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NEPTEL ONLINE CERTIFIACTION COURSE

Nonlinear Progrmming

Lec 15
Dynamic Programming II

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Welcome to lecture 15 in non linear programing. So in the last lectures we have seen that if we have a complecated network how can we solve it using forward or backward correction. That is not a dynamic programming approach. Now how can we solve the same problem, the same network which we have discuss in the last lecture using dynamic programming approach okay. You see that we have already fid out the minimum path from the node one to node to node 10 is to 20 units.

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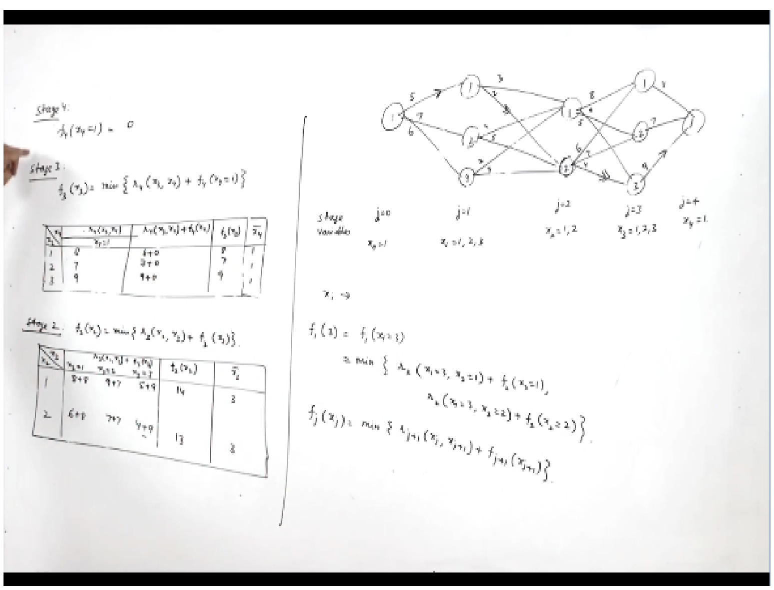

And the path is 1 to 2, 2 to 6, 6 to 9 and 9 to 10 is a minimum path, and the minimum path line is 20 units. Now how can we solve this problem into using dynamic programming approach? So in dynamic programming we devide the larger problem into sub problem which we are calling is stages. We devide this problem into stages, we find out the solution of each stage. And use the solution of the previous stage in next stage till we get the last stage.

And whatever we obtain the last stage, that will give the optimal solution okay. So how can we solve this problem, let us see. In a dynamic programming we definded some terms, some variables first. First variables is stage variable okay. We are calling this as stage 0 suppose for which we are denoting by j. stage variable we are denotig by j, so these are the stage variables. The first stage we are calling as 0 stage oklay.

Now this stage we are calling as stage 1, this is stage we are calling as stage 2 and this stage we are calling as stage 3, and this is stage 4 okay. Now in this stage we are having only one variable because we are having only one node. So we are calling that is state variable okay. So here x is 1. What are x is—what are x size are nothing but state variables okay. Now for—for this corresponding stage how may nodes we are having three.

So we will be having 3 state variable cross points to this stage. So that is x1, so x1 may be 1, 2, 3 so we are calling it 1, we are calling it as 2, and we are calling it as 3 okay. This is stage 1 and then in this stage there are three nodes, so we are calling each node as 1x_1 I mean x_1 =1 means this node x1=2 means this node x1=3 mean this node. Now for this particular stage x2 we already having only 2 nodes, so 1 and 2, so this is 1 and this is 2.

For this stage we are again having 3 are nodes, I mean 3 node so we are callinh it 1, 2, 3. So it is 1, it is 2, it is 3 and this only having 1 okay. So first we defining a stage variable a stage variables are 0, 1, 2, 3 and 4 corners point to each stage we are defining a state variables here only one node, so we called as 0 is 1. Here we are having 3 stage 3 node so x1 is 1, 2 or 3 it is 1 or 2 1, 2 or 3 and then it is 1.

So it is 1 okay. So we have calssified we have distriputed in the entire network into stages and its state okay. We have sub devided you see this complicated network this is a very simple example this a simple example how can we solve any complicated network using dynamic programming. We have define we have devided this problem into some stages okay. And corrospond to each stage we defind each variables okay.

Suppose we are talking about f1= 2 that means that means this node. Suppose we are talking about x3 =3—x3 =3 that means this node is clear okay. Next we define certain terms first is fj xj.

