Integral Equations, Calculus of Variations and their Applications
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Lecture 52
Isoperimetric Problem - 2

Hello friends welcome to today's lecture we will continue our discussion over the Isoperimetric Problem. So as we have discussed in Isoperimetric Problem we are finding the extremal of a some functional provided some some conditions on the extremal curve is given in terms of in other functional.

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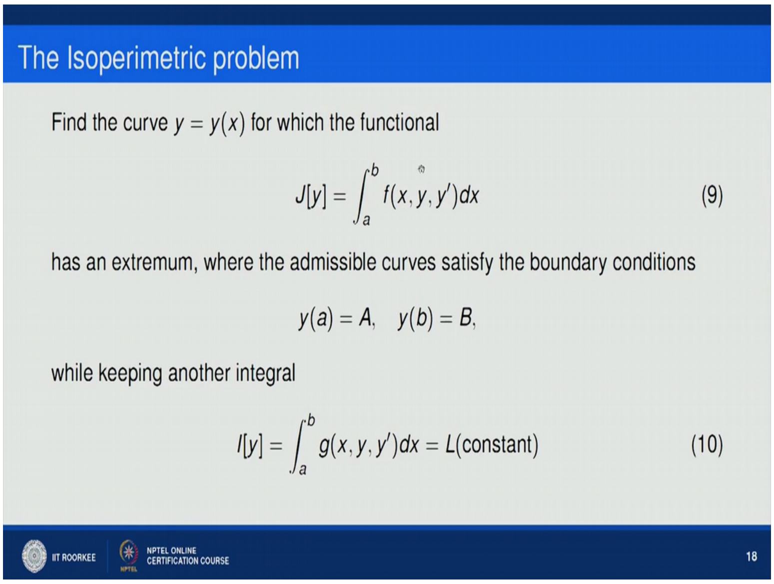

So let us come to this so what we have shown here that we want to find out the curve for which the functional is having an extremum and this extremal curve satisfy the boundary condition y of a equal to A and y of b equal to capital B and this extremal curve y equal to y of x keep this another integral as a constant value. So in previous lecture we have seen the working of this.

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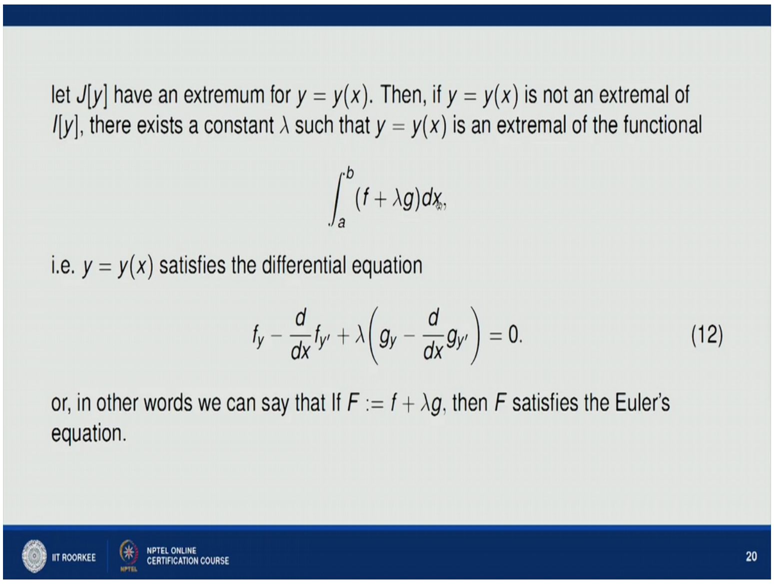

Here to find out the condition on this extremal curve is to show that the extremal will be an extremal of the another functional that is a to b f plus lambda g dx provided this extremal curve that is y equal to y of x is not an extremal of the other functional that is I of y. So here we have seen that if y equal to y x is an extremal curve for J of y and if y equal to y of x is not an extremal of another functional that is I of y then they exist a constant lambda such that y equal to y of x is an extremal of the functional a to b f plus lambda g d of x.

So here if I denote this integrand f plus lambda g as another integrand that is capital F here then you can say that F satisfy the Eulers Equation and in next in previous class we have proved the theorem using variational derivative and here in this lecture now let us try to generalized the concept which we have discussed in previous lecture.

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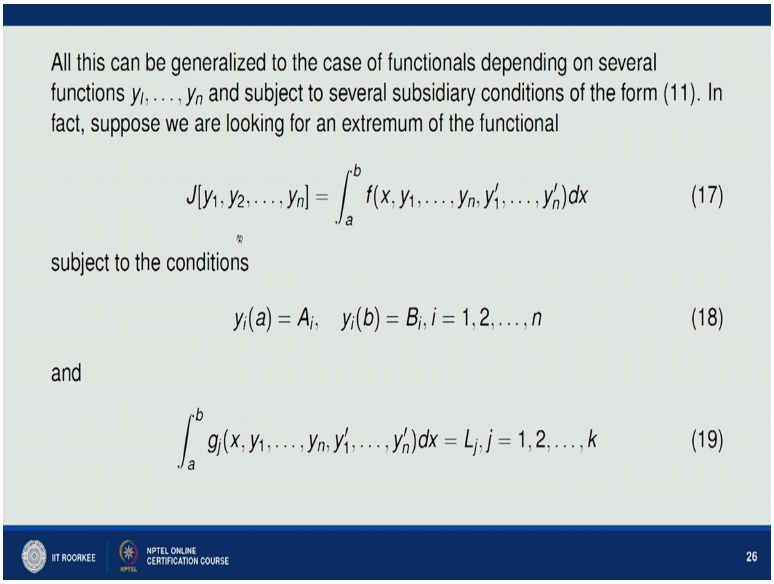

So what we try to do here we want to find out say extremal of the functional J of y 1 to y n which is given as a to b f of x, y 1 to y n y 1 dash to y n dash d of x. Subject to the boundary condition y i a equal to capital A i, y i b equal to capital B of i, for each i equal to 1 to n. And the condition given that this a to b g j x, y 1 to y n, y 1 dash to y n dash d of x is equal to L of j, where j is equal to 1 to k.

So here we want to find out say extremal curves for this functional keeping that these k integrals are at a constant value that is L j.

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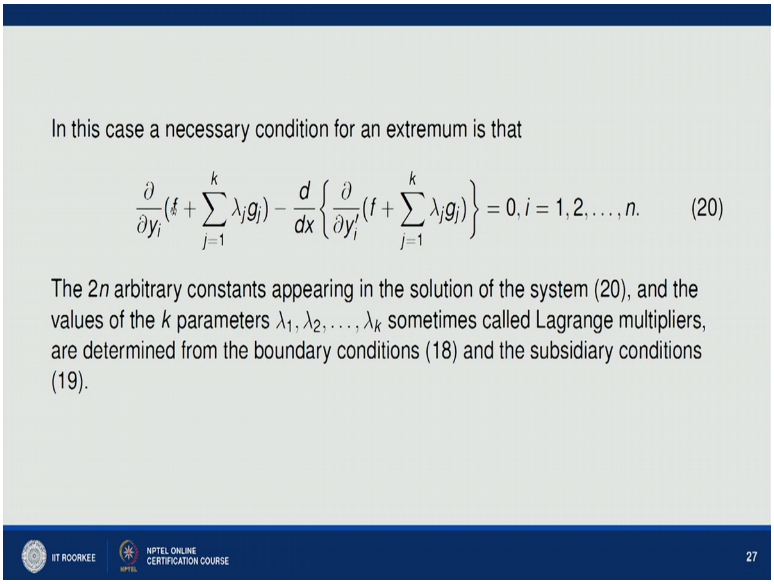

So the proof is not given here in fact to show the extremal here what we try to do here we consider another functional another integrand that is f plus summation j equal to 1 to k, lambda j g of j and we try to show that this f plus summation j equal to 1 to k lambda j g j satisfy the Eulers Equation it means that deba by deba y i f plus summation j equal to 1 to k lambda j g j minus d by dx deba by deba y i dash f plus summation j equal to 1 to k lambda j g j is equal to 0 for each i equal to 1 to n.

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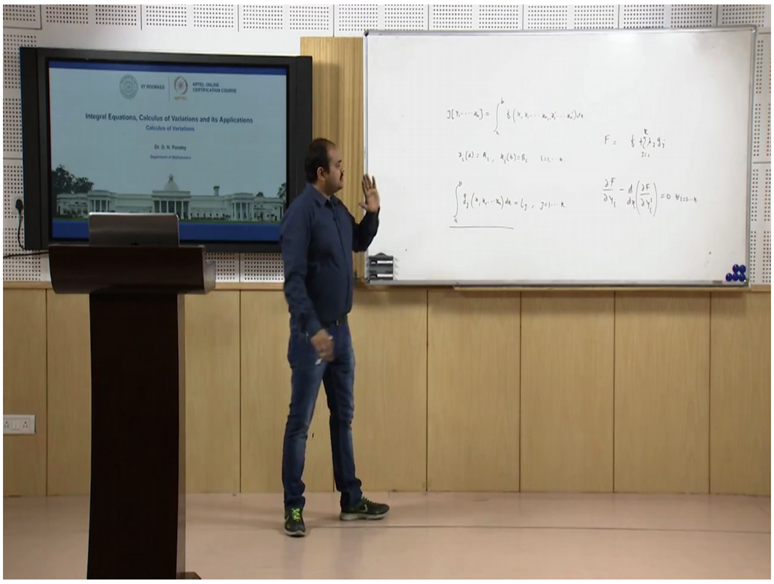

Let me write the thing here say here we have say a functional which is depending on these functions y 1 to y n given as a to b and here we have f of x, y 1 to say y n and y 1 dash to say y n dash d of x and the condition on y i is at y of i a equal to capital A i and y of i b is equal to capital B i, i is equal to 1 to n here and we not only extremizing this function but it also satisfy the condition a to b g j x, y 1 to say y n d of x equal to L j for each j equal to 1 to k.

So here we have unknowns are y 1 to y n curves are unknown which extremizing this functional provided that these integrals k integrals are also kept as constant value. So for that as we have done in a previous case we consider a new function capital F as f plus lambda summation lambda j g of j, j is equal to 1 to k and we try to show we can show in a similar manner that if y 1 to y n in extremal curves for this functional such that this value is kept at constant such that this y 1 to y n are not the extremal of this not extremal of these functional this.

Then this F will satisfy the Eulers Equation in terms of this, so F satisfy the Eulers Equation for so it means that deba F by deba y i minus d by dx, deba F by deba y i dash is equal to 0 for each i equal to 1 to n.

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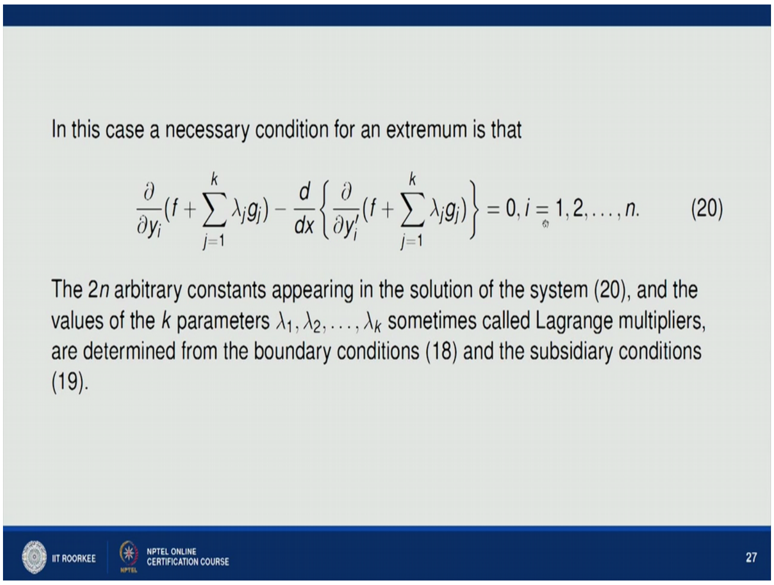

So this proof is similar to the previous proof so we are leaving it and we are simply writing the condition that deba by deba y i f plus summation j equal to 1 to k, lambda j g j minus d by dx of deba by deba y i dash f plus summation j equal to 1 to k lambda j g j is equal to 0 for each i equal to 1 to n.

So here if you solve this differential equation then we have 2n arbitrary constant appearing in the solution and this constant lambda j these parameters lambda j, lambda 1 to lambda k sometimes called Lagrange multipliers and these we can obtain these 2n arbitrary constant and these parameter we can obtain.

