.

So it is 1 okay. So we have classified we have distributed in the entire network into stages and its state okay. We have sub divided you see this complicated network this is a very simple example this a simple example how can we solve any complicated network using dynamic programming. We have define we have divided this problem into some stages okay. And correspond to each stage we define each variables okay.

Suppose we are talking about $f_1 = 2$ that means that means this node. Suppose we are talking about $x_3 = 3$ — $x_3 = 3$ that means this node is clear okay. Next we define certain terms first is $f_j(x_j)$.

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$f_j(x_j)$ = min path from state x_j in stage j to state $x_4 = 1$ in stage 4.
 $f_1(3) = f_1(x_1 = 3)$ = min-path from state $x_1 = 3$ in stage 1 to $x_4 = 1$ in stage 4.
 $f_4(x_4 = 1) = 0$.
 $r_{j+1}(x_j, x_{j+1})$ = length of the arc(path) from state x_j in stage j to state x_{j+1} in stage $j + 1$.
 $r_3(2, 1) = r_3(x_2 = 2, x_3 = 1) = 6$.
 $f_j(x_j) = \min_j \{r_{j+1}(x_j, x_{j+1}) + f_{j+1}(x_{j+1})\}$
 $f_1(x_1 = 3) = \min \{r_2(x_1, x_2 = 1) + f_2(x_2 = 1), r_2(x_1, x_2 = 2) + f_2(x_2 = 2)\}$

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What is $f_j(x_j)$ is minimum path from a state x_j in a stage j to it state $x_4 = 1$ in stage j . suppose you want $f_1(3)$, $f_1(3)$ means $f_1(x_1)=3$ okay. Because it is, it denotes this 1 denotes a stage okay. This 1 denotes stage $j=1$ stage. And 3 means f_1 is 3 okay, so this means from this stage from this stage is 1 and state variable 3. The minimum path from this to this. This is $f_1(3)$ okay, so what to find we have to find—we have to find basically $f_0(1)$ this has to find.

The minimum path from this 0th state 0th stage to $f_4 = 1$ this to find out the minimum path line okay. Now next we defined $r_{j+1}(x_j, x_{j+1})$ it is length of the arc from a state x_j in stage j to state x_{j+1} in stage $j+1$. Suppose $r_3(2,1)$ okay suppose we have to find $r_3(2,1)$ so $r_3(2,1)$ means okay $j+1$ is 3 that means j is 2, so that means $x_2 = 2$ $f_3 = 1$. It is a arc length basically from $x_2 = 2$. x_2 go to this and $x_3=1$ is this, so the arc length is 6 units, so it is equal to 6.

Like Dij what we having in the backward of forward dictation in the same way here we define like $r_{j+1}(x_j, x_{j+1})$ okay. The arc length from x_j to x_{j+1} , x_j in j^{th} stage to f_{j+1} in $j+1^{\text{th}}$ stage okay. Now how to find x_j , f_j okay so suppose you have to find $f_1(3)$ like this now suppose you want to find $f_1(3)$ so $f_1(3)$ $f_1, x_1(3)$ that means this thing. So first you comoute this length that is r_2 of $x_1 = 3$ and $x_2 = 1$ $+f_2(x_2=1)$ you see this arc length plus this the minimum what we are obtaining at this node to this node okay.

Then this 3 that is $r_2(x_1=3, x_2=2) + f_2(f_2=2)$ the minimum of these 2 the minimum of these 2 is simply gave $f_1(x_1)=3$. So in general how can we define as $f_j(x_j)$. It is it will be minimum of $r_{j+1}(x_j, x_{j+1}) + f_{j+1}(x_{j+1})$ that will be that the minimum of this will give $f_j(x_j)$ okay. Now let us find out the minimum distance this a certain terminology you should define before the starting dynamic approach.

First we define a stages, a stages means we define the entire problem into sub problems, as we already discussed it. The sub problems are is stage $j = 0, j=1, j=2, j=3$ and $j=4$ in each stage we defined state variables, a state variable here are 1, for this particular stage there are 3 nodes 1, 2 and 3 because there are 3 nodes and similarly others then we defined—then we defined $f_j(x_j)$. $f_j(x_j)$ is nothing but the minimum distance from node x_j in j th state to node $x_4=1$ in fourth stage.