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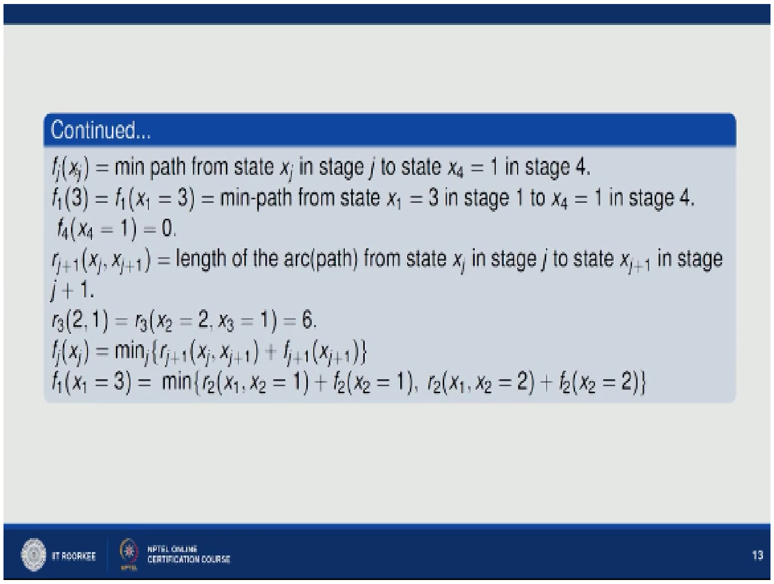

What is fj(xj) is minimum path from a state xj in a stage j to it state x4 =1 in stage j. suppose you want f1(3), f1(3) means f1(x1)=3 okay. Because it is, it denotes this 1 denotes astage okay. This 1 denotes stage j=1 stage. And 3 means f1 is 3 okay, so this means from this stage from this stage is 1 and state variable 3. The minimum path from this to this. This is f1 (3) okay, so what to find we have to find—we have to find basically f0(1) this has to fiond.

The minimum path from this 0^{th} state 0^{th} stage to f4 = 1 this to find out the minimum path line okay. Now next we defined $r_{j+1}(x_j,x_{j+1})$ it is length of the arc from a state x_j in stage j to state x_{j+1} in stage j+1. Suppose r3(2,1) okay suppose we have to find r3(2,1) so r3(2,1) means okay j+1 is 3 that means j is 2, so that means x2= 2 2f3 =1. It is a arc length basically from x2 =2. x 2 go to this and x3=1 is this, so the arc length is 6 units, so it is equal to 6.

Like Dij what we having in the backward of forward dictation in the same way here we define like rj+1(xj,xj+1) okay. The arc length from xj to xj+1, xj in j^{th} stage to fj+1 in $j+1^{th}$ stage okay. Now how to find xj, fj okay so suppose you have to find f1(3) like this now suppose you want to find f1(3) so f1(3) f1, x1(3) that means this thing. So first you comoute this length that is r2 of x1= 3 and x=1 +f2(x2=1) you see this arc length plus this the minimum what we are obtaining at this node to this node okay.

Then this 3 that is r2 (x1=3, x2=2)+ f2(f2=2) the minimum of these 2 the minimum of thes 2 is simply gave f1(x1)=3. So in general how can we defind as fj xj. It is it will be minimum of rj+1(xj, xj+1)+fj+1(xj+1) that will be that the minimum of this will give fj(xj) okay. Now let us find out the minimum distance this a certain tremalogy you should define before the starting dynamic approach.

First we define a stages, a stages means we define the entire problem into sub problems, as we already discussed it. The sub problems are is stage j = 0 j=1 , j=2, j=3 and j=4 in each stage we defined state variables, a state variable here are 1, for this particular stage there are 3 nodes 1, 2 and 3because there are 3 nodes and similarly others then we defined—then we defined fj(xj). fj(xj) is nothing but the minimum distance from node xj in jth state to node x4=1 in fourth stage.

And rj+1 (xj+1) is nothing but the arc length from xj in the j th stage to xj+1 in j+1 th stage is a arc length okay. And the minimum of this minimum of j will give fj(xj). So let us now find out the minimum distance from this to this how we can start. First will take stage 4, again we are using backward recurssion because this stage 4 upto here. Upto here it is stage 3, upto here it is stage 3 then stage 1 and then stage 0.

