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Nonlinear Programming - 1

Lec – 14
Dynamic Programming - I

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So hello friends welcome to lecture series on nonlinear programming now our next topic is dynamic programming so what dynamic programming is and how it is useful to solve some nonlinear optimization problem let us say in this topic so what dynamic programming problem first.

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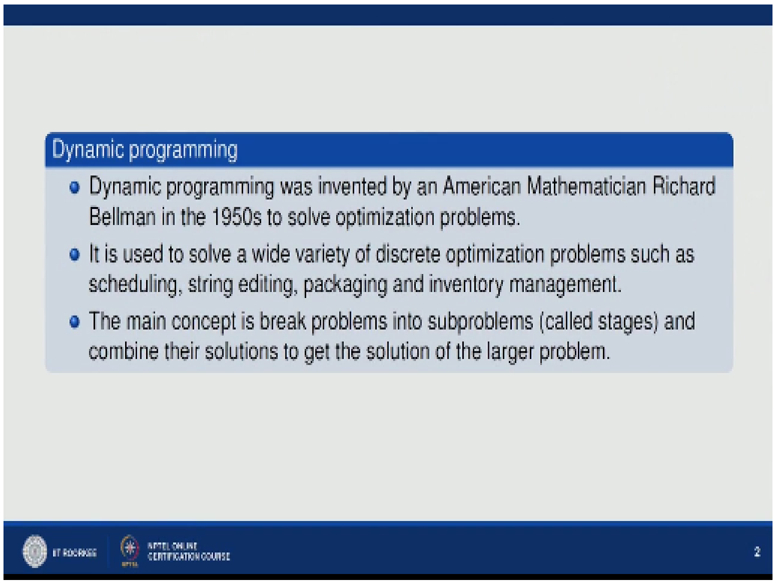

Let us see so dynamic programming was invented by an American mathematician Richard bellman in the1950s to solve some optimization problems it is used to solve a wide variety of discrete optimization problems such as scheduling, string editing packaging and inventory management, so there are many other many more applications of dynamic programming which is used to solve some discrete nonlinear optimization problems what is the main concept of this approach of this problems.

The main concept is break problems into sub problems which we are called as stages and combine their solution to get the solution of the larger problems basically what we have suppose we have a larger problem larger optimization problem so we divide that larger problem into sub problems and solve the last sub problem use the data of that sub problem into the next step and similarly when a when we solve that problem that is a problem we use optimal solution or the solution obtained from that stage to the next stage. In this way we can get solve a larger problem okay.

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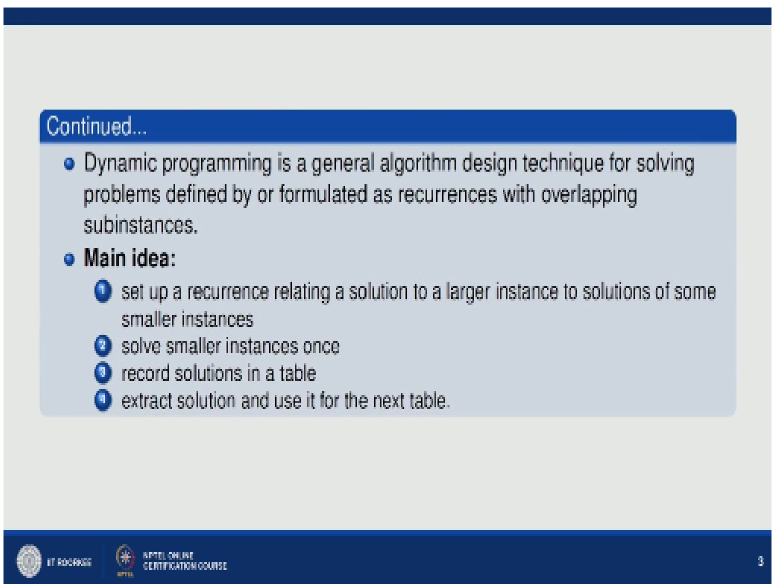

Now dynamic programming is a general algorithm design technique for solving problems defined by or formulated as recurrences with overlapping sub instances, so what does it mean we will discuss be it by an example know what the main idea main idea is set up an equation relating our solution to a lager instance a solution of some smaller instances okay solve smaller instances smaller vessel means sub problems you break the problem into sub problems solve smaller problems recall the solutions recall a solution of that sub-problem or that small problem and then extract solution and use it for the next table okay.

