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Nonlinear Programming - 1

Lec – 13
Geometric Programming- III

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Hello friends, so welcome series on in the linear programming so we are dealing with the geometric programming problems, we have seen that if we have highly non linear problem having negative powers and the powers infraction then how can we find the optimal solution of such problems what we have seen that if we have a un consist optimization I mean the problem without any constraint then how can we find optimer solution and how can we find optimer solution for the problems with constraint optimer solution and the equality optimer solution of the constraint and deal with.

Now in all those problem the coefficient we have assumed they are positive all the coefficients and all that terms were assumed that they are positive now how can we solve those positive normal having the negative coefficient, so that will deal in this lecture first we will see how can we solve appropriate problem begin the disability more than the zero it is that one suppose that is okay.

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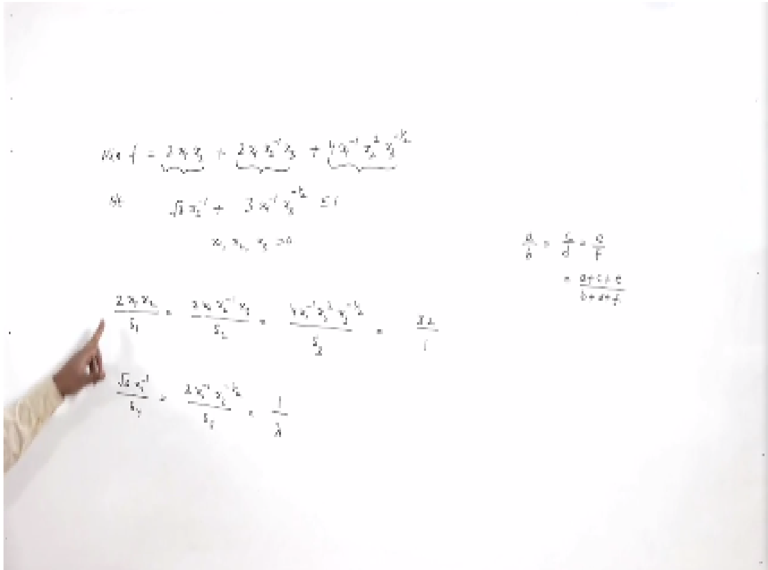

It is the suppose the problem is the minimization of the F= it goes to and x_1 x_2 +it is 2 x 1 and the x 2^{-1} and it is to be okay the +4 x_1, x_2^{-1} x $_2$ and the x $_3$ – ^{½} subject to the constraint is under the 3 x_1 x_2 – 1 +3 x_1 x_2 _1 3 $x_2^{-1/2}$ and it = to 1 and x1 x2 x3 and greater than 0, So let us consider the problem okay now in this problem how many terms we are having we are having 1 2, 3, 4, 5.

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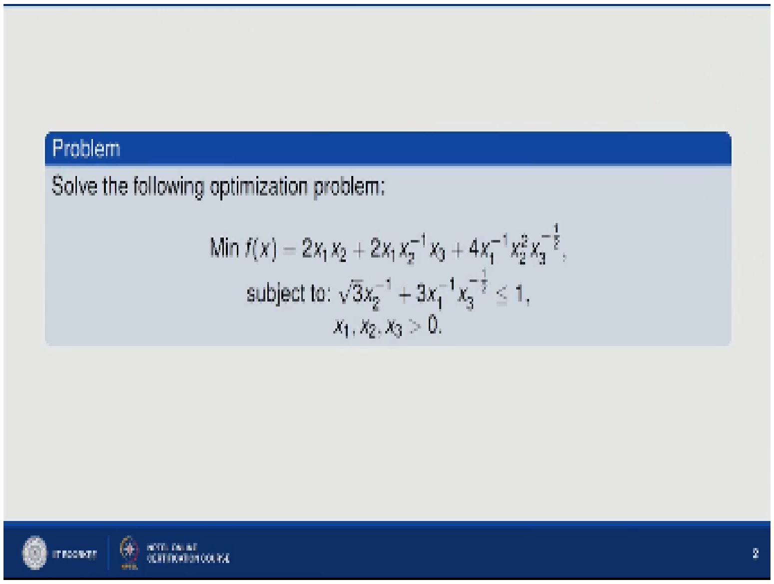