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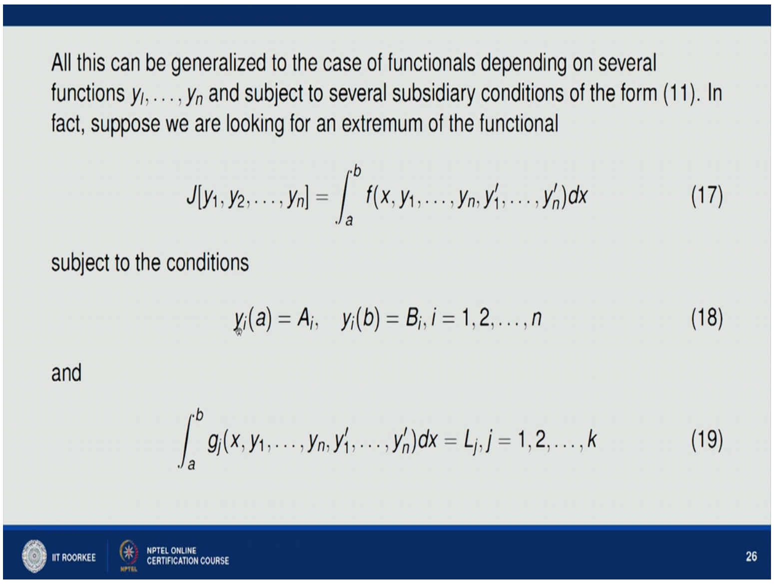

By these boundary condition and the condition which is given here. So 2n arbitrary constant can be obtained by this and k arbitrary parameters lambda 1 to lambda k can be obtained by this condition that a to b g j this thing dx equal to lambda j and L j.

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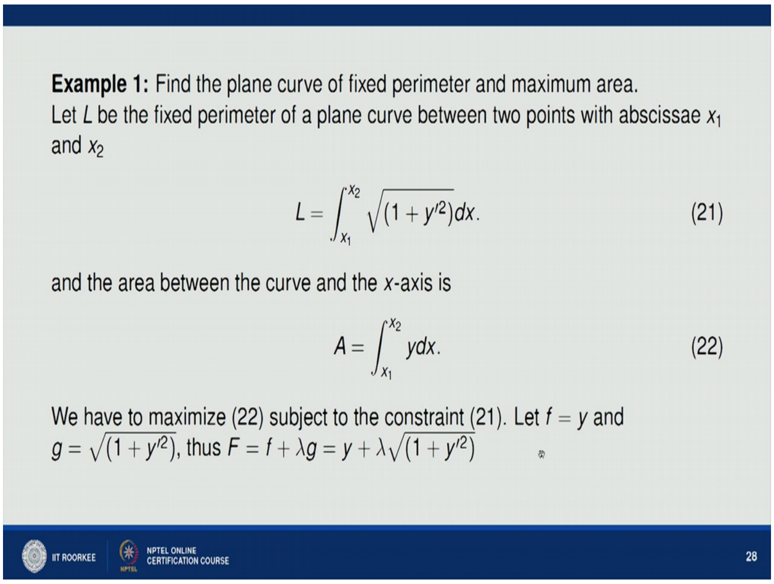

So let us now consider the example based on the theory which we have presented here. So first example is again a triple example which we have already discussed but just for the sake of completeness we are discussing it again. So here we have the problem is find the plane curve of fixed parameter and maximum area so it means that we have a curve call it y and define say bounded between say point x 1 to x 2 and which has fixed parameter that is L equal to x 1 to x 2 under root 1 plus y dash square dx, it is a constant value that constant value we are keeping it as L and the area between the curve and the x axis which is given by A it is x 1 to x 2 y of dx.

So problem is to find out a curve y for which this functional is kept as constant value and it is maximizing this functional A. So for this what we try to do here? So we let us take f is equal to y here and g is this under root 1 plus y dash square. So with the help of theory discussed earlier let us define capital F which is f plus lambda g, so f is small f is y and small g is under root 1 plus y dash square. So here capital F is equal to y plus lambda under root 1 plus y dash square. Now the condition which we have obtained is that this capital F satisfy the Eulers Equation.

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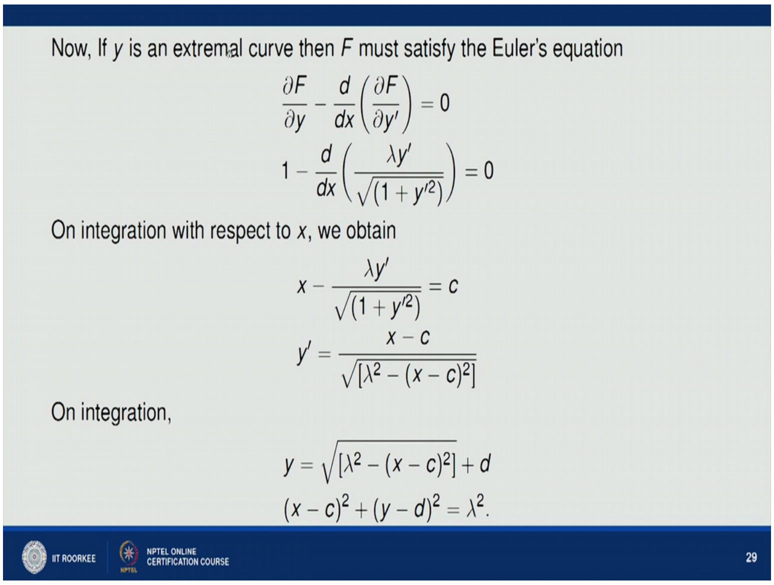

So if y is an extremal curve then F must satisfy the Eulers Equation which is nothing but deba F by deba y minus d by dx of deba F by deba y dash is equal to 0.

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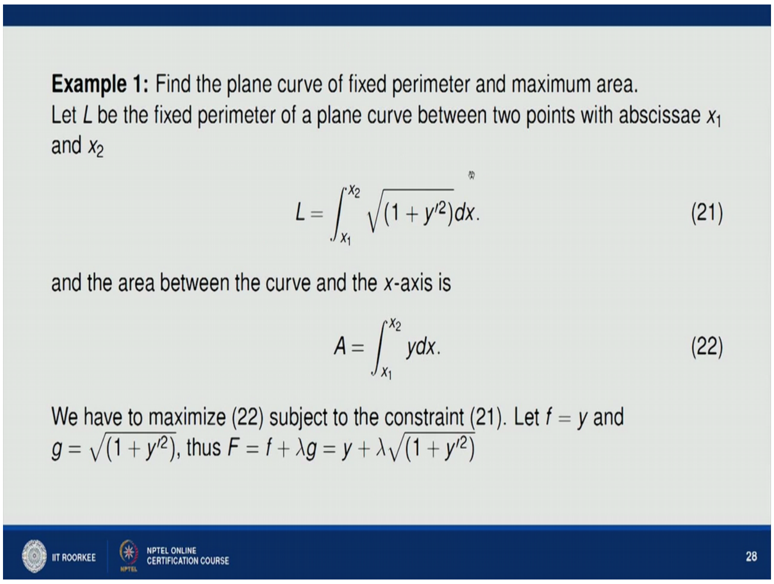

Which is here F is is given already as this so f y is only 1 and f y dash you can find out by differentiating this.

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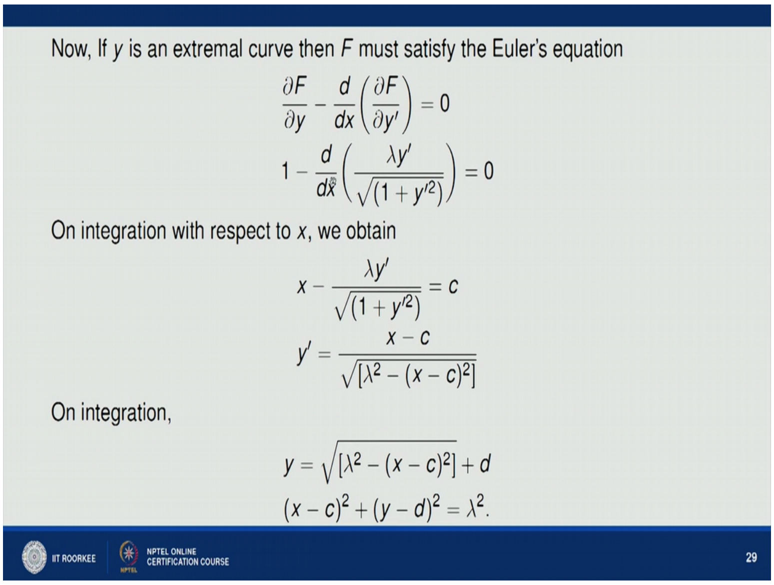

So here we have 1 minus d by dx of lambda y dash upon under root 1 plus y dash square is equal to 0. So here we can solve it by simply differentiating it and converting into second order differential equation and solving that but here we may solve it in a in an alternative way that is you just integrate this with respect to x and when you integrate with respect to x you have 1 is integration of 1 is x minus lambda y dash divided by 1 plus y dash square is equal to c.

Now this constant c can be obtained by the boundary condition we have. So now when you simplify when you simplify what you will get let me write it here.

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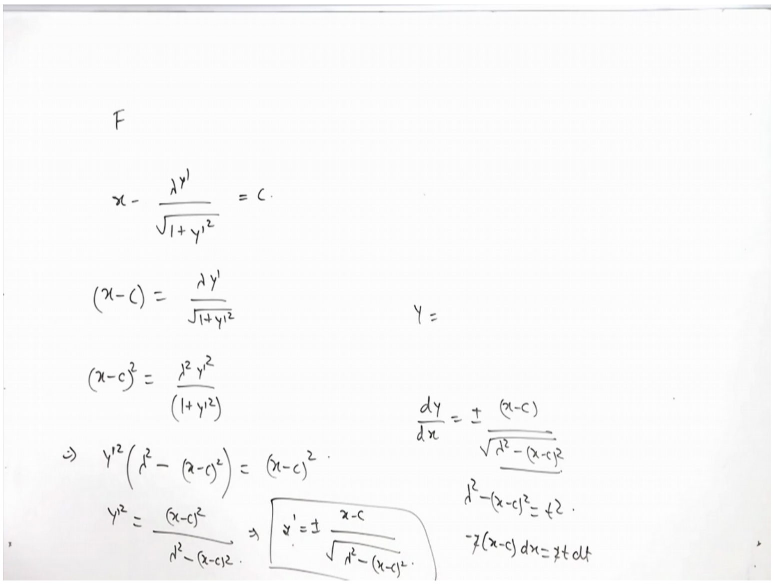

So here we have seen that since F satisfy the Eulers Equation then we have x minus lambda y dash divided by under root 1 plus y dash square is equal to c and we can simplify by saying that x minus c is equal to lambda y dash upon under root 1 plus y dash square we can square it out and we have x minus c square equal to lambda square y dash square divided by 1 plus y dash square and you can simplify and you can get y dash square that is lambda square minus x minus c whole square equal to x minus c whole square.

So we can say that v dash square is equal to x minus c whole square divided by lambda square minus x minus c whole square. So we can take the square root here and you can say that y dash is equal to x minus c divided by this is plus minus under root lambda square minus x minus c whole square and y dash this is y dash not y 1 these are all y dash, so here y dash square this is y dash, okay so here we have.

So now we have y dash equal to x minus c upon under root lambda square minus x minus c whole square and this is quite easy to solve in terms of y dash so it is what dy upon d of x is plus minus x minus c divided by under root lambda square minus x minus c whole square. So if you assume this denominator term this lambda square minus x minus c whole square as t square then you can simplify and you can get this as y equal to you simply integrate with respect to let us say assume that lambda square minus x minus c whole square equal to t square and you can get minus 2 of x minus c here you can get 2 x minus c d of x equal to 2t dt so these two will cancel out and you can get and you can integrate, is it okay.