You take a solution of that small problem mode or that sub problem and use the solution of-problem into the next table in this way we will get back to the solution of the larger problem okay.

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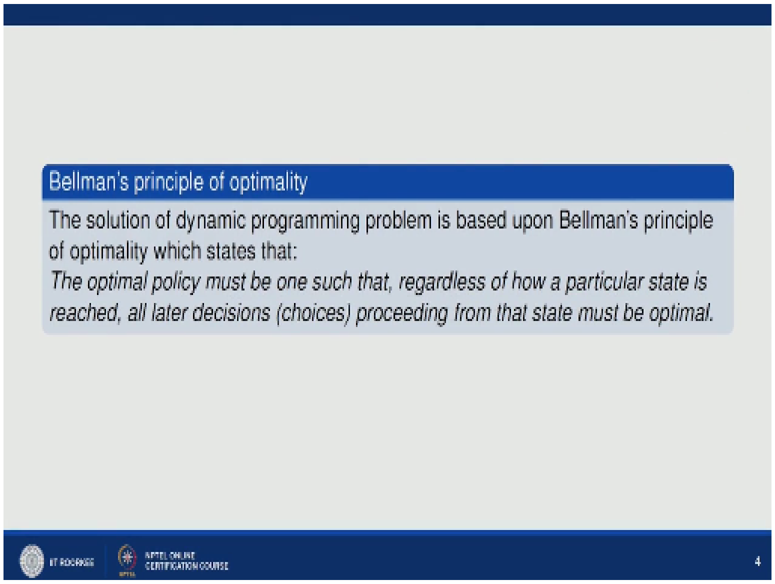

Now how does it come it comes from bellman principle of optimality what does it states it is states that the optimal policy must be one such that regardless of how a particular state is reached all later decisions or choices proceeding from that state must be optimal, so that is Bellman's principle what does it state it is states that whatever the solution we obtained from the sub problems and use it in the next table then if you repeat the same process until we get back to the primary table then the solution remains optimal okay.

So let us discuss dynamic programming first by a shortest path problem okay let us discuss software path problem first and from this we really have an idea what dynamic programming problems are and how it is useful to solve some non-linear problems.

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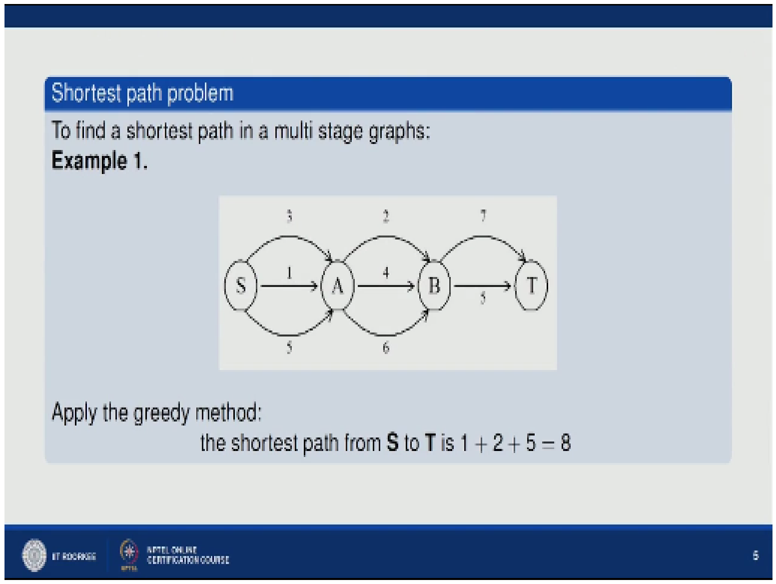

Now first use to find the shortest part in the multi stage graphs you see here in this example number one we have stages we have 4 nodes S,A,B,T these are the nodes or the cities or the station now these arc indicates the distance from this stage or the stage, it may be a distance it may be a cost or it may be some other venue okay say from S to a there are three different paths or three different cost it may be it is these are three one and five from A to B again we are having three different paths 2 4 and 6 and say okay from B to P we are having two different paths 7 units and 7 units.