And there are five terms okay, and how many variables we are having we are having three variables X1 X2 X3, so the degree of the difficulty for this problem is the total number of terms in the objective function as well as in the constraints that is 5 1 2 3 4 5, how many variables are three that is -3 -1 this we have already shown in the last lecture the % of the difficulties is equal to and the total number of the terms in the objective function and in the all the constraints – number of the variables and the number of the unknowns in the next one that will be the % of the difficulty, and the % of the difficulty is for this problem is 1 .

Now the % of difficulty is this problem if you solve it will involve the less number of that gap of the variables and the constraints will be 1 that means the constraint which we are having with the respect to enter and with us we will be having many solutions okay so how we will form gap of the problem that will be maximize the G Δ okay. now the first coefficient is 2 it is the two appoint Δ will be the 1 and the Δ1 it is the two appoint the Δ2 will the power is the Δ2the third term is the 4 upon the Δ3 and the next term is under the 3 upon the Δ3 is the Δ4 and the next term is 3 which is 3 upon the Δ and the 5 is the Δ5 and the σ is the power σ is Δ4 and the Δ 5.

And the σ is the constraint and the constraint it will the Δ 4 and the Δ 5. So it is the σ =to Δ 4 and Δ 5 and it is okay. subject to the what are the condition now see the condition okay the corresponding to the X 1 is we take the Δ1, so the power of here is Δ 1 here it is Δ 2 ant it is the 1 and the 1 *Δ2 is the Δ 2 here it is -1power it is -1 Δ 1 3 s o it is Δ1 and the Δ 1 from here + Δ2 – Δ3 and it is the –Δ 4 and –Δ 5 = 0 so that will be the first constrain okay that will be the first constraint now it is X 2 it this problem it is X 2 and it is the – 1 so Δ$ will not be here will not be in the constrain okay now cropper the X 2 it is the Δ1 and it is okay.

Then it is the from the next term what we are having in the – Δ2 okay here it is the +2 Δ3 then here it – Δ4 =0 okay now from the net construction corresponding through the X 3 X 3 there is no term here it is the 1 term so it is the Δ2 and then here it is -1 / 2 and Δ3 there here is no term of ΔX 3 and here it is – ½ and it is -1 / 2 Δ5 = o and off course Δ1 + Δ2 + Δ3 =1 corresponding in the constraint in the objective function and the corresponding the objective function we are have 123 terms and the Δ1 Δ2 Δ3, so sum of these three in the Δ1+ Δ2 + Δ3 and the all Δ is must be all the 0 for 1.

Tthese are the system of the equation which we are having now in these equation we are having the 5 unknownsΔ1 Δ2 Δ 3 Δ4 Δ5 and we are having the four equations so it will be having many solutions as we have already seen by the finding of the difficulty so it is the difficulty 1 so we will be having the less equation and more unknown infinitely many solutions now what we will do we will the save this exercising many solutions and we will try to express all the Δ of the one Δ and the Δ 5 okay so we have solve this problem you can see here we have first find the g Δ .

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Solution

Degree of difficulty of the problem is

$$d = 5 - 3 - 1 = 1.$$

$$\text{Max } g(\delta) = \left(\frac{2}{\delta_1}\right)^{\delta_1} \left(\frac{2}{\delta_2}\right)^{\delta_2} \left(\frac{4}{\delta_3}\right)^{\delta_3} \left(\frac{\sqrt{3}}{\delta_4}\right)^{\delta_4} \left(\frac{3}{\delta_5}\right)^{\delta_5} \lambda^{\lambda},$$

subject to:

$$\begin{gathered} \delta_1 + \delta_2 - \delta_3 - \delta_5 = 0, \\ \delta_1 - \delta_2 + 2\delta_3 - \delta_4 = 0, \\ \delta_2 - \frac{1}{2}\delta_3 - \frac{1}{2}\delta_5 = 0, \\ \delta_1 + \delta_2 + \delta_3 = 1, \\ \delta_j > 0 \; \forall j. \end{gathered}$$

(Here $\lambda = \delta_4 + \delta_5$). So, we are having 4 equations and 5 unknowns.