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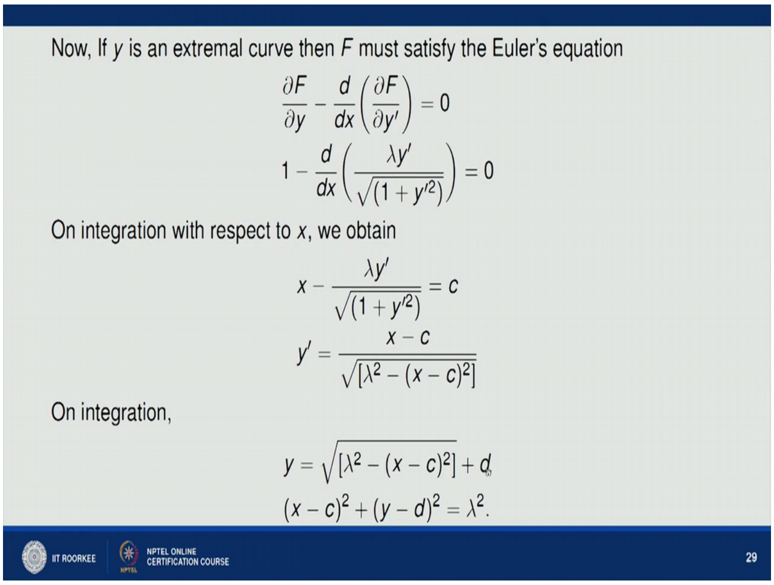

So here your integration is coming out to be y equal to under root lambda square minus x minus t whole square plus d. So when you simplify you can square it and you can get x minus c whole square plus y minus d whole square equal to lambda square. Here the constant c and d which we can obtain by the boundary condition.

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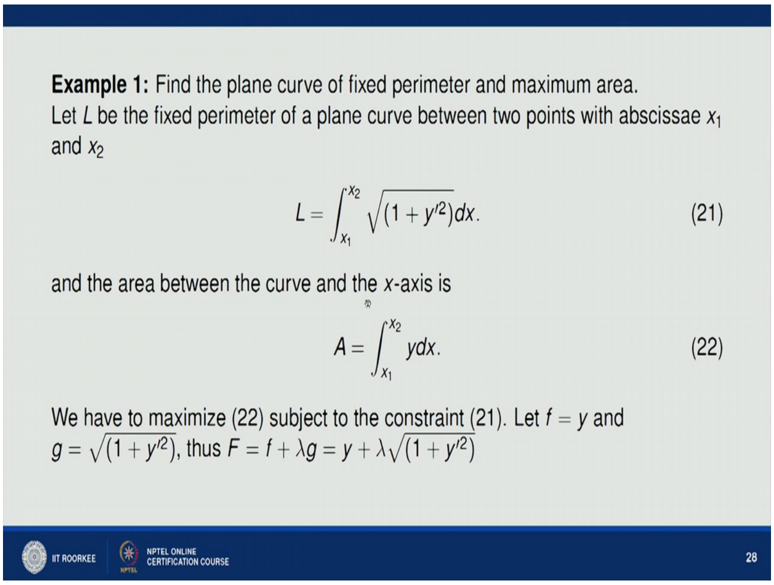

That y of x 1 is equal to y 1 let us call it y 1 and y of x 2 is suppose another value that is y 2.

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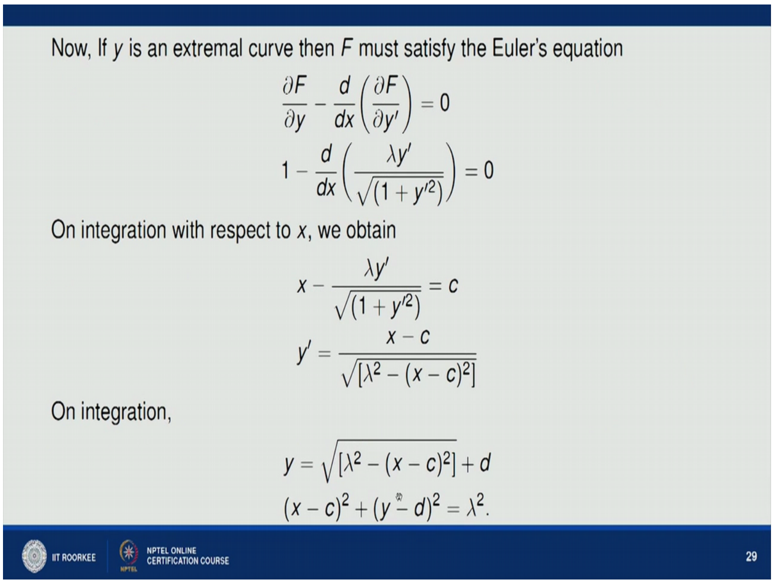

So given the boundary condition you can find out the value of c and d.

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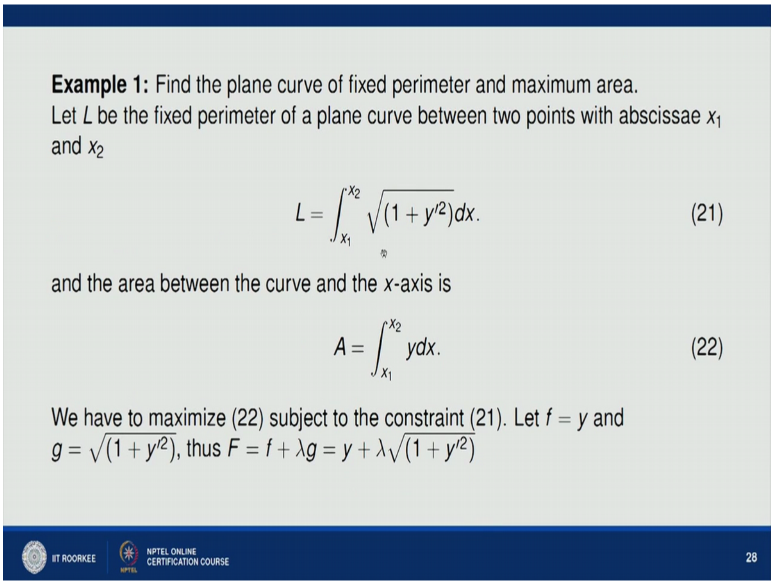

And which says that the extremal of this problem which is the the problem that a curve having fixed parameter which maximize the area is nothing but a circle which is given by this your c and d can be obtained by the by the boundary condition.

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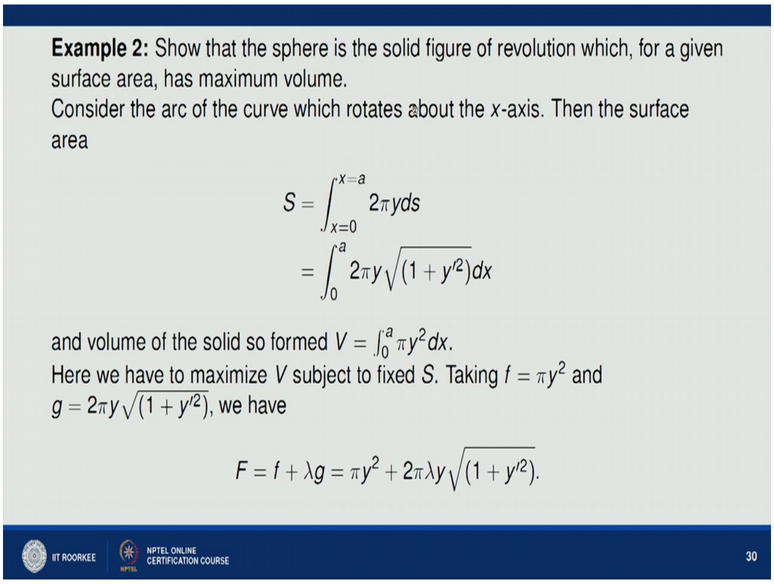

So now moving on to second example here we want to show that the sphere is the solid figure of revolution which for a given surface area has maximum value. So it means that we have a problem where the we need to find out a extremal curve which keep this functional as a constant value that is the surface area x equal to 0 to between x equal to 0 to x equal to a, 2 pi y ds as a constant value but maximizing the volume so formed that is V equal to 0 to a pi of y square d of x.

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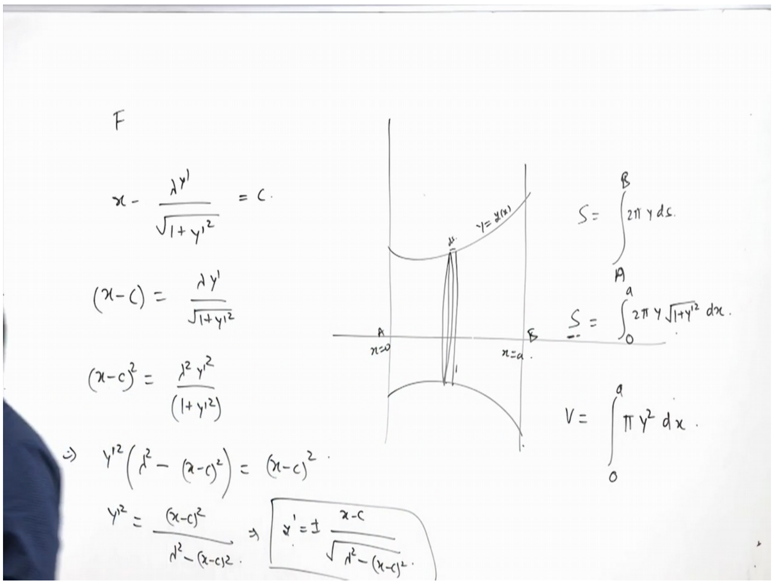

Let me do it here, okay. So here what is surface area surface area is your 0 to a 2 pi here you can find out the surface area by you you just take the surface element here let us say this is a d of S so it is 2 pi y d of S between this point call it say a to b, so here we have a to b 2 pi y ds and which we can write it 0 to a 2 pi and y and ds you can write it (under) 1 plus y dash square d of x.

So here we want to find out a curve y equal to y of x such that this S fixed value that is capital S and the area then the volume of the say formed by this revolution is is maximum. So what is that 0 to a here you can find out say volume of that shape as pi of y square and integrating between a to b, so here you can say that it is a to b pi of y square and d of x here. So here we want to maximize the volume here keeping the surface area integral as constant.

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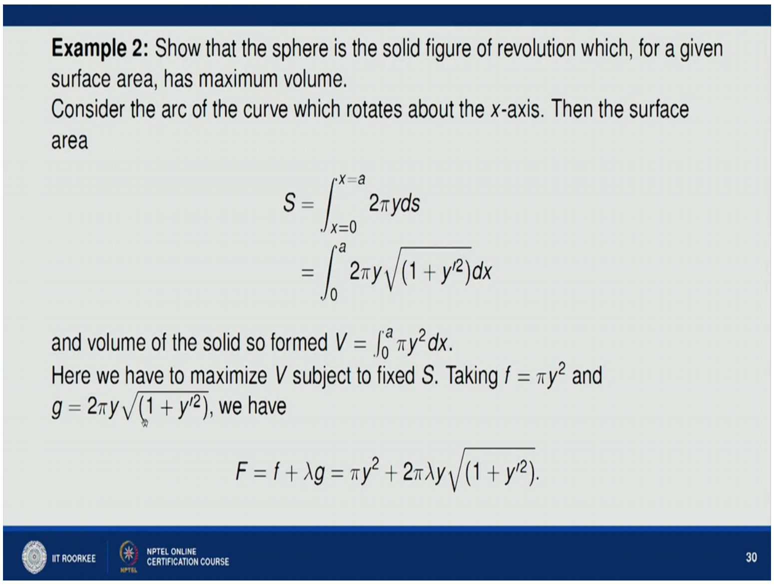

So here here your f is pi y square so taking f equal to pi y square and g is equal to 2 pi y under root 1 plus y dash square we want to maximizing this V keeping this integral as constant. So for that you define capital F which is given as small f plus lambda g and small f is defined as pi of y square and small g is given by 2 pi lambda y under root 1 plus y dash square. Now this capital F satisfy the Eulers Equation if y extremizing the functional be here.

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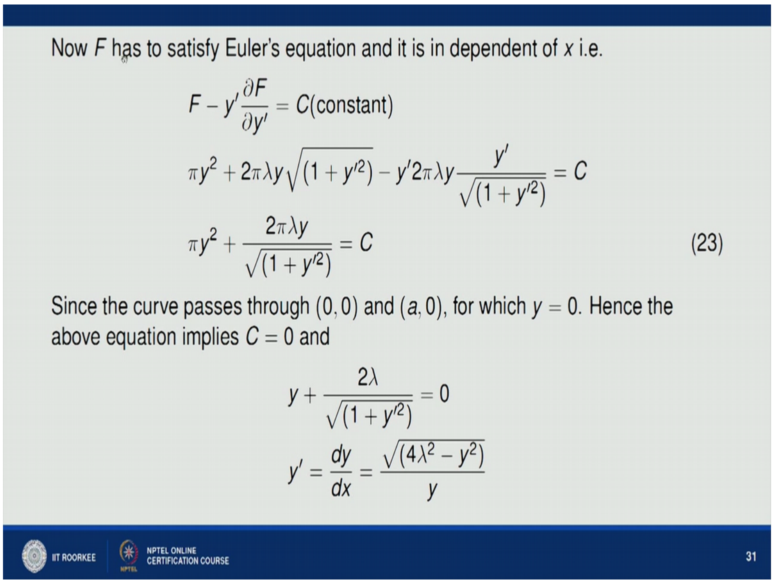

So now F has to satisfy the Eulers Equation and it is independent of x.