Now what is the problem, problem is to find the shortest path from S to T which path we should adopt so that the distance from S to T is minimum how to find that path so the easiest method is greedy method what greedy method is you see if you start from s and to come to point A or node A so from S to A there are 3 parts or three units one unit and 5 unit what is the meaning our main aim is to find out the minimum distance or the minimum path from S to A out of these 3 paths which one is minimum.

Minimum is this one from S to A left straight path okay, so let us choose it one then from A to B we have 3 different paths to four and 6 units out of these 3 which one is minimum this two this two is minimum okay from A to B this one so from S to B which is minimum there is A straight path of one unit and from this path of two units that is total is 3 units and from B to P we are having only two paths which 5 is minimum okay. So this part is minimum that means if a person want to travel from node S to node T.

The minimum distance will be S, S to A which is one unit A to B which is 2 units B to T which is 5 minutes total is 1 + 2 + 3 + 5 that is 8 units, so 8 units the minimum path or the shortest path from S to T so this is ready method what we do method does greedy method takes minimum path from one node to another okay, so in this way we cover all the nodes and find out the minimum path or a minimum distance, however this method is not applicable for all type of networks you see if you take example number 2.

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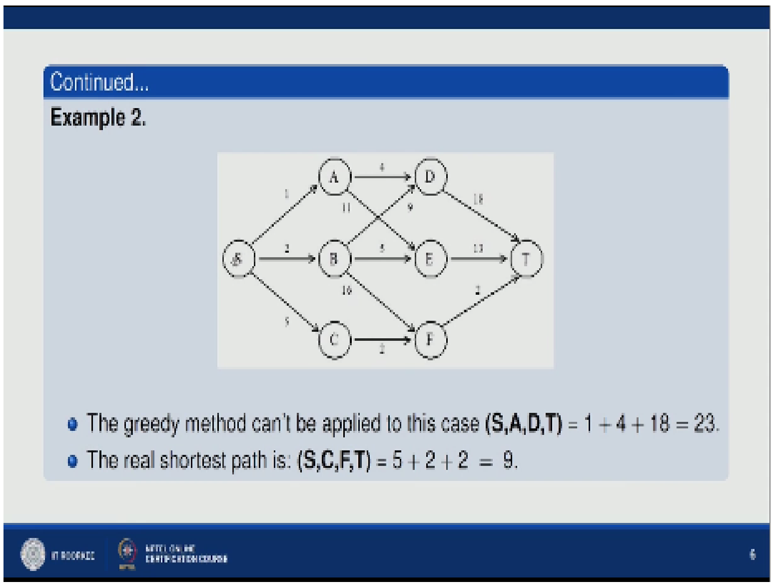

Say you have this network from S to T we are having we have to find out the minimum path again from S to T okay, if we apply a greedy method then from S to S we are having 3 different nodes here A B and C okay, which one is minimum which one is giving the minimum distance or the minimum path this one S to A if we apply a greedy method okay, now we came now we come to point node A from A to this stage we are having A to D or A to E only two paths which one is minimum A to D is 4 units A to is 11 units minimum is 4 units okay that means from S to D it is 1 + 4 that is 5 units, now from D to T we are having only one path that is 18 units, so total will be 18 + 4 + 1 that is 23 by the greedy method. However this is not the shortest path if you see this path SCFT, S to C 5 units, C to F 2 units and F to T 2 units that is 5 + 2+ 2 that is 9 units this is minimum, this is much lower than 23.

So that means greedy method fails here that means greedy method may be applicable for some simple networks but may not be applicable for complicated networks, so how can we find out the shortest distance if we have a complicated Network if we have this type of problem, how to find out the minimum distance or the shortest distance from node A to node P or from node X node Y okay.