And the G Δ is the same expression and which we have discussed here the subject to these number of the equations so we are having the four equation and the 5 unknowns here.

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Hence, the system have many solutions. Express all the variables (δ_i) in terms of one δ_i (say δ_5).

Then

$\delta_1 = \delta_2 = \frac{1+\delta_5}{4}$, $\delta_3 = \frac{1-\delta_5}{2}$, $\delta_4 = 1-\delta_5$.

Substituting these values of δ_i's in the expression of $g(\delta)$, taking logarithm and then the derivative of $g(\delta)$ w.r.t. δ_5 (to maximize $g(\delta)$), we get $\delta_5 = \frac{1}{2}$.

Therefore:

$\delta_1 = \delta_2 = \frac{3}{8}$, $\delta_3 = \frac{1}{4}$, $\delta_4 = \frac{1}{2}$ and min $f(x) = \max g(\delta) = 32$.

From the equality condition,

$$\frac{2x_1x_2}{\delta_1} = \frac{2x_1x_2^{-1}x_3}{\delta_2} = \frac{4x_1^{-1}x_2^2x_3^{-1/2}}{\delta_3} = 32,$$

and $\frac{\sqrt{3}x_2^{-1}}{\delta_4} = \frac{3x_1^{-1}x_3^{-1/2}}{\delta_5} = 1$, we obtain $x_1 = \sqrt{3}$, $x_2 = 2\sqrt{3}$ and $x_3 = 12$.

So we will express all that Δ in the Δ5 so after solving in the spectrum solving in the will be obtain the Δ1 and the Δ2 is the 1 +1 /5 and the and the Δ 5/ 2 and the Δ /4 = 1- Δ5 okay this we can obtain and the shoving this equation now then we can sub seed all the values of the Δ and the Δ1 and Δ2 and Δ 3 and Δ 4 in the G Δ okay in this expression in this expression we will substitute all the values of the Δ okay then this function will be in the one variable that is Δ 5 so it is the maximize in the order to maximize we are only now this function and this Δ is one variable and take the log both the sides differentiate the spectrum the data 5 find the maximum value of the G Δ where we do all this process we get the Δ 5 = to ½ okay as we got the Δ5 = 1/2substuting this value in this all this expression over here we will get Δ1 Δ2 Δ3 Δ4 .

So the minimum value of the F which is = to the maximum value of the G Δ which we already know is nothing but the 32 how we obtain 32 by subtitling the Δ1 Δ2 Δ3 Δ4 Δ5 over here and the σ over here when we substitute all this value we got the maximum value of the G which is the same as the medium value of the F okay now how to find where that in that now use the equality and the condition okay from the equality and the condition that we know that it this is the one you one and this you too and this is the U 3 so we know that from the equating constraint that U 1 upon the Δ 1 should be = to U 2 upon the Δ2 should be equal to U 3 upon the Δ3 and that should be = to is the a upon the b = to is the as the upon the b = c upon the d = e upon the f that is also = to a +b +c upon c +e +f so that is also = to this term +this term +this term which is the nothing but the minimum value of the F which is 32 and the violent obtain and the denominator the Δ1+ Δ2 + Δ3 is the one so it is 1 okay from the next condition from is the constrain the under the root do the 3 x is the – 1 upon the Δ4 =to 3 X 1 X 2 -1 and – ½ and the Δ 5 = to now this +this as a upon b = to a upon D = to A upon as the B + D is equal to this + this which is = to 1 because this optima the optimer solution will be the optimer is the equality holes so it is equal to 1 and the denominator of the Δ4 is Δ 5 which is σ okay now using this equality constrain this can easily find out the value and the X 1 X2 and X 3 which are X 1 = to 1 /3 X 2 is the two under the 3 and the X 3 = to 3 L.