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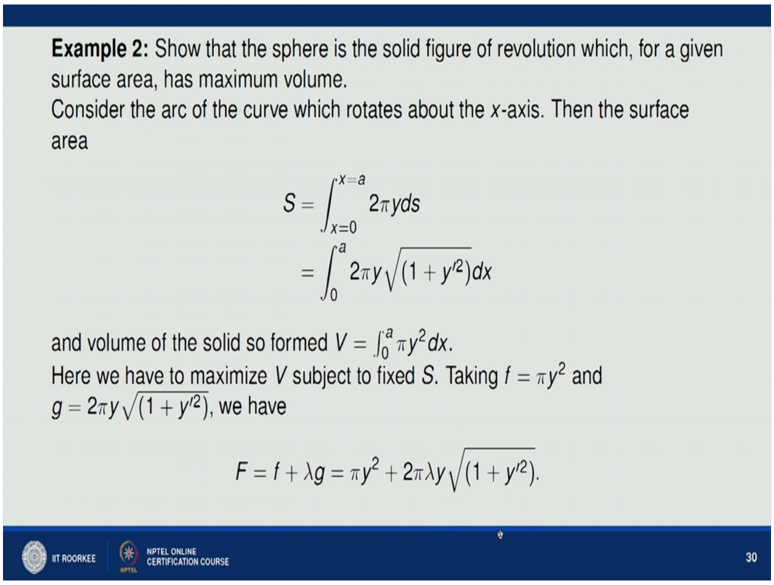

So since here this integral is independent of x.

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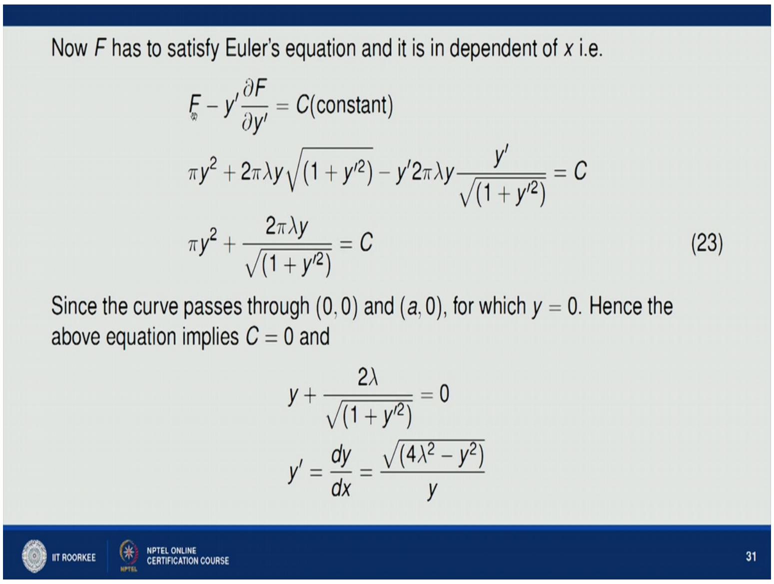

So here we can say that Eulers Equation reduced to this F minus y dash deba F by deba y dash equal to constant. This is the subcase 5 which we have considered at the end of Eulers Equation. So F is given as pi y square plus 2 pi lambda y under root 1 plus y dash square. Now when you calculate y dash deba F by deba y dash you will get this equation and when you simplify you will get pi of y square plus 2 pi lambda y under root 1 plus y dash square equal to C.

Now to find out this constant C we have to look at the boundary condition, what is the boundary condition here? The boundary condition is that this curve passes through origin and and passes through this point a, comma 0.

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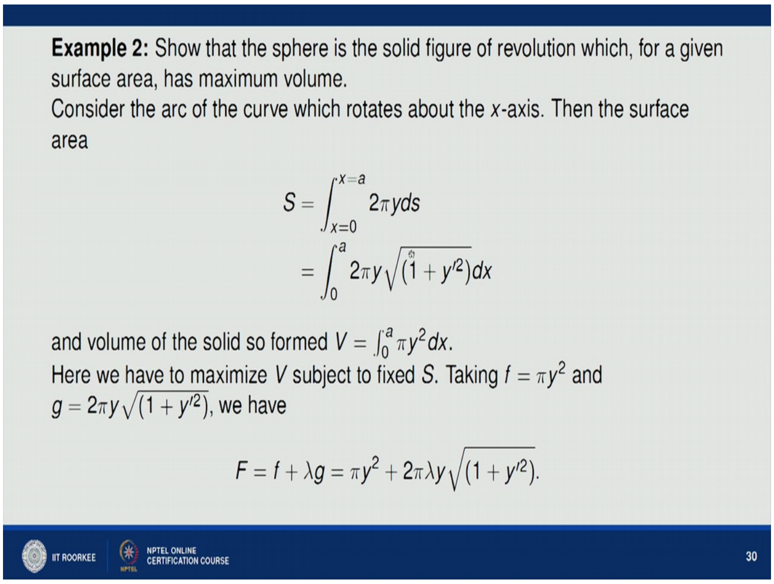

So here the condition is given as that y passes through the origin here.

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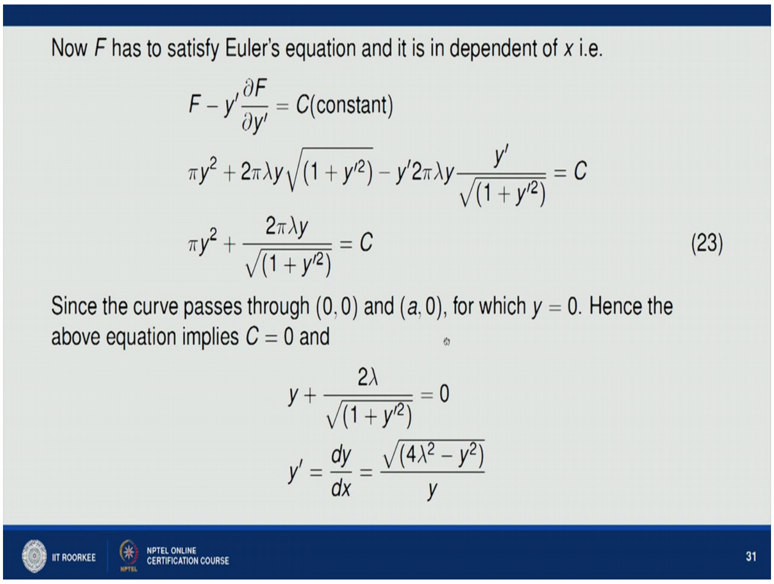

So if we impose this condition that the curve passing through origin and pass through this point a, 0 for which we have y equal to 0 for at x equal to 0, y equal to 0, at x equal to a, y equal to 0. So we can say that that the only value of C which satisfy this condition is that C is equal to 0 and if we take C equal to 0 then this equation reduce to y plus 2 lambda upon under root 1 plus y dash square equal to 0, here we simply taken out this pi of y.

So we have y equal to y plus 2 lambda upon under root 1 plus y dash square equal to 0 which is a simple not simple but it is a differential equation in terms of y dash. So you can find out y dash by squaring it out and solving for y dash so y dash is coming out to be dy by dx equal to under root 4 lambda square minus y square divided by y, so this you can solve.

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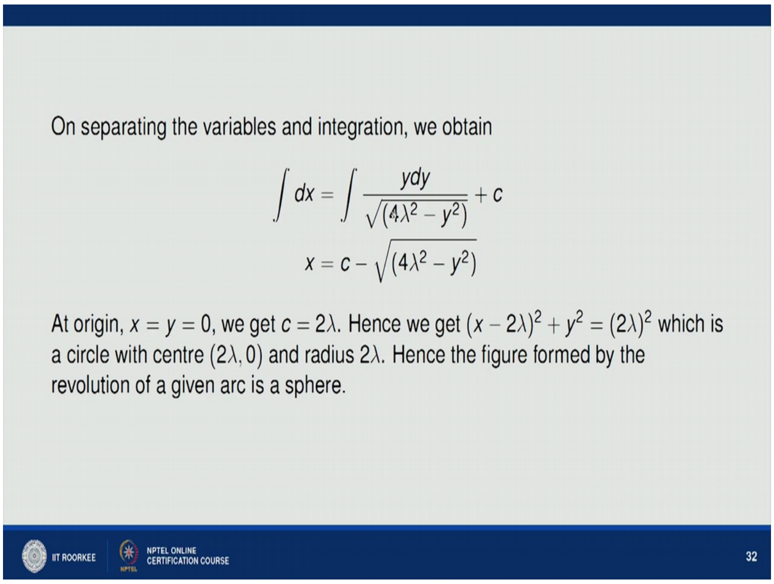

And you can write integration of dx is equal to integration of y dy under root 4 lambda square minus y square plus c. When you integrate you can assume 4 lambda square minus y square as some t square and you can simply solve and in this way you can get x equal to c minus under root 4 lambda square minus y square. Now we have boundary condition given that curve passes through origin that is x equal to 0, y equal to 0, we can get our condition c equal to 2 lambda and hence we can say that our extremal curve will be what x minus 2 lambda whole square plus y square equal to 2 lambda whole square which is nothing but a circle with centre 2 lambda equal to 0 and radius is 2 lambda and hence the figure formed by the revolution of a given arc is a sphere. So that is what we have given here. So here this is in general the problem is.

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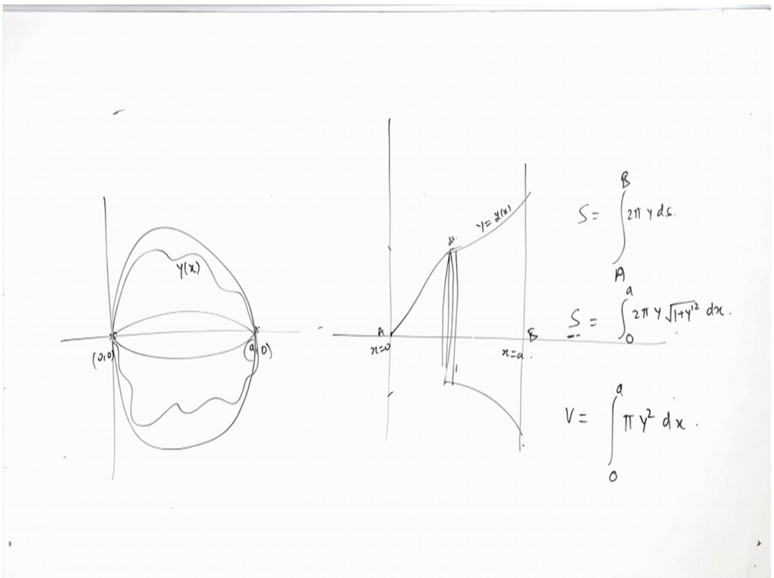

Now you can say problem is given that at this point 0, 0 and a, 0 your curve is passing through so it means that it is something like curve like this and when you revolve you will get a shape of revolution. And then we are saying that all those curves for which your parameter is fixed but volume is maximum that is coming out to be a sphere which is obtained by revolving the circle passing through this point 0, 0 and a, 0, right. So here we have shown that that sphere is a solid figure of revolution which for a given surface area has maximum volume.

Now let us try to generalize the Isoperimetric Problem where the condition that the constant is not given in terms of function but it is given in terms of simple condition.