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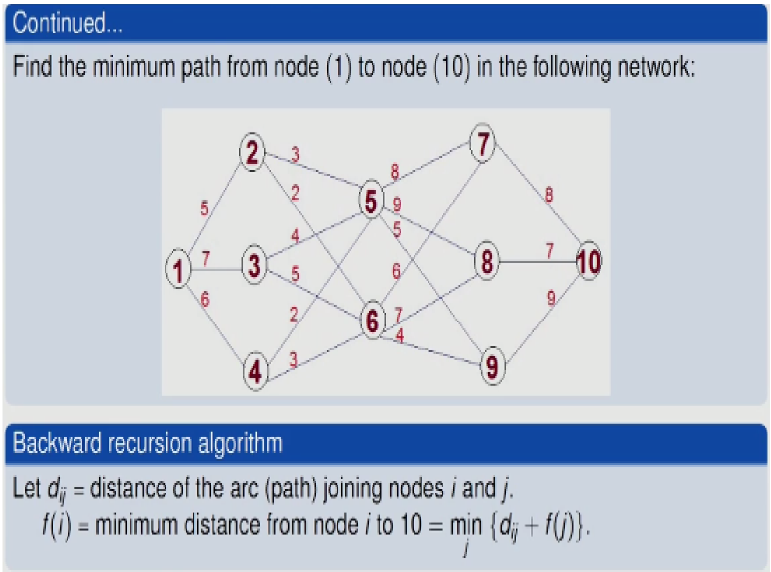

So suppose we have this path we have to find out the minimum path from node 1 to node 10 in the following network okay, here is node 1 this is node 1 and this is node 10 we have to find out the minimum distance or a minimum paths from this node to this node, so how can we proceed for this problem, so let us see.

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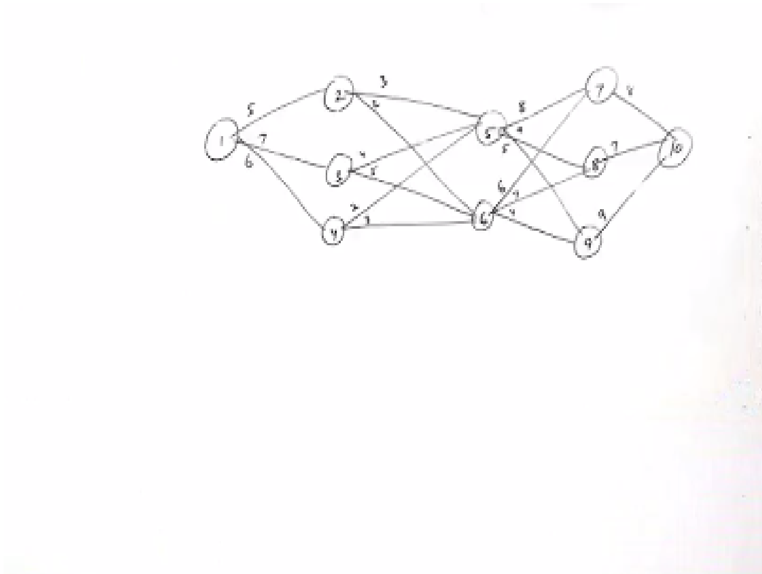

Here in this network we have to find out the minimum distance from node 1 to node 10 and the path is given to you say from 1 to 2 node 1 to node to node 2 the minimum the path is of 5 units node 1 to node 3 the distance is the path length is 7 units similarly if you see from path from node 7 node 5 to node 7 the path is of 8 units okay, now how to find out the minimum distance from node 1 to node 10.