Stage0 will give the minimum from this to this okay. So first to find stage 4, why stage 4 it will be f4(x4) okay. f4(x4) now from now fj(xj) is the minimum distance from nodei to node x4=1. So from node x4 =1 to itself the minimum distance is 0, so it is 0 okay. Now stage 3, the stage 3 means f3(x3) that will be minimum of r4(x3,x)+f4(x4=1) of course okay. Now haw can I make a table for this, the table is very simple for this you see, and we are having x3, x 4 okay.

x3 are 1 to 3 here x4 is only one, so x4 is only one. From 1to 1 when x3(1f4) is 1, when x3 is 1 and f4 is 1 the arc length is 8, so it is 8. That is r4(x3, x4) okay. r3(x3, x4) is 8. Then 2 to 1, the length is 7 units. So it is 7. Then 3 to 1, 3 to 1 when x3 is 3 it is 1 it is 9, so it is 9 oaky. Now r4(x3, x4)+f4(x4), f4(x4) is 0, ao add 0. It is 8+0, 7+0 ansd 9+0 and it is f3(x3) here there is only one term many of these minimum is remain 8.

The minimum is 7, minimum is 9 and at which f3 this 8 is obtaining f3 is 1, this 8 is obtaining so it is x4 it is 7 and it is 9—oh it is 2 and it is 3—oh sorry it is 4. It is 1,1. At x4 =1 we are obtaining all three minimums 1, 1, 1 okay so this stage is over. Now we go to stage 2, a stage 2 means f2(x2) which is minimum of r3(x2, x3) +f2(x2) +f3(x3) okay. a stage 2 means from upto from here to here okay.

That means you vary these two nodes and find out the minimum from this stage to this stage okay. So it is again you make a table for this you will make it x2 and x3. For x2 we are only having 2 values one in 2, for x3 we are having value 1, value2, and value3. 1 to 1 x21it is 8 then 9 then 5. It is 8, 9, 5. These are the arc length okay then it is 6, 7, 4 here it is 6 here it is 7 and here it is 4.

These are what these are r3(x2, x3) now you want to add f3(x3) also okay. We want to add f3(x3) also, you see f3(x3) are 8, 7 and 9 okay. So it is 8, 7 and 9. First we wright the value of the arc length this 8, 9, 5 6, 7, 4 value arc length and then we have to add f3(x3) F3(x3) our previous table. From the previous table we have already obtain the minimum of this stage to this stage many of this stage to this stage we will add the two arc length from node to this stage okay.

Now here the minimum is 8, 7, and 9 so you have to add 8, 7, 9. So what are the minimum that will give f2(x2) here it is 16 16 40 so minimum is 40 add which x3 to see. Then this 14 you are obtaining x=3. Now this 14, 14, 13, so it is minimum is 13 and this 13 we are obtain when x3=3 so x3 is 3 oaky. So this is a second stage which we have obtained that shown in a table also.

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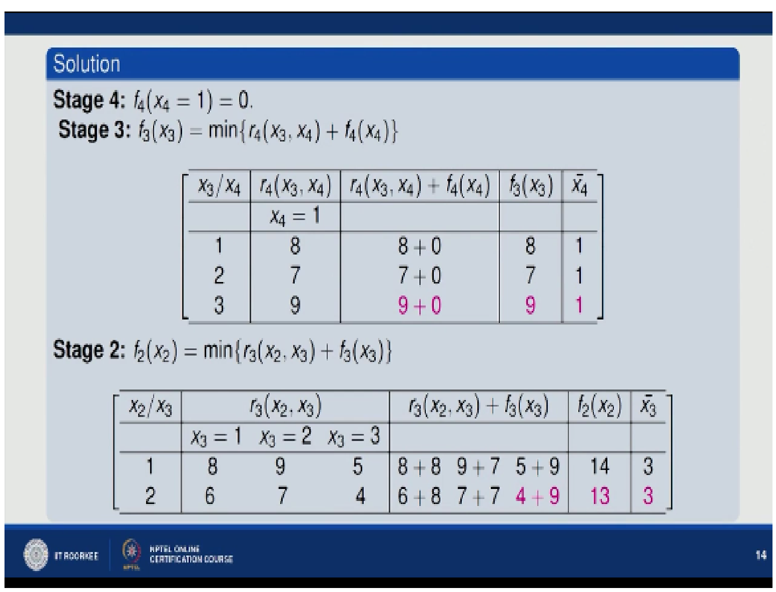

So here in the ppt, now similarly we go for a stage 1. A stage 1 will be refering but f1(x1) as shown innthe ppt minimum of r2(x1,x2) +f2(x2) we shown here x1, x2 and then r2 which the arc length from x1 to x2 . then we add f2(x2) which we already obtain innthe last stage from a stage 2. f2(x2) we have already obtained and then f1(x1) you see, now f1(x1) we are having three variables 1, 2 and 3. To it is 1, 2, 3 and x2 having only 2 values that it 1 and 2 so it is 1 and 2.