So this will be the optimer solution of this problem so that is how were can solve those problem having difficult and the 0 if the difficulty is the 0 in that case we will be having the suppose the n number of the variables and n number of the equation this is the unique solution in that is case of no need of the maximization in that the G Δ since the value of the G Δ is unique so that itself with the value of minimum of the F however the % of the difficult is more than the 0 so we have to find the maximum value of the G Δ/ differentiating with the respect to the unknown variable okay now how to solve a general possible nominal is mean.

How to solve a problem having negative confident so we have the generalize polymer optimization applicable for the problems involving negative coefficient also it will solve such problems we introduce the signal function so that the terms of the coefficient reduces with the problem with the positive coefficient okay to solve such problems having negative coefficients so those terms involve negative coefficient we involve a term a signal function so that term will become positive okay suppose in this problem okay so how to solve this problem let we see so this problem is minimum of the F that is equal to.

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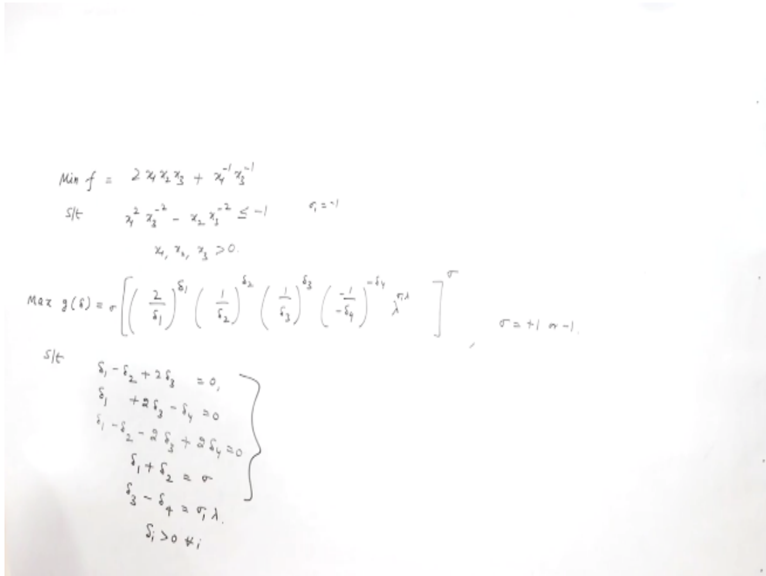

And it is – 2 l x_1 x_2^{-3} x_2^{2} + it is 4 x_1^{-3} x_2^{-3}+ and it is sx_1^{-1} $^{-4}$ and x2 ^{2} from subject to x1, x2 greater than 0 so it is it is unconstraint optimization okay ns this term is negative this term is having negative coefficient, now what a degree or difficulty of this problem no we are having 3 terms 1 , 2, 3, 3 terms we are having and how many un knows two unknowns x1 and x2 so degree of difficulty will be 3 – 2- 1 that is 0 degree of difficulty is 0 that means we will be having unique solution for g Δ and we are having unique solution g Δ means that value of g Δ will be the value of minimum of a f okay.

Now to solve such problem let us see we first find maximum of g Δ which is given as now it is having negative coefficient which is -12 so in -12 we will take – Δ to encounter this negative side to make this positive okay so for this term we are taking negative Δ1 – Δ1 okay this is positive coefficient so we take Δ2 + Δ2 this is positive coefficients so we take + Δ3 okay so this we will do only for those terms having negative coefficients okay so it is – Δ1 then 4 / Δ2 $^{\Delta 2}$ and 3/ Δ^3 now to encounter this – Δ because we are introducing it –Δ to make -12 positive so what should be change in this function the change is only this that we multiply this by P and rational power P where P is either +1 or -.