So we can we have two methods either we can use backward recursion or we can use forward recursion suppose we use backward recursion how we solve, so first we define certain terms so first is $D_{ij.}$

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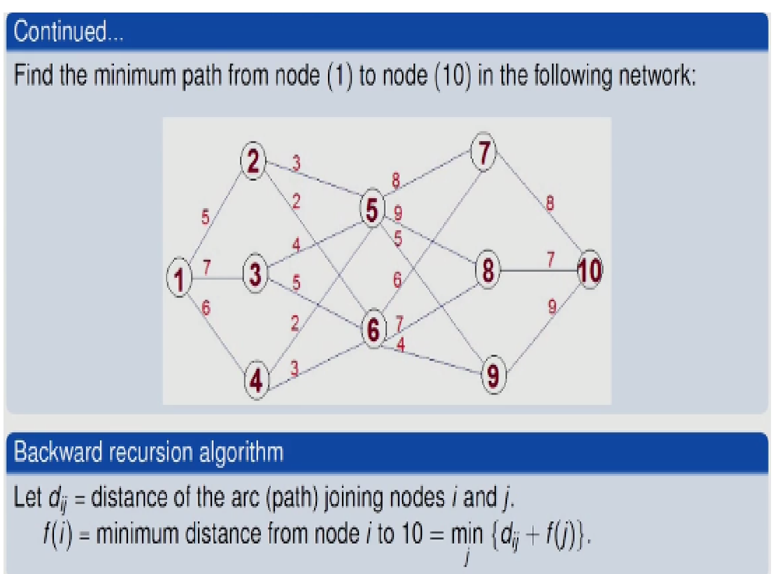

Is the distance of the arc joining the nodes I and J suppose you want to find out.

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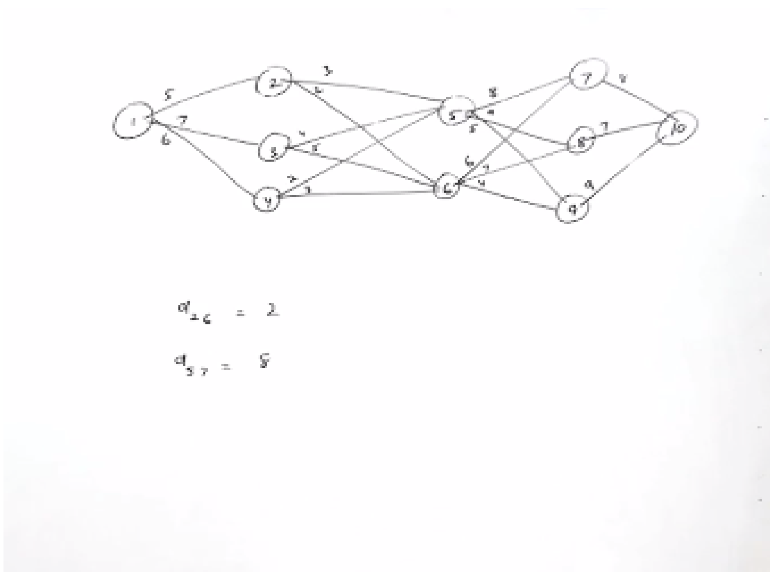

Suppose d_{26} d2 says the distance from node 2 to node 6 node 2 to node 6 the distance is 2 units, so this is then this is 2. Suppose D 17 or D7 D_{57} okay D_{57} , D_{57} is the distance from nodes 5 to node 7 it is a weight unit so it is 8. So D_{ij} simply stands for the distance from the node I to node J okay, now if you come to FI.

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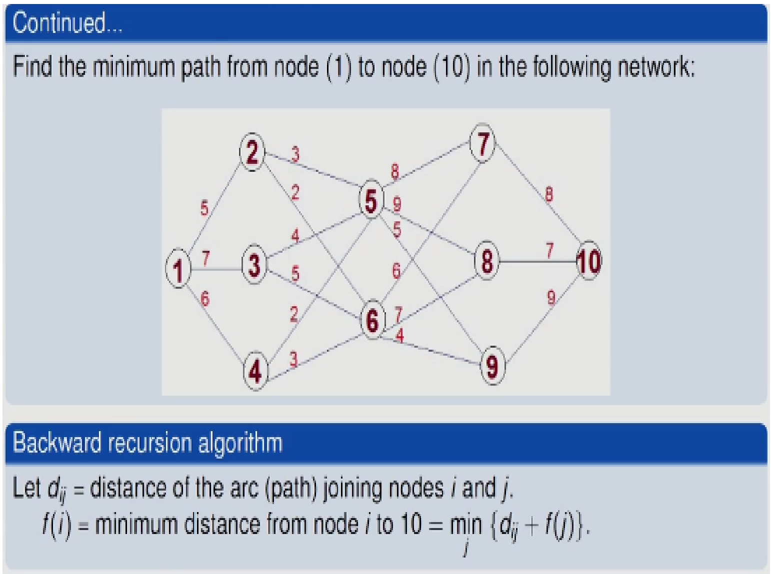