And $r_{j+1}(x_{j+1})$ is nothing but the arc length from x_j in the j th stage to x_{j+1} in $j+1$ th stage is a arc length okay. And the minimum of this minimum of j will give $f_j(x_j)$. So let us now find out the minimum distance from this to this how we can start. First will take stage 4, again we are using backward recursion because this stage 4 upto here. Upto here it is stage 3, upto here it is stage 3 then stage 1 and then stage 0.

Stage 0 will give the minimum from this to this okay. So first to find stage 4, why stage 4 it will be $f_4(x_4)$ okay. $f_4(x_4)$ now from now $f_j(x_j)$ is the minimum distance from node i to node $x_4=1$. So from node $x_4=1$ to itself the minimum distance is 0, so it is 0 okay. Now stage 3, the stage 3 means $f_3(x_3)$ that will be minimum of $r_4(x_3, x_4) + f_4(x_4=1)$ of course okay. Now how can I make a table for this, the table is very simple for this you see, and we are having x_3, x_4 okay.

x_3 are 1 to 3 here x_4 is only one, so x_4 is only one. From 1 to 1 when $x_3(1)$ is 1, when x_3 is 1 and f_4 is 1 the arc length is 8, so it is 8. That is $r_4(x_3, x_4)$ okay. $r_3(x_3, x_4)$ is 8. Then 2 to 1, the length is 7 units. So it is 7. Then 3 to 1, 3 to 1 when x_3 is 3 it is 1 it is 9, so it is 9 okay. Now $r_4(x_3, x_4) + f_4(x_4)$, $f_4(x_4)$ is 0, so add 0. It is $8+0, 7+0$ and $9+0$ and it is $f_3(x_3)$ here there is only one term many of these minimum is remain 8.

The minimum is 7, minimum is 9 and at which f_3 this 8 is obtaining f_3 is 1, this 8 is obtaining so it is x_4 it is 7 and it is 9—oh it is 2 and it is 3—oh sorry it is 4. It is 1,1. At $x_4=1$ we are obtaining all three minimums 1, 1, 1 okay so this stage is over. Now we go to stage 2, a stage 2

means $f_2(x_2)$ which is minimum of $r_3(x_2, x_3) + f_2(x_2) + f_3(x_3)$ okay. a stage 2 means from upto from here to here okay.

That means you vary these two nodes and find out the minimum from this stage to this stage okay. So it is again you make a table for this you will make it x_2 and x_3 . For x_2 we are only having 2 values one in 2, for x_3 we are having value 1, value2, and value3. 1 to 1 x_2 it is 8 then 9 then 5. It is 8, 9, 5. These are the arc length okay then it is 6, 7, 4 here it is 6 here it is 7 and here it is 4.

These are what these are $r_3(x_2, x_3)$ now you want to add $f_3(x_3)$ also okay. We want to add $f_3(x_3)$ also, you see $f_3(x_3)$ are 8, 7 and 9 okay. So it is 8, 7 and 9. First we write the value of the arc length this 8, 9, 5 6, 7, 4 value arc length and then we have to add $f_3(x_3)$ $F_3(x_3)$ our previous table. From the previous table we have already obtain the minimum of this stage to this stage many of this stage to this stage we will add the two arc length from node to this stage okay.

Now here the minimum is 8, 7, and 9 so you have to add 8, 7, 9. So what are the minimum that will give $f_2(x_2)$ here it is 16 16 40 so minimum is 40 add which x_3 to see. Then this 14 you are obtaining $x=3$. Now this 14, 14, 13, so it is minimum is 13 and this 13 we are obtain when $x_3=3$ so x_3 is 3 okay. So this is a second stage which we have obtained that shown in a table also.

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Solution

Stage 4: $f_4(x_4 = 1) = 0$.

Stage 3: $f_3(x_3) = \min\{r_4(x_3, x_4) + f_4(x_4)\}$

x_3/x_4	$r_4(x_3, x_4)$	$r_4(x_3, x_4) + f_4(x_4)$	$f_3(x_3)$	\bar{x}_4
	$x_4 = 1$			
1	8	8 + 0	8	1
2	7	7 + 0	7	1
3	9	9 + 0	9	1