Okay to encounter this effect we multiply raise4 to power by rho where P is either +1 or -1 now subject to what are the conditions again for this term we are having – Δ1 for this term we are having + Δ2 and for this term we are having + Δ3 now correct the coefficient of x1 1x1 it is 3δ1 because -3δ1 – 3 x – δ1 that is 3δ1 here it is – 3 δ2 because δ2 is positive + 2 and here it is – 4 δ3

= 0, now for the second term for the second term variable x2 we are having – 2 δ1 because it is 2 x – δ1 because correspond to this term we are having – 2 – δ1 okay and for this term we are having + 3 δ2 +2 δ3 = 0 and of course – δ1 +δ2 + δ3 instead of δ1 we are having – δ1 okay.

Now this is σ which may be + 1 or – 1 okay and all δ is r strictly > 0 now here we are having 3 equations with 3 unknowns, unique solution and that unique solution will give the value of maximum z maximum of dδ = minimum of f and using the equality constraint where an easily obtain the value of the unknowns okay, so solution is solution is here you see that we can find.

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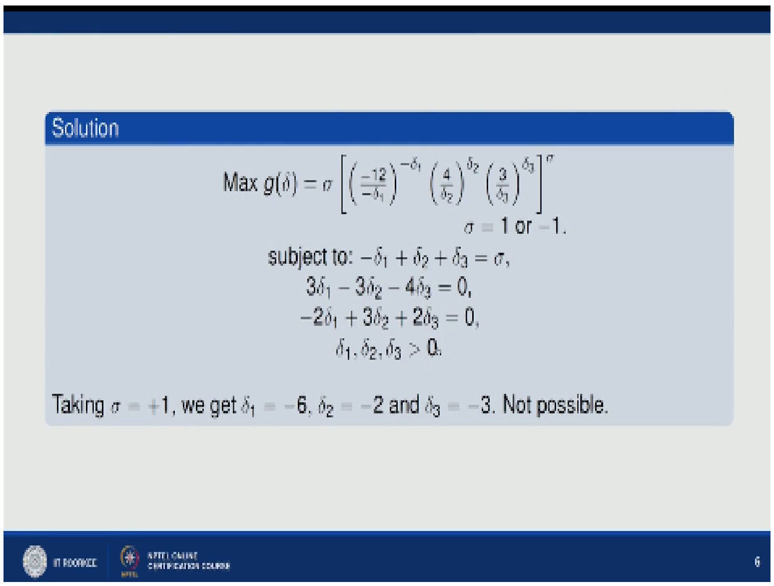

Maximum of gδ which is given by this expression where row is + 1 or – 1 subject to these conditions now we can take ∂s either = 1 or – 1 suppose it is + 1 then we get δ1 as – 6 δ2 is – 2 and δ3 is – 3 that we can easily verify okay you can substitute ∂ as 1 here and solve this e equations you can simply see that these are values of δ is we are obtaining which is not possible why it is not possible because over δ is must > 0 as per this condition okay.

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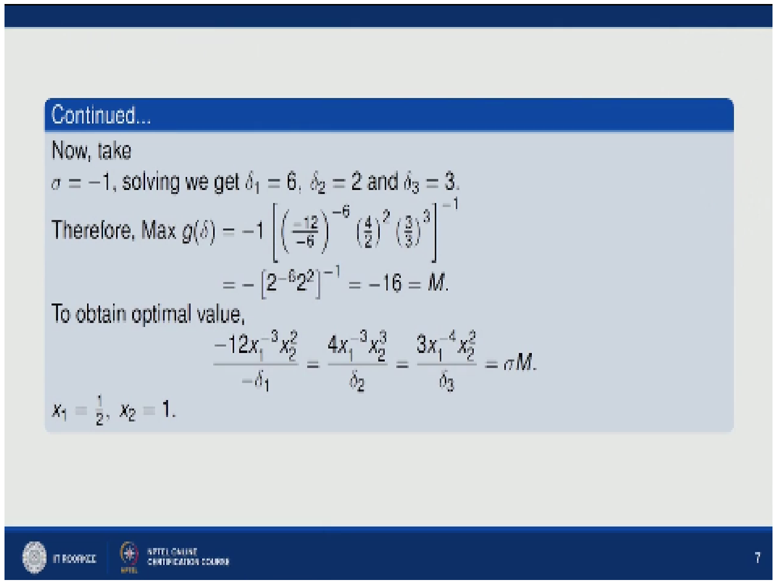