FI is a minimum distance from node I to node 10, okay from node I to node 10.

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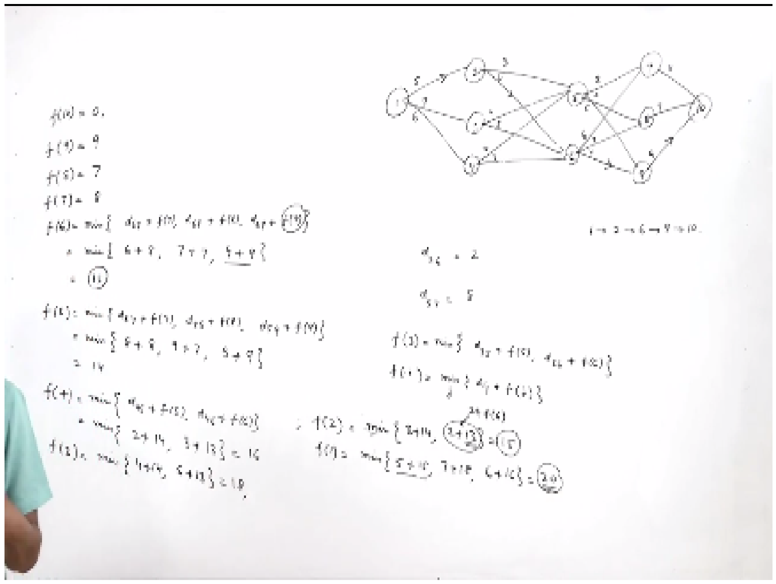

Suppose you want to find out suppose you want to find out F(10) okay that means the minimum distance from node 10 to itself that is 0, of course 0,suppose we want to find out F(9), f(9) is a minimum distance from node 9 to node 10 as per definition of F(i), f(i) is minimum distance from node I to node 10, if you want to find out F(9) that means minimum distance from node 9 to node 10, from node 9 to know 10 there is only one path.

So the distance minimum is 9 only okay, f(a) suppose we want to find out so f(a) is what, effect will be the minimum distance from node 8 to node 10 from node 9 to node 10 there is only one path so the distance minimum is 9 only okay. f(a) suppose we want to find out so f(a) is what f(a) will be in the minimum distance of node 8 to node 10 there is only one path here so it is of 7 units okay, now suppose we want to find out f(3) okay, f(3) f(3) will be what f(3) will be the minimum path minimum distance from node 3 to node 10 now from 3 to 10 we have to find out a minimum path okay.

Now 3 is connected to these two modes 5 and 6okay, so we can say that it is nothing but it is distance between 3 and 5 +f(5) and distance between 3 and 6+f6 and the minimum of these two will be f(3) of course you see this is 3 5+f(5) f(5) will be a minimum distance for 5 to 10 that will give the minimum that will be a distance from 3 to 5 the minimum of these two I mean distance between 3 and 5+f5 okay, distance between 3 to 6+f(6) the minimum of these two will give the minimum distance from 3 to 10 okay.

So in general how can we define f(5), f(5) how can we find f(5) it is nothing but or f(i) how can you find it f(i) it is minimum of dij+f(j) okay, minimum over j that will be the minimum, that will be give the f(i) the minimum distance from node i to node 10 okay, now follow the same thing what will be f(7) again f(7) will be a medium distance or node 7 to node 10 from node 7to node 10 there is only one path, so that will be a minimum that is 8 units, 8.