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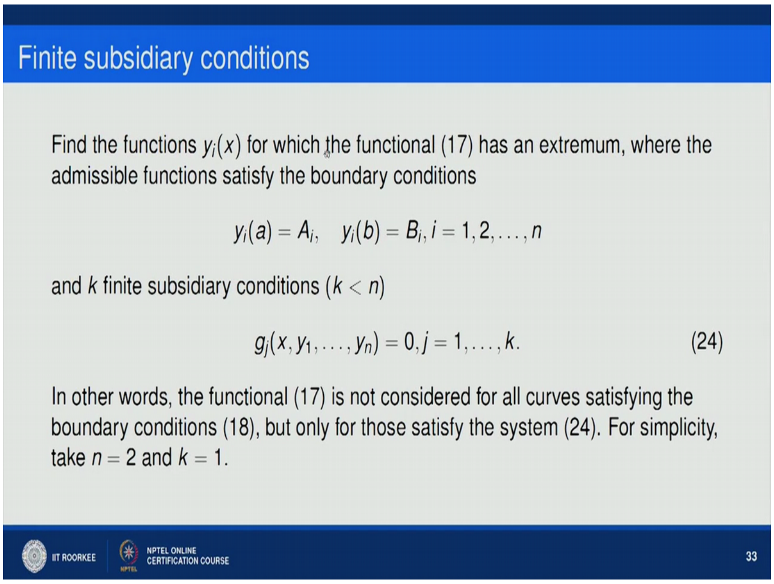

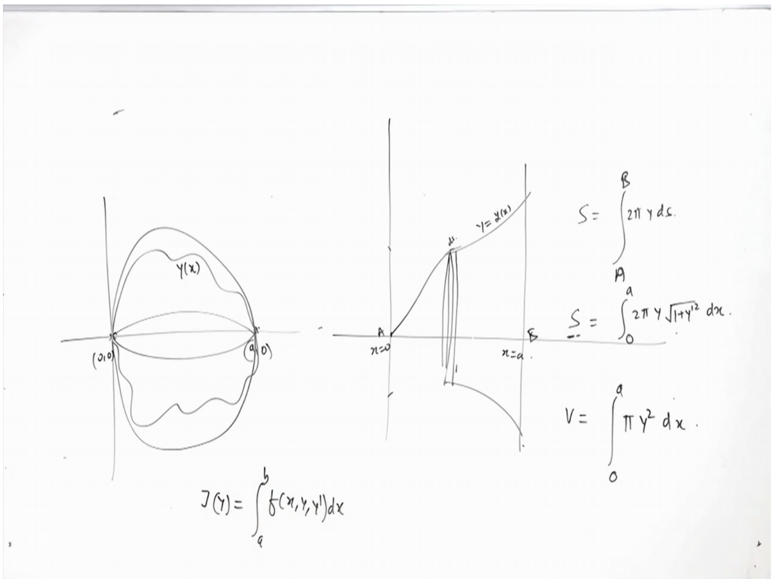

So here we want to find the functions y i x for which the functional 17 which is functional J of y, J of y is equal to a to b f of x, y, y dash d of x. This is this has an extremum value and where the admissible function satisfy the boundary condition y i a equal to capital A of i, y i b equal to capital B of i, i equal to 1 to n and k finite subsidiary condition is given in terms of y 1 to y n that is g j x, y1 to y n equal to 0 for J equal to 1 to k here.

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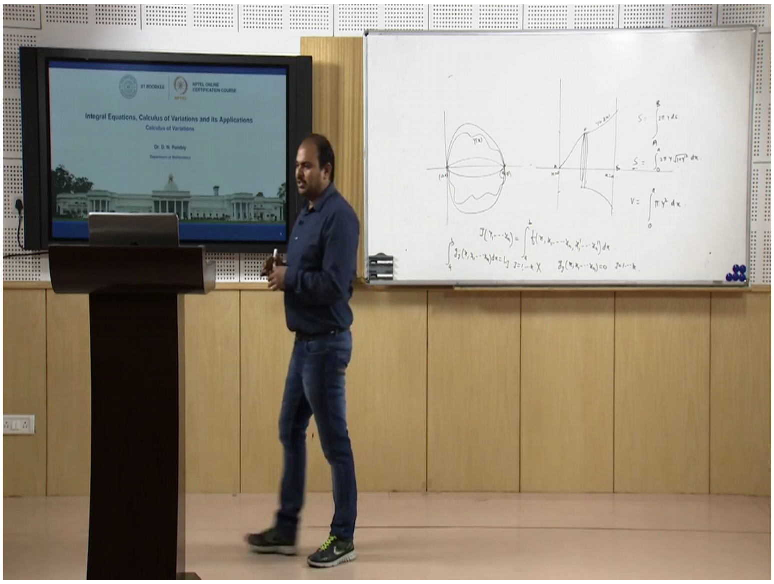

So here we are considering the generalize problem. So it means that problem is this not this, so functional is J of y 1 to say y n is equal to a to b f of x, y 1 to say y n and y 1 dash to y n dash d of x. So we have seen Isoperimetric Problem generalization of Isoperimetric Problem where these curves y 1 to y n satisfy the condition that another integral a to b g j x, y 1 to y n d of x is equal to some L j for J equal to 1 to k.

But when this condition is replaced by finite equation not in terms of integral it means that now your condition is reduced to this so in place of this now the condition is given as that g of j your x y 1 to say y n is equal to 0 your j is equal to 1 to k. So these conditions are known as finite equation so it means your constant is given in terms of finite equation rather than given in a functional form.

So now we want to find out say extremal curve which extremizing these functional here provided it satisfy the finite subsidiary equation like this. So one such example is to find out say geodesics on a given surface. So it means that we can say that we have a surface and there we want to find out say minimum distance between two point. So you can consider that geodesics as a particular case of the case we which we are considering here.

So now for simplicity we are just taking n equal to 2, so here we are assuming that only y 1 and y 2 are given and k equal to 1 means only 1 subsidiary equation is given here, right.

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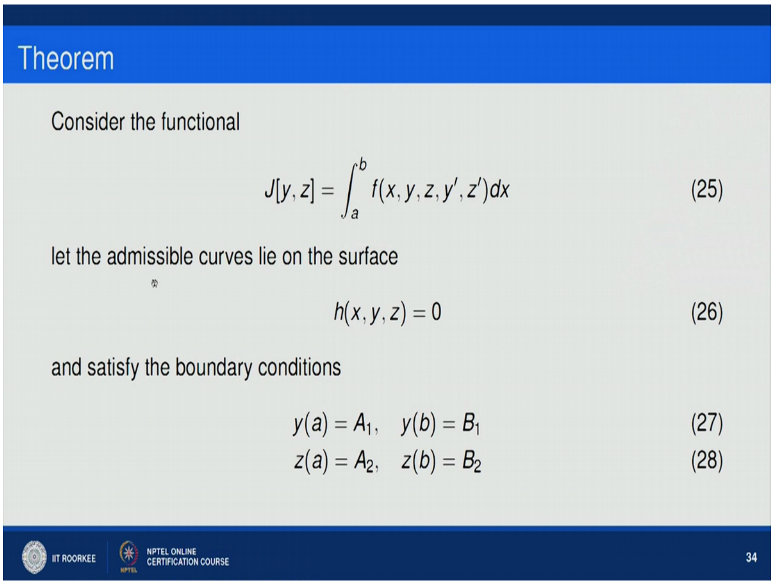

So let us consider the theorem. So here we want to extremize the functional J of y of z y comma z, a to b f of x, comma y, comma z y dash, comma z dash dx and here the admissible curve lie in this surface h of x, comma y, comma z equal to 0. So here it means we need to find out say curve y of x and z of x which extremizing this provided this y x and z of x will lie on this surface here and the boundary conditions are given here that y of a equal to capital A 1 and y of b is equal to capital of B 1. Similarly z satisfy the same boundary condition we want to extremizing we want to find out the condition such that y and z extremizing the function given at (5) 25.

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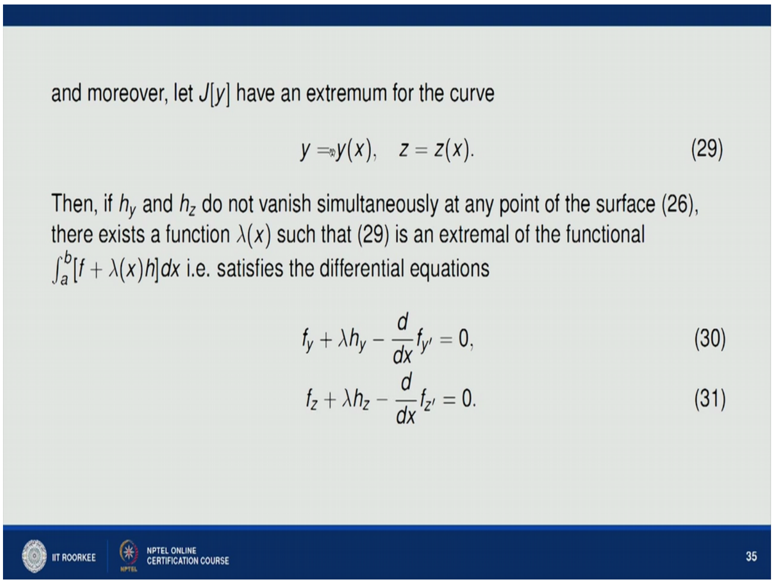

So here we simply say that let J y have an extremum for the curve y equal to y of x and z equal to z of x and also we assume that h y and h z do not vanish simultaneously or you can say simultaneously at any point of the surface 26.

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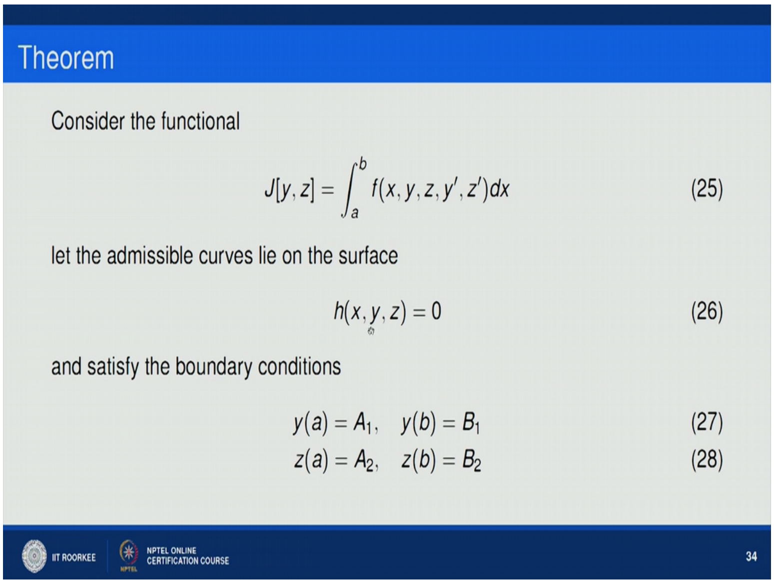

So here this surface h x, y, z has non zero partial derivative with respect to y and z here.

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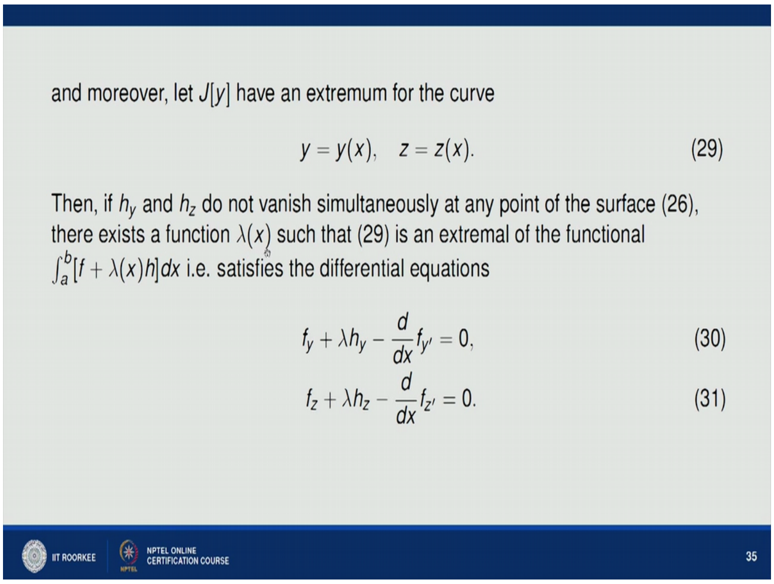

Then they exist a function lambda of x such that this curves y equal to y of x and z equal to z of x is an extremal of the functional a to b f plus lambda x h d of x. So it means that provided that h has non zero partial derivative with respect to y and z then your then they exist a lambda x such that this f plus lambda x h satisfy the Eulers Equation it means that f y plus lambda h y minus d because y dx f y dash equal to 0 and f z plus lambda h z minus d by dx of f z dash equal to 0. So here you just look at here we can say that this a to b f plus lambda x h x dx satisfy the differential equation f y plus lambda h y minus d by dx f y dash equal to 0 and similarly for z f z plus lambda h z minus d by dx f z dash equal to 0.