Now take ∂ = - 1 because ∂ can take only two values either +1 or -1 + 1 does not give a visible point visible solution so we take another option that is we take ∂ as – 1 if we take ∂ is – 1 and solve these three equation we will get δ1 as 6 δ2 is 2 and δ3 as 3, okay we substitute these values in the expression of gδ we can obtain the maximum value of gδs – 60 which is same as minimum value of f okay, now how we will obtain the optimal solution, by applying equality condition now here what will be equality condition let us see. Here what is the first term, first term is -12 $x1^{-3}$ $x2^{2}$ upon now corresponding to this term we are having –Δ1 so it is –Δ1 = it is 4$x1^{-3}$ $x2^{3}$ /Δ2 because correspond this term we are having +Δ2.

And for the third term it is 3$x1^{-4}$ $x2^{2}$ /Δ3 again this is equals to this is something a/b = c/d = e/f = a + c + e/ that is some of the numerators another some of the denominators. Some of the numerators equals to minimum of f which is -16 as we already obtained and –Δ1 + Δ2 + Δ3 is ρ and ρ is -1, okay. For -1 we are obtaining the solution we are getting the solution so this is nothing but 16.

So we are having this Δ1, Δ2, Δ3 already know we already know the values of Δ1, Δ2 and Δ3 so solving this and put it equal to 16 we can easily find the values of x1 and x2 so x1 is ½ and x2 is 1 you can easily check okay so in this way we can easily get the optimal solution of such problems. Now this is unconstraint posynomial optimization with negative coefficients how can we solve our problem having negative coefficient and having constrained also that is constrained posynomial optimization with negative coefficients, so what is the problem let us discuss the problem which is minimum of f which is equals to it is 2x1x2x3+ is a $x1x2^{-1}$ $x3^{-1}$ subject ton it is $x1^2x3^{-2}$ –x2 and $x3^{-2}$ ≤-1 and x1x2x3=0.

So I am simply giving the outline of how to solve the problem okay, and you can simply solve the same equations and you can easily find out the values of δs when you find out the value of δ this will give the value of maximum g(δ) which is equals to minimum of f and using the equality condition you can easily find the values of the unknowns, okay. Now here this is negative okay, how many terms we are having 1,2,3,4, 4 terms we are having how many unknowns 3 unknowns so degree of difficulty will be 4-3-1 that is 0.

So degree of difficulty is 0 that means unique solution okay, now how to solve it so we find maximum of gδ which is equals to the first coefficient is 2 so it is $(2/\delta1)^{\delta1}$ it is positive, so we take +δ1 now it is also positive 1so it is $(1/\delta2)^{\delta2}$ the third term is again positive coefficient so it is $1/\ \delta_3{}^{\delta3}$ now it is -1 so we take – δ_4 correspond in to this tem okay, so it is $-1\ /\ -\ \delta_4{}^{-\delta4}$ now this is -1 so what we do we take ρ_1 as -1 where ρ_1 is this term okay we assume. Now this in to σ^{σ} where σ is either +1 or -1 and for the constraint positive nominal optimization we are also having a term δ^{δ} okay so δ here it is over δ because this is directive oaky.