Now suppose you want to find f(6) okay, what will be f(6), f(6) will be minimum of now from 6 to 10 we have first we have here these three nodes or 6 is connected to first with DC nodes and then with 10 okay, so first we find d67+f(7) then d68+f(8)+ then d69+f(9) so the minimum of these 3 will give f(6) which is the minimum distance from node 6 node 10 that is further equal to a minimum of d67 is 6 and f(7) is 8, d68 is 7 and f(8) is 7, d69is 4 and f(9) is 9, so it is 13, it is 14, it is14 so minimum is 13, okay.

So from node 6 to node 10 the minimum path length is 13 units and how to, how we can travel, how can we get it 13 from node 6 to node 10 you see 13 we are getting over here that means 6 to 9 and then 9 to 10 that is in 4+9 is 13 okay, because 13 this 13 we are getting from this point and this is f 9 that means 6 s square t 2 9 and 9 to 10 that is 4 plus sign is 30 okay but we have to find some node one to node ten okay so in the same way we will move backward then you find f5, f5 again will be minimum of it is D again it is f5 7 + f7 then d_{58} d_{58} + f_8 then d_{59} + f_9 it is refers to a minimum of what is d_{57} it is 8 f_7 is 8 d_{58} is 7 f_8 okay d_{57} also d_{58} it is 9 okay.

So it is 9 and f_8 is 7 which is 7 d_{59} is of five minutes + f_9 is 9 so it is 16 and 16 and 14 so minimum is 14 so minimum distance from node 5 node to10 is14 units okay now f_4 you find $f_{4,}f_4$ will be minimum of now from 4 to 10 first we have these three these two nodes that is 5 and 6 so you first find d_{45} + f_5 and d_{46} + f_6 d_{45} is distance between node 4 and node 5 that is 2 2 +$F_{5,}$ F_5 is 14 then d_{46} is 3, 3+ F_6 is 13 so it is13.

So it is 16 minimum is 16 at both the points it also stated all 16 now similarly f_3 okay so f_3 will be minimum of minimum of this 4+f_5 so 4+$f_{5,}$ $f_5$is 14 so 4+40 and then it is 5 + f6 so 5+f_6 plus is 13 so it is 18 okay now what is F_2 will be equal to minimum of now F_2 it is 3 +f_5 first 3 +F_5, F_5 is 14 so 3+ 40 and then it is it is 2+ $F_{6.}$
So 2+ F_6 is 13 so it is 13 so minimum is 15 minimum is over here so that means from node 2 to node 10 the minimum path length is 15 units now finally F_1because F_1 is to find out $f_1$is a minimum distance from node 1 to node 10 this is to find out okay so f_1 is minimum all now F_1 it

is 5 this is 5 + F_2 5 +F_2 F_2 15 and then 7 +$F_{3,}$ F_3 is 18 then 7 + 18and then 6 plus F_4 6+ F_4 then 6+ F_4 is 16 so it is 16 so minimum of these three is that is 20 it is 25 it is 22 so minimum is 20 so the minimum path length, so the minimum path length from node 1 to node 10 is 20 units what is the part of 20 units? Now how to find that you see, this is 20 units this is a minimum distance minimum path length from node 1node10. We are getting this 20 over here this is 20 okay, now this 15 we are getting from 2, so this 15 we are getting from here, this f_2, this is this was f_2 this was f3this was a 4 okay.

So this 15 we are getting from here now this 15 we are getting from here okay, now this is minimum of this f_2 okay this f_2 minimum of 3 + f_5 and 2 + f_6 so it is a f_6, 2+ f_6 okay now come to f_6 is here , we are starting this certain is over here, so first we move to 2 this is f_2 okay first we move to 1 to 2 and then from 2 to 6 from 2to 6 okay and then from 6, now if this 13 we are getting from here and this is a f_9 from 6 we are getting to 9 and from 9 we are get you 10 there is only one path.