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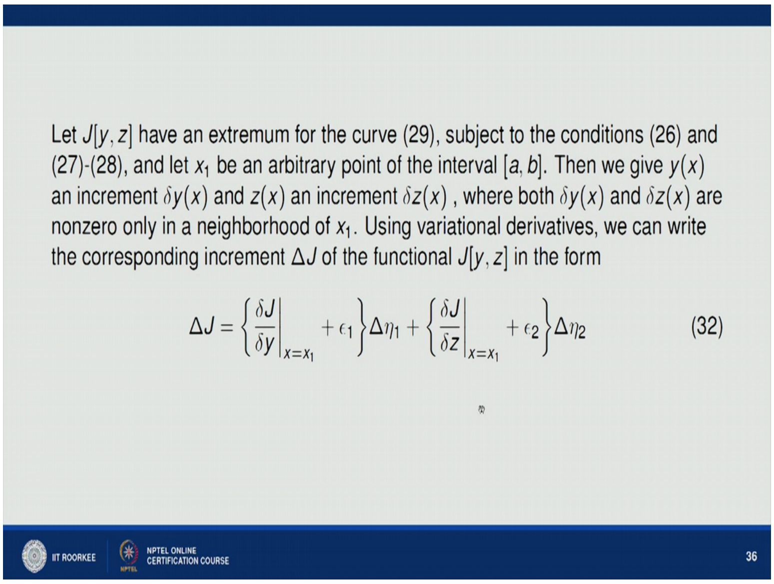

So to prove this we will follow the proof given on earlier and let us see how it is, so let J y, z have an extremum for the curve 29.

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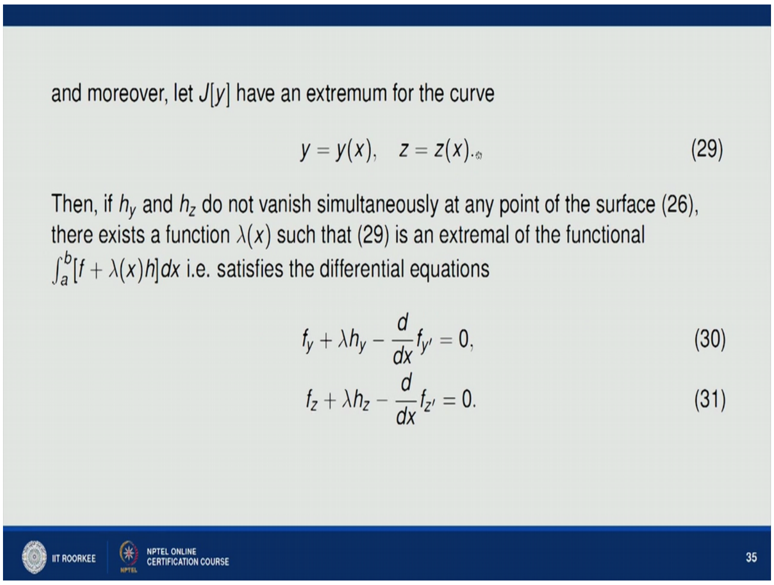

29 is this y equal to y of x and z equal to z of x.

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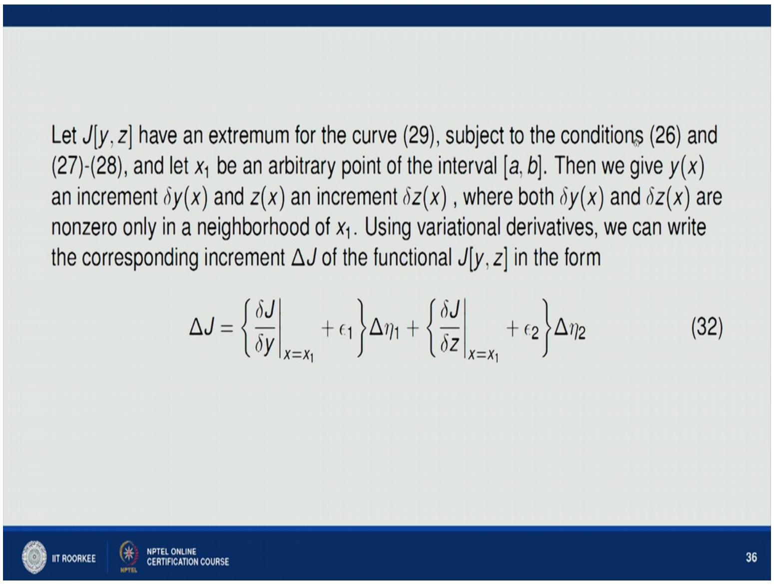

Subject to the condition 26 that it satisfy the condition.

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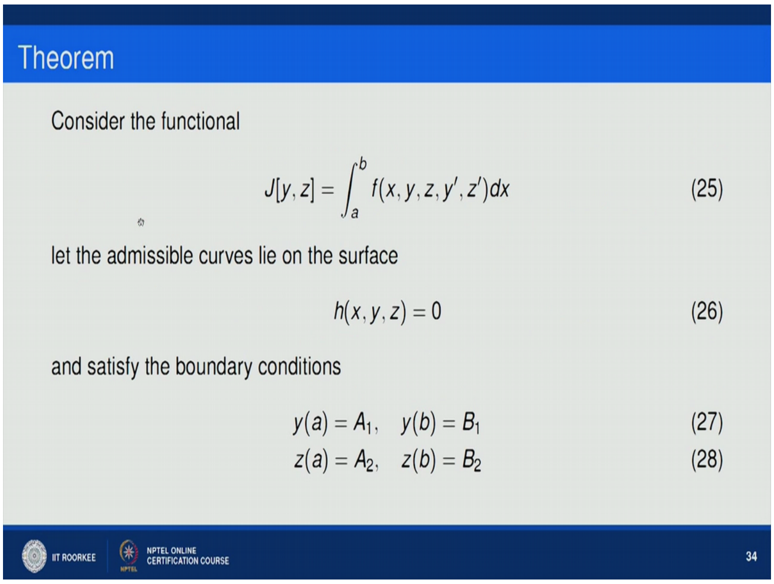

And 27, 28 is this boundary condition that is satisfy the boundary condition.

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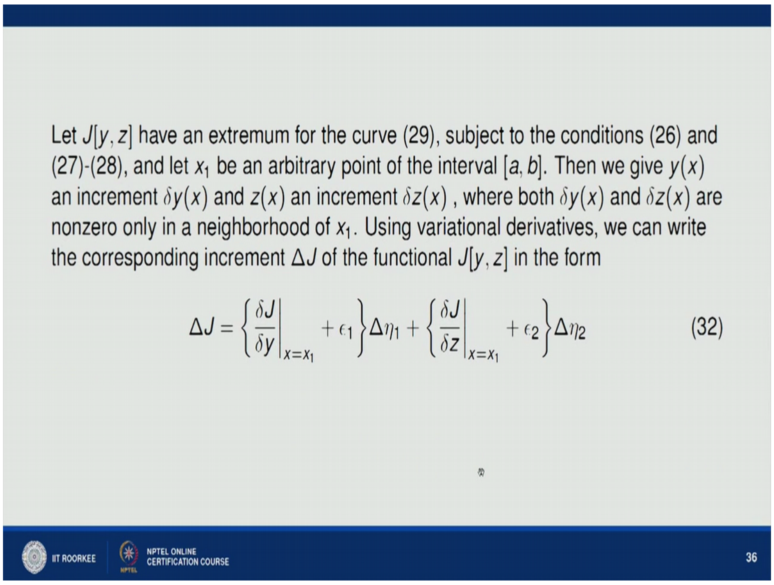

And let x 1 be an arbitrary point of the interval a to b then we give y of x an increment of delta y x and similarly z x is given as an increment delta z x, where both delta y x and delta z x are non-zero only in a neighbourhood of x1 rest it is all 0.

So now using variational derivative we can write down the corresponding increment delta J as this, so delta J is delta J upon delta y evaluated at x equal to x 1 plus epsilon 1 delta eta 1 plus similarly you can write down for z also plus delta z by delta J by delta z at x equal to x 1 plus epsilon 2 delta eta 2. So here as delta eta 1 and delta eta 2 tending to 0 your epsilon 1 and epsilon 2 is tending to 0.

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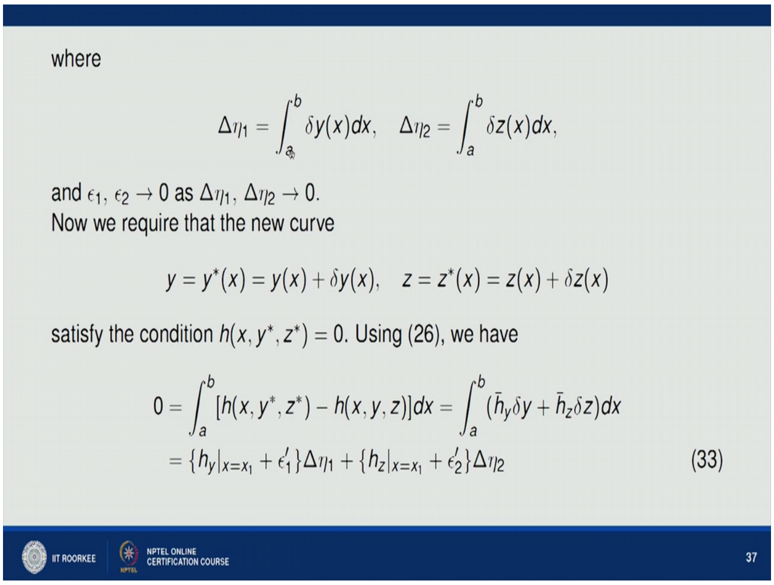

Where delta eta 1 is given by the area a to b delta y x dx and delta eta 2 is equal to a to b delta z x dx and this is what we have done the only thing is now it is given for both y and z so everything we are doing in a same manner. So similarly we can define your y y star x that is y of x plus delta y of x similarly z star x is defined as z x plus delta z of x and now here the only difference is that they are functional has constant value here this function h of x, y star, z star has same value that is 0. So it means that a to b h of x, y star, z star minus h x, y, z dx will be equal to 0.

So this we can write it using the Taylors expansion Taylors theorem we can write this as a to b h y delta y plus h z delta z d of x. So this we can write it in this form in terms of a derivative here that h y evaluated at x equal to x 1 plus epsilon 1 dash delta eta 1 plus h z evaluated at x equal to x 1 plus epsilon 2 eta dash delta eta 2. So this is also similar to your previous theorem so here we try since this quantity is equal to 0, so you can calculate delta eta 2 in terms of delta eta 1 that you can do by assuming that h z is non-zero since this is already known because we already know that a partial derivative of h and h with respect to y and z are non-zeros, so this is non-zero so you can write it delta eta 2.

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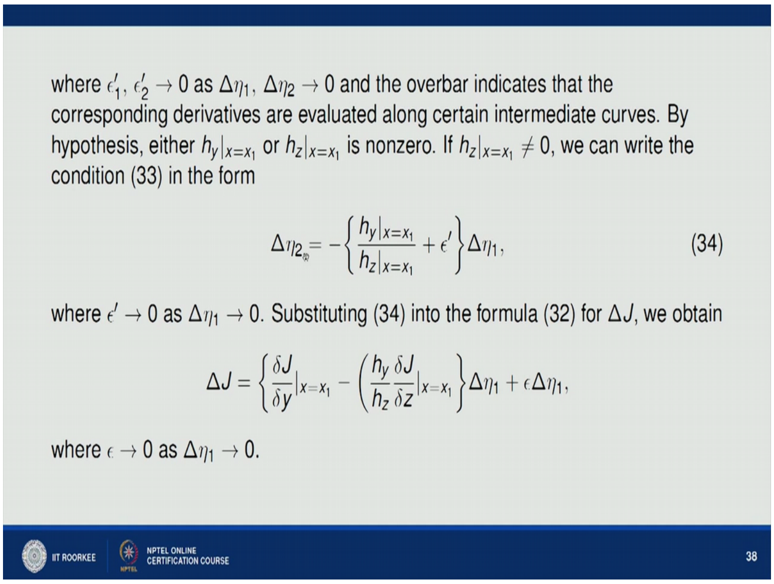

In terms of delta eta 1 that is this. So using the value of delta eta 2 you write down this expression for delta J which is given as delta J is equal to delta J by delta y evaluated at x equal to x 1 minus h y upon h z delta J by delta z at evaluated at x equal to x 1 into delta eta 1 plus epsilon delta eta 1. So here epsilon is tending to 0 as delta eta 1 is tending to 0. So it is also in a similar manner.