Whatever term we are having it is +1 so it is +1 and if it is -1 so it is -1 okay and subject to what are the condition let us see the condition also, so coefficient that is power of x_1 power of x_1 here is δ_1 here it is – δ_2 here it is +2 δ_3 and here no tem of x_1 so it is equal to 0. Now for x_2, x_2 is δ_1 no term here and here it is +2 δ_3 and here it is – δ_4 because here we are taking term as – δ4 then $x_3{}^{x3}$ is $\delta_1 - \delta_2 -2\ \delta_3 +2\ \delta_4 = 0$ as this is -2 δ-2 multiplied by $-\Delta_4$ which is plus $2\Delta_4$ that is equals to 0 and $\Delta_1+\Delta_2$ is equals to 0 here the row is+ 1 or -1 $\Delta_1+\Delta_2$ and $\Delta_3 +\Delta_4$ is equals to λ were row 1 is -1 okay and all Δ is are 0 Δ_1-Δ_4 because it is $-\Delta_4$ cross point to this okay.

Now if you see these equations okay so this are four equations four unknowns okay that means unique solution solving these first you take row has suppose +1 you find the values of Δ if all Δ, to be positive that means you are getting the solution okay if Δ is come of sort to be negative then we take row has -1 and against from the problem okay.

So all this systems of equations so for once you get the values of Δ all positive values you substitute it over here you get the values of maximum over gΔ that will be equals to minimum of f and use the equality constraint you get optimal solution of the problem okay.

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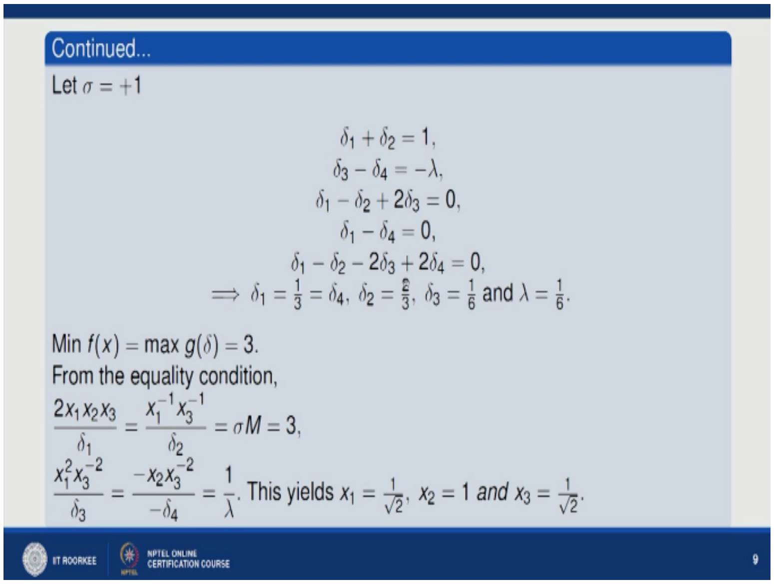

So that's what the solution gives now when you let row has +1 so you get the values of Δ has Δ_1=1/3 which is equal to Δ_4 δ2 = 2/3, δ3 1/6, and λ is also 1/6 hence verified and minimum of fx = maximum of g δ = 3 okay. Now from the equality condition, now you see, if you see the equality condition that u2/ δ1 = u2 δ2 that must be equal to the sum of numerator and sum of denominator , sum of numerator is m, m is the minimum value of f okay we are calling it M which is 3 and sum of δ δ1+ δ2 is ρ okay, so it will be M / ρ you see it is when you apply here it is 2x1, x2, x3/ δ1 x2 x1 – 3 / δ2 which is = M/ρ the sum of numerator and sum of denominator.

Which is also = ρ M / ρ^2 and ρ^2 is always 1 because row is either +1 or – 1, so it is nothing but ρ M and M is here for this example M is 3. Now similarly take the term code constraint you take the sum of the numerator, sum of the numerator is -1 here and sum of denominator is – λ, so – cancel out it is 1/ λ, λ you already know it is 1/6 so you will be having this system of equations by which you can easily find out the value of x1, x2, x3, which are 1/√2, so that will be the optimal solution of this problem.

So that is how if we have complicated non linear programming problems, so some of the problems can be solved using geometric approach, if we have some fresh in power or negative power or higher powers constraint or unconstraint problems, so that can be solved simply by using arithmetic geometric mean inequality, so that is how we can easily solve some non linear problems using this approach so thank you very much.

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