So that is this part now, so path is 1 to 2 then 2 to 6 then 6 to 9 and a 9 to 10 you take this path it is 5 it is to 5 + 2 7, 7 + 4 11, 11 – 20, so that is a minimum path length is 20 and the path is 1 2, 2 to 2 6,6 to 9, 9 to 10, so this is how for such networks we can easily find out the minimum path length from node to node 10 okay. Now this is backward recursion because we started from node10 and come to node 1, now same lines on the same lines we can also use forward recursion.

We can let fi as minimum path from node 1 to node I, if you use if we define fi like this, so instead of following them as a backward direction we will move in the forward direction okay, so same thing is given in the 50s also.

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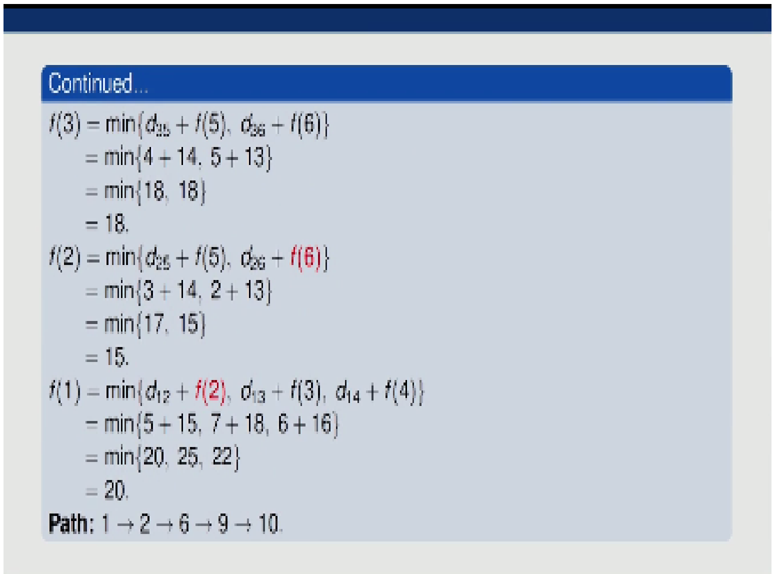

you see there is a path length is 1 2, 2 to 6, 6 to 9, 9 to 10 and that we obtain from here f 2 then F 6 then f 9 okay and f9 is 9 from there we are getting.

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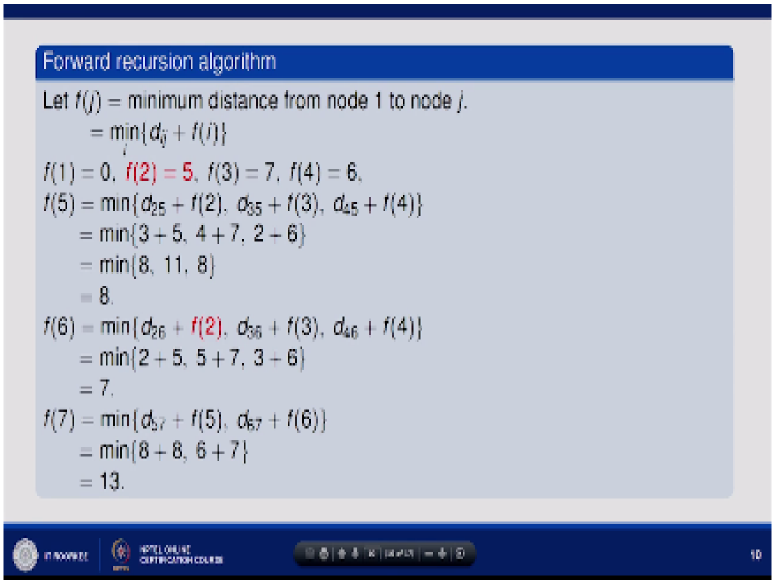

And this is forward recursion algorithm for the question whether we define the minimum distance from node 1 to node G like this and using the same concept we can easily find out the path is 1 to 2, 2 to 6, 6 to 9, 9 to 10. So this is just an illustration that how can we solve such a complicated networks using forward or backward recursion but this is not an animal programming approach, so how can we solve this type of problem for problem of shortest path using dynamic programming approach, so that we will see in the next lecture that how can we solve such problems using dynamic programming approach so thank you very much.

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