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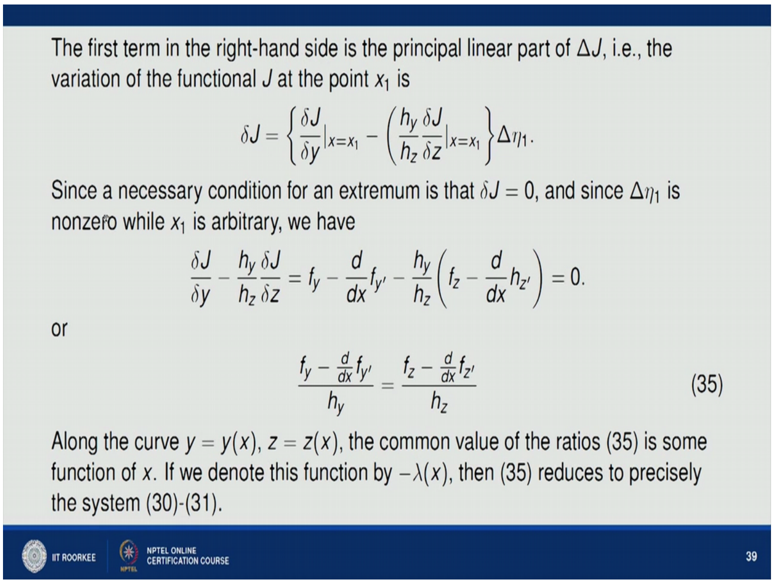

So we can say that here a necessary condition for an extremum is that delta J is tending to 0. Now since delta eta 1 is non-zero while x 1 is arbitrary we can say that this quantity is going to be 0. So this quantity is going to be 0 means your delta J by delta y minus h y upon h z delta J by delta z equal to this quantity is what delta J by delta y? This is f y minus the by dx of f y dash minus h y upon h z this is delta J by delta z is given by f z minus d by dx of h z dash.

When you simplify this you can write it in this form that ratio of f y minus d by dx of f y dash divided by h y and f z minus d by dx of f z dash by h of z has to be equal. Now you can say that this is a function of x, this is also a function of x. So you can say that this is equal to some function which is we are denoting here as minus of lambda x and if we assume this as minus of lambda x then our theorem statement.

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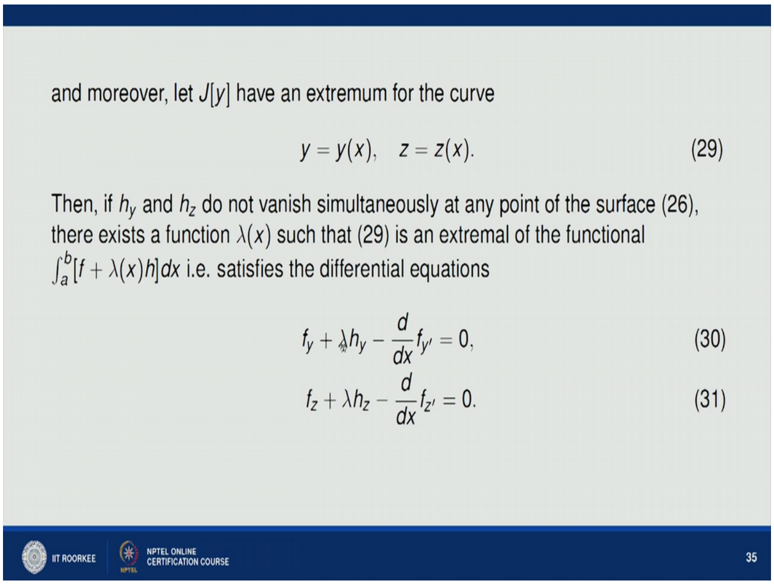

That is this that f of y plus lambda h y minus d by dx of f y dash is equal to 0 and f z plus lambda h z minus d by dx of f z dash is equal to 0 is true. So now let us take a particular example based on this then we can understand the only thing is we have to understand here that delta J by delta y minus h y upon h z delta J by delta z is equal to 0 we remember this thing.

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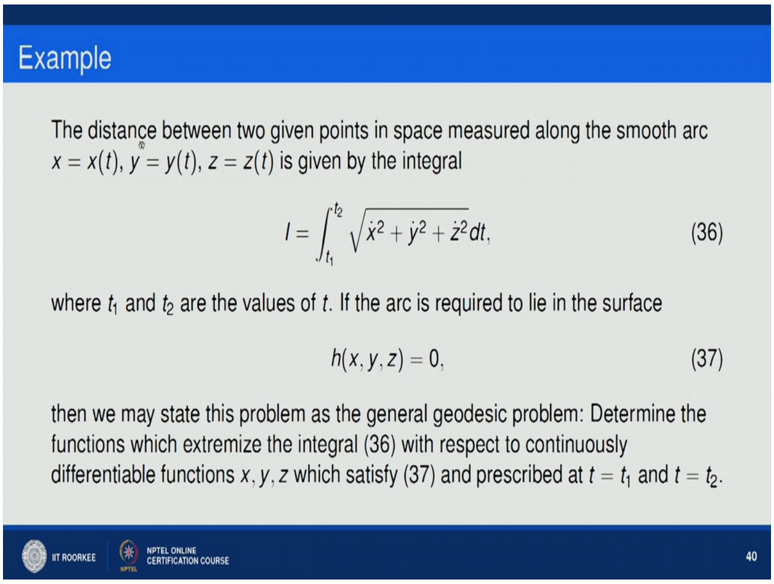

So let us take an example, here example is that the distance between two given points in space measured along the smooth arc x equal to x t, y equal to y t, z equal to z t is given by this integral, right this is the distance between two points evaluated at t equal to t 1 and t equal to t 2 so that represent the distance between these two points. And arc is lying on the surface h of x, y, z equal to 0.

So here we want to consider we want to minimize this functional here. Now this is the same problem which we can consider as geodesics problem. So determine the function which extremizing the integral 36 with respect to continuously differentiable function x, y, z which satisfy the condition 37 means we have to find out the curve on this surface having a minimum distance between the point t 1 and t 2.

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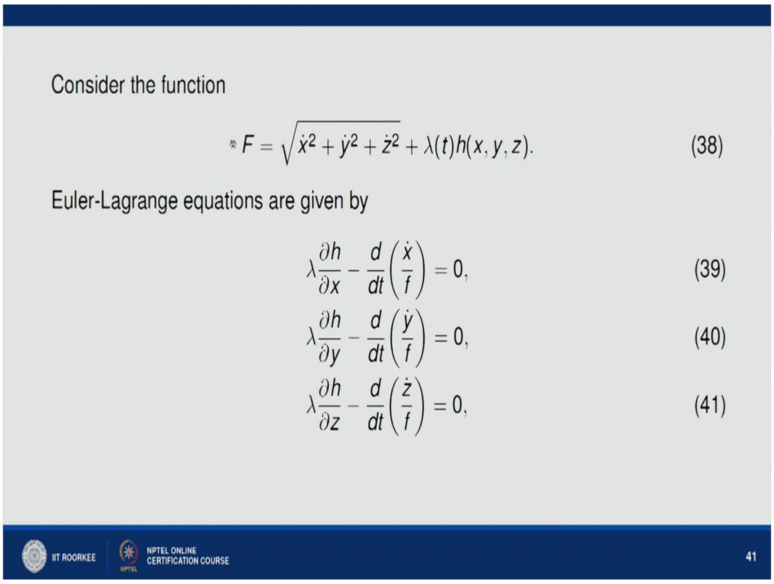

So so here your capital F is given as under root x dash square plus y dash square plus z dash square plus lambda t h of x, comma y, comma z. So by previous theorem your Eulers Equation which is also known as Euler-Lagrange equations are given by this. So simply you find out deba f by deba y minus d by dx of deba f by deba y dash. So we are simply generalizing this concept. So lambda deba h by deba x minus d by dt here because independent variable is t so minus d by dt x dash by f. Now what is x dash here? So you find out say since this this is not involving any derivative here derivatives are only here.

So here we simply say lambda deba h by deba x minus d by dt of now when you differentiate this with respect to x dash what you will get 1 upon 2 under root x dash square plus y dash square plus z dash square and then in numerator you will have two x dash so you can write it this is nothing but x dash by f equal to 0 and since it is symmetric with respect to x, y and z so you can have 39, 40 and 41 as a relation. Now when you simplify this you can find out say value of lambda in each equation 39, 40, 41.

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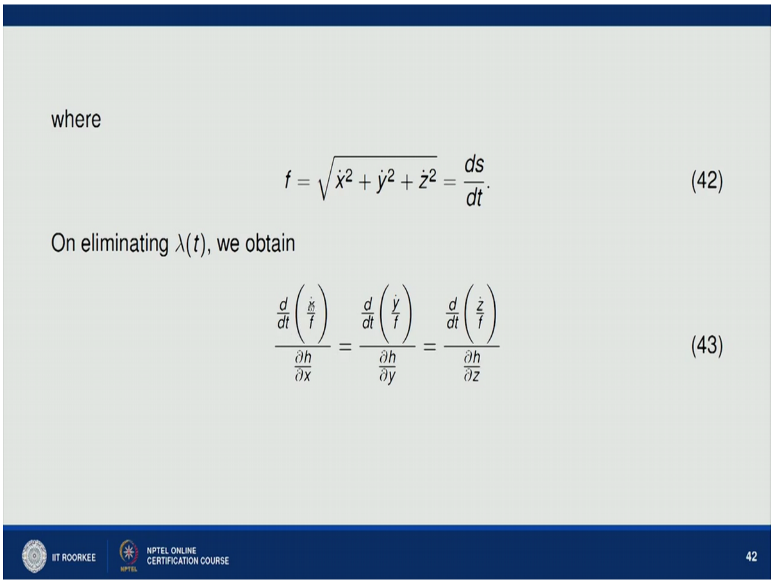

And you can write it d by dt of x dash by f equal to divided by deba h by deba x equal to this.

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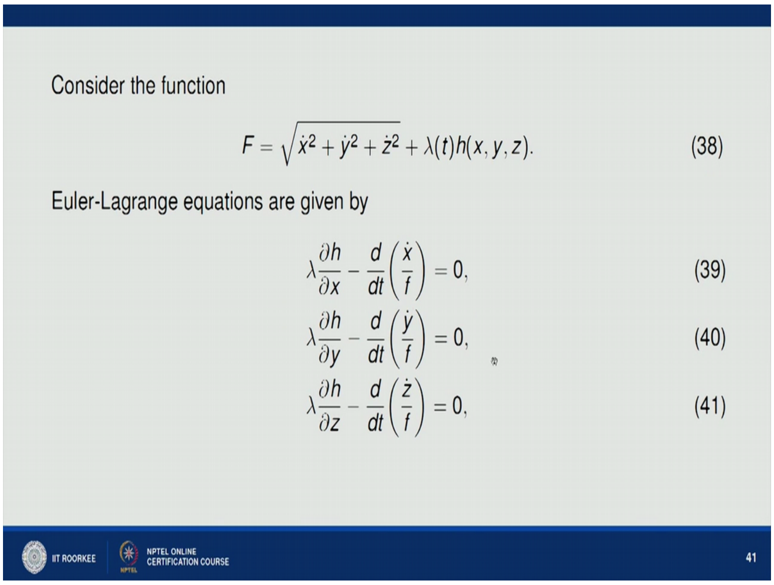

So all these are equal which is nothing but the value of lambda which you have evaluated from these three equation.

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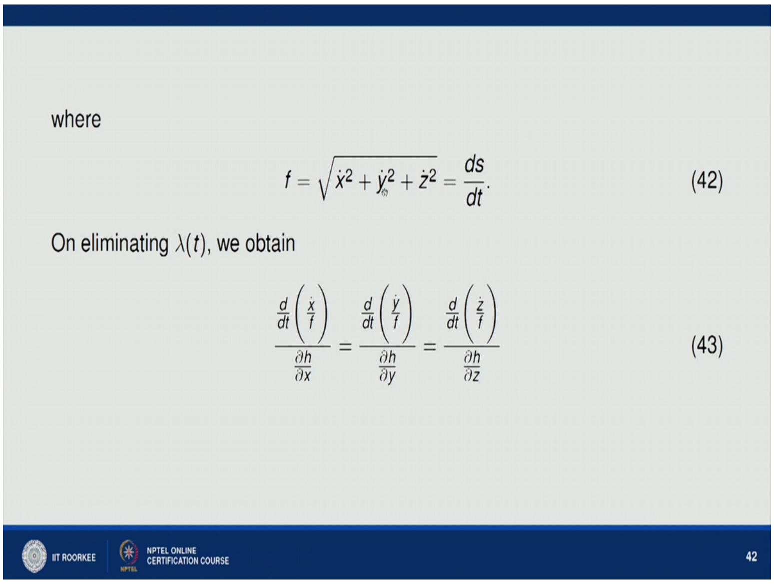

So here small f is given by under root x dash square plus y dash square plus z dash square which is commonly known as ds by dt, right.