Stage 2: $f_2(x_2) = \min\{r_3(x_2, x_3) + f_3(x_3)\}$

x_2/x_3	$r_3(x_2, x_3)$			$r_3(x_2, x_3) + f_3(x_3)$	$f_2(x_2)$	\bar{x}_3
	$x_3 = 1$	$x_3 = 2$	$x_3 = 3$			
1	8	9	5	8 + 8 9 + 7 5 + 9	14	3
2	6	7	4	6 + 8 7 + 7 4 + 9	13	3

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So here in the ppt, now similarly we go for a stage 1. A stage 1 will be referring but $f_1(x_1)$ as shown in the ppt minimum of $r_2(x_1, x_2) + f_2(x_2)$ we shown here x_1, x_2 and then r_2 which the arc length from x_1 to x_2 . then we add $f_2(x_2)$ which we already obtain in the last stage from a stage 2. $f_2(x_2)$ we have already obtained and then $f_1(x_1)$ you see, now $f_1(x_1)$ we are having three variables 1, 2 and 3. To it is 1, 2, 3 and x_2 having only 2 values that it 1 and 2 so it is 1 and 2.

Now 1 to 1 that is $x_1, x_2 = 1$ so the value is I mean the distance is $f=1$ is $x_1=1$ $x_1=1$ to $x_2 = 1$ there are one part that is three units and from this to this it is 2 units so it is 3 and 2. Similarly from $x_1 = 2$ to $f_1= 1$ in to it is 4 and 5 then 2 and 3. And whatever we have obtained in the last stage. That is stage 2 $f_2(x_2)$ which we already obtain. Here it is 14 and 13 okay, you simply add, you simply add it here. 14, 13—14 and 13.

Now for this x_1 the minimum is obtaining at here it is 15 and $x_2=2$ this we are obtaining 18, 18 at both the values are 18. So it is obtaining a $x_2=1$ or 2, now here it is 16, 16 we a obtaining when $x_1= 1$ and 2 again 1 and 2. Now the last final stage is stage 0, the stage 0 is the value 0 which is the minimum of this plus this. Here x_0 is only one variable $x_0=1$ and $x_1=1, 2, 3$ in x_1 are 1, 2, 3. Hence are 5, 7 and 6 so it is 5, 7 and 6 are shown in the picture also.

And $f_1(x_1)$ is the minimum path obtained minimum value obtain in the stage 1, which is 15, 18 and 16 so you add 15, 18 and 16. Now minimum of these three will give $f_0(x_0)$ which you have to find out, so the minimum is 20 now okay. And $x_1 = 1$, so $x_1=1$ $x_1= 1$ means this one this node this to this. Path is this to this and it is obtain at 15 and 15 we are getting from this point that is $x_2=2$. You go now to know the minimum distance is 20 we are obtaining from here.

What are the paths to know the path we will go from last stage to the first stage backward direction. We go to the backward direction to find out the path, you see it is 20 at $x_1=1$ 20 we are obtaining from this point 15, 15 we are getting from here okay when x_2 is 2, x_2 is 2 means this one that means this part okay now this 13 this 13 from where getting from where we are getting 13. This 13 we are getting from here okay.

When x_3 is 3 so x_3 is 3 that mean this one okay x_3 is 3 this one and the last one is of course there is only one part so x_4 is 1 that means the thing. So this will give the path I mean the path also.

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Continued...

Minimum distance from node 1 to 10 is 20.
 $x_0 = 1 \rightarrow x_1 = 1 \rightarrow x_2 = 2 \rightarrow x_3 = 3 \rightarrow x_4 = 1.$

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So the minimum distance of node 1 to node 10 is 20. The path is x from x_0 to 1 then x_1 to 1 x_2 to 2 x_3 to 1 $x_3 = 3$ and then $x_4 = 1$. So this is how using dynamic programming approach we can easily find out the minimum distance from node 1 to node 10 the minimum path and the path also. The path length and the path also using dynamic programming approach. So what we have done here we have defined, we have divided a larger problem into sub problems okay.

We are divided the problem into stages and whatever solution we are getting from the stage 3 we are using in the next stage and then the whatever we obtaining from here we are using the next stage. And the same processes we repeat then we get 0. So this is basically dynamic programming approach. We divide a larger problem into sub problem or a stage we defined a stage variables and then we whatever we are obtaining in the previous stage. The solution whatever we are obtaining we are using that solution in the next stage till we get the last stage okay. so this is how we can find out the optimal solutions of such problems using dynamic

programming approach. In the next lecture we will solve some more problems based on dynamic programming. So thank you very much.

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