Now 1 to 1 that is x1, x2 =1 so the value is I mean the distance is f=1 is x1=1 x1= 1 to x2 = 1 there are one part that is three units and from this to this it is 2 units so it is 3 and 2. Similarly from x1 = 2 to f1= 1 in to it is 4 and 5 then 2 and 3. And whatever we have obtained in the last stage. That is stage 2 f2(x2) which we already obtain. Here it is 14 and 13 okay, you simply add, you simply add it here. 14, 13—14 and 13.

Now for this x1 the minimum is obtaining at here it is 15 and x2=2 this we are obtaining 18, 18 at both the values are 18. So it is obtaining a x2=1 or 2, now here it is 16, 16 we a obtaining when x1= 1 and 2 again 1 and 2. Now the last final stage is stage0, the stage 0 is the value 0 which is the minimum of this plus this. Here x0 is only one variable x0=1 and x1r1, 2, 3 in x1 are 1, 2, 3. Hence are 5, 7 and 6 so it is 5, 7 and 6 are shown in the picture also.
And f1(x1) is the minimum path obtained minimum value obtain in the stage 1, which is 15 , 18 and 16 so you add 15, 18 and 16. Now minimum of these three will give f0(x0) which you have to find out, so the minimum is 20 now okay. And x1 = 1, so x1=1 x1= 1 means this one this node this to this. Path is this to this and it is obtain at 15 and 15 we are getting from this point that is x2=2. You go now to know the minimum distance is 20 we are obtaining from here.

What are the paths to know the path we will go from last stage to the fiorst stage backward direction. We go to the backward direction to find out the path, you see it is 20 at x1=1 20 we are obtaining from this point 15, 15 we are getting from here okay when x2 is 2, x2 is 2 means this one that means this part okay now this 13 this 13 from where getting from where we are getting 13. This 13 we are getting from here okay.

When x3 is 3 so x3 is 3 that mean this one okay x3 is 3 this one and the last one is of course there is only one part so x4 is 1 that means the thing. So this will give the path I mean the path also.

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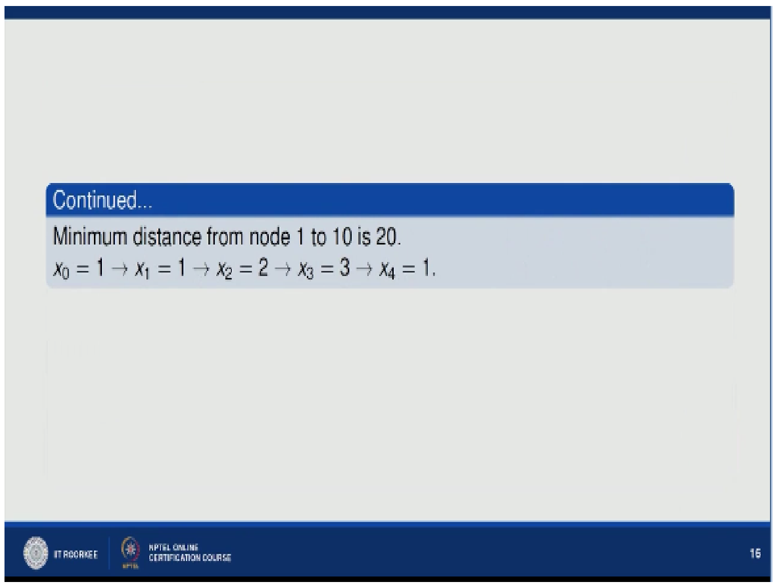

So the minimum distance of node 1 to node 10 is 20. The path is x from x0 to1 then x1 to 1 x2 to 2 x3 to 1 x3 =3 and then x4 = 1. So this is how using dynamic programming opproach we can easily find out the minimum distance from node 1 to node 10 the minimum path and the path also. The path length and the path also using dynamic progrqamming approach. So what we have done here we have defined, we have devided a larger problem into sub problems okay.

We are devided the problem into stages and whatever solution we are getting from the stage 3 we are using in the next stage and then the whatever we obtaining from here we are using th next stage. And the same processes we repeat then we get 0. So this is basically dynamic programming approach. We devide a larger problem into sum problem or a stage we defined a stage variables and then we whatever we are obtaining in the previous stage. The solution whatever we are obtaining we are using that solution in the next stage till we get the last stage okay. so this is how we can find out the optimal solutions of such problems using dynamic programming approach. In the next lexture we will solve some more problems based on dynamic programming. So thank you very much.

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