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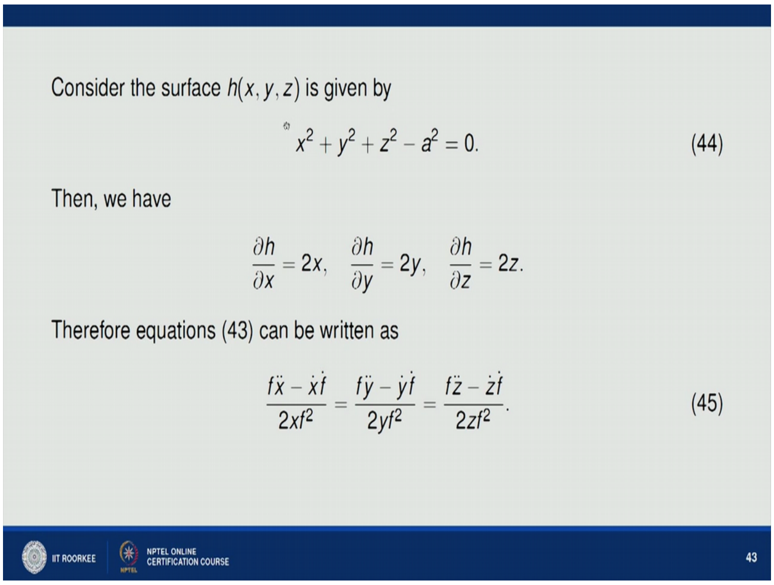

So here we take a particular example let us take the example that the surface is given by sphere of radius a whose centre is given at origin. So here if we take h x, y, z as this sphere x square plus y square plus z square equal to a square. Then you can find out h x, h y, h z.

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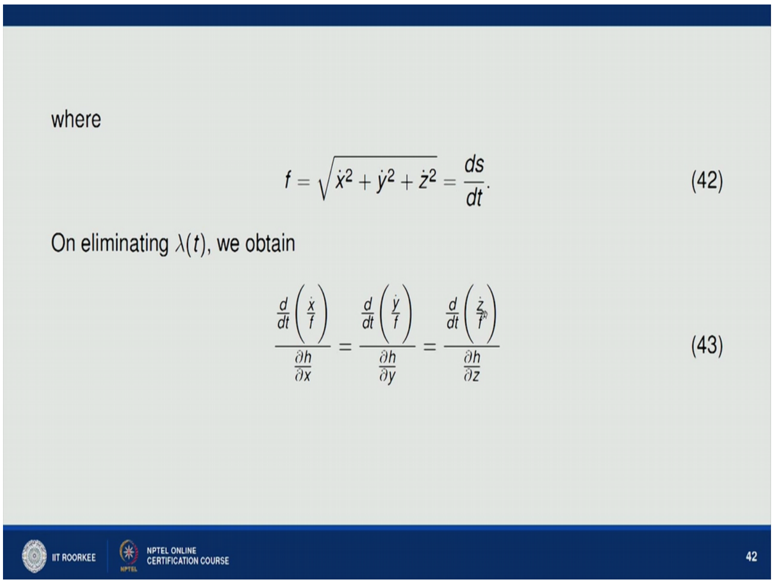

And if you keep the value of h x, h y, h z in this equation number 43 here so by putting this value you will have I am solving this d by dt of of x dash upon f.

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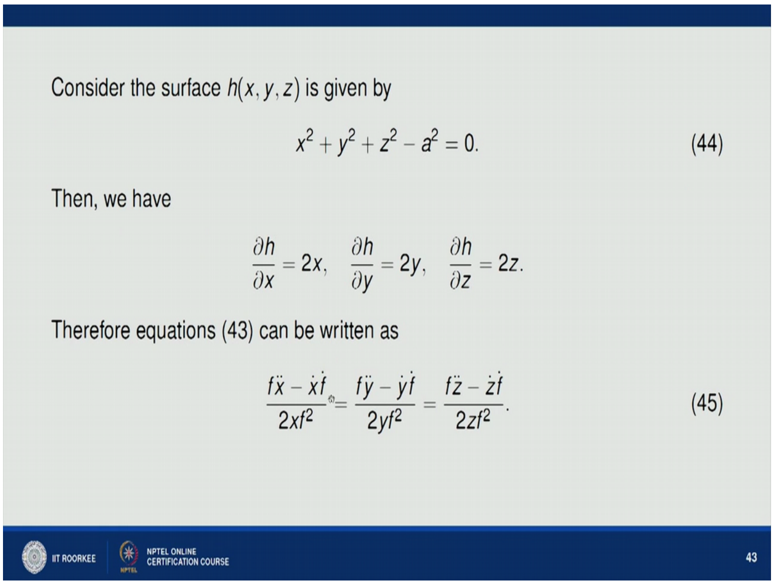

You will see that it will be reduced to this equation equation number 45. So here we have f of x double dash minus x dash f dash divided by 2xf square equal to fy double dash minus y dash f dash divided by 2yf square in fact it is symmetric with respect to x, y, z. So once we have for x dash you can similarly evaluate for y dash y and z component. So from here this is you can simplify further simplify and you can write down this.

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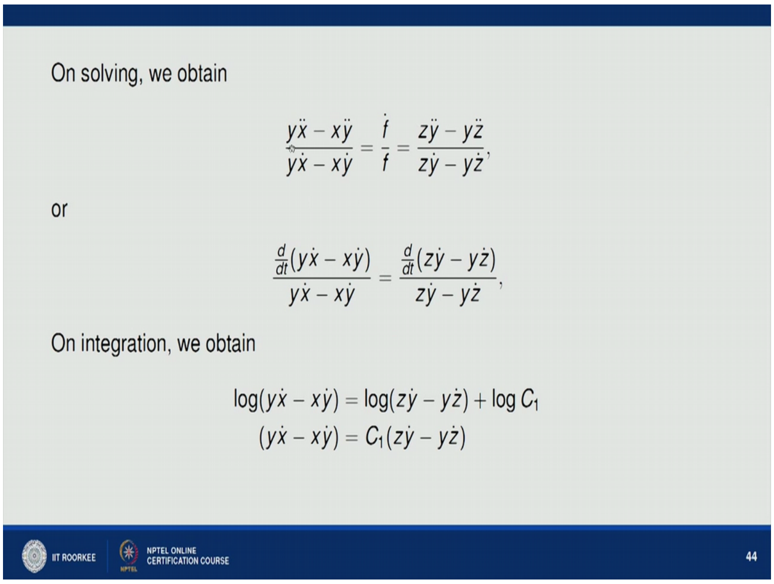

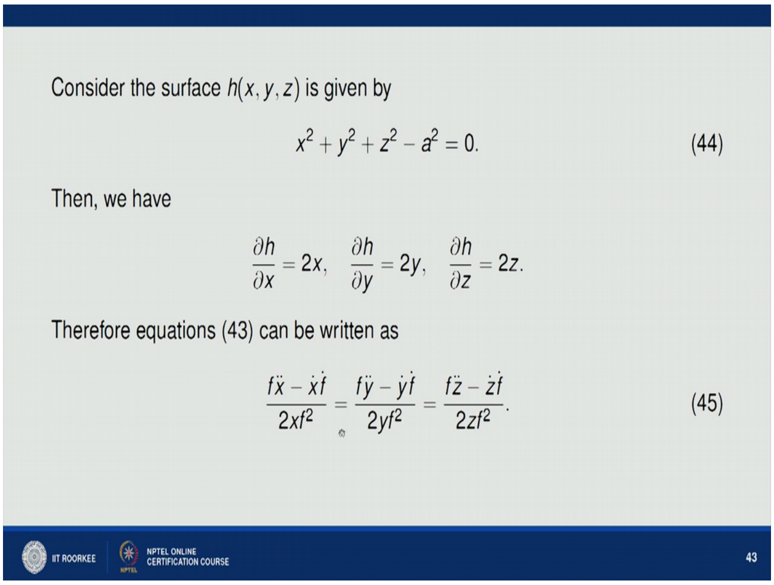

In terms of this, so what you try to do take these two equation find out the ratio of f dash upon f. Similarly take these two find out the ratio of f dash upon f. Similarly the first and third you will get value of f dash upon f.

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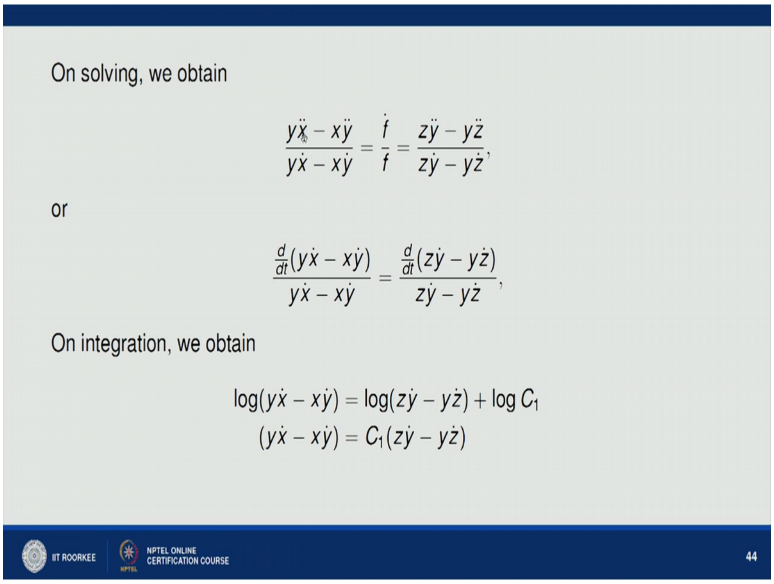

So here we have this y of yx double dash minus xy double dash divided by yx dash minus xy dash equal to f dash upon f. So this you can get so here you forget this f dash upon f and then we have this these two equation yx double dash minus xy double dash divided by yx dash minus xy dash is equal to zy double dash minus yz double dash divided by zy dash minus yz dash, if you look at the numerator is nothing but derivative of the denominator.

So taking these thing in mind we can write this as d by dt of derivative of the denominator here. So if you do it then we can integrate and we can get this log of y of x dash minus x of y dash equal to log of zy dash minus yz dash plus some integration constant that is log of C 1. So we will get this condition yx dash minus xy dash equal to C 1 zy dash minus yz dash.

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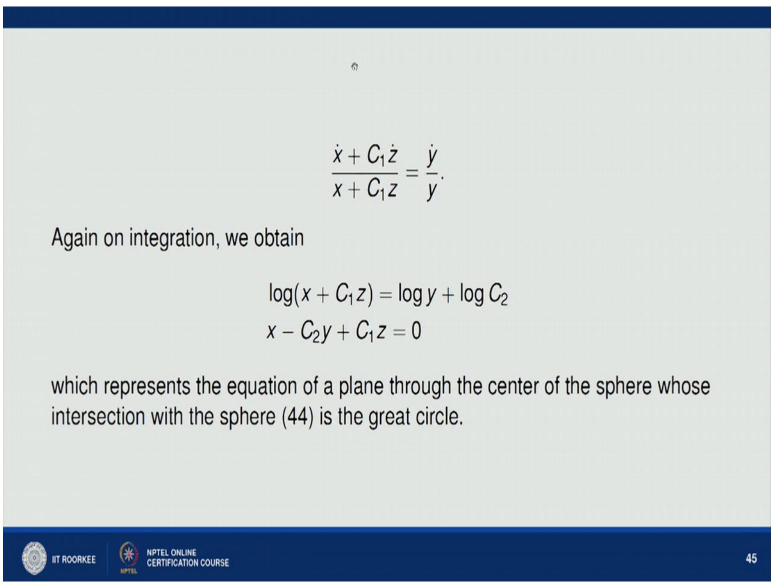

Now this again we can simplify and we can write it x dash plus C 1 z dash upon x plus C 1 z equal to y dash upon y. Now again we can get that this numerator is derivative of the denominator. So you can further integrate both the side and you can get log of x plus C 1 z equal to log of y plus another integration constant that is log of C 2 here. When you simplify this you will get that x minus C 2 y plus C 1 z is equal to 0.

So this is the equation of the curve which gives you say which which extremizing the given functional and which represent the equation of a plane through the centre of the sphere whose intersection with the sphere is the great circle. So this we have already shown that geodesics on a sphere is coming out to be a great circle. So this we have already proved but we are proving it showing it again with the help of theory which we have developed in this lecture. So here we end our discussion and in next class we will discuss some more problem based on calculus of variation. So thank you for listening us